Dependence Analysis

Delivered by

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Dependence Problem

Example:

```
Do i =1,N
        X(f(i)) =
                    = X(g(i))
EndDo
```

Dependence Types in loop

1. The flow dependence of T on S consists of all instance pair (S(i),T(j)) such that T(j) is flow dependent on S(i).
2. The anti-dependence of T on S consists of all instance pair (S(i),T(j)) such that T(j) is anti-dependent on S(i).
3. The output dependence of T on S consists of all instance pair (S(i),T(j)) such that T(j) is output dependent on S(i).
4. The input dependence of T on S consists of all instance pair (S(i),T(j)) such that T(j) is input dependent on S(i).

Data Dependence

In general we need to know if two usages of an array access the same memory location and what type of dependence helpful as this can be done relatively cheaply for simple programs

- General dependence is intractable at $X(f(i)) = X(g(i))$ for arbitrary f, g

- ***Theorem:*** *Consider any two statements S and T in the loop L. Let X(f(I)) denote a variable of S and X(g(I)) a variable of T, where X is a one-dimensional array. If these variables cause a dependence between S and T, then the equation*

$$f(p + \theta\, \hat{\imath}) - g(p + \theta\, \hat{\jmath}) = 0 \qquad (2.1)$$

$$0 \le \hat{\imath} \le \hat{q} \quad and \quad 0 \le \hat{\jmath} \le \hat{q}, \qquad (2.2)$$

where $\hat{q} = \lfloor (q - p)/\theta \rfloor$. In each of the following special cases, the solution satisfies an additional condition:

(a) *If $S < T$ and $S\, \delta\, T$, then $\hat{\imath} \le \hat{\jmath}$;*

(b) *If $S = T$ and $S\, \delta\, S$, then $\hat{\imath} < \hat{\jmath}$;*

(c) *If $S < T$ and $T\, \delta\, S$, then $\hat{\imath} > \hat{\jmath}$.*

$$f(i) = g(j)$$

- $i = p + \theta\, \hat{\imath}$ and $j = p + \theta\, \hat{\jmath}$
- $f(p + \theta\, \hat{\imath}) = g(p + \theta\, \hat{\jmath})$
- The dependence equation then

$$f(i) = g(j)$$

The index value i and j satisfy the condition

1. $(i - p)/\theta$ is an integer
2. $0 <= (i - p)/\theta <= (q-p)/\theta$
3. $(j - p)/\theta$ is an integer
4. $0 <= (j - p)/\theta <= (q-p)/\theta$

- **Theorem:** *Suppose that Equation (2.1)has an integer solution* (î, ĵ) *that satisfies the conditions in (2. 2).*

(a) *If* $\hat{\imath} < \hat{\jmath}$, *then* $S\,\overline{\delta}\,T$.

(b) *If* $\hat{\imath} > \hat{\jmath}$, *then* $T\,\overline{\delta}\,S$.

(c) *If* $\hat{\imath} = \hat{\jmath}$ *and* $S < T$, *then* $S\,\overline{\delta}\,T$.

Example

$$
\begin{array}{lll}
L: & \textbf{do } I = 1, 100, 2 & \\
 & \quad \vdots & \\
\quad S: & \quad X(2I) = \cdots & \\
 & \quad \vdots & \\
\quad T: & \quad \cdots = \cdots X(3I+1) \cdots & \\
 & \quad \vdots & \\
 & \textbf{enddo} &
\end{array}
$$

Let us test for dependence between the statements S and T caused by the two variables shown. Here, we have $p = 1, q = 100, \theta = 2$, $f(I) = 2I$ and $g(I) = 3I + 1$. Since

$$
\begin{aligned}
f(p + \theta\hat{\imath}) - g(p + \theta\hat{\jmath}) &= 2(p + \theta\hat{\imath}) - 3(p + \theta\hat{\jmath}) - 1 \\
&= 2(1 + 2\hat{\imath}) - 3(1 + 2\hat{\jmath}) - 1 \\
&= 4\hat{\imath} - 6\hat{\jmath} - 2,
\end{aligned}
$$

- the dependence equation

$$4\hat{\imath} - 6\hat{\jmath} = 2$$

Since $\hat{q} = \lfloor (100 - 1)/2 \rfloor = 49$,

$$0 \le \hat{\imath} \le 49 \qquad \text{and} \qquad 0 \le \hat{\jmath} \le 49.$$

$T\,\overline{\delta}\,S$ holds and that $S\,\overline{\delta}\,T$ does not.