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MODULE ParReachProofs
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This module contains TLAPS checked proofs of the safety properties asserted in module ParReach-namely, the invariance of Inv and that the parallel alorithm implements the safety part of Misra's algorithm under the refinement mapping defined there.

EXTENDS ParReach, Integers, TLAPS

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LEMMA TypeInvariant \stackrel{\triangle}{=} Spec \Rightarrow \Box Inv
\langle 1 \rangle 1. Init \Rightarrow Inv
      BY RootAssump DEF Init, Inv, ProcSet
\langle 1 \rangle 2. Inv \wedge [Next]_{vars} \Rightarrow Inv'
    BY SuccAssump DEF Inv, Next, vars, ProcSet, p, a, b, c
\langle 1 \rangle 3. QED
   BY \langle 1 \rangle 1, \langle 1 \rangle 2, PTL DEF Spec
THEOREM Spec \Rightarrow R!Init \wedge \Box [R!Next]_R!vars
\langle 1 \rangle 1. Init \Rightarrow R!Init
      By ProcsAssump def Init, R! Init, pcBar, vrootBar, ProcSet
\langle 1 \rangle 2. Inv \wedge [Next]_{vars} \Rightarrow [R!Next]_R!vars
   \langle 2 \rangle suffices assume Inv,
                                       [Next]_{vars}
                         PROVE [R!Next]_R!vars
      OBVIOUS
   (2) USE DEF Inv, Next, vars, R! Next, R! vars, vrootBar, pcBar
   \langle 2 \rangle 1. Assume new self \in Procs,
                          a(self)
           PROVE [R!Next]_R!vars
      \langle 3 \rangle 1. Assume vroot \neq \{\}
              PROVE UNCHANGED R! vars
          BY \langle 2 \rangle 1, \langle 3 \rangle 1 DEF a
      \langle 3 \rangle 2. Assume vroot = \{\}
              PROVE [R!Next]_R!vars
         \langle 4 \rangle 1. Assume vrootBar = \{\}
                 PROVE [R!Next]_R!vars
           BY \langle 2 \rangle 1, \langle 3 \rangle 2, \langle 4 \rangle 1 DEF a, R!a
         \langle 4 \rangle 2. Assume vrootBar \neq \{\}
                 PROVE UNCHANGED R!vars
            \langle 5 \rangle 1. \quad \exists \ q \in Procs \setminus \{self\} : pc[q] \neq \text{"Done"}
               BY \langle 4 \rangle 2, \langle 3 \rangle 2, \langle 2 \rangle 1 DEF a
            \langle 5 \rangle 2. pcBar' \neq "Done"
               By \langle 5 \rangle 1, \langle 3 \rangle 2, \langle 2 \rangle 1 def a
            \langle 5 \rangle.QED
              BY \langle 5 \rangle 2, \langle 3 \rangle 2, \langle 2 \rangle 1 DEF a
         \langle 4 \rangle 3. QED
           BY \langle 4 \rangle 1, \langle 4 \rangle 2
      \langle 3 \rangle 3. QED
         BY \langle 3 \rangle 1, \langle 3 \rangle 2 DEF R!Next
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\langle 2 \rangle 2. \text{ Assume new } self \in Procs, \\ b(self) \\ \text{prove } [R!Next]_R!vars \\ \text{by } \langle 2 \rangle 2 \text{ def } b, R!a \\ \langle 2 \rangle 3. \text{ Assume new } self \in Procs, \\ c(self) \\ \text{prove } [R!Next]_R!vars \\ \text{by } \langle 2 \rangle 3 \text{ def } c \\ \langle 2 \rangle 4. \text{Case unchanged } vars \\ \text{by } \langle 2 \rangle 4 \\ \langle 2 \rangle 5. \text{ qed} \\ \text{by } \langle 2 \rangle 1, \langle 2 \rangle 2, \langle 2 \rangle 3, \langle 2 \rangle 4 \text{ def } Next, p \\ \langle 1 \rangle 3. \text{ qed} \\ \text{by } \langle 1 \rangle 1, \langle 1 \rangle 2, \textit{TypeInvariant, } \textit{PTL def Spec} \\ \text{Spec} \\ \text{
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 $[\]backslash * \ {\it Modification History}$

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