Dependence Problem: Distance & Direction Vectors

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Dependence Concepts

Two statements S and T, if there is an instance S(i) of S, an instance T(j) of T, and a memory location M, such that

1. Both S(i) and T(j) reference (read or write) M
2. S(i) is executed before T(j) in the sequential execution of the program
3. During sequential execution, the location M is not written in the time period from the end of execution of S(i) to the beginning of execution of T(j).

Let d denote the distance from S(i) to T(j)

Let *б* = sig(d) and *L* = lev(d)

Where,

d is a distance vector,

б a direction vector, and

L a dependence level for the dependence of T on S.

Dependence Distance Vector

Definition:

Suppose that there is a dependence from S on iteration $\boldsymbol{i}$ to T on iteration $\boldsymbol{j}$; then the *dependence distance vector* $\mathbf{d}(\boldsymbol{i}, \boldsymbol{j})$ is defined as

$$\boldsymbol{d}(\boldsymbol{i}, \boldsymbol{j}) = \boldsymbol{j} - \boldsymbol{i} \quad \boldsymbol{or} \quad \boldsymbol{d}(\boldsymbol{i}, \boldsymbol{j})_k = \boldsymbol{j}_k - \boldsymbol{i}_k$$

Example:

```
  DO I = 1, 3
    DO J = 1, I
S     A(I+1,J) = A(I,J)
    ENDDO
  ENDDO
```

True dependence between S and itself on
$\boldsymbol{i} = (1,1)$ and $\boldsymbol{j} = (2,1)$: $\boldsymbol{d}(\boldsymbol{i}, \boldsymbol{j}) = (1,0)$
$\boldsymbol{i} = (2,1)$ and $\boldsymbol{j} = (3,1)$: $\boldsymbol{d}(\boldsymbol{i}, \boldsymbol{j}) = (1,0)$
$\boldsymbol{i} = (2,2)$ and $\boldsymbol{j} = (3,2)$: $\boldsymbol{d}(\boldsymbol{i}, \boldsymbol{j}) = (1,0)$

Dependence Direction Vector

Definition: Suppose that there is a dependence from S on iteration $\boldsymbol{i}$ and T on iteration $\boldsymbol{j}$; then the *dependence direction vector* $\mathbf{D}(\boldsymbol{i}, \boldsymbol{j})$ is defined as

$$\boldsymbol{D}(\boldsymbol{i},\boldsymbol{j})_k = \begin{cases} \text{“<” if } \boldsymbol{d}(\boldsymbol{i},\boldsymbol{j})_k > 0 \\ \text{“=” if } \boldsymbol{d}(\boldsymbol{i},\boldsymbol{j})_k = 0 \\ \text{“>” if } \boldsymbol{d}(\boldsymbol{i},\boldsymbol{j})_k < 0 \end{cases}$$

By using direction vector, we can find the level of dependence i.e

1 → inner loop

2 → outer loop

3 → involve both loops

Direction vector

The instance T(j1, j2) depends on the instance S(i1, i2). Let (d1, d2) denote the distance from S(i1, i2) to T (j1, j2).

Let (б1,б2) = (sig(d1), sig(d2)) and L= lev (d1, d2).

$$\sigma_1 = \begin{cases} 1 & \text{if } d_1 > 0 \\ -1 & \text{if } d_1 < 0 \\ 0 & \text{if } d_1 = 0, \end{cases}$$

$$\sigma_2 = \begin{cases} 1 & \text{if } d_2 > 0 \\ -1 & \text{if } d_2 < 0 \\ 0 & \text{if } d_2 = 0, \end{cases}$$

$$\ell = \begin{cases} 1 & \text{if } d_1 > 0 \\ 2 & \text{if } d_1 = 0 \text{ and } d_2 > 0 \\ 3 & \text{if } d_1 = 0 \text{ and } d_2 = 0. \end{cases}$$

(d1, d2) is a distance vector, (б1,б2) is a direction vector, and L is a dependence level for the dependence of T on S.

