National University of Singapore

Adaptive Control Systems

EE5104

Model Reference Adaptive Control

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Abstract

This report explores a Model Reference Adaptive Control System($MRAS$) for a second order system with relative degree one. The report explores the effects of certain adaptive control parameters, and shows a comparison between the implementation of an adaptive control strategy versus an exact control strategy. The system is also tested for several reference signals and the performance evaluated.

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Introduction

THE traditional approach to develop a control system design is to develop a linear system model for the given range of parameters and then develop a control mechanism having constant parameters. This systems is preferred when a thorough model of the process can be developed. When it is not possible to develop a well formulated model due to lack of knowledge of the system or due to economical constraints, *Adaptive Control* systems provide a good alternative. In this report a Model Reference Adaptive Control($MRAS$) is deployed to a second order system given by

$$\frac{Y(s)}{U(s)} = \frac{-0.11s - 1}{s^2 + 0.29s + 7.8} \tag{1.1}$$

In the physical setup only *y(t)*(the plant output)and *u(t)*(the input to the plant) are measurable. The exact values of the transfer function coefficients are not known. It is only known that b_0 is a negative number, and that the plant has no zeros in the right half of the s-plane assuming the second order system is defined

$$\frac{Y(s)}{U(s)} = \frac{b_0 s + b_1}{s^2 + a_1 s + a + 2} \tag{1.2}$$

The system can be written as

$$(s^2 + 0.2s + 7.8)y(s) = (-0.11)(s + 9.09)u(s) \tag{1.3}$$

Comparing the above equations to the general form

$$R_p y(s) = k_p Z_p u(s) \tag{1.4}$$

we have

$Z_p \ : \ s + 9.09$

R_p : $s^2 + 0.29s + 7.8$

k_p : -0.11

n : 2

m : 1

n^* : n-m=1

Since open loop tests reveal that the system is lightly damped with a natural frequency of 2.5 rad/s, we shall incorporate it into the development of the observer polynomial given by T.

$$\begin{aligned} T &= s^2 + 2\zeta\omega_n + \omega_n^2 \\ &= s^2 + 2 * 0.1 * 2.5 + 2.5^2 \\ &= s^2 + 0.5s + 6.25 \end{aligned} \tag{1.5}$$

assuming a damping factor $\zeta = 0.1$

The next step to be performed is to choose a suitable reference model for the plant to follow. For a system with relative degree one the reference model can be chosen as

$$\frac{Y_m(s)}{U_m(s)} = \frac{a_m}{s + a_m} \tag{1.6}$$

Let us assume a stable system with one pole in the left half of the s-plane at $s = 5$. This gives the reference model a time constant of 0.2s. The reference model can then be given as

$$\frac{Y_m(s)}{U_m(s)} = \frac{5}{s + 5} \tag{1.7}$$

From the above model we determine that

R_m : s+5

Z_m : 1

k_m : 5

1.1 Exact Controller Gains

Before an adaptive controller is implemented, let us implement a controller to be able to compare the performance of the adaptive controller with that of the exact controller, assuming we have complete knowledge about the system.The diophatine equation for the given system is described by

$$T(s)R_m = R_pE_p + F_p \tag{1.8}$$

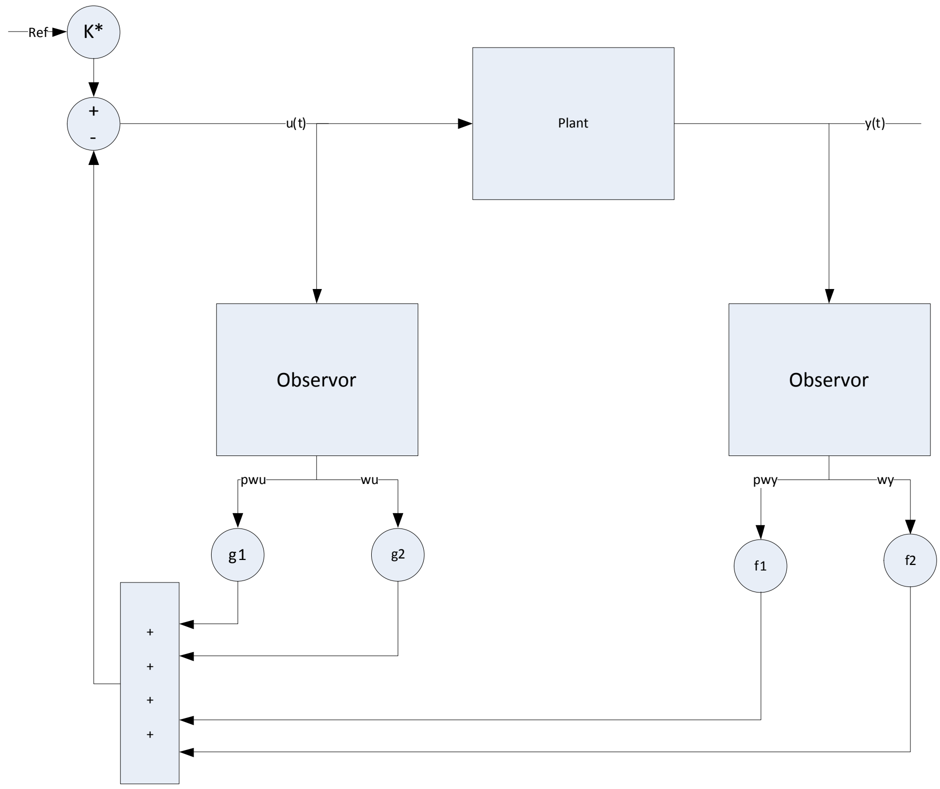

Figure 1.1: Structure of Exact Controller

Since E is a monic polynomial of degree *(n-m)*, it assumes the form s+a and F is polynomial of degree *(n-1)* assumes the form bs+c. Substituting this and other information derived in Section1 of this chapter, we have

$$(s^2 + 0.5s + 6.25)(s + 5) = (s^2 + 0.29s + 7.8)(s + a) + (bs + c) \tag{1.9}$$

$$\begin{aligned} s^3 + 5.5s^2 + 8.75s + 31.25 &= s^3 + s^2(a + 0.29) + s(0.29a + 7.8) + 7.8a + bs + c \\ &= s^3 + s^2(a + 0.29) + s(0.29a + 7.8 + b) + 7.8a + c \end{aligned} \tag{1.10}$$

Solving the above equations for a,b and c we get

$$E = s + 5.21 \tag{1.11}$$

$$F = -0.5609s - 9.388 \tag{1.12}$$

To find the feedforward gain term $k*$

$$k^* = \frac{k_m}{k_p} = -45.4545 \tag{1.13}$$

Next we calculate feedback gain f_1,f_2,g_1,g_2

$$\overline{F} = f_1 s + f2$$
$$\overline{F} = \frac{F}{k_p} \tag{1.14}$$
$$\tag{1.15}$$

$$G_1 = g_1 s + g2$$
$$G_1 = T - \overline{G} \tag{1.16}$$

Solving Eq: 1.15 and Eq 1.16 for emphf_1,f_2,g_1,g_2,we have

$$\overline{F} = 5.0991s + 85.3455 \tag{1.17}$$
$$G_1 = 13.8s + 41.1089 \tag{1.18}$$

For the purposes of this study, 3 input cases have been recommended,

- A step input of magnitude 1
- A square input of chosen period and magnitude(Freq = 2.2rad/s, Amplitude =1)
- A sine wave of amplitude 10 and freq = 2rad/s

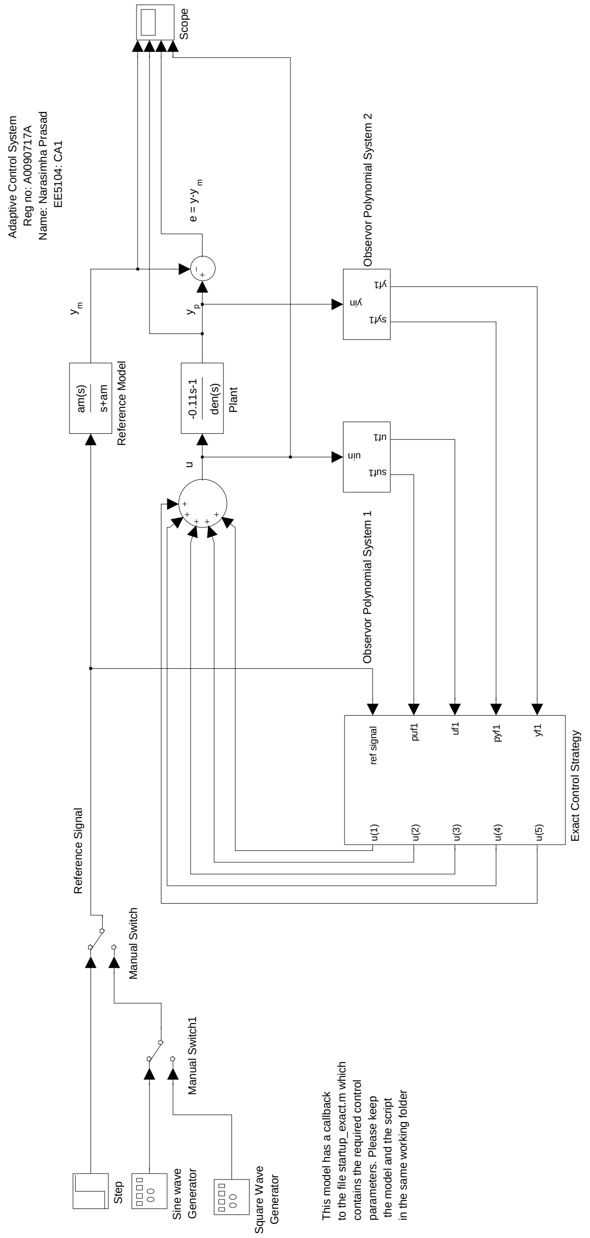

Figure 1.2: Overview of controller system with exact gains

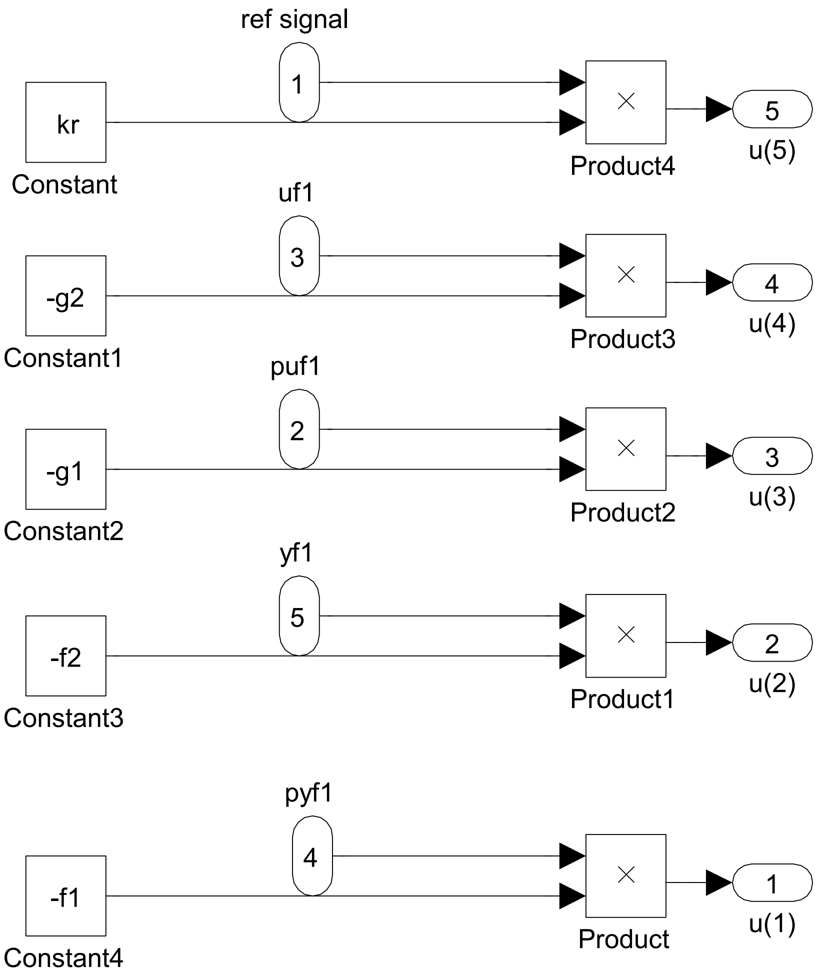

Figure 1.3: Exact Controller Gain Strategy

The exact gain(traditional) controller is as shown in fig 1.2 and fig 1.3. Where {*-f1,-f2,-g1,-g2,kr*} are the gains calculated above. The next section details the performance of the traditional controller.

1.2 Exact Gain Controller Results

Here we see the output for all three reference inputs. It is clear from Fig: 1.4,1.5 and 1.6 that the plant is able to follow the reference model quite easily without much load on the controller.

