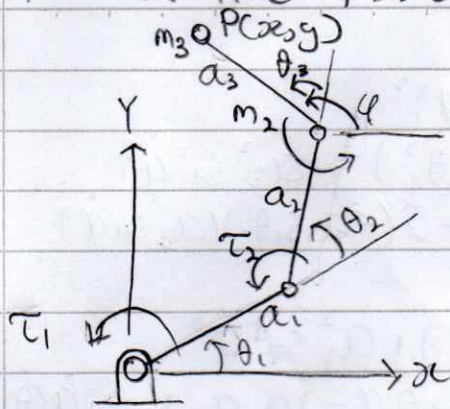


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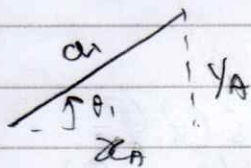


$$a_1 = 0.8\text{m}, a_2 = 1.1\text{m}, a_3 = 0.5\text{m}$$

$$m_1 = 15\text{kg}, m_2 = 12\text{kg}, m_3 = 5\text{kg}$$

$$\phi = \theta_1 + \theta_2 + \theta_3$$

①



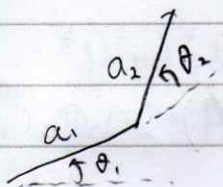
$$x_A = a_1 c_1$$

$$y_A = a_1 s_1$$

$$\dot{x}_A = -a_1 s_1 \dot{\theta}_1$$

$$\dot{y}_A = a_1 c_1 \dot{\theta}_1$$

②



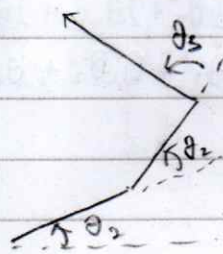
$$x_B = a_1 c_1 + a_2 c_{12}$$

$$y_B = a_1 s_1 + a_2 s_{12}$$

$$\dot{x}_B = -a_1 s_1 \dot{\theta}_1 - a_2 c_{12} (\dot{\theta}_1 + \dot{\theta}_2)$$

$$\dot{y}_B = a_1 c_1 \dot{\theta}_1 + a_2 s_{12} (\dot{\theta}_1 + \dot{\theta}_2)$$

③



$$x_C = a_1 c_1 + a_2 c_{12} - a_3 c_{123}$$

$$y_C = a_1 s_1 + a_2 s_{12} + a_3 s_{123}$$

$$\dot{x}_C = -a_1 s_1 \dot{\theta}_1 - a_2 s_{12} (\dot{\theta}_1 + \dot{\theta}_2) + a_3 s_{123} (\dot{\phi})$$

$$\dot{y}_C = a_1 c_1 \dot{\theta}_1 + a_2 c_{12} (\dot{\theta}_1 + \dot{\theta}_2) + a_3 c_{123} (\dot{\phi})$$

Simplify $c_{123} = c_4$, $s_{123} = s_4$

$$V_A^2 = \dot{x}_A^2 + \dot{y}_A^2$$

$$= (-a_1 s_1 \dot{\theta}_1)^2 + (a_1 c_1 \dot{\theta}_1)^2$$

$$= a_1^2 \dot{\theta}_1^2 (s_1^2 + c_1^2)$$

$$V_A^2 = a_1^2 \dot{\theta}_1^2$$

$$V_B^2 = \dot{x}_B^2 + \dot{y}_B^2$$

$$= (-a_1 s_1 \dot{\theta}_1 - a_2 s_{12} (\dot{\theta}_1 + \dot{\theta}_2))^2 + (a_1 c_1 \dot{\theta}_1 + a_2 c_{12} (\dot{\theta}_1 + \dot{\theta}_2))^2$$

$$= a_1^2 \dot{\theta}_1^2 (c_1^2 + s_1^2) + 2a_1 a_2 s_1 s_{12} \dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2) + a_2^2 s_{12}^2 (\dot{\theta}_1^2 + 2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2)$$

$$+ 2a_1 a_2 c_1 c_{12} \dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2) + a_2^2 c_{12}^2 (\dot{\theta}_1^2 + 2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2)$$

$$= a_1^2 \dot{\theta}_1^2 + 2a_1 a_2 (s_1 s_{12} + c_1 c_{12}) \dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2) + a_2^2 (s_{12}^2 + c_{12}^2) (\dot{\theta}_1^2 + 2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2)$$

$$= a_1^2 \dot{\theta}_1^2 + 2a_1 a_2 c_2 (\dot{\theta}_1^2 + \dot{\theta}_1 \dot{\theta}_2) + a_2^2 (\dot{\theta}_1^2 + 2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2)$$

$$V_c^2 = \dot{x}_c^2 + \dot{y}_c^2$$

$$\begin{aligned}\dot{x}_c^2 &= (-a_1 \dot{s}_1 \dot{\theta}_1 - a_2 s_{12} (\dot{\theta}_1 + \dot{\theta}_2) + a_3 s_4 \dot{\phi})^2 \\ &= a_1^2 s_1^2 \dot{\theta}_1^2 + a_2^2 s_{12}^2 (\dot{\theta}_1^2 + 2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2) + a_3^2 s_4^2 \dot{\phi}^2 \\ &\quad + 2(-a_1 s_1 \dot{\theta}_1)(-a_2 s_{12} (\dot{\theta}_1 + \dot{\theta}_2)) + 2(-a_1 s_1 \dot{\theta}_1)(a_3 s_4 \dot{\phi}) \\ &\quad + 2(-a_2 s_{12} (\dot{\theta}_1 + \dot{\theta}_2))(a_3 s_4 \dot{\phi}) \\ &= a_1^2 s_1^2 \dot{\theta}_1^2 + a_2^2 s_{12}^2 (\dot{\theta}_1^2 + 2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2) + a_3^2 s_4^2 \dot{\phi}^2 \\ &\quad + 2a_1 a_2 s_1 s_{12} \dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2) - 2a_1 a_3 s_1 s_4 \dot{\theta}_1 \dot{\phi} - 2a_2 a_3 s_{12} s_4 \dot{\phi} (\dot{\theta}_1 + \dot{\theta}_2)\end{aligned}$$