Example 1

```
       DO I = 1, N
          DO J = 1, M
             DO K = 1, L
S1              A(I+1,J,K-1) = A(I,J,K) + 10
             ENDDO
          ENDDO
       ENDDO
```

direction vector (<,=,>)

Dependence cannot exist if it has a direction vector whose leftmost non = component is not “<” because that would mean that the sink of the dependence occurs before the source, which is impossible.

Example 2

In a perfect nest of loops, a particular loop can be parallelized at the outermost level if and only if the column of the direction matrix for that nest contains only "=" entries.

```
DO I = 1, N
   DO J = 1, M
      DO K = 1, L
         A(I+1,J,K) = A(I,J,K) + X1
         B(I,J,K+1) = B(I,J,K) + X2
         C(I+1,J+1,K+1) = C(I,J,K) + X3
      ENDDO
   ENDDO
ENDDO
```

Since there are no columns with all "=" entries, none of the loops can be parallelized at the outermost level.

The direction matrix for this nest is

$$\begin{bmatrix} < & = & = \\ = & = & < \\ < & < & < \end{bmatrix}$$

Example 3

```
DO I = 2, N+1
   DO J = 2, M+1
      DO K = 1, L
         A(I,J,K) = A(I,J-1,K+1)+ A(I-1,J,K+1)
      ENDDO
   ENDDO
ENDDO
```

This code has a direction vector in which all the directions in the inner-most loop are ">":

$$\begin{bmatrix} = & < & > \\ < & = & > \end{bmatrix}$$

Example 3 continue..

- The aforementioned loop can be reversed by applying loop reversal to parallel some loops

- Reverse the direction of iteration of the inner loop (i.e. run from L to 1 by −1), which reverses the directions for that loop in every dependence, now move the loop to the outermost position

Now the direction vectors will be

$$\begin{bmatrix} < = < \\ < < = \end{bmatrix}$$

Since all dependences are now carried by the outer loop, running it sequentially allows the two inner loops to be run in parallel. The code that results is

```
DO K = L, 1, -1
   PARALLEL DO I = 2, N+1
      PARALLEL DO J = 2, M+1
         A(I,J,K) = A(I,J-1,K+1)+ A(I-1,J,K+1)
      END PARALLEL DO
   END PARALLEL DO
ENDDO
```

Example 4

$$
\begin{array}{lll}
L_1: & \textbf{do } I_1 = 10, 100, 2 & \\
L_2: & \quad \textbf{do } I_2 = 16, 11, -2 & \\
\quad S: & \qquad X(I_1 + 2, I_2) = \cdots & \\
\quad T: & \qquad \cdots = \cdots X(I_1, I_2 - 4) \cdots & \\
\quad U: & \qquad \cdots = \cdots X(I_1 + 3, I_2 + 2) \cdots & \\
\quad V: & \qquad X(I_1, I_2) = \cdots & \\
 & \quad \textbf{enddo} & \\
 & \textbf{enddo} &
\end{array}
$$