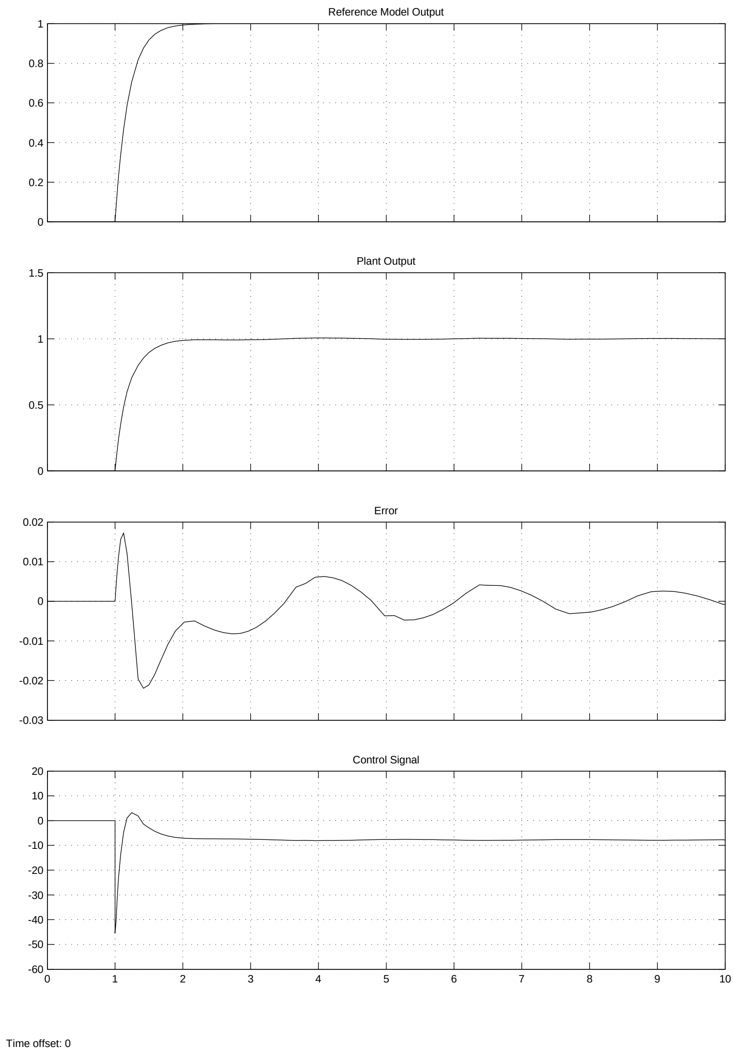

Figure 1.4: System response to step input of magnitude 1

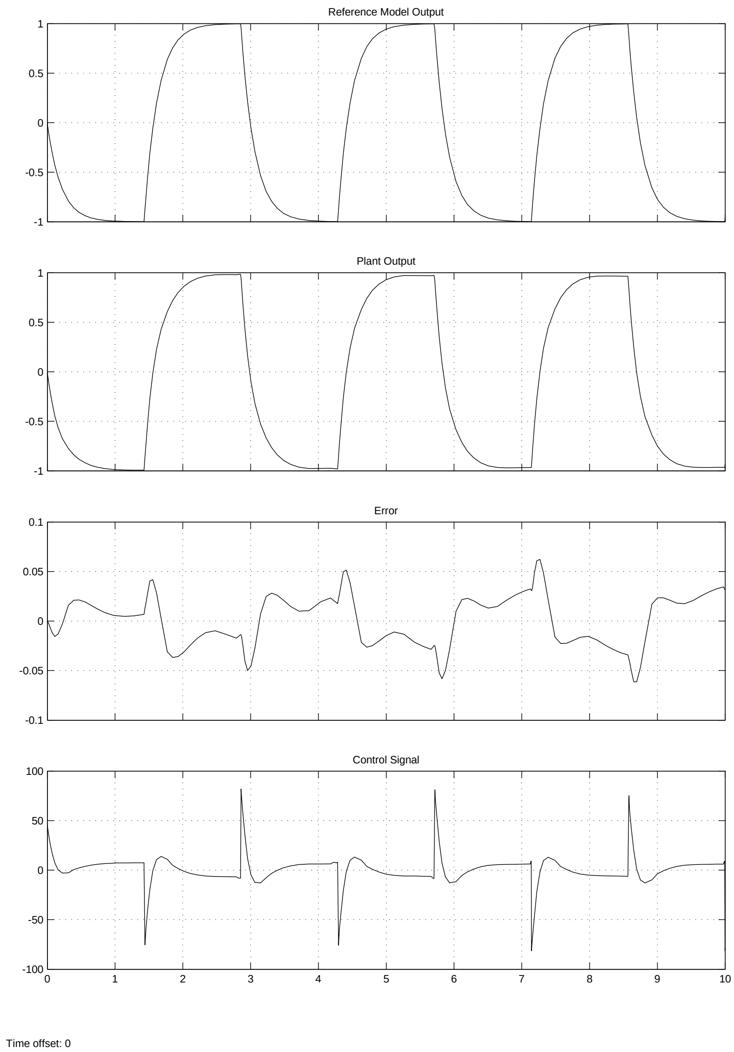

Figure 1.5: System response to square input with magnitude = 1; period = 2.2rad/s

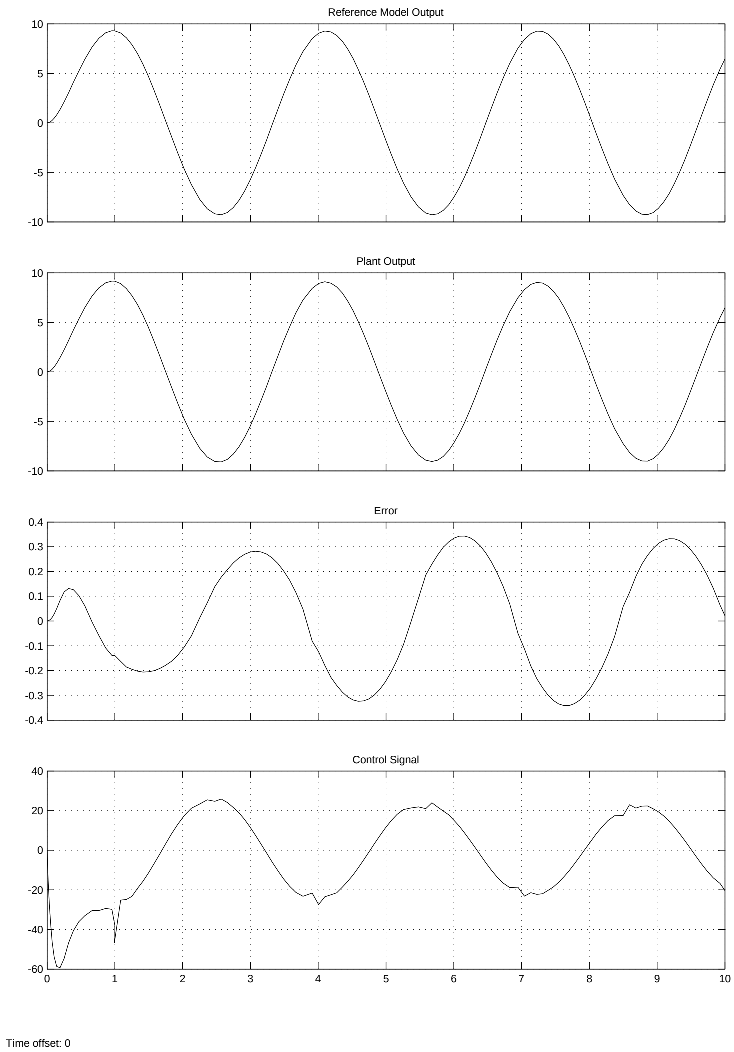

Figure 1.6: System response to sine input of form magnitude = 10; period = 2rad/s

2

Model Reference Adaptive Control

For the given plant and reference model chosen in Ch 1:Sec 1. Let us assume an adaptive control strategy of

$$u(t) = \overline{\theta}(t)^T \overline{w}(t) \tag{2.1}$$

As opposed to the controller discussed in the previous chapter, the gains of the adaptive controller are time varying. The adaptive law for the chosen system is given by

$$\dot{\overline{\theta}} = -sgn(k_p)\Gamma \tag{2.2}$$

when the error considered is given by

$$e_1 = y - y_m \tag{2.3}$$

The state realization of $\omega(t)$ and $\overline{\theta^*}$ is given by

$$\omega(t) = \left[y^{f1}, \ldots, p^{n-2}y^{f1}, p^{n-1}y^{f1}, u^{f1}, \ldots, p^{n-1}u^{f1}, p^{n-2}u^{f1} \right] \tag{2.4}$$

$$\overline{\theta^*} = \left[-f_1 \quad -f_2 \quad -g_1 \quad -g_2 \quad k^*]^T \right] \tag{2.5}$$

where $y^{f1} = \frac{1}{T}y$, $u^{f1} = \frac{1}{T}u$ and $T(p) = p^2 + t_1 p + t_2$ represents the observer polynomial.

For the given controller we deduce the following

- Order of R_p, n =1
- Z_p is stable since the plant has no zeros in the right half plane
- $sgn(k_p)$ = -1

If we assume a Lyapunov candidate given by

$$V(e,\phi) = e^T P e + \overline{\phi}^T \Gamma^{-1} \overline{\phi} \tag{2.6}$$

with the choice of $\dot{\overline{\theta}} = \dot{\overline{\phi}} = -sgn(k^*)\Gamma\overline{\omega}e_1 = sgn(k_p)\Gamma\overline{\omega}e_1$ we have the $\dot{V} = e^T Q e < 0$. Since the Lyapunov function is always decreasing the boundedness of $||e||,||\overline{\phi}||$ is guaranteed. The next step is prove the non-existance of pulses in the error function. We do this by proving the boundedness of the derivative of e($\dot{e}$). Since

$$\int_0^\infty e(\tau)^T Q e(\tau) d\tau \leq c_1 \tag{2.7}$$

$$\lim_{t\to\infty} e(t) = 0 \tag{2.8}$$

The general structure of the adaptive controller along with the Simulink implementation of the system and the adaptive control strategy is shown in Figures : 2.1,2.2 and 2.3.