$$\begin{aligned}\dot{y}_c^2 &= (a_1 c_1 \dot{\theta}_1 + a_2 c_{12} (\dot{\theta}_1 + \dot{\theta}_2) + a_3 c_4 \dot{\phi})^2 \\ &= a_1^2 c_1^2 \dot{\theta}_1^2 + a_2^2 c_{12}^2 (\dot{\theta}_1^2 + 2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2) + a_3^2 c_4^2 \dot{\phi}^2 \\ &\quad + 2(a_1 c_1 \dot{\theta}_1)(a_2 c_{12} (\dot{\theta}_1 + \dot{\theta}_2)) + 2(a_1 c_1 \dot{\theta}_1)(a_3 c_4 \dot{\phi}) \\ &\quad + 2(a_2 c_{12} (\dot{\theta}_1 + \dot{\theta}_2))(a_3 c_4 \dot{\phi}) \\ &= a_1^2 c_1^2 \dot{\theta}_1^2 + a_2^2 c_{12}^2 (\dot{\theta}_1^2 + 2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2) + a_3^2 c_4^2 \dot{\phi}^2 \\ &\quad + 2a_1 a_2 c_1 c_{12} \dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2) + 2a_1 a_3 c_1 c_4 \dot{\theta}_1 \dot{\phi} + 2a_2 a_3 c_{12} c_4 \dot{\phi} (\dot{\theta}_1 + \dot{\theta}_2)\end{aligned}$$

$$\begin{aligned}V_c^2 &= a_1^2 \dot{\theta}_1^2 + a_2^2 (\dot{\theta}_1^2 + 2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2) + a_3^2 (\dot{\theta}_1^2 + \dot{\theta}_2^2 + \dot{\theta}_3^2 + 2\dot{\theta}_1 \dot{\theta}_2 + 2\dot{\theta}_1 \dot{\theta}_3 + 2\dot{\theta}_2 \dot{\theta}_3) \\ &\quad + 2a_1 a_2 c_2 (\dot{\theta}_1^2 + \dot{\theta}_1 \dot{\theta}_2) + 2a_1 a_3 (c_1 c_{123} - s_1 s_{123}) (\dot{\theta}_1^2 + \dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_1 \dot{\theta}_3) \\ &\quad + 2a_2 a_3 c_{123} (\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_1 \dot{\theta}_3 + \dot{\theta}_2 \dot{\theta}_3)\end{aligned}$$

$$c_{12} c_{123} - s_{12} s_{123} = c_{123}$$

$$c_1 c_{123} - s_1 s_{123} = c(2\theta_1 + \theta_2 + \theta_3)$$

$$(\theta_1 + \theta_2)(\theta_1 + \theta_2 + \theta_3) = \theta_1^2 + \theta_2^2 + 2\theta_1 \theta_2 + \theta_1 \theta_3 + \theta_2 \theta_3$$

$$(\theta_1 + \theta_2 + \theta_3)(\theta_1 + \theta_2 + \theta_3) = \theta_1^2 + \theta_2^2 + \theta_3^2 + 2\theta_1 \theta_2 + 2\theta_1 \theta_3 + 2\theta_2 \theta_3$$

Kinetic Energy

$$\begin{aligned}KE_1 &= \frac{1}{2} m_1 V_A^2 \\ &= \frac{1}{2} m_1 a_1^2 \dot{\theta}_1^2\end{aligned}$$

$$\begin{aligned}KE_2 &= \frac{1}{2} m_2 V_B^2 \\ &= \frac{1}{2} m_2 [a_1^2 \dot{\theta}_1^2 + 2a_1 a_2 c_2 (\dot{\theta}_1^2 + \dot{\theta}_1 \dot{\theta}_2) + a_2^2 (\dot{\theta}_1^2 + 2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2)]\end{aligned}$$

$$\begin{aligned}KE_3 &= \frac{1}{2} m_3 V_c^2 \\ &= \frac{1}{2} m_3 [a_1^2 \dot{\theta}_1^2 + a_2^2 (\dot{\theta}_1^2 + 2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2) + a_3^2 (\dot{\theta}_1^2 + \dot{\theta}_2^2 + \dot{\theta}_3^2 + 2\dot{\theta}_1 \dot{\theta}_2 + 2\dot{\theta}_1 \dot{\theta}_3 + 2\dot{\theta}_2 \dot{\theta}_3) \\ &\quad + 2a_1 a_2 c_2 (\dot{\theta}_1^2 + \dot{\theta}_1 \dot{\theta}_2) + 2a_1 a_3 (c_1 c_{123} - s_1 s_{123}) (\dot{\theta}_1^2 + \dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_1 \dot{\theta}_3) \\ &\quad + 2a_2 a_3 c_{123} (\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_1 \dot{\theta}_3 + \dot{\theta}_2 \dot{\theta}_3)]\end{aligned}$$