Iteration Point	Iteration of loop Nest	
(0,0)	$S(10,16):$	$X(12,16) = \cdots$
	$T(10,16):$	$\cdots = \cdots X(10,12) \cdots$
	$U(10,16):$	$\cdots = \cdots X(13,18) \cdots$
	$V(10,16):$	$X(10,16) = \cdots$
(0,1)	$S(10,14):$	$X(12,14) = \cdots$
	$T(10,14):$	$\cdots = \cdots X(10,10) \cdots$
	$U(10,14):$	$\cdots = \cdots X(13,16) \cdots$
	$V(10,14):$	$X(10,14) = \cdots$
(0,2)	$S(10,12):$	$X(12,12) = \cdots$
	$T(10,12):$	$\cdots = \cdots X(10,8) \cdots$
	$U(10,12):$	$\cdots = \cdots X(13,14) \cdots$
	$V(10,12):$	$X(10,12) = \cdots$
(1,0)	$S(12,16):$	$X(14,16) = \cdots$
	$T(12,16):$	$\cdots = \cdots X(12,12) \cdots$
	$U(12,16):$	$\cdots = \cdots X(15,18) \cdots$
	$V(12,16):$	$X(12,16) = \cdots$
(1,1)	$S(12,14):$	$X(14,14) = \cdots$
	$T(12,14):$	$\cdots = \cdots X(12,10) \cdots$
	$U(12,14):$	$\cdots = \cdots X(15,16) \cdots$
	$V(12,14):$	$X(12,14) = \cdots$
(1,2)	$S(12,12):$	$X(14,12) = \cdots$
	$T(12,12):$	$\cdots = \cdots X(12,8) \cdots$
	$U(12,12):$	$\cdots = \cdots X(15,14) \cdots$
	$V(12,12):$	$X(12,12) = \cdots$
(2,0)	$S(14,16):$	$X(16,16) = \cdots$
	$T(14,16):$	$\cdots = \cdots X(14,12) \cdots$
	$U(14,16):$	$\cdots = \cdots X(17,18) \cdots$
	$V(14,16):$	$X(14,16) = \cdots$
(2,1)	$S(14,14):$	$X(16,14) = \cdots$
	$T(14,14):$	$\cdots = \cdots X(14,10) \cdots$
	$U(14,14):$	$\cdots = \cdots X(17,16) \cdots$
	$V(14,14):$	$X(14,14) = \cdots$
(2,2)	$S(14,12):$	$X(16,12) = \cdots$
	$T(14,12):$	$\cdots = \cdots X(14,8) \cdots$
	$U(14,12):$	$\cdots = \cdots X(17,14) \cdots$
	$V(14,12):$	$X(14,12) = \cdots$

- Here, statement T is flow dependent on statement S.
- For example, the instance T(12, 16) depends on the instance S(10, 12).
- The iteration points corresponding to the index points (12, 16) and (10, 12), are (1, 0) and (0, 2), respectively.
- Distance vector is (dl, d2) = (1,-2), and the direction vector is (б1,б2) = (1,-1).
- It implies that T depends on S at level 1, since d1 > 0. This dependence is carried by the outer loop L1, it is not carried by the inner loop L2, and there is no loop-independent part.
- The pattern is repeated several times.
- For example, T(14, 16) depends on S(12, 12), T(16, 16) depends on S(14, 12), and so on. We do not get any new distance or direction vectors.

- Statement V is anti-dependent on statement T.
- Indeed, V(10, 12) depends on T(10, 16), V(12, 12) depends on T(12, 16), etc. For this dependence, the unique distance vector, direction vector, and level are (0, 2), (0, 1), and 2, respectively.
- This dependence is carried by the inner loop L2, but not by L1.

- Statement V is output dependent on statement S, since V(12, 16) depends on S(10, 16), V(12, 14) depends on S(10, 14), etc.
- In this case, the distance vector is (1,0), the direction vector is also (1, 0), and the level of dependence is 1.
- This dependence is carried by L1, but not by L2.

- There is no dependence between statement U and any other statement in the loop nest.

Example 5

dependence between statements and their instances in the triple loop

$$
\begin{array}{lll}
L_1: & & \textbf{do } I_1 = 1, 4, 2 \\
L_2: & & \quad \textbf{do } I_2 = 10, 5, -3 \\
L_3: & & \quad\quad \textbf{do } I_3 = 1, 3, 1 \\
& S: & \quad\quad\quad X(I_1 + I_2 + I_3) = \cdots \\
& T: & \quad\quad\quad \cdots = \cdots X(I_2 + I_3) \cdots \\
& & \quad\quad \textbf{enddo} \\
& & \quad \textbf{enddo} \\
& & \textbf{enddo}
\end{array}
$$