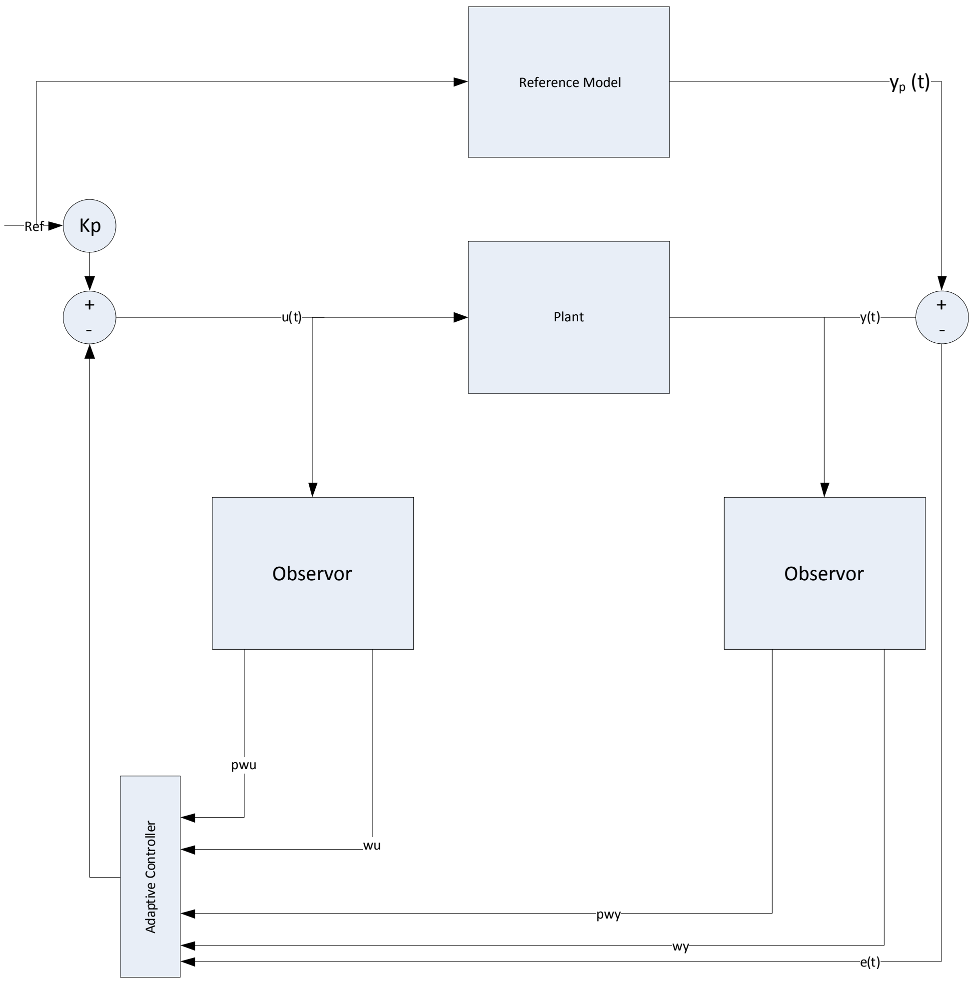

Figure 2.1: Adaptive Controller Structure

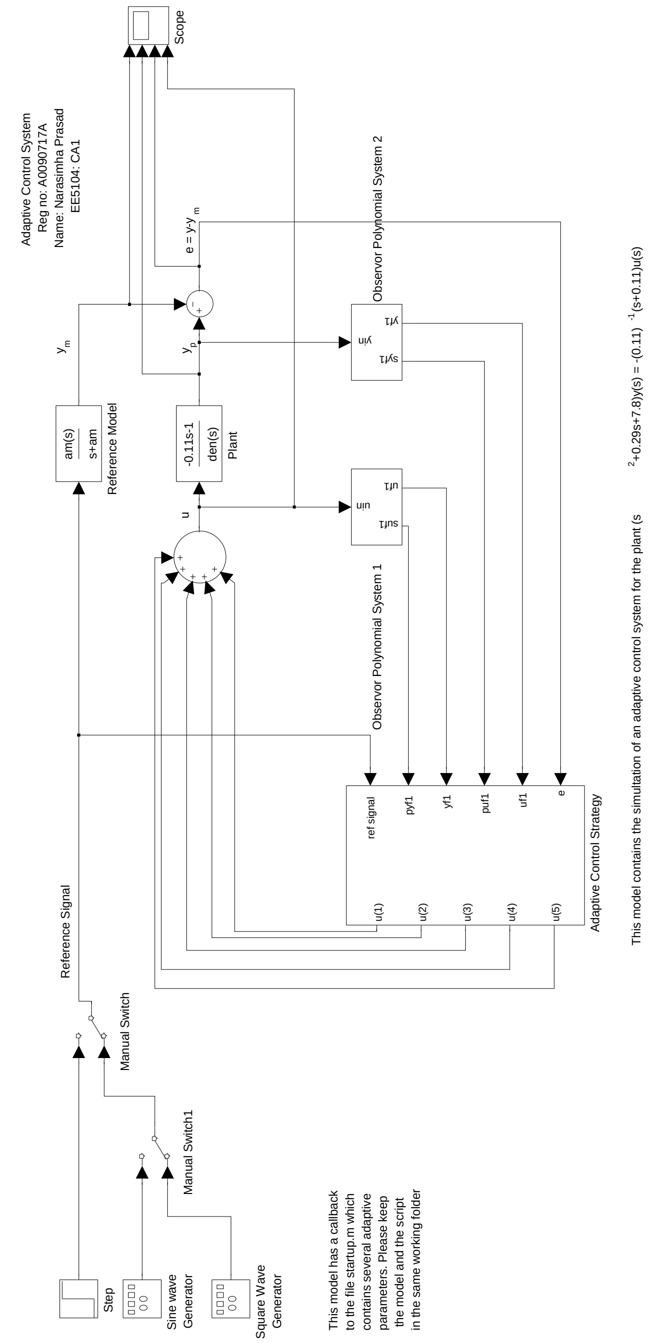

Figure 2.2: Adaptive Controller

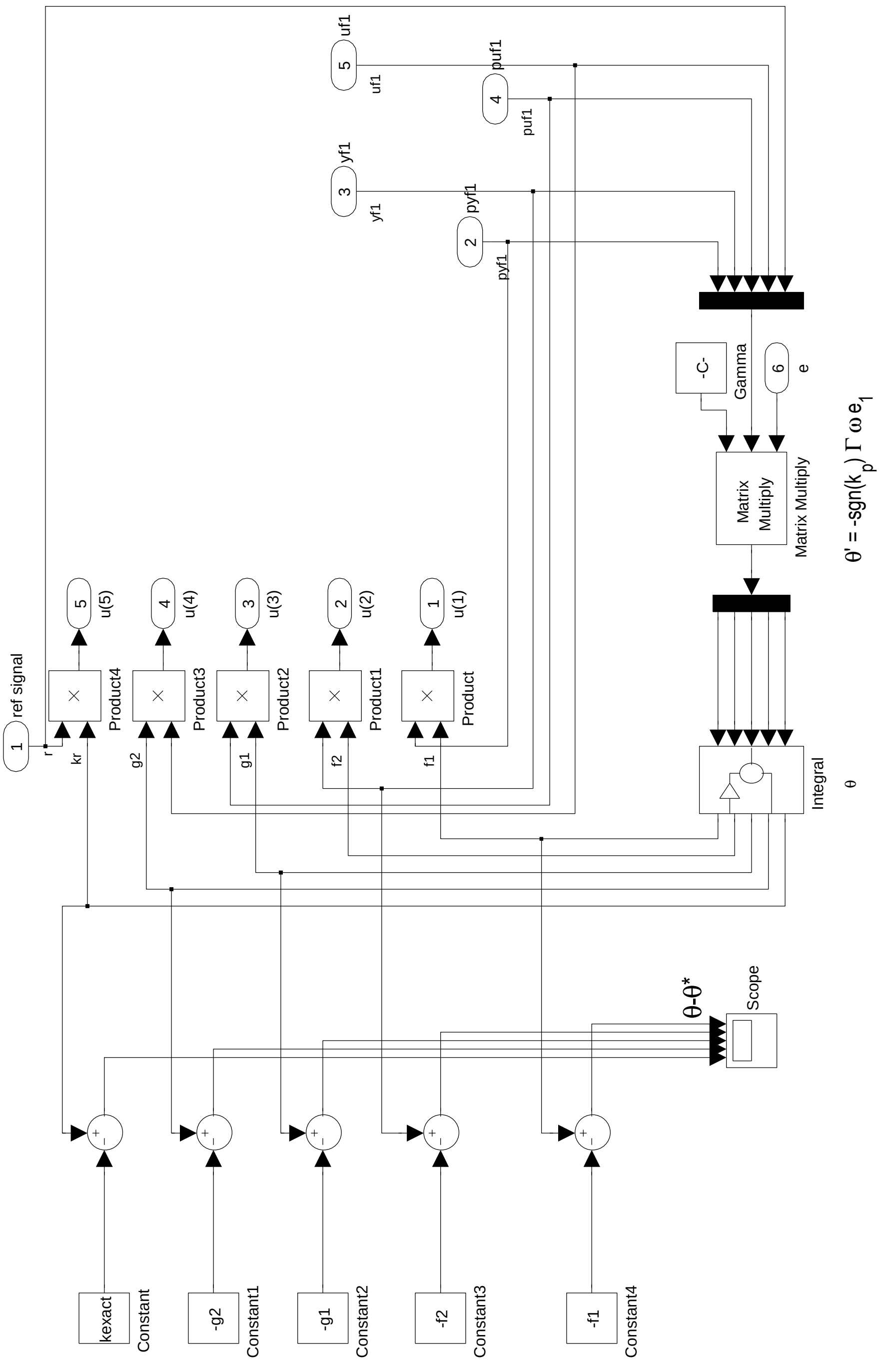

Figure 2.3: Adaptive Control Strategy

The following sections will explore the effectiveness of this adaptive control law for the three cases explored in the previous chapter, viz. step input, square wave input and a sine input and also the effect in variation of the adaptation gain Γ and the observer polynomial $T(p)$ for each of these inputs.

2.1 Input Response

This section analyses the system response of the plant with the developed adaptive control law for the best observed scenario, with the following controller variables

$$\Gamma = 5000 * \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$
$$T(p) = s^2 + 0.5s + 6.25$$

These choices of Γ and $T(p)$ offer good tracking capabilities and do not change the controller input too often, which in a real case scenario might be important to consider, given the inability of physical controller to change rapidly.

2.1.1 Step Input

In the first scenario a step input of magnitude 1 is applied at t=1s. From Figure 2.4 we see that the plant is able to track the model reference plant quite well, except for the initial stages, when the step input is applied. But all in all the tracking capabilities of the plant do not seem to be affected as it settles in the same time range as that of the reference model. From Figure: 2.5 we see that even though the plant is able to track the model reference system, the gains do not tend towards the perfect gain values.

2.1.2 Square Wave Input

Next we explore the performance of the adaptive controller with a continuously changing excitation input, which in this case is a square wave of magnitude 1 and a frequency of 2.2rad/s.

An interesting point to notice is that the controller seems to perform better as time goes on. In the first period of excitation the plant does have an element of disturbance to its output, but is still able to track the reference model. The control signal required is also of higher magnitude and changes more rapidly to try and bring the plant to the required input. But during subsequent periods of excitation, the controller adapts to the situation, and is able to control the plant much more effectively and the control signal required is both of a lesser magnitude and has significantly lesser switching.In the third period, the small disturbances present in the system are effectively negated

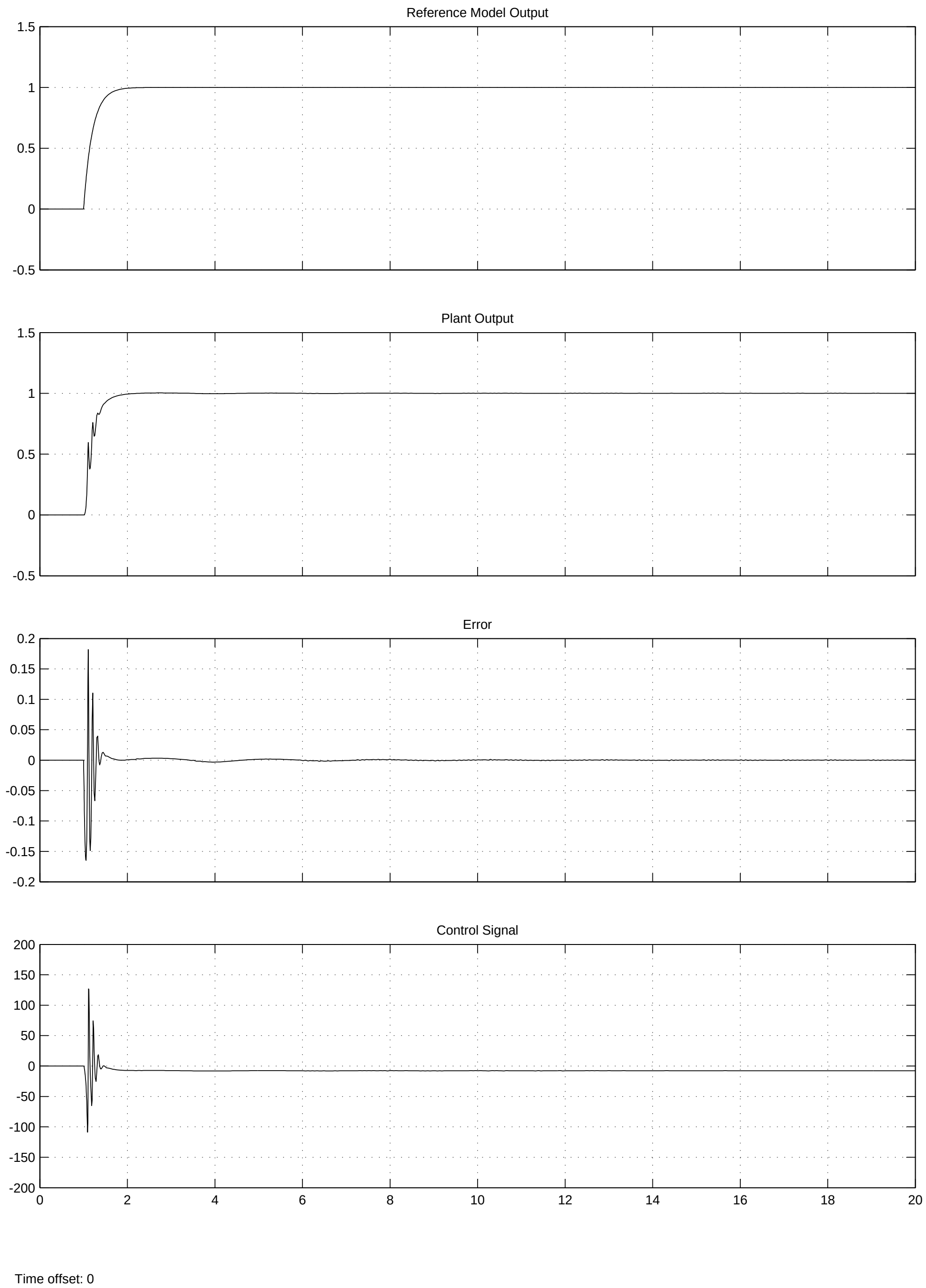

Figure 2.4: Step Input Performance

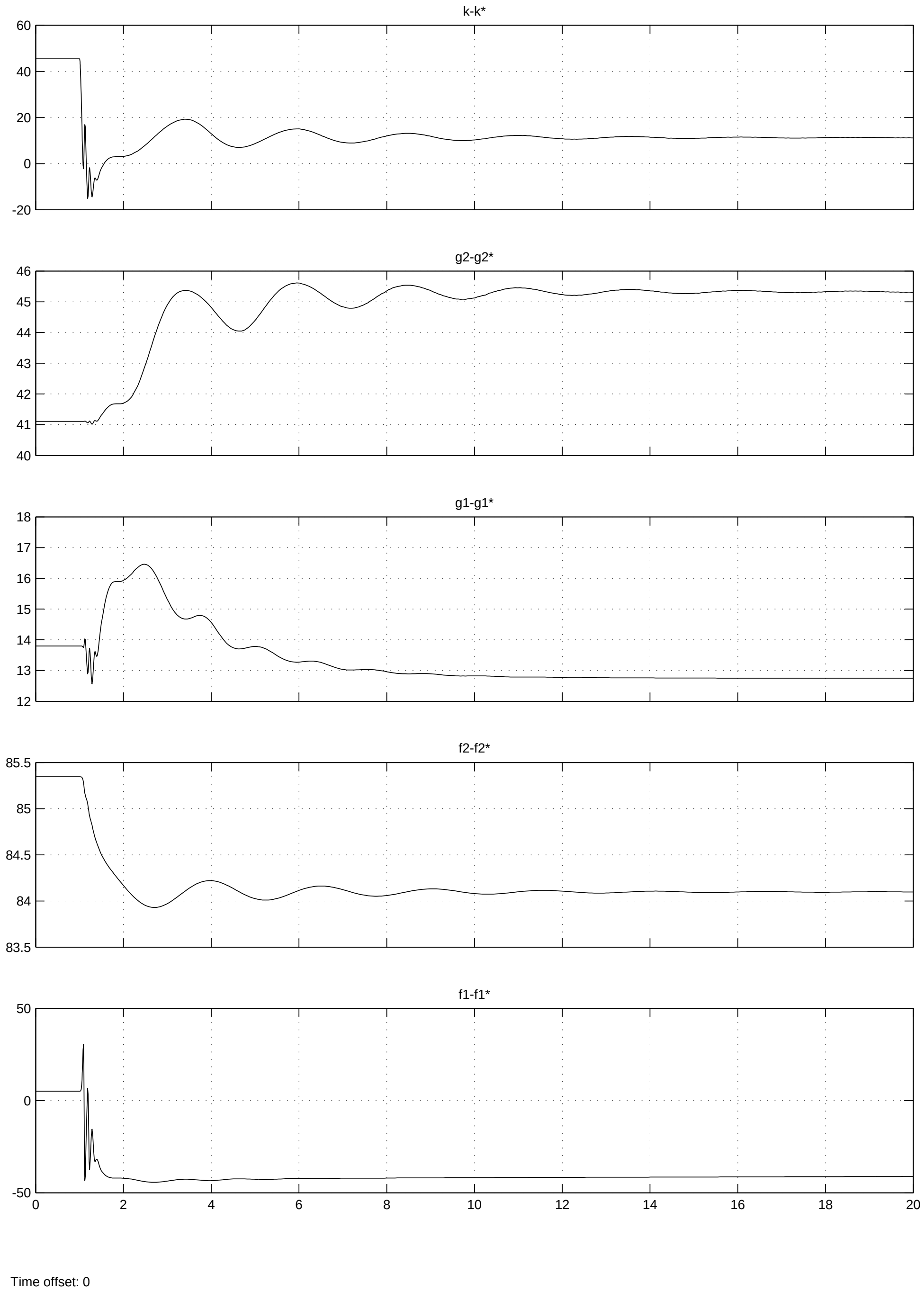

Figure 2.5: Step Input Control Parameters

and the system follows the reference model as required. This is also evidenced from the error output, where we see the maximum magnitude of error in each period decreasing. As in the previous section, the adaptive controller gains do not tend towards the exact controller gains.

2.2 Gamma Variation

This section explores the effects of the adaptation gain Γ. To clearly see the effects of Γ on the performance of the adaptive controller, I have chosen two scenarios,

- $\frac{\Gamma}{10}$
- $\Gamma * 10$

with respect to the best Γ considered in the previous section of this chapter.

2.2.1 Gamma/10

The value of Γ has been reduced to a tenth of the best value considered in the previous section to now become

$$\Gamma = 500 * \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Figures: 2.8 and 2.10 show the performance of the adaptive controller while Figures: 2.9 and 2.11 show the controller gain variation for the new value of Γ considered. It is clear from these figures that Γ controls speed of the plants ability to track the reference signal. There are significantly more number of oscillations in the plant output. For a step input, the settling time doubles, while a square wave reference takes upto 2 periods more to settle down. But, the tradeoff occurs in the form of switching rate in the control signal. This might be of some importance when the physical system used to control the plant is not capable of achieving fast switching rates. The reference gains also tend towards the same values, albeit at a slower rate.