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Potential Energy

$$P_1 = m_1 g h_1$$

$$= m_1 g (a_1 s_1)$$

$$P_2 = m_2 g h_2$$

$$= m_2 g (a_1 s_1 + a_2 s_{12})$$

$$P_3 = m_3 g h_3$$

$$= m_3 g (a_1 s_1 + a_2 s_{12} + a_3 s_{123})$$

Total Kinetic Energy

$$KE_{123} = KE_1 + KE_2 + KE_3$$

$$= \frac{1}{2} [m_1 a_1^2 \dot{\theta}_1^2 + m_2 [a_1^2 \dot{\theta}_1^2 + 2a_1 a_2 c_2 (\dot{\theta}_1^2 + \dot{\theta}_1 \dot{\theta}_2) + a_2^2 (\dot{\theta}_1^2 + 2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2)]$$

$$+ m_3 [a_1^2 \dot{\theta}_1^2 + a_2^2 (\dot{\theta}_1^2 + 2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2) + a_3^2 (\dot{\theta}_1^2 + \dot{\theta}_2^2 + \dot{\theta}_3^2 + 2\dot{\theta}_1 \dot{\theta}_2 + 2\dot{\theta}_1 \dot{\theta}_3 + 2\dot{\theta}_2 \dot{\theta}_3)$$

$$+ 2a_1 a_2 c_2 (\dot{\theta}_1^2 + \dot{\theta}_1 \dot{\theta}_2) + 2a_1 a_3 (c_{123} - s_1 s_{123}) (\dot{\theta}_1^2 + \dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_1 \dot{\theta}_3) +$$

$$2a_2 a_3 c_{123} (\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_1 \dot{\theta}_3 + \dot{\theta}_2 \dot{\theta}_3)]]$$

Total Potential Energy

$$P_{123} = P_1 + P_2 + P_3$$

$$= (m_1 + m_2 + m_3) g a_1 s_1 + (m_2 + m_3) g a_2 s_{12} + m_3 g s_{123}$$

Lagrange Method

$$L = KE_{123} - P_{123}$$

$$= \frac{1}{2} (m_1 + m_2 + m_3) a_1^2 \dot{\theta}_1^2 + \frac{1}{2} (m_2 + m_3) (2a_1 a_2 c_2 (\dot{\theta}_1^2 + \dot{\theta}_1 \dot{\theta}_2) + a_2^2 (\dot{\theta}_1^2 + 2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2))$$

$$+ \frac{1}{2} m_3 a_2^2 (\dot{\theta}_1^2 + \dot{\theta}_2^2 + \dot{\theta}_3^2 + 2\dot{\theta}_1 \dot{\theta}_2 + 2\dot{\theta}_1 \dot{\theta}_3 + 2\dot{\theta}_2 \dot{\theta}_3) + m_3 a_1 a_3 (c_{123} - s_1 s_{123}) (\dot{\theta}_1^2 + \dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_1 \dot{\theta}_3)$$

$$+ m_3 a_2 a_3 c_{123} (\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_1 \dot{\theta}_3 + \dot{\theta}_2 \dot{\theta}_3) - (m_1 + m_2 + m_3) g a_1 s_1 -$$

$$(m_2 + m_3) g a_2 s_{12} - m_3 g s_{123}$$

$$\begin{aligned} \frac{dL}{d\dot{\theta}_1} = & (m_1+m_2+m_3)a_1^2\dot{\theta}_1 + 2(m_2+m_3)(a_1a_2c_2\dot{\theta}_1) + \\ & (m_2+m_3)(a_1a_2\dot{\theta}_2) + (m_2+m_3)a_2^2\dot{\theta}_1 + (m_2+m_3)a_2^2\dot{\theta}_2 \\ & + m_3a_3^2\dot{\theta}_1 + m_3a_3^2\dot{\theta}_2 + m_3a_3^2\dot{\theta}_3 + 2m_3a_1a_3(c_1c_{123}-s_1s_{123})\dot{\theta}_1 \\ & + m_3a_1a_3(c_1c_{123}-s_1s_{123})\dot{\theta}_2 + m_3a_1a_3(c_1c_{123}-s_1s_{123})\dot{\theta}_3 + \\ & 2m_3a_2a_3c_{123}\dot{\theta}_1 + 2m_3a_2a_3c_{123}\dot{\theta}_2 + m_3a_2a_3c_{123}\dot{\theta}_3 \end{aligned}$$

$$\begin{aligned} \frac{dL}{d\dot{\theta}_2} = & (m_1+m_2)(a_1a_2c_2\dot{\theta}_1) + (m_1+m_2)(a_2^2\dot{\theta}_1) + 2(m_1+m_2)(a_2^2\dot{\theta}_2) \\ & + m_3a_3^2\dot{\theta}_2 + m_3a_3^2\dot{\theta}_1 + m_3a_3^2\dot{\theta}_3 + m_3a_3(c_1c_{123}-s_1s_{123})\dot{\theta}_1 \\ & + 2m_3a_2a_3c_{123}\dot{\theta}_2 + 2m_3a_2a_3c_{123}\dot{\theta}_1 + m_3a_2a_3c_{123}\dot{\theta}_3 \end{aligned}$$

$$\begin{aligned} \frac{dL}{d\dot{\theta}_3} = & m_3a_3^2\dot{\theta}_3 + m_3a_3^2\dot{\theta}_1 + m_3a_3^2\dot{\theta}_2 + m_3a_1a_3(c_1c_{123}-s_1s_{123})\dot{\theta}_1 \\ & + m_3a_1a_3c_{123}\dot{\theta}_1 + m_3a_2a_3c_{123}\dot{\theta}_2 \end{aligned}$$