Iteration Point	Iteration of loop Nest
(0, 0, 0)	$S(1,10,1): \quad X(12) = \cdots$ $T(1,10,1): \quad \cdots = \cdots X(11) \cdots$
(0, 0, 1)	$S(1,10,2): \quad X(13) = \cdots$ $T(1,10,2): \quad \cdots = \cdots X(12) \cdots$
(0, 0, 2)	$S(1,10,3): \quad X(14) = \cdots$ $T(1,10,3): \quad \cdots = \cdots X(13) \cdots$
(0, 1, 0)	$S(1,7,1): \quad X(9) = \cdots$ $T(1,7,1): \quad \cdots = \cdots X(8) \cdots$
(0, 1, 1)	$S(1,7,2): \quad X(10) = \cdots$ $T(1,7,2): \quad \cdots = \cdots X(9) \cdots$
(0, 1, 2)	$S(1,7,3): \quad X(11) = \cdots$ $T(1,7,3): \quad \cdots = \cdots X(10) \cdots$
(1, 0, 0)	$S(3,10,1): \quad X(14) = \cdots$ $T(3,10,1): \quad \cdots = \cdots X(11) \cdots$
(1, 0, 1)	$S(3,10,2): \quad X(15) = \cdots$ $T(3,10,2): \quad \cdots = \cdots X(12) \cdots$
(1, 0, 2)	$S(3,10,3): \quad X(16) = \cdots$ $T(3,10,3): \quad \cdots = \cdots X(13) \cdots$
(1, 1, 0)	$S(3,7,1): \quad X(11) = \cdots$ $T(3,7,1): \quad \cdots = \cdots X(8) \cdots$
(1, 1, 1)	$S(3,7,2): \quad X(12) = \cdots$ $T(3,7,2): \quad \cdots = \cdots X(9) \cdots$
(1, 1, 2)	$S(3,7,3): \quad X(13) = \cdots$ $T(3,7,3): \quad \cdots = \cdots X(10) \cdots$

After Unrolling

From Instance	To Instance	Type	Distance	Direction	Level
$S(1,10,1)$	$T(1,10,2)$	Flow	$(0,0,1)$	$(0,0,1)$	3
$S(1,10,1)$	$T(3,10,2)$	Flow	$(1,0,1)$	$(1,0,1)$	1
$S(1,10,1)$	$S(3,7,2)$	Output	$(1,1,1)$	$(1,1,1)$	1
$T(1,10,1)$	$S(1,7,3)$	Anti-	$(0,1,2)$	$(0,1,1)$	2
$S(1,7,3)$	$T(3,10,1)$	Flow	$(1,-1,-2)$	$(1,-1,-1)$	1
$T(3,10,1)$	$S(3,7,1)$	Anti-	$(0,1,0)$	$(0,1,0)$	2
$S(1,7,3)$	$S(3,7,1)$	Output	$(1,0,-2)$	$(1,0,-1)$	1
$S(1,10,2)$	$T(1,10,3)$	Flow	$(0,0,1)$	$(0,0,1)$	3
$T(1,10,3)$	$S(3,7,3)$	Anti-	$(1,1,0)$	$(1,1,0)$	1
$S(1,10,2)$	$S(3,7,3)$	Output	$(1,1,1)$	$(1,1,1)$	1
$S(1,10,3)$	$S(3,10,1)$	Output	$(1,0,-2)$	$(1,0,-1)$	1
$S(1,7,1)$	$T(1,7,2)$	Flow	$(0,0,1)$	$(0,0,1)$	3
$S(1,7,1)$	$T(3,7,2)$	Flow	$(1,0,1)$	$(1,0,1)$	1
$S(1,7,2)$	$T(1,7,3)$	Flow	$(0,0,1)$	$(0,0,1)$	3
$S(1,7,2)$	$T(3,7,3)$	Flow	$(1,0,1)$	$(1,0,1)$	1

Dependence structure of (L1, L2, L3)

1. There is a flow dependence of T on S. The distance vectors of this dependence are (0,0,1), (1,0,1), and (1,-1,-2) . The direction vectors of this dependence are (0, 0, 1), (1, 0, 1), and (1, -1 , - 1) . The dependence levels are 1 and 3. No part of this dependence is carried by the loop L2.

2. There is an anti-dependence of S on T. The distance vectors of this dependence are (0, 1,2), (0, 1,0), and (1, 1,0). The direction vectors are (0, 1, 1), (0, 1, 0), and (1, 1, 0). The dependence levels are 1 and 2. No part of this dependence is carried by L3.

3. There is an output-dependence of S on itself. The distance vectors of this dependence are (1, 1, 1) and (1, 0, − 2). The direction vectors are (1, 1, 1) and (1, 0, − 1). The only dependence level is 1. No part of this dependence is carried by either L2 or L3.