2.2.2 Gamma*10

The value of Γ is then increased tenfold

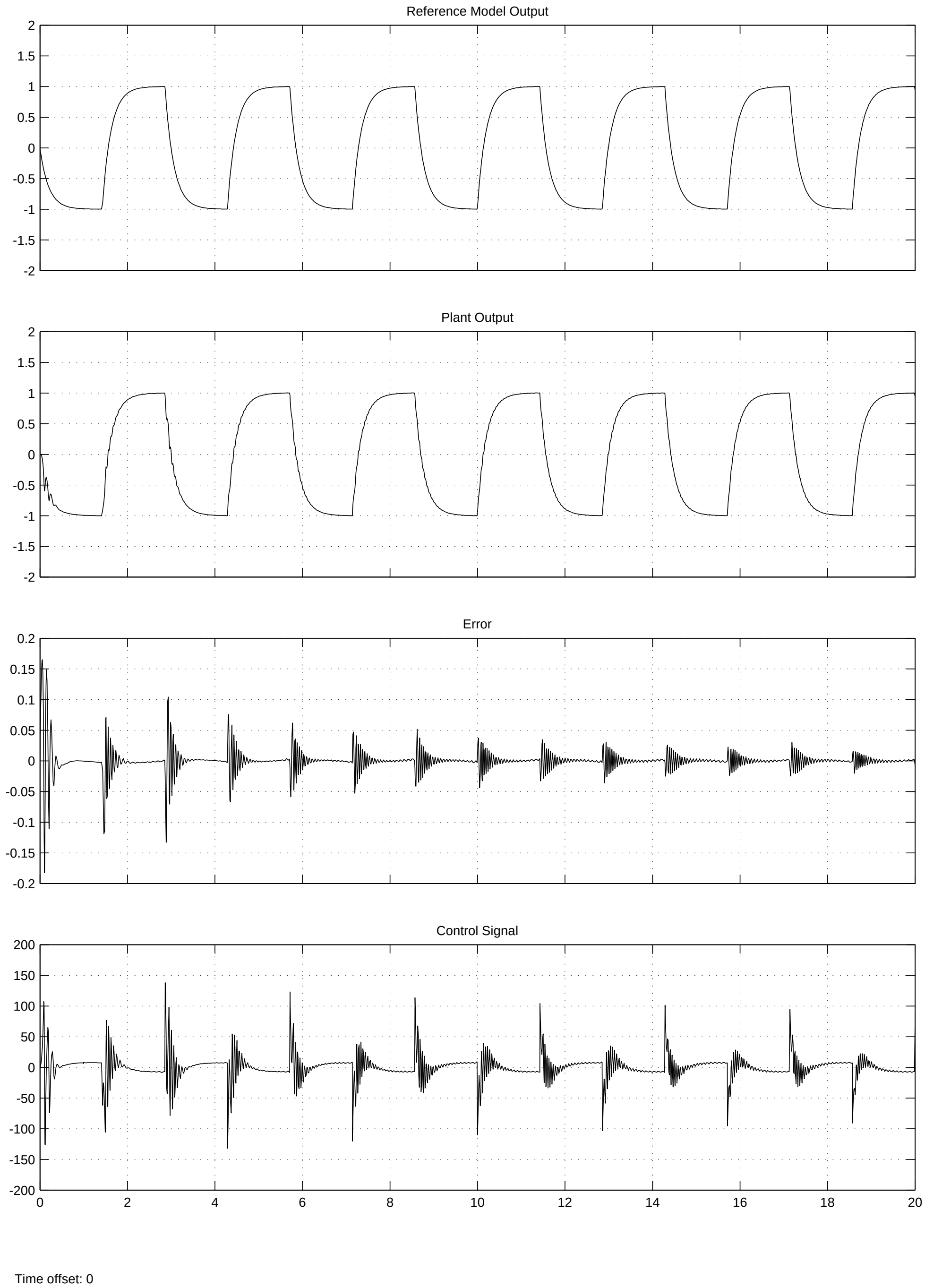

Figure 2.6: Square Input Performance

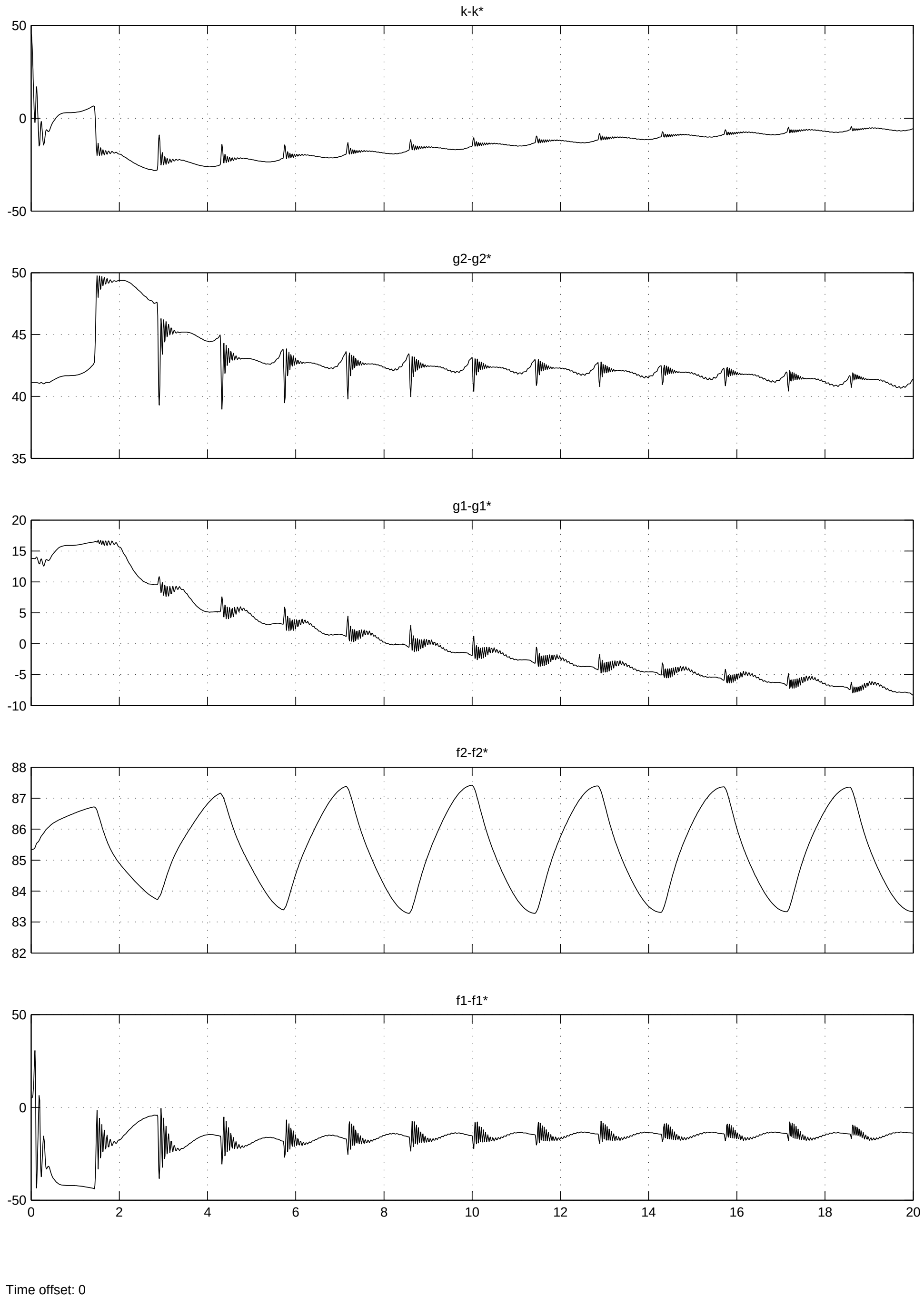

Figure 2.7: Square Input Control Parameters

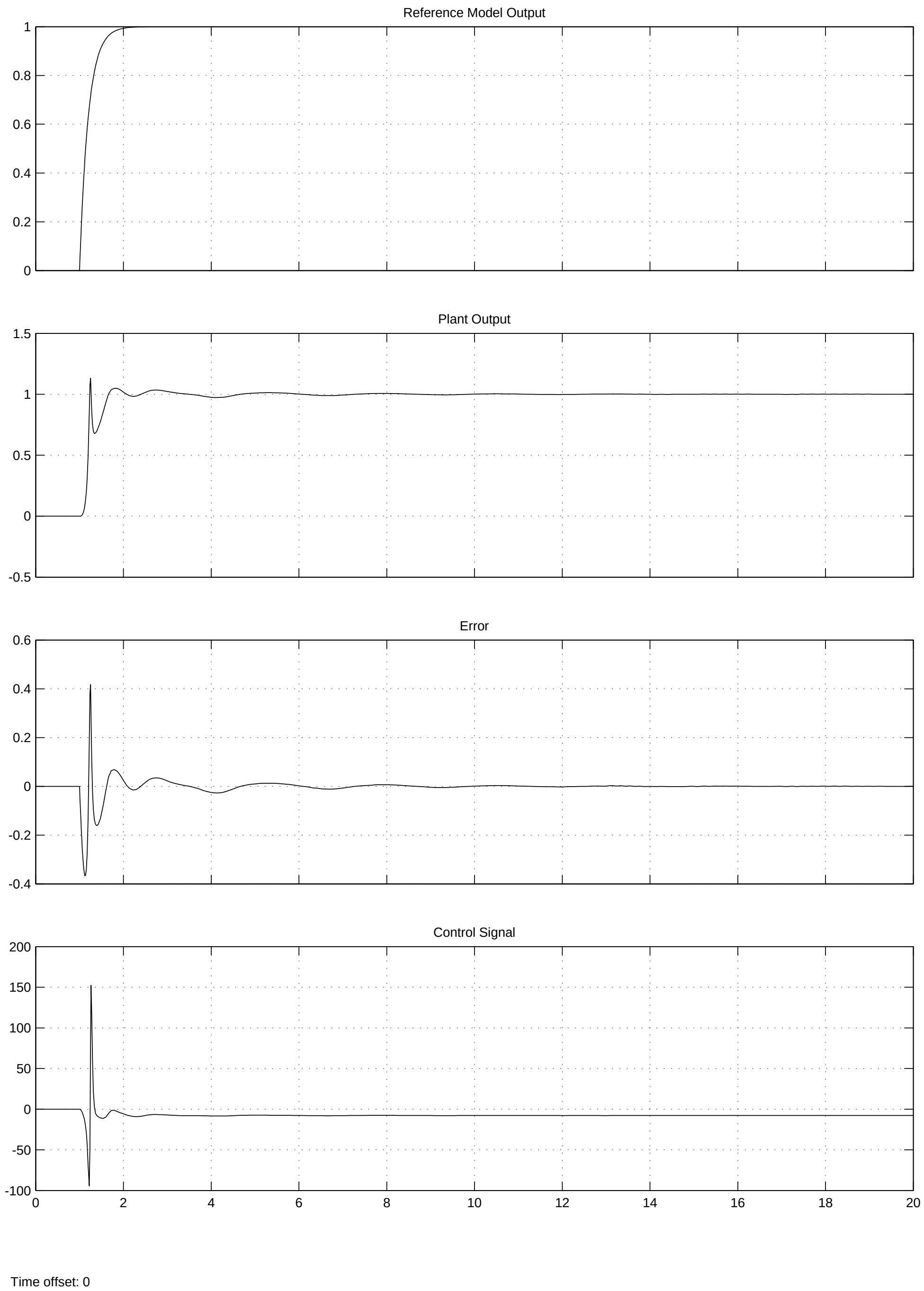

Figure 2.8: Step Input Control Parameters, Gammma = Gamma/10

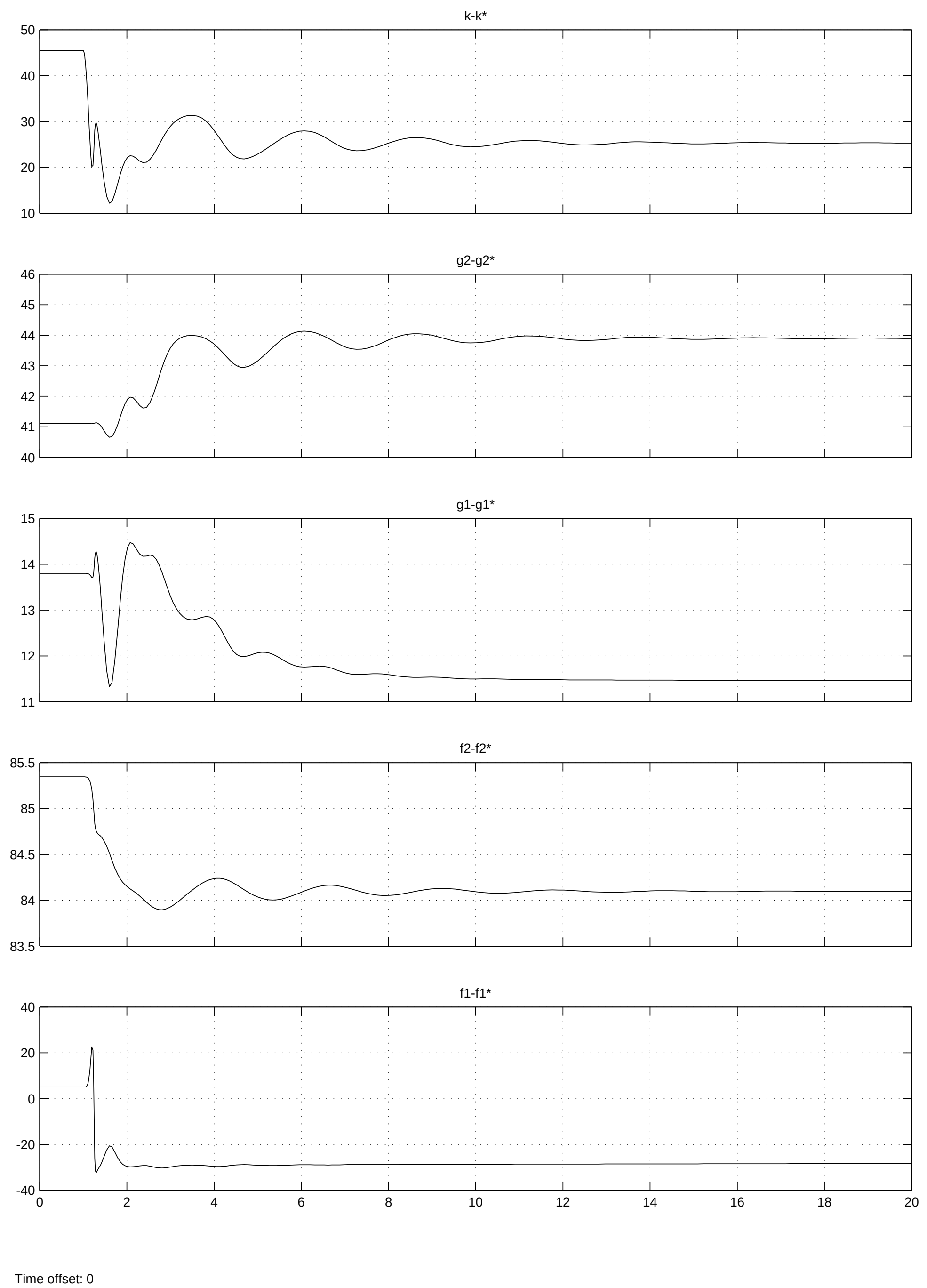

Figure 2.9: Step Input Control Parameters, Gammma = Gamma/10

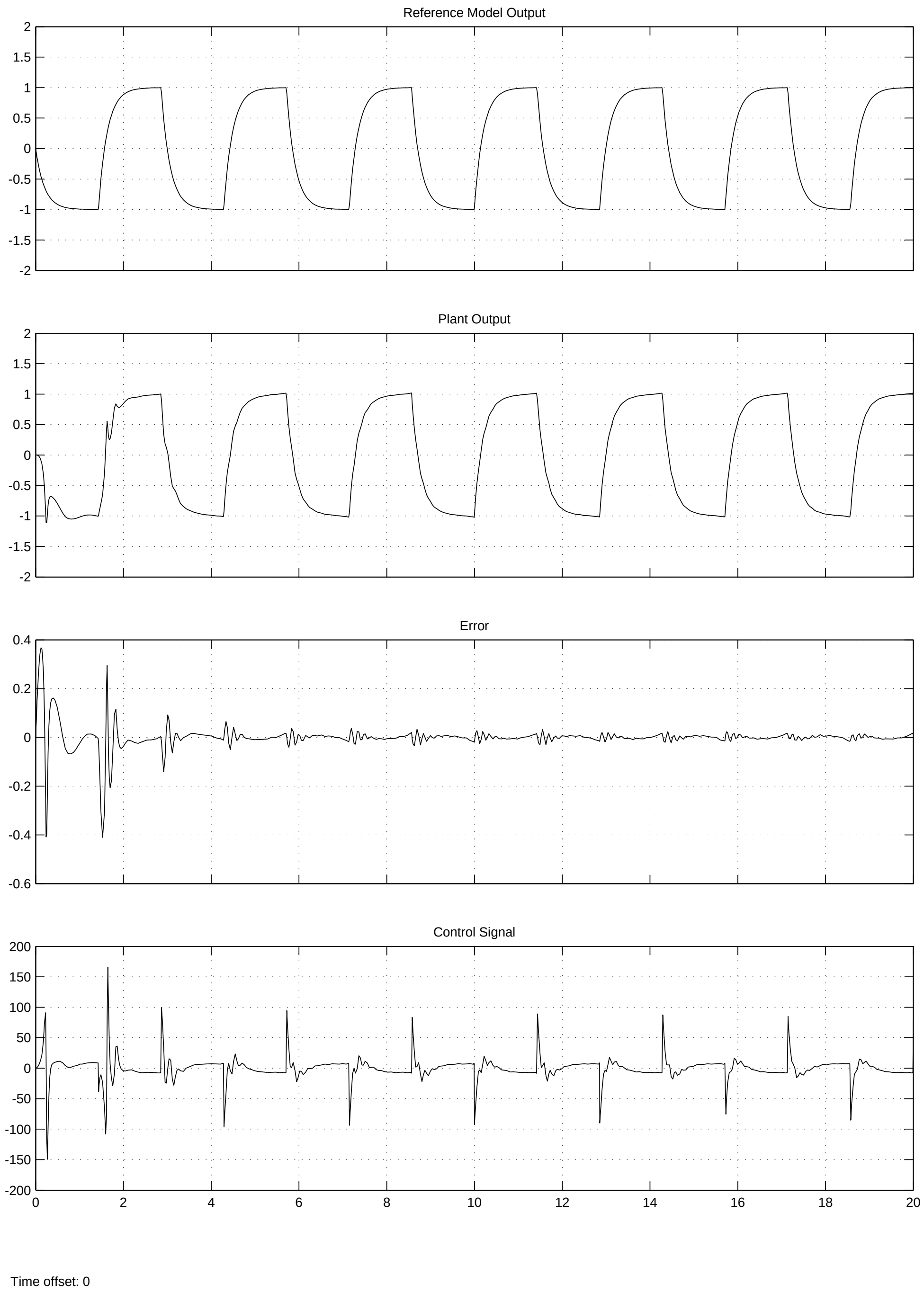

Figure 2.10: Square Input Control Parameters, Gammma = Gamma/10

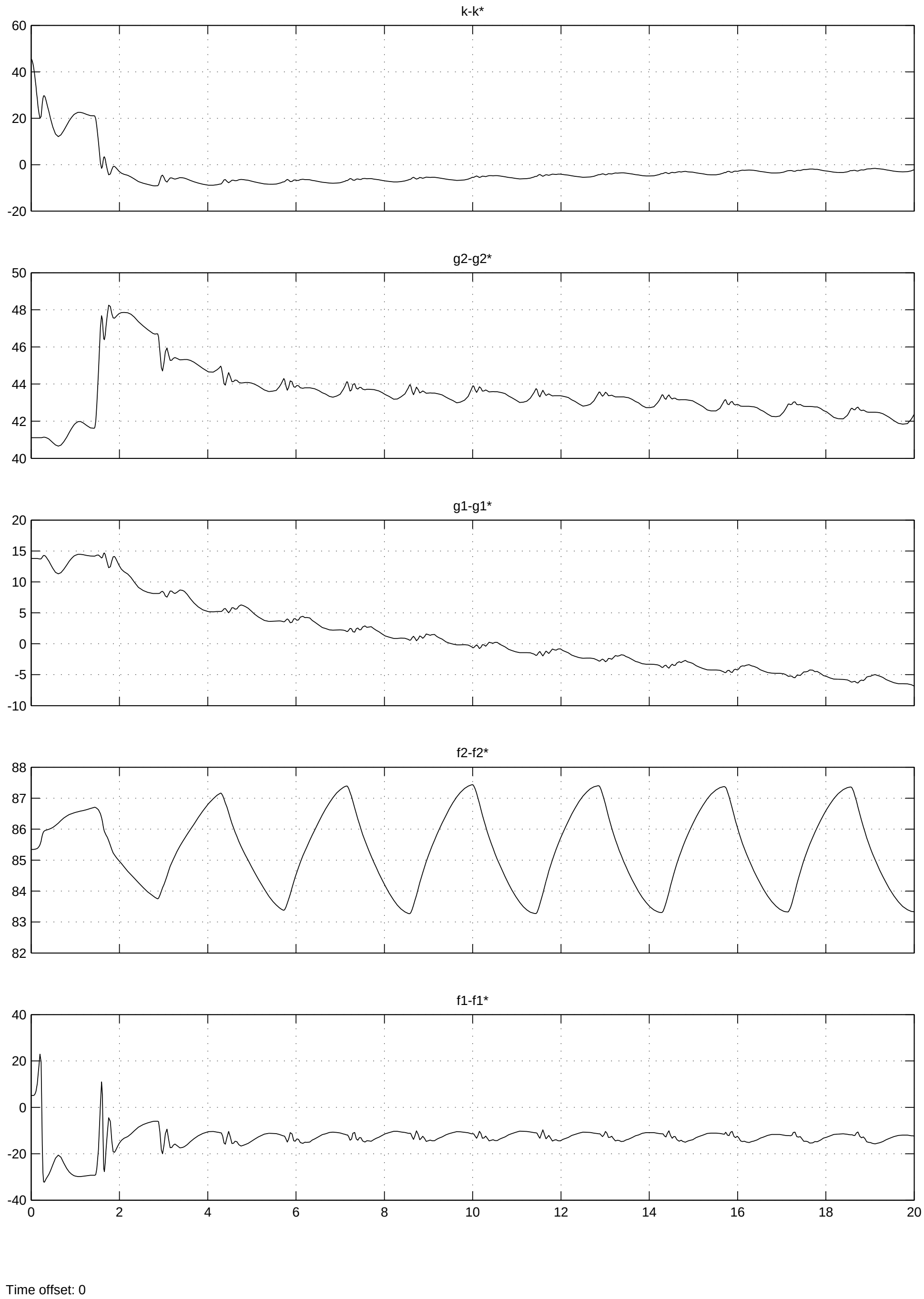

Figure 2.11: Square Input Control Parameters, Gammma = Gamma/10

$$\Gamma = 5000 * \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Figures :2.12 and 2.14 show the output of the plant for an increase in Γ. Gamma from the previous section was found to control the rate of adaptation of the plant, and as expected an increase in Γ results in much better tracking performance. But the tradeoff again occurs in the rate of switching of the control signal .An increased rate of switching might lower the lifetime of actuator device and might not be a desirable quality altogether. It is especially prominent when a square wave reference is given ,with the control signal switching rapidly to compensate. But the magnitude of the control signal does not change and remains to be around the same magnitude for all values of Γ

2.3 Observer Polynomial Variation

In this section, we shall explore the effect of the observer polynomial on the performance of the system. The polynomial system, upto now had been assumed as a good representation of the plant, i.e. to say the the states of the $\omega(t)$ would have been accurate estimations. In this section, we shall vary both the damping factor and the natural frequency of the observer polynomial, by increasing and decreasing both the factors. Tests were performned on both step and square wave inputs, but since the performance of the controller for step inputs did not vary much, only the effects of the variation on a square wave reference will be shown. The variations that were tested in simulations were

- $5 * \omega_n$
- $\frac{1}{5} * \omega_n$
- $5 * \zeta$
- $\frac{1}{5} * \zeta$

2.3.1 Variation in Natural Frequency

The effects of both a decreased ω_n and increased ω_n are shown in Figures: 2.16 and 2.17 respectively. Both cases show to have detrimental effects on the performance of the controller. If we assume a natural frequency lesser than that of the system, high frequency oscillation are setup in the plant output. This is because the controller assumes the plant to operate at a much lower rate and over actuates the system to compensate for