$$\begin{aligned} \frac{d}{dt}\left(\frac{dL}{d\dot{\theta}_1}\right) = & (m_1+m_2+m_3)a_1^2\ddot{\theta}_1 + 2(m_2+m_3)(a_1a_2c_2\ddot{\theta}_1) \\ & + (m_2+m_3)(a_1a_2\ddot{\theta}_2) + (m_2+m_3)a_2^2\ddot{\theta}_1 + (m_2+m_3)a_2^2\ddot{\theta}_2 + m_3a_3^2\ddot{\theta}_1 \\ & + m_3a_3^2\ddot{\theta}_2 + m_3a_3^2\ddot{\theta}_3 + 2m_3a_1a_3\ddot{\theta}_1(c_1c_{123}-s_1s_{123}) \\ & - 2m_3a_1a_3\ddot{\theta}_1(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)c_1s_{123} - 2m_3a_1a_3\ddot{\theta}_1s_1s_{123} \\ & - 2m_3a_1a_3\ddot{\theta}_1^2c_1s_{123} - 2m_3a_1a_3\ddot{\theta}_1(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)s_1c_{123} + m_3a_1a_3\ddot{\theta}_2c_1c_{123} \\ & - m_3a_1a_3\ddot{\theta}_2\dot{\theta}_1s_1c_{123} - m_3a_1a_3\ddot{\theta}_2(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)c_1s_{123} - m_3a_1a_3\ddot{\theta}_2s_1s_{123} \\ & - m_3a_1a_3\ddot{\theta}_2\dot{\theta}_1c_1s_{123} - m_3a_1a_3\ddot{\theta}_2(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)s_1c_{123} + m_3a_1a_3\ddot{\theta}_3c_1c_{123} \\ & - m_3a_1a_3\ddot{\theta}_3\dot{\theta}_1s_1c_{123} - m_3a_1a_3\ddot{\theta}_3(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)c_1s_{123} - m_3a_1a_3\ddot{\theta}_3s_1s_{123} \\ & - m_3a_1a_3\ddot{\theta}_3\dot{\theta}_1c_1s_{123} - m_3a_1a_3\ddot{\theta}_3(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)s_1c_{123} \\ & + 2m_3a_2a_3\ddot{\theta}_1c_{123} - 2m_3a_2a_3\ddot{\theta}_1(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)s_{123} \\ & + 2m_3a_2a_3\ddot{\theta}_2c_{123} - 2m_3a_2a_3\ddot{\theta}_2(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)s_{123} \\ & + 2m_3a_2a_3\ddot{\theta}_3c_{123} - 2m_3a_2a_3\ddot{\theta}_3(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)s_{123} \end{aligned}$$

$$\begin{aligned} \frac{d}{dt}\left(\frac{dL}{d\dot{\theta}_2}\right) = & (m_1+m_2)(a_1a_2c_2\ddot{\theta}_1) - (m_1+m_2)(a_1a_2\ddot{\theta}_2)c_2 + (m_1+m_2)(a_2^2\ddot{\theta}_2) \\ & + 2(m_1+m_2)(a_2^2\ddot{\theta}_1) + m_3a_3^2\ddot{\theta}_2 + m_3a_3^2\ddot{\theta}_1 + m_3a_3^2\ddot{\theta}_3 \\ & + m_3a_3\ddot{\theta}_1c_1c_{123} - m_3a_3\ddot{\theta}_1^2s_1c_{123} - m_3a_3\ddot{\theta}_1(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)c_1s_{123} \\ & - m_3a_3\ddot{\theta}_1s_1s_{123} - m_3a_3\ddot{\theta}_1c_1s_{123} - m_3a_3\ddot{\theta}_1(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)s_1c_{123} \\ & + 2m_3a_2a_3\ddot{\theta}_2c_{123} - 2m_3a_2a_3\ddot{\theta}_2(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)s_{123} \\ & + 2m_3a_2a_3\ddot{\theta}_3c_{123} - 2m_3a_2a_3\ddot{\theta}_3(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)s_{123} \\ & + 2m_3a_2a_3\ddot{\theta}_3c_{123} - 2m_3a_2a_3\ddot{\theta}_3(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)s_{123} \end{aligned}$$

$$\begin{aligned} \frac{d}{dt}\left(\frac{dL}{d\dot{\theta}_3}\right) = & m_3a_3^2\ddot{\theta}_3 + m_3a_3^2\ddot{\theta}_1 + m_3a_3^2\ddot{\theta}_2 + m_3a_1a_3\ddot{\theta}_1c_1c_{123} \\ & - m_3a_1a_3\ddot{\theta}_1^2s_1c_{123} - m_3a_1a_3\ddot{\theta}_1(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)c_1s_{123} - m_3a_1a_3\ddot{\theta}_1s_1s_{123} \\ & - m_3a_1a_3\ddot{\theta}_2c_1s_{123} - m_3a_1a_3\ddot{\theta}_2(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)s_1c_{123} + m_3a_2a_3\ddot{\theta}_1c_{123} + \dots \end{aligned}$$

$$\frac{d}{dt} \left(\frac{dL}{d\dot{\theta}_3} \right) = \dots - m_3 a_2 a_3 \ddot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3) S_{123} + m_3 a_2 a_3 \ddot{\theta}_2 C_{123} \\ - m_3 a_2 a_3 \ddot{\theta}_3 (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3) S_{123}$$

$$\frac{dL}{d\dot{\theta}_1} = - (m_1 + m_2 + m_3) g a_1 C_1 + (m_2 + m_3) g a_2 C_{12} + m_3 g a_3 C_{123}$$

$$\frac{dL}{d\dot{\theta}_2} = (m_2 + m_3) a_1 a_2 S_2 (\dot{\theta}_1^2 + \dot{\theta}_1 \dot{\theta}_2) + m_3 a_1 a_3 S_{23} (\dot{\theta}_1^2 + \dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_1 \dot{\theta}_3) \\ + (m_2 + m_3) g a_2 C_{12} + m_3 g a_3 C_{123}$$