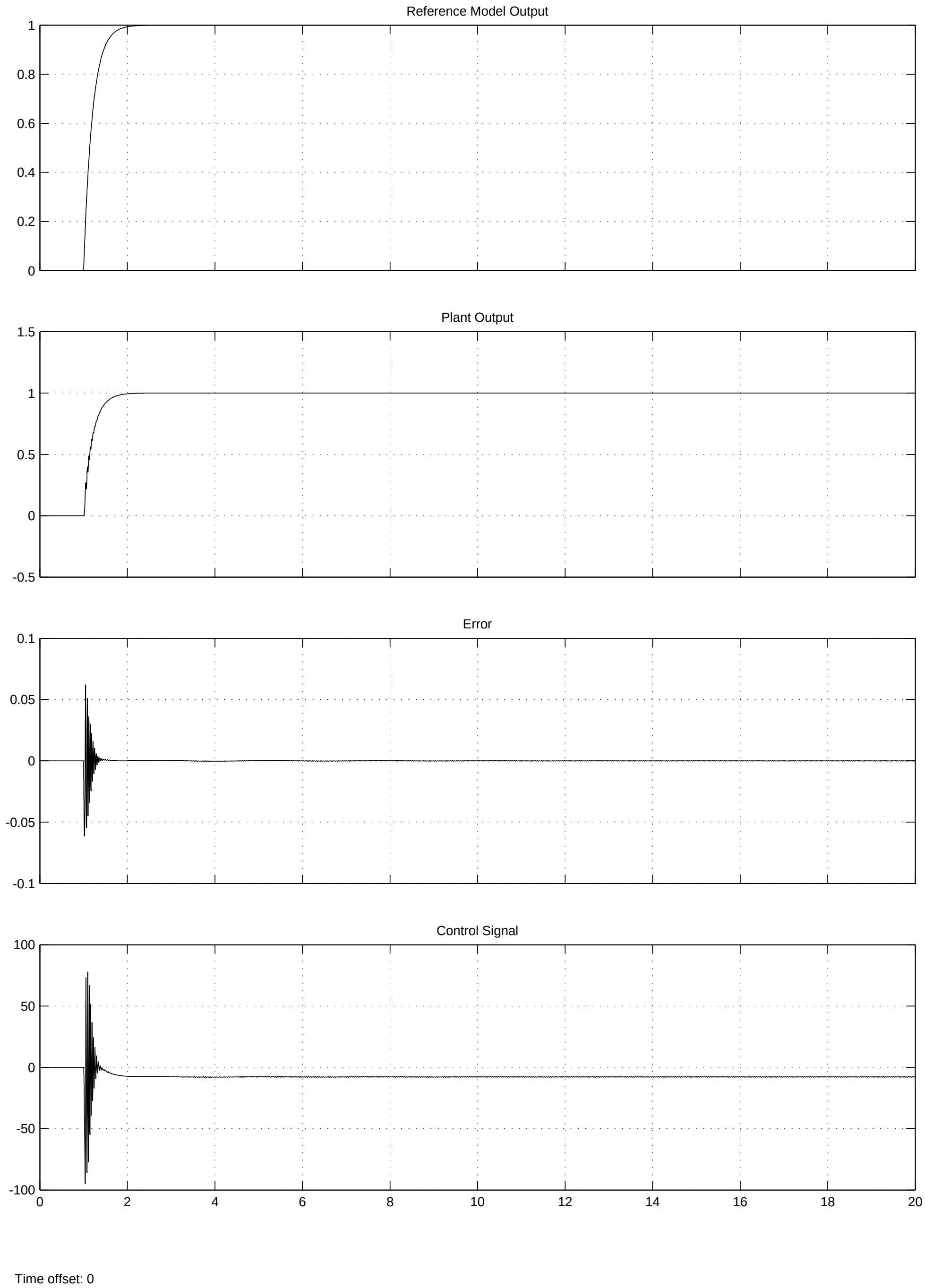

Figure 2.12: Step Input Control Parameters, Gammma = Gamma*10

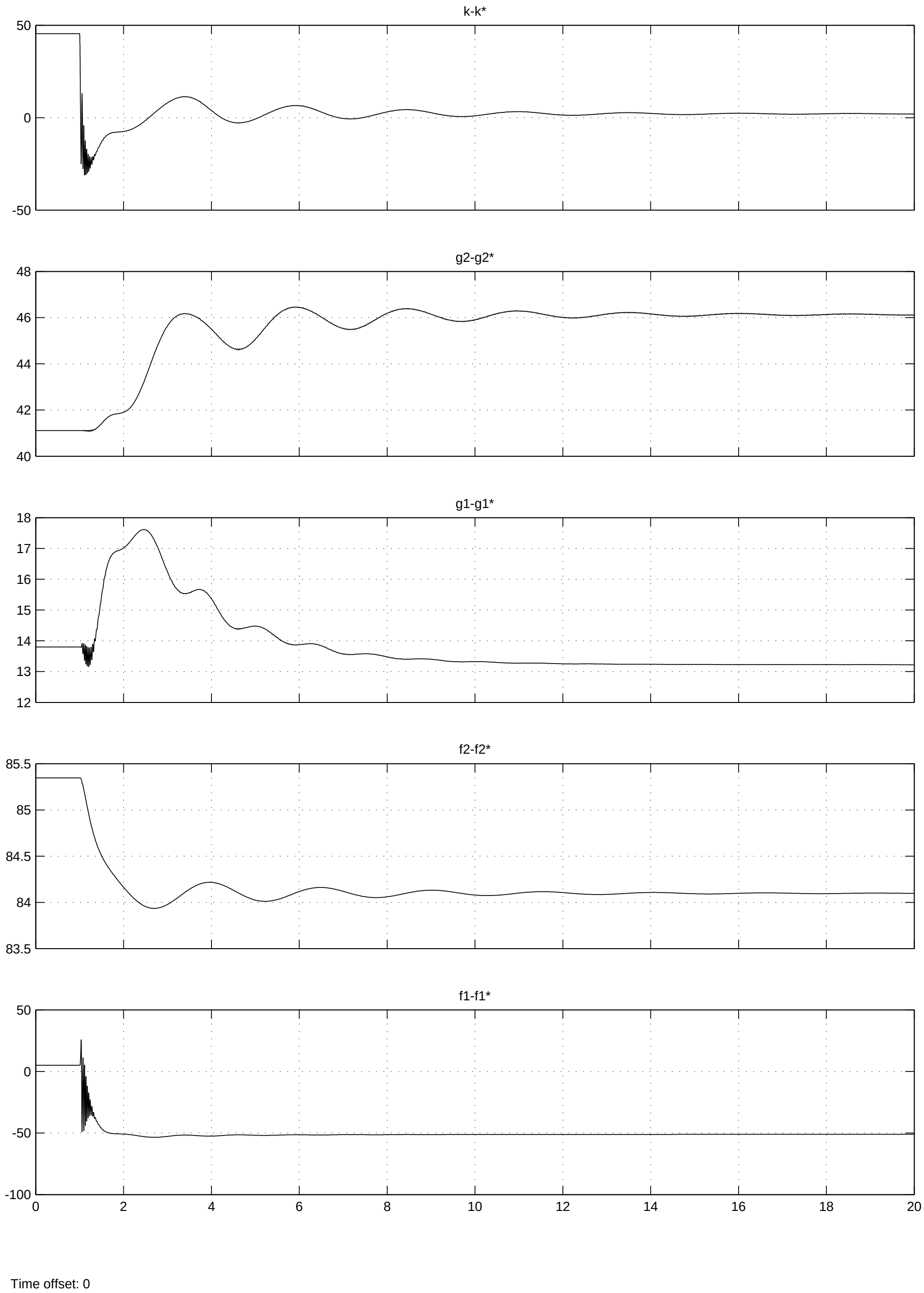

Figure 2.13: Step Input Control Parameters, Gammma = Gamma*10

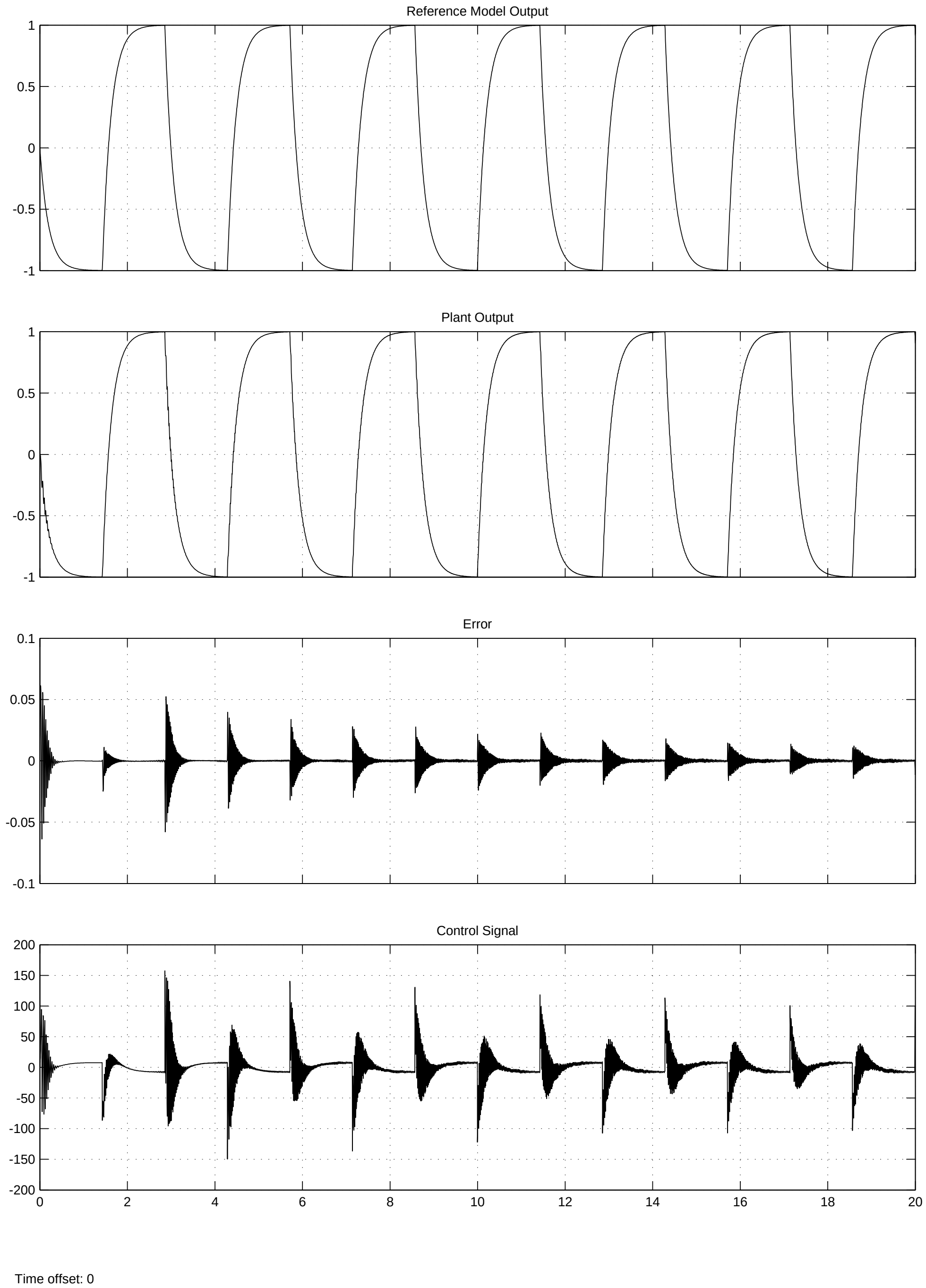

Figure 2.14: Square Input Control Parameters, Gammma = Gamma*10

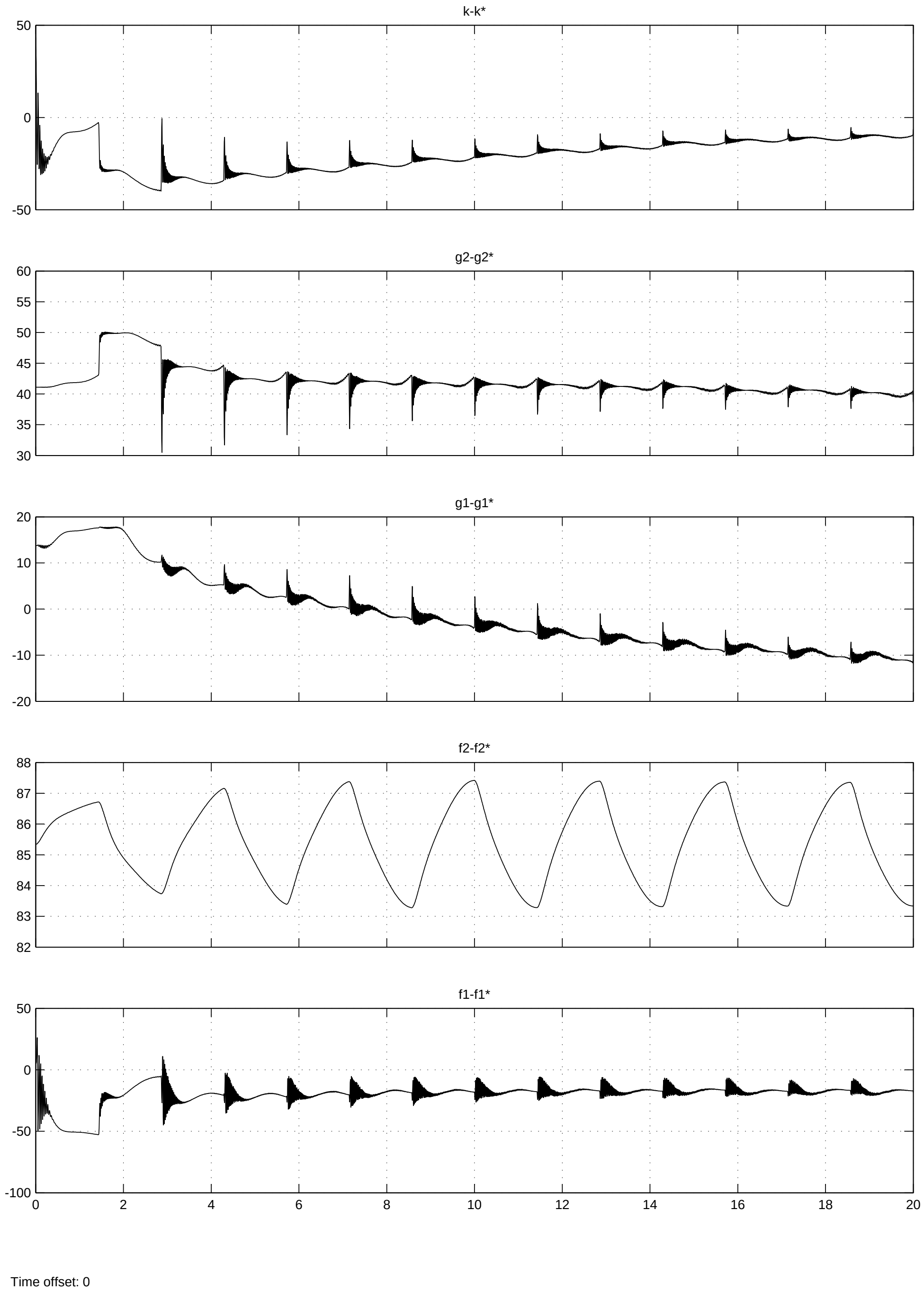

Figure 2.15: Square Input Control Parameters, Gammma = Gamma*10

the rise/dip of the reference. The other assumption of a much higher natural frequency than the system actually posses proves to be very detrimental to the performance of the controller. Since the controller assumes the plant to be able to operate at a much higher speed than what is achievable, it actuates the plant thusly. But the plant is unable to follow this cover as it is significantly higher than its natural frequency as a result of which, it oscillates wildly.

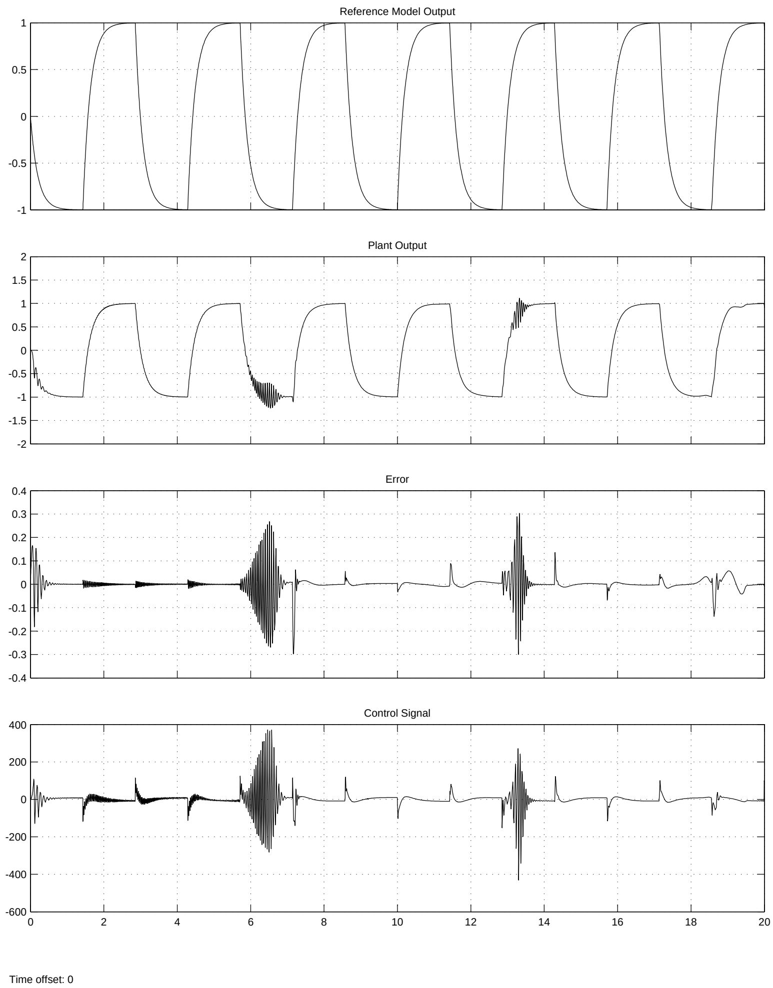

Figure 2.16: Plant Performance for Decreased ω_n

2.3.2 Variation in Damping

Now, we shall assume variations of the actual damping ratio of the system. The original system had a damping factor of 0.1, on the assumption that it was lightly damped.

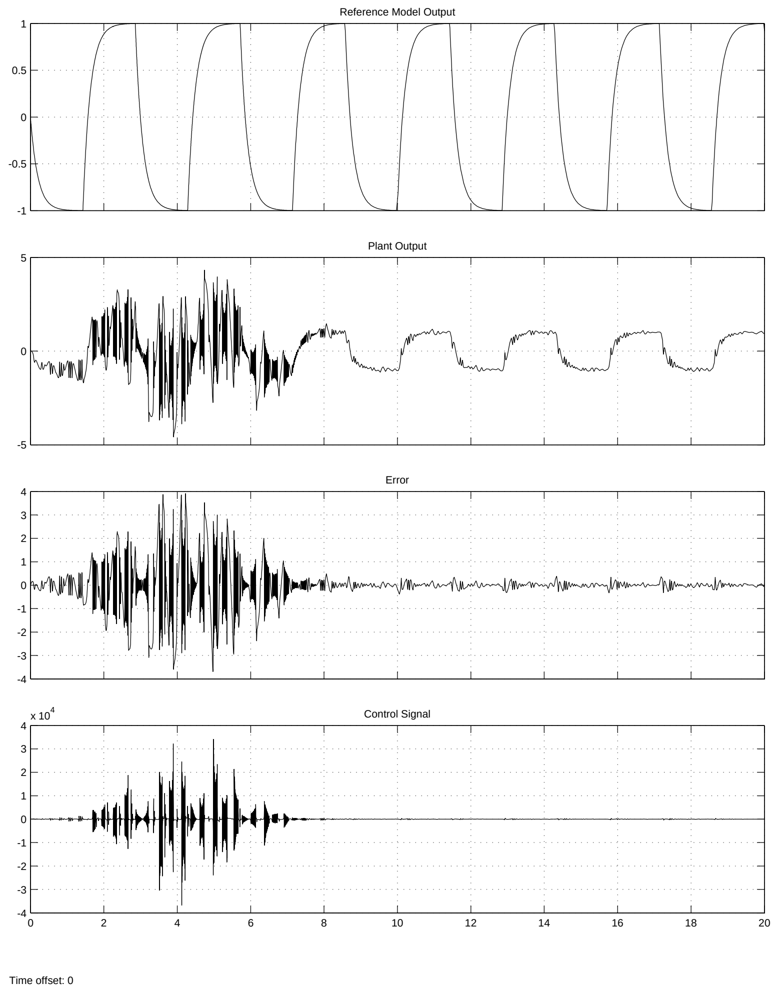

Figure 2.17: Plant Performance for Increased ω_n

In this section the damping factor will be both decreased and increased by a factor of 5. Damping factor does not seem to have a huge impact on the performance of the controller. When a lower damping factor was assumed, the controller had to compensate for its initial error and had to correct for the inaccuracy, which it did by switching more frequently. On the other hand when the damping factor was assumed to be higher than what the plant actually exhibited, the controller's performance dropped, but not significantly. It was still able to track the signal, but allowed the plant output to drop on each cycle and then compensated to reach the required reference trajectory.

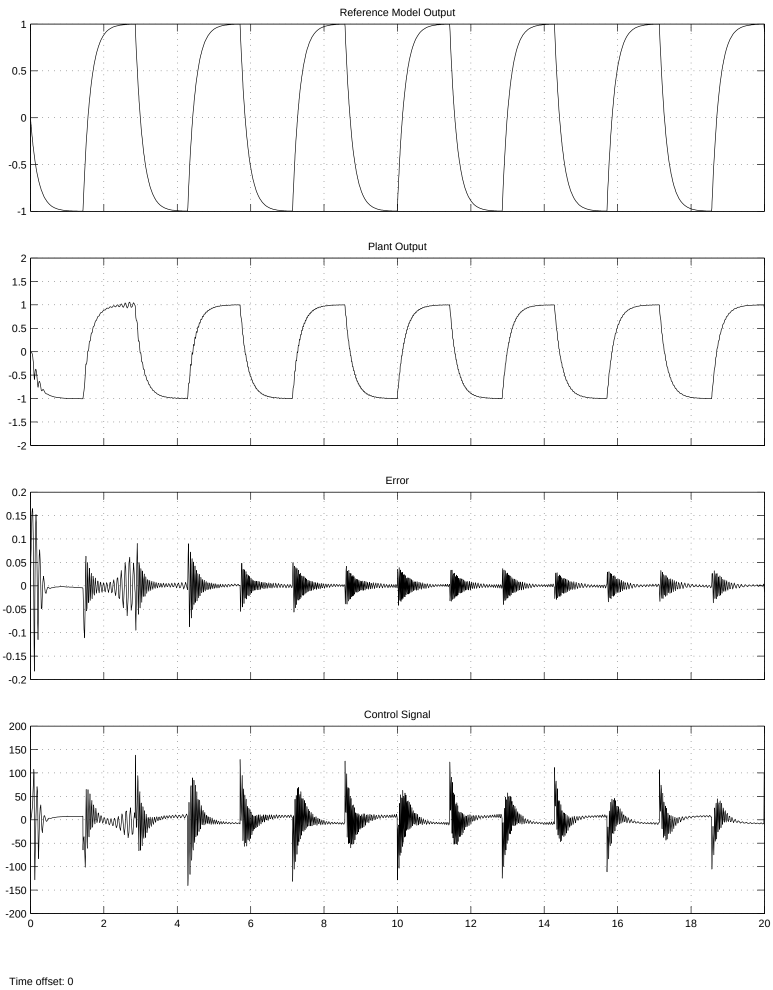

Figure 2.18: Plant Performance for Decreased ζ

In conclusion, the natural frequency of the system seems to have the most impact in terms of both stability and robustness of the system. While an inaccurate estimation of the damping factor of the system did prove to be bad for the effectiveness of the controller,

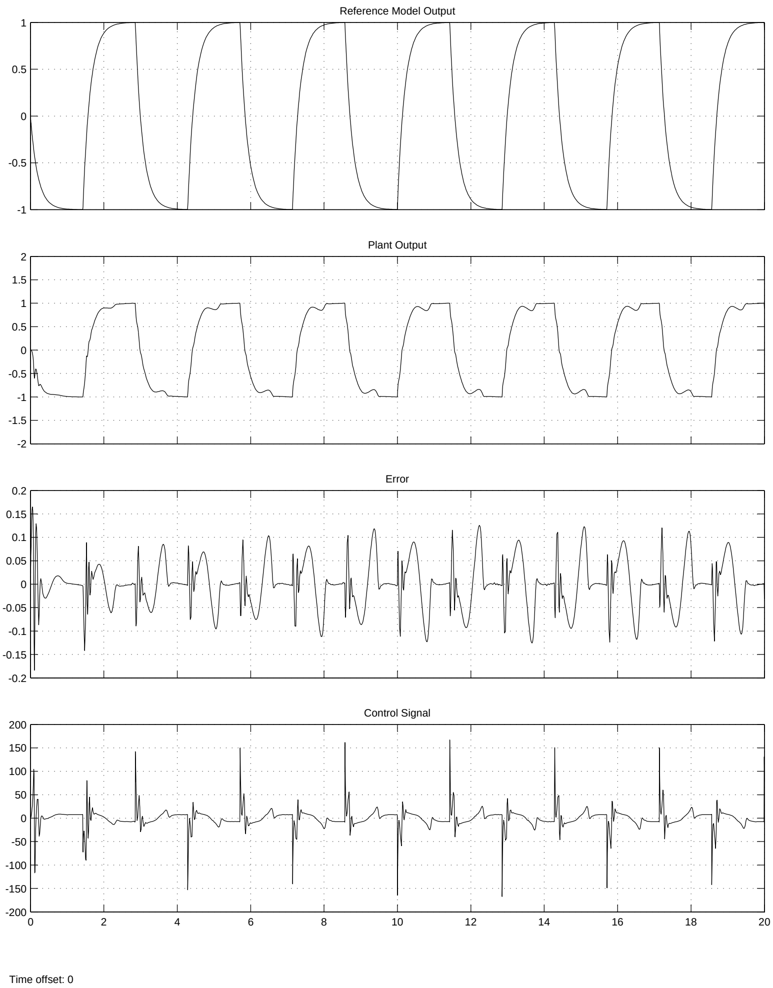

Figure 2.19: Plant Performance for Increased ζ

it did not hinder its ability to track the reference signal completely. When the natural frequency of the system was assumed to be higher than what the system exhibited, the system went to large oscillations during the first two periods of the reference trajectory, and continued to show symptoms of oscillations further on during the operation.

2.4 Sinusoidal Reference Signal

In this section we apply a sinusoidal input of

$$r(t) = 10sin(2t) \tag{2.9}$$

with the following control parameters

$$\Gamma = 5000 * \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$
$$T(p) = s^2 + 0.5s + 6.25$$

Figures: 2.20 and 2.21 show the performance of the plant the variation in the controller gains for the given reference signal. There are two points of interest in this simulations. One that the error does not tend towards zero as the simulation progresses. This is explained by the other point of interest wherein the controller gain do not converge but rather showcase sustained oscillations.

But the output of the plant shows us that the plant is able to track the reference signal quite well. From this we can deduce that the the non-convergence of the controller gains does not necessarily imply the failure of the controller. But from previous sections we know that convergence of the controller gains does guarantee the performance of the controller.

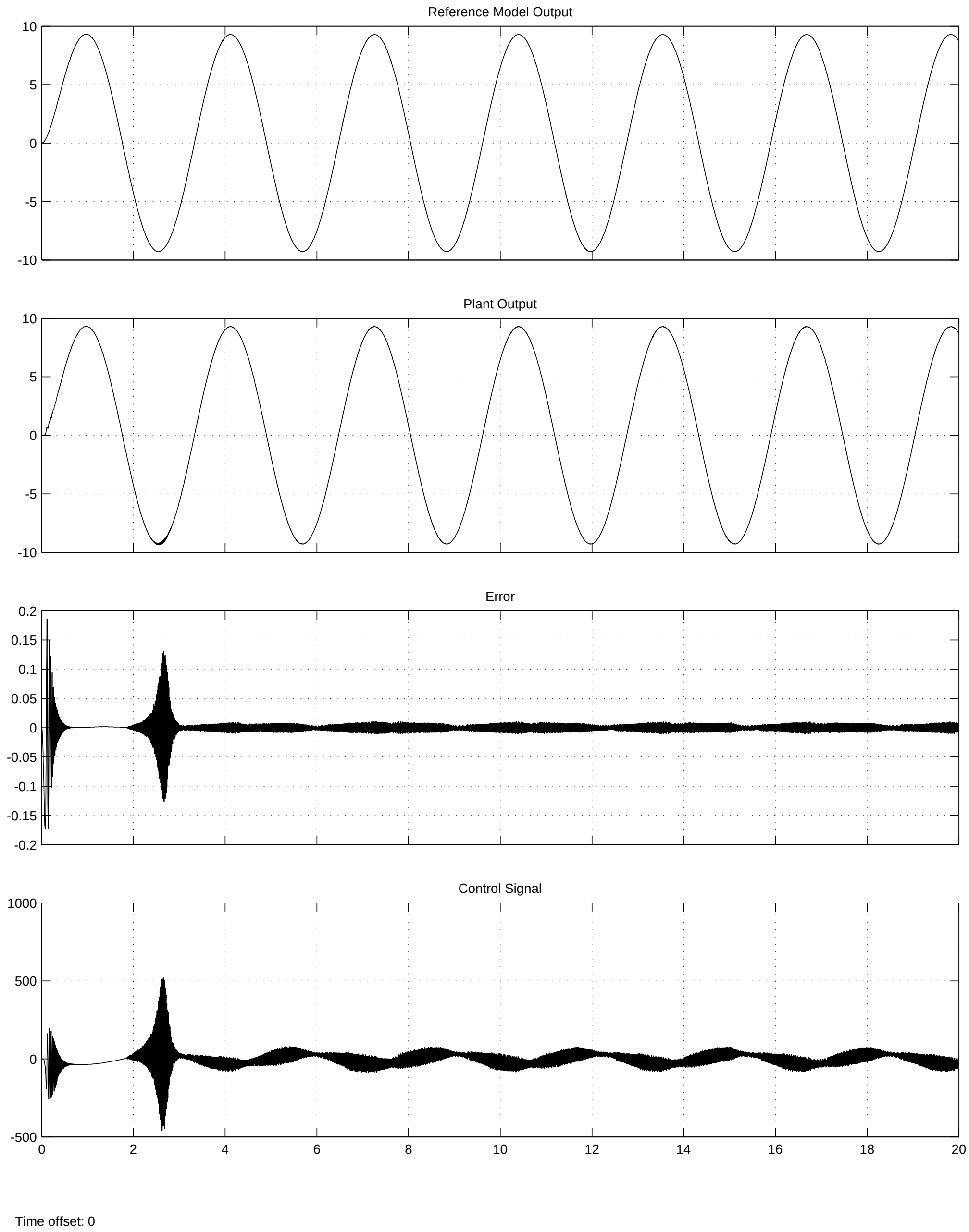

Figure 2.20: Sine Input Plant Performance

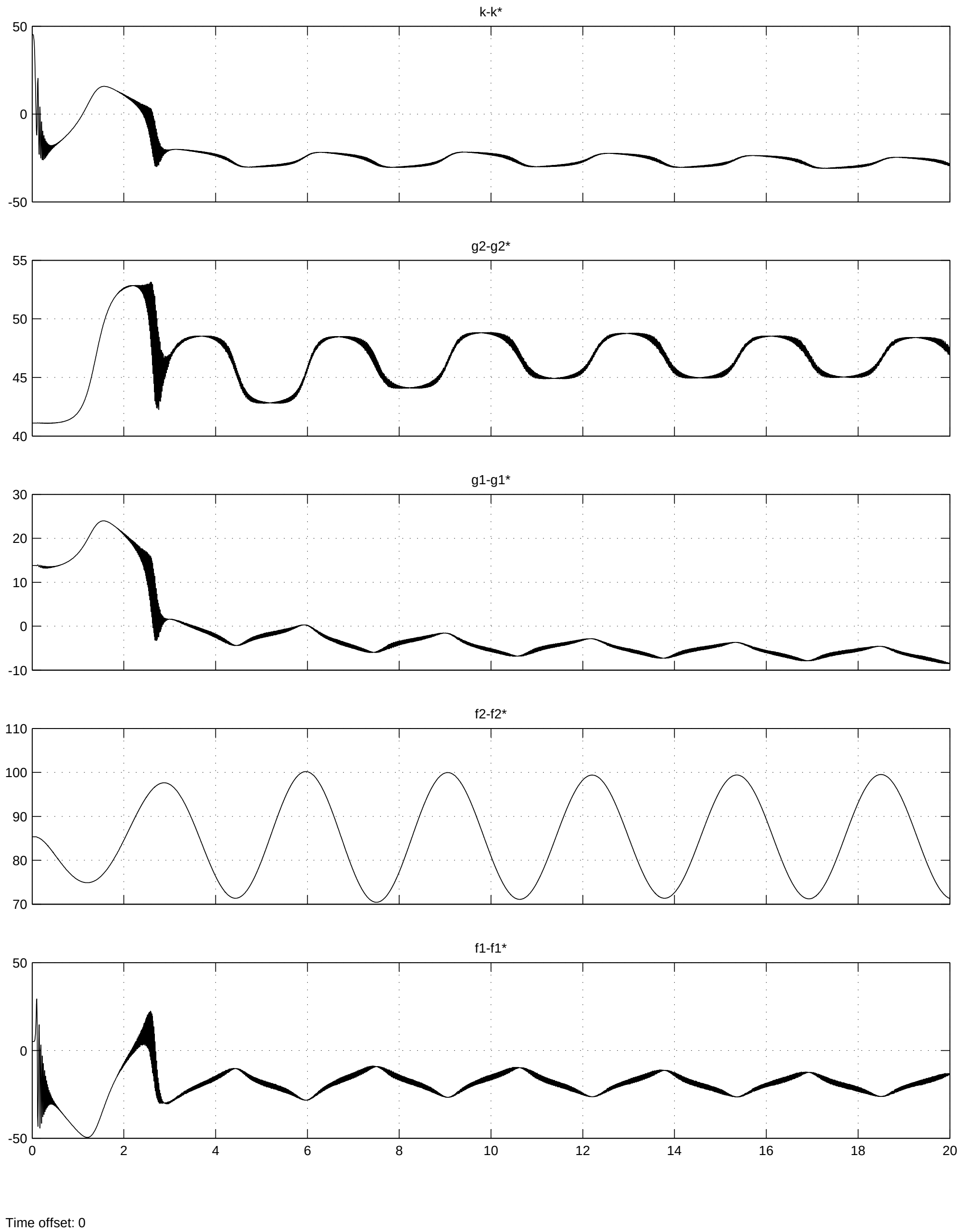

Figure 2.21: Sine Input Control Parameters

3

Conclusion

In this study, a Model Reference Adaptive Controller has been designed for a continuous time system with unmeasurable output states. The system

$$\frac{Y(s)}{U(s)} = \frac{-0.11s - 1}{s^2 + 0.29s + 7.8}$$

was simulated and controllers, both exact and adaptive were designed and the performance of each evaluated for exact observers.