$$\frac{dL}{d\dot{\theta}_3} = m_3 a_1 a_3 (\dot{\theta}_1^2 + \dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_1 \dot{\theta}_3) S_{23} + m_3 a_2 a_3 (\dot{\theta}_1 + \dot{\theta}_2) (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3) S_3 \\ + m_3 g a_3 C_{123}$$

$$\begin{aligned} \tau_1 = & [(m_1 + m_2 + m_3) a_1^2 + 2(m_2 + m_3) (a_1 a_2 C_2) + (m_2 + m_3) a_2^2 + m_3 a_3^2 \\ & + 2m_3 a_1 a_3 C_{123} - 2m_3 a_1 a_3 S_1 S_{123} + 2m_3 a_2 a_3 C_{123}] \ddot{\theta}_1 \\ & + [(m_2 + m_3) (a_1 a_2) + (m_2 + m_3) a_2^2 + m_3 a_3^2 + 2m_3 a_1 a_3 C_{123} \\ & - 2m_3 a_1 a_3 S_1 S_{123} + 2m_3 a_2 a_3 C_{123}] \ddot{\theta}_2 \\ & + [m_2 a_3^2 + 2m_3 a_1 a_3 C_{123} - 2m_3 a_1 a_3 S_1 S_{123} + 2m_3 a_2 a_3 C_{123}] \ddot{\theta}_3 \\ & + [-2m_3 a_1 a_3 S_1 C_{123} - 2m_3 a_1 a_3 C_1 S_{123} - 2m_3 a_1 a_3 C_{123} - 2m_3 a_2 a_3 S_{123}] \dot{\theta}_1^2 \\ & + [-m_3 a_1 a_3 S_1 C_{123} - m_3 a_1 a_3 C_1 S_{123} - 2m_3 a_2 a_3 S_{123}] \dot{\theta}_2^2 \\ & + [-m_3 a_1 a_3 S_1 C_{123} - m_3 a_1 a_3 C_1 S_{123} - 2m_3 a_2 a_3 S_{123}] \dot{\theta}_3^2 \\ & + [-4m_3 a_1 a_3 C_1 S_{123} - 4m_3 a_1 a_3 S_1 C_{123}] \dot{\theta}_1 \dot{\theta}_2 \\ & + [-4m_3 a_1 a_3 C_1 S_{123} - 4m_3 a_1 a_3 S_1 C_{123}] \dot{\theta}_1 \dot{\theta}_3 \\ & + [-2m_3 a_1 a_3 C_1 S_{123} - 2m_3 a_1 a_3 S_1 C_{123}] \dot{\theta}_2 \dot{\theta}_3 \\ & - (m_1 + m_2 + m_3) g a_1 C_1 - (m_2 + m_3) g a_2 C_{12} + m_3 g a_3 C_{123} \end{aligned}$$

$$\begin{aligned} \tau_2 = & [(m_1 + m_2) (a_1 a_2 C_2) + (m_1 + m_2) a_2^2 + m_3 a_3^2 + m_3 a_3 C_{123} - m_3 a_3 S_1 S_{123} \\ & + 2m_3 a_2 a_3 C_{123}] \ddot{\theta}_1 + [2(m_1 + m_2) a_2^2 + m_3 a_3^2 + 2m_3 a_2 a_3 C_{123}] \ddot{\theta}_2 \\ & + [m_3 a_3^2 + 2m_3 a_2 a_3 C_{123}] \ddot{\theta}_3 + [-(m_1 + m_2) (a_1 a_2 C_2) - m_3 a_3 C_{123} \\ & - m_3 a_3 S_1 C_{123} - 4m_2 a_2 a_3 S_{123}] \dot{\theta}_1 \dot{\theta}_2 + [-m_3 a_3 C_{123} - m_3 a_3 S_1 C_{123} \\ & - 4m_2 a_2 a_3 S_{123}] \dot{\theta}_1 \dot{\theta}_3 + [-4m_3 a_2 a_3 S_{123}] \dot{\theta}_2 \dot{\theta}_3 \\ & + [m_3 a_3 C_{123} - m_3 a_3 S_1 C_{123} - 2m_2 a_2 a_3 S_{123}] \dot{\theta}_1^2 \\ & + [-2m_2 a_2 a_3 S_{123}] \dot{\theta}_2^2 + [-2m_2 a_2 a_3 S_{123}] \dot{\theta}_3^2 - (m_2 + m_3) (a_1 a_2 S_2) (\dot{\theta}_1^2 + \dot{\theta}_1 \dot{\theta}_2) \\ & - m_3 a_1 a_3 S_{23} (\dot{\theta}_1^2 + \dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_1 \dot{\theta}_3) - (m_2 + m_3) g a_2 C_{12} + m_3 g a_3 C_{123} \end{aligned}$$

$$\begin{aligned} \tau_3 = & [m_3 a_3^2 + m_3 a_1 a_3 C_{123} - m_3 a_1 a_3 S_1 S_{123}] \ddot{\theta}_1 + [m_3 a_3^2 + m_3 a_2 a_3 C_{123}] \ddot{\theta}_2 \\ & + [m_3 a_3^2] \ddot{\theta}_3 + [-2m_3 a_1 a_3 S_1 C_{123} - m_3 a_1 a_3 C_1 S_{123} - 2m_3 a_2 a_3 S_{123}] \dot{\theta}_1^2 \\ & + [-m_3 a_2 a_3] \dot{\theta}_2^2 + [-m_3 a_1 a_3 C_{123} - m_3 a_1 a_3 S_1 S_{123} - 2m_3 a_2 a_3 S_{123}] \dot{\theta}_1 \dot{\theta}_2 \\ & + [-m_3 a_1 a_3 C_1 S_{123} - m_3 a_1 a_3 S_1 C_{123} - m_3 a_2 a_3 S_{123}] \dot{\theta}_1 \dot{\theta}_3 + [-m_3 a_2 a_3 S_{123}] \dot{\theta}_2 \dot{\theta}_3 \\ & + m_3 a_1 a_3 (\dot{\theta}_1^2 + \dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_1 \dot{\theta}_3) S_{123} - m_3 a_2 a_3 (\dot{\theta}_1 + \dot{\theta}_2) (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3) S_3 \end{aligned}$$