The impact of both adaptation gain Γ and the observer polynomial $T(s)$ was studied. It was found that an increase in adaptation gains leads to better convergence of the controller gains but also increases the switching in of the control signal, which may not be particularly desirable in a real life scenario. The controller was able to perform at its best when the observer polynomial closely matched the plant, which is obvious as the ability of the controller depends on its ability to gather data from the output of the observer. The natural frequency of the observer seemed to have the most impact on the ability of the controller, with the controller having to work extra hard when the natural frequency was assumed to be higher than that of the plant.

Lastly the system was tested for a sinusoidal reference. The simulation results show that even though the plant is able to track the reference signal quite well, the controller gains do not converge but rather oscillate which leads us to formulate that the non-convergence of the controller gains does not imply a controllers inability to control the plant, but the convergence of the gains definitely proves its ability to do the same.

A

Appendix

This section contains all the models and callback functions used in the simulation
startup_adaptive.m: Initialization function for all parameters required for the adaptive controller
startup_exact.m: Initialization function for all parameters required for the exact gain controller
Exact Gain Controller
Adaptive Controller

Listing A.1: startup_adaptive

```
% This Program contains the initialization parameters for the adaptive
% control system.
% Name: Narasimha Prasad Prabhu
% Reg No. A0090717
% Submission for EE5104 CA1

clear

% Observor Characteristics
zeta = 0.1; % Damping factor of the observor system
wn = 2.5; % Natural frequency for the observor system
T = [1 2*zeta*wn wn^2]; % Observor Polynomial

% Adaptive Parameters
gamma = 50000*[1 0 0 0 0;
          0 1 0 0 0;
          0 0 1 0 0;
          0 0 0 1 0;
          0 0 0 0 1]; % Gamma is a symmetric Positive definite matrix
ulim = inf; % Upper Limit for integrals in "adaptive control strategy"
llim = -inf; % Lower Limit for integrals in "adaptive control strategy"

% Reference Model System Parameter
am = 5; % Reference model system is given by G = am/(s+am)

% Exact Controller Parameters
% Note: These have been calculated only to view progression of \theta and
```

```
% are not required in the control mechanism
g1 = 13.8;
g2 = 41.1089;
f1 = 5.0991;
f2 = 85.3455;
kexact = -45.4545;
```

Listing A.2: start_exact

```
% This Program contains the initialization parameters for the adaptive
% control system.
% Name: Narasimha Prasad Prabhu
% Reg No. A0090717
% Submission for EE5104 CA1

clear
% Observor Characteristics
zeta = 0.1; % Damping factor of the observor system
wn = 2.5; % Natural frequency for the observor system
T = [1 2*zeta*wn wn^2]; % Observor Polynomial


% Reference Model System Parameter
am = 5; % Reference model system is given by G = am/(s+am)

% Exact Controller Parameters
g1 = 13.8;
g2 = 41.1089;
f1 = 5.0991;
f2 = 85.3455;
kexact = -45.4545;
```

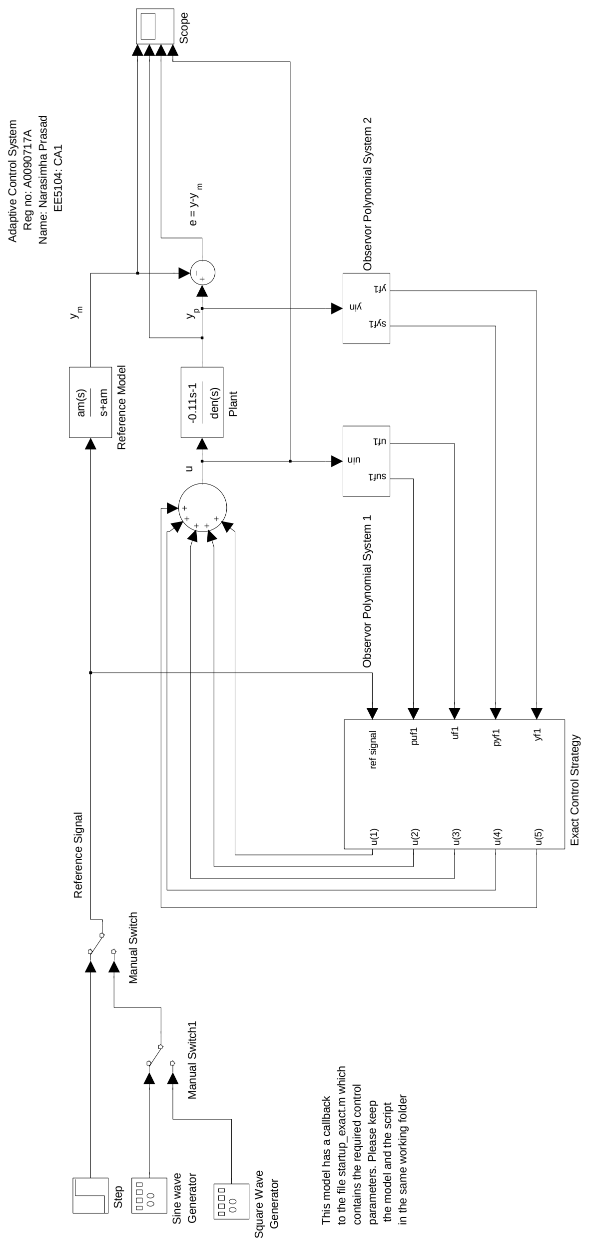

Figure A.1: Overview of controller system with exact gains

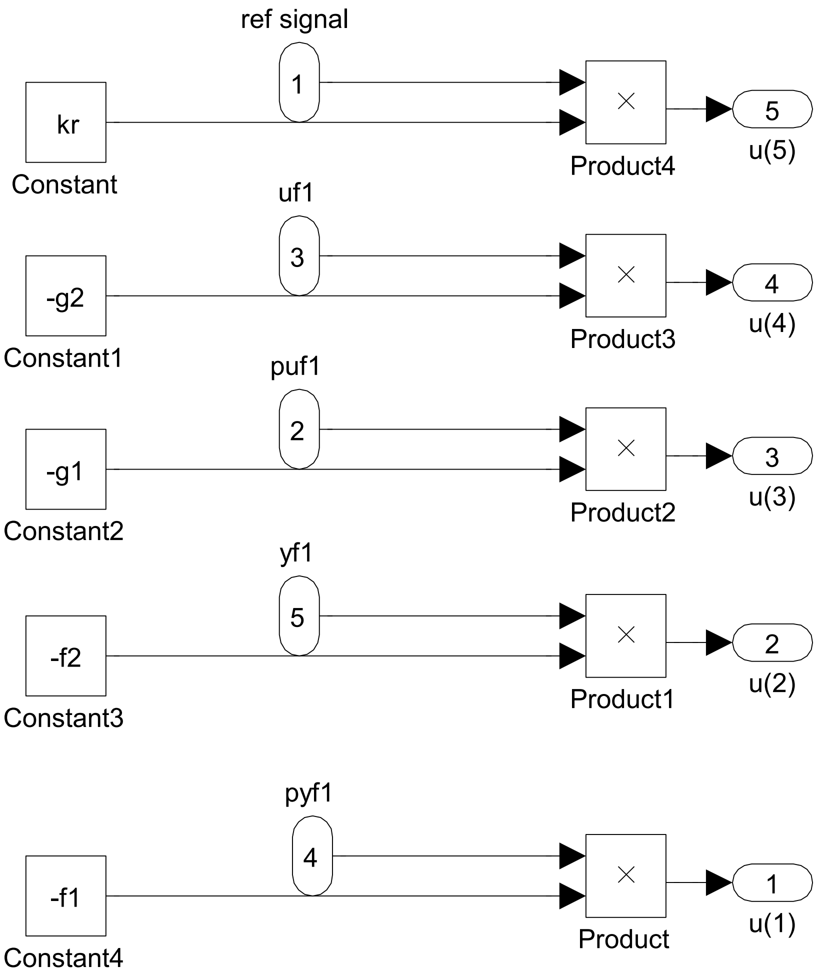

Figure A.2: Exact Controller Gain Strategy

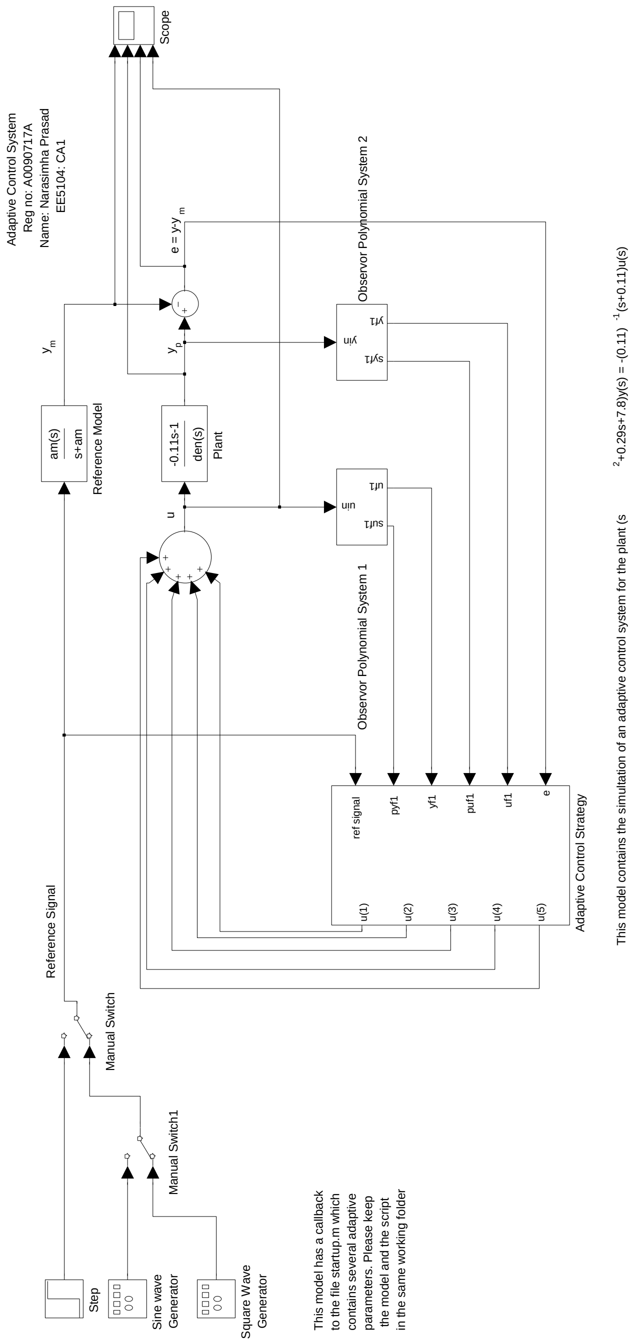

Figure A.3: Adaptive Controller

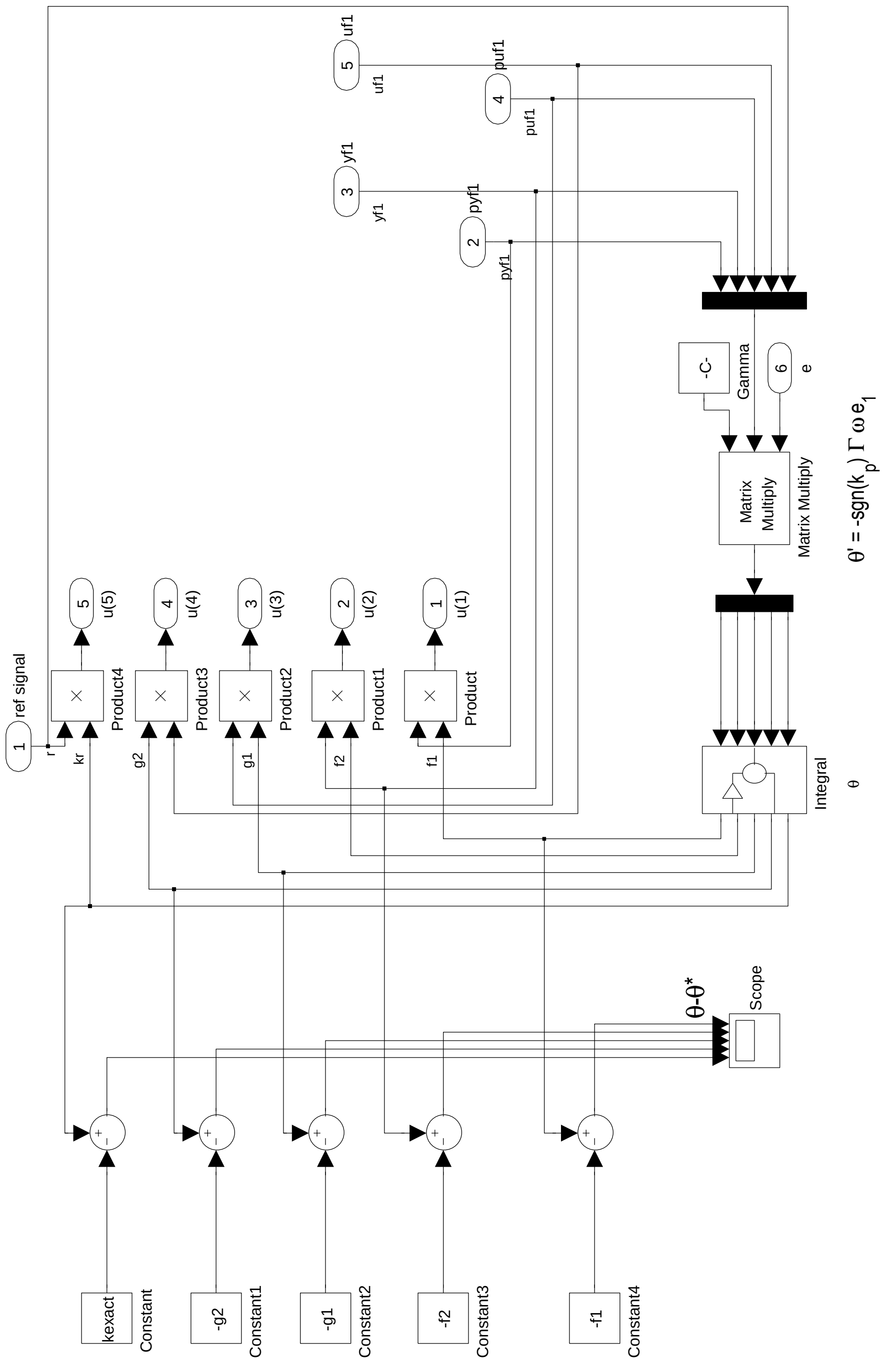

Figure A.4: Adaptive Control Strategy

Bibliography

[1] B. M. Chen, Z. Lin, Y. Shamash, Linear systems theory, 2004.