$$\vec{\tau} = [D(\vec{q})] \ddot{\vec{q}} + [C(\dot{\vec{q}}) \cdot \dot{\vec{q}}] \dot{\vec{q}} + \vec{g}(\vec{q})$$

$$T_{123} = \begin{bmatrix} (m_1 + m_2 + m_3) \dot{q}_1^2 + 2(m_2 + m_3)(\dot{q}_1 \dot{q}_2) \\ + (m_2 + m_3) \dot{q}_2^2 + m_3 \dot{q}_3^2 + 2m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} \\ - 2m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} + 2m_2 \dot{q}_2 \dot{q}_3 \dot{q}_{123} \\ (m_2 + m_3)(\dot{q}_1 \dot{q}_2) + (m_2 + m_3) \dot{q}_2^2 + m_3 \dot{q}_3^2 \\ + 2m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} - 2m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} \\ - 2m_3 \dot{q}_2 \dot{q}_3 \dot{q}_{123} \\ m_2 \dot{q}_3^2 + 2m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} \\ - 2m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} \\ + 2m_3 \dot{q}_2 \dot{q}_3 \dot{q}_{123} \end{bmatrix}$$

$$+ \begin{bmatrix} -2m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} - 2m_3 \dot{q}_1 \dot{q}_{123} \\ -m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} - 2m_3 \dot{q}_2 \dot{q}_3 \dot{q}_{123} \\ -m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} - m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} \\ -2m_3 \dot{q}_2 \dot{q}_3 \dot{q}_{123} \\ -m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} - m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} \\ -2m_3 \dot{q}_2 \dot{q}_3 \dot{q}_{123} \end{bmatrix}$$

$$+ \begin{bmatrix} -4m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} - 4m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} \\ -4m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} - 4m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} \\ -4m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} - 4m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} \\ -2m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} - 2m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} \end{bmatrix}$$

$$+ \begin{bmatrix} -(m_1 + m_2 + m_3) \dot{q}_1 \dot{q}_1 - (m_2 + m_3) \dot{q}_2 \dot{q}_2 + m_3 \dot{q}_3 \dot{q}_{123} \\ - (m_2 + m_3) \dot{q}_2 \dot{q}_2 + m_3 \dot{q}_3 \dot{q}_{123} \\ - m_3 \dot{q}_3 \dot{q}_{123} \end{bmatrix}$$

$$T_2 = \begin{bmatrix} (m_1 + m_2)(\dot{q}_1 \dot{q}_2) + (m_1 + m_2) \dot{q}_2^2 \\ + m_3 \dot{q}_3^2 + m_3 \dot{q}_2 \dot{q}_{123} - m_3 \dot{q}_3 \dot{q}_{123} \\ + 2m_3 \dot{q}_2 \dot{q}_3 \dot{q}_{123} \\ 2(m_1 + m_2) \dot{q}_2^2 + m_3 \dot{q}_3^2 + \\ 2m_3 \dot{q}_2 \dot{q}_3 \dot{q}_{123} \\ m_3 \dot{q}_3^2 + 2m_3 \dot{q}_2 \dot{q}_3 \dot{q}_{123} \end{bmatrix}$$

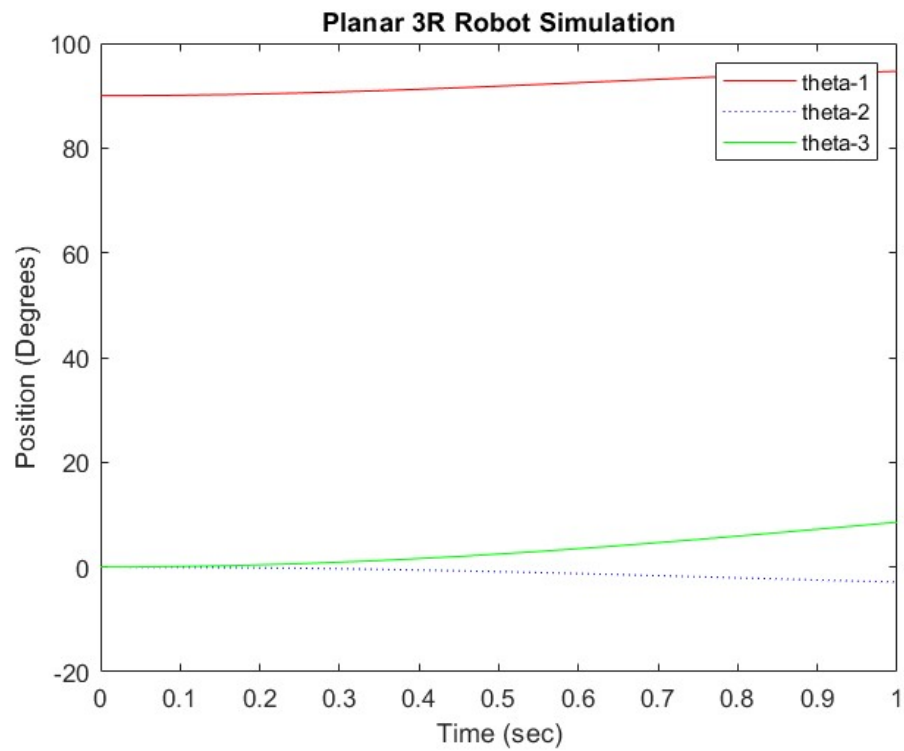
$$T_3 = \begin{bmatrix} m_3 \dot{q}_3^2 + m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} \\ - m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} \\ m_3 \dot{q}_3^2 + m_3 \dot{q}_2 \dot{q}_3 \dot{q}_{123} \\ m_3 \dot{q}_3^2 \end{bmatrix}$$

$$\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} \begin{bmatrix} -2m_3 \dot{q}_1 \dot{q}_2 \dot{q}_{123} - m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} \\ -2m_3 \dot{q}_2 \dot{q}_3 \dot{q}_{123} - m_3 \dot{q}_1 \dot{q}_3 \\ -m_3 \dot{q}_2 \dot{q}_3 \\ -m_3 \dot{q}_1 \dot{q}_3 \end{bmatrix}$$

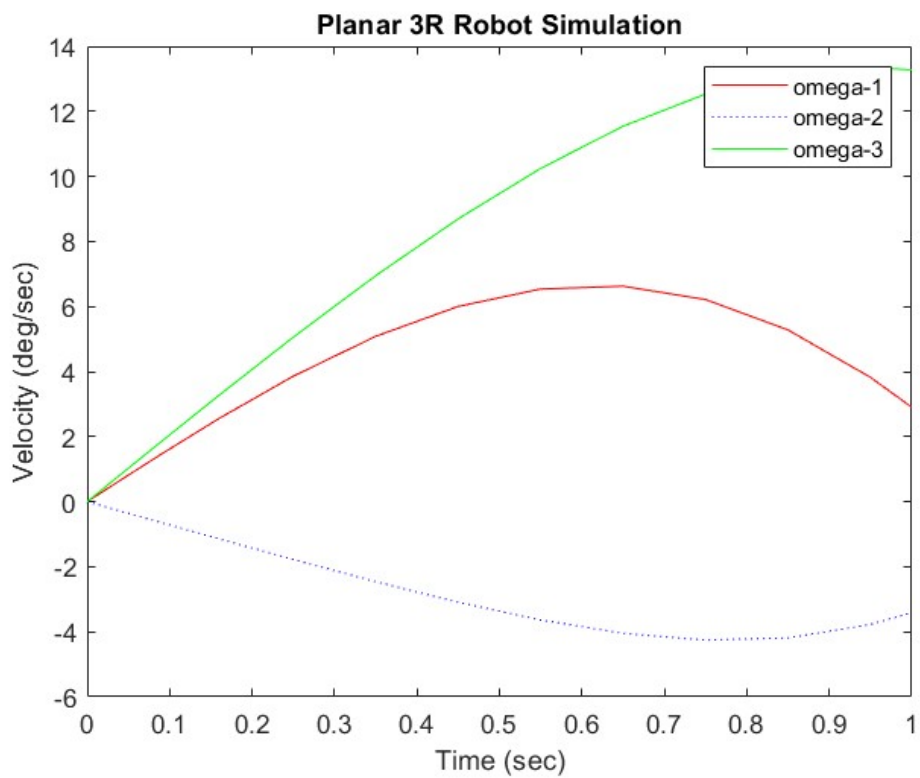
$$\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} \begin{bmatrix} -2m_3 \dot{q}_1 \dot{q}_2 \dot{q}_{123} - 2m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} \\ -m_3 \dot{q}_2 \dot{q}_3 - m_3 \dot{q}_1 \dot{q}_3 \\ -4m_3 \dot{q}_2 \dot{q}_3 \dot{q}_{123} - m_3 \dot{q}_1 \dot{q}_3 \\ -m_3 \dot{q}_2 \dot{q}_3 - m_3 \dot{q}_1 \dot{q}_3 \end{bmatrix}$$

$$\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} \begin{bmatrix} -2m_3 \dot{q}_1 \dot{q}_2 \dot{q}_{123} - 2m_3 \dot{q}_1 \dot{q}_3 \dot{q}_{123} \\ -m_3 \dot{q}_2 \dot{q}_3 - m_3 \dot{q}_1 \dot{q}_3 \\ -4m_3 \dot{q}_2 \dot{q}_3 \dot{q}_{123} - m_3 \dot{q}_1 \dot{q}_3 \\ -m_3 \dot{q}_2 \dot{q}_3 - m_3 \dot{q}_1 \dot{q}_3 \end{bmatrix}$$

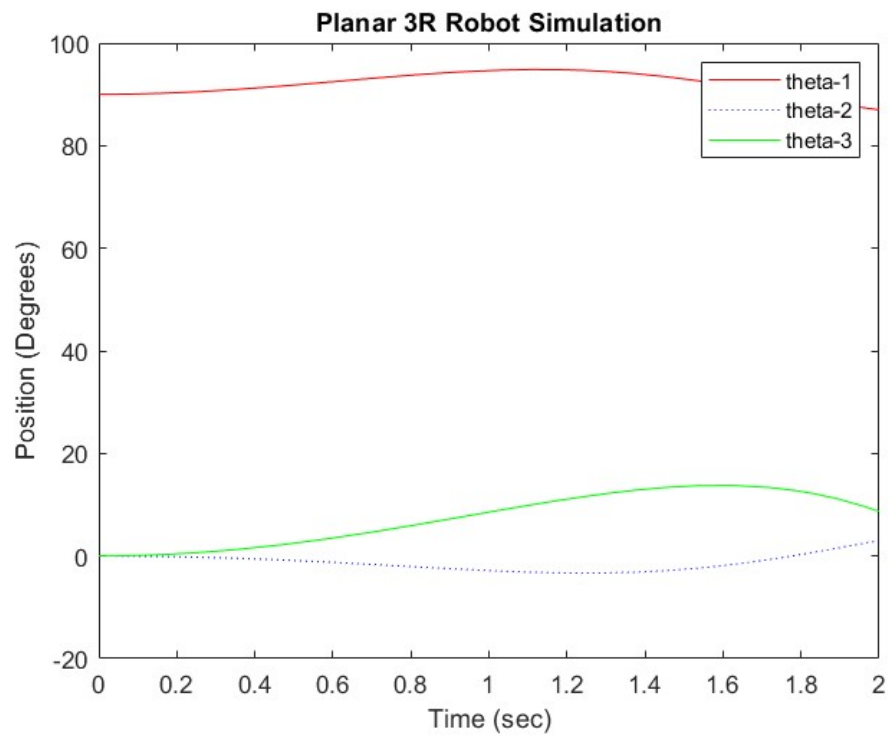
1a. Variables versus time (1 second)



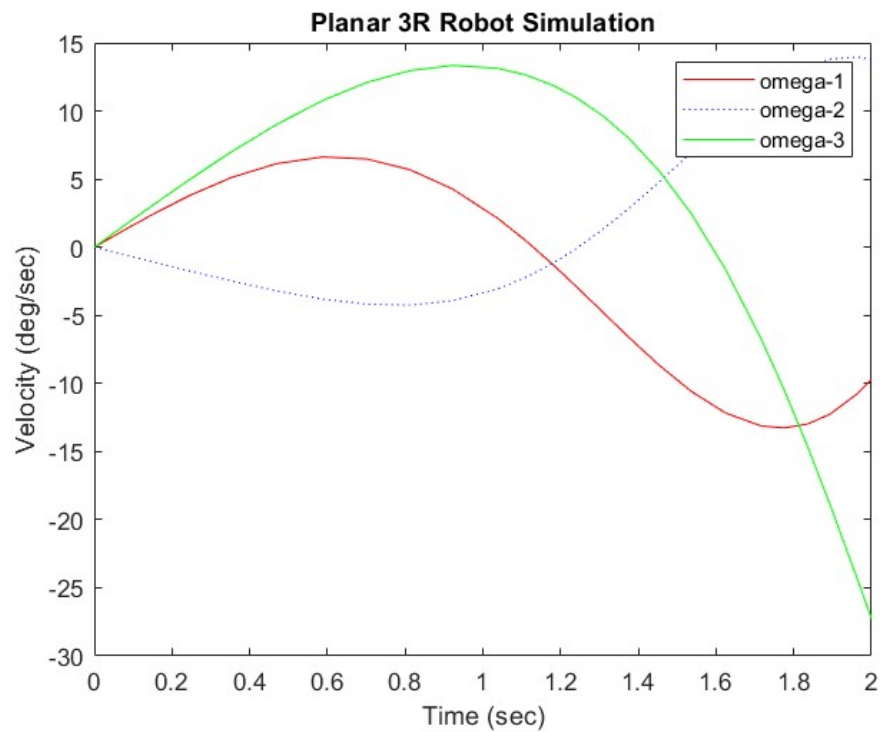
1b. Speed versus time (1 second)



2a. Variables versus time (2 seconds)

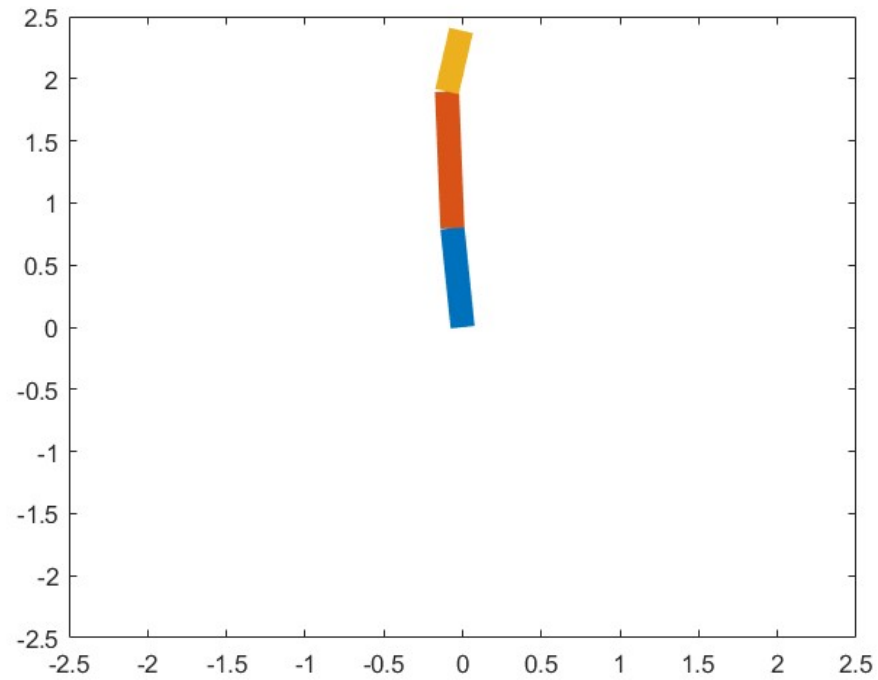


2b. Speed versus time (2 seconds)



3. PlanarRRR motion

1 Second



2 Second

