102716 - DORIS MTAE 11/11/2020

MAT3201 - SIMULATION AND MODELLING.

1. Solution.

a) Simulation model is a set of assumptions concern the operation of the system which is expressed as mathematical, logical and symbolic expression between the objects of interest of the system
while
Mathematical model uses symbolic notations and mathematical equations to represent the system.

b) Deterministic model is a representation of a system with no random inputs and no random output
while
Stochastic model is a representation of a system with random inputs and random outputs.

c) Monte Carlo Simulation and Dynamic Simulation.
Monte Carlo simulation is a broad class of computation algorithms that rely on repeated random sampling to obtain numerical results
while
~~Dynamic simulation relies on sampling to to check over time the most effective on the system.~~

Dynamic simulation refers to constructing of a mathematical model of some real world system and analyzing its behaviour through computer based experiment overtime to check the most effective on the system.

2. Solution.

i) no callouts

For Poisson distribution $= Pr(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$

$\lambda = 5.3$

$x = 0$

$$\frac{5.3^0 e^{-5.3}}{0!}$$

$\therefore = 0.004992$

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2i.

ii.) 1 callout.

$\lambda = 5.3$
$x = 1$

$$\frac{5.3^{1} e^{-5.3}}{1!}$$

$\therefore = 0.0265$

iii.) 2 callout

$\lambda = 5.3$
$x = 2$

$$\frac{5.3^{2} e^{-5.3}}{2!}$$

$\therefore = 0.0701$

iv.) 3 callout

$\lambda = 5.3$
$x = 3$

$$\frac{5.3^{3} e^{-5.3}}{3!}$$

$\therefore = 0.1239$

v.) 4 callout

$\lambda = 5.3$
$x = 4$

$$\frac{5.3^{4} e^{-5.3}}{4!}$$

$\therefore = 0.1641$

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vi.) more than 4 callouts.

$X>4 \Rightarrow 1 - [P(0) + P(1) + P(2) + P(3) + P(4)]$

$1 - [0.004992 + 0.0265 + 0.0701 + 0.1239 + 0.164]$

$1 - 0.389592 = 0.610408$

$\therefore = 0.610408$

3. Solution.

i.) No particles

$\lambda = 10.5$

$x = 0$

For Poisson distribution $Pr(X=x) = \dfrac{\lambda^x e^{-\lambda}}{x!}$

$\Rightarrow \dfrac{10.5^0 e^{-10.5}}{0!} = 0.00002754$

$\therefore = 0.00002754$

ii.) 2 particles

$\Rightarrow \lambda = 10.5$

$x = 2$

$\Rightarrow \dfrac{10.5^2 e^{-10.5}}{2!} = 0.0015179$

$\therefore = 0.0015179$

iii.) atleast 5 particles.

$\Rightarrow 1 - [P(0) + P(1) + P(2) + P(3) + P(4)]$

$P(1) = \dfrac{10.5^1 e^{-10.5}}{1!} = 0.0002891$

$P(4) = \dfrac{10.5^4 e^{-10.5}}{4!} = 0.01395$

$P(3) = \dfrac{10.5^3 e^{-10.5}}{3!} = 0.005313$

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iii) $1 - (0.00002754 + 0.0015179 + 0.0002891 + 0.005313 + 0.01315)$

$1 - 0.02109754 = 0.9789$

$\therefore = 0.9789$

4. Solution.

i) Po 0

Poisson distribution $= Pr(X=x) = \dfrac{\lambda^x e^{-\lambda}}{x!}$

$\lambda = 6$

$x = 0$

$\Rightarrow \dfrac{6^0 e^{-6}}{0!} = 0.002479$

$\therefore = 0.002479$

ii) 1

$\lambda = 6$

$x = 1$

$\Rightarrow \dfrac{6^1 e^{-6}}{1!} = 0.0149$

$\therefore = 0.0149$

iii) 2

$\lambda = 6$

$x = 2$

$\Rightarrow \dfrac{6^2 e^{-6}}{2!} = 0.0446$

$\therefore = 0.0446$

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4 iv) 3

$\lambda = 6$
$x = 3$

$$\Rightarrow \frac{6^3 e^{-6}}{3!} = 0.0892$$

$\therefore = 0.0892$

v.) More than 3 bacteria

$1 - [P(0) + P(1) + P(2) + P(3)]$

$1 - [0.002479 + 0.0149 + 0.0446 + 0.0892]$

~~$\therefore = 0.848$~~

$\therefore = 0.848821$

5. Solution.

a) $P[X = x] = \binom{n}{x} p^x q^{n-x}$

$\Rightarrow p = \frac{1}{6}$ $\quad n = 8$

$q = \frac{5}{6}$ $\quad x = 4$

$$\Rightarrow \binom{8}{4}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^4 = 0.02605$$

$\therefore = 0.02605$

b) Never
$x = 0$

$$= \binom{8}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^8$$

$\therefore = 0.23257$

5.

c) Atleast five times.

$$1 - [P(0) + P(1) + P(2) + P(3) + P(4)]$$

$X = 1$

$\Rightarrow \binom{8}{1}\left(\frac{1}{6}\right)^{1}\left(\frac{5}{6}\right)^{7} = \cancel{0.04651} \quad 0.37208$

$X = 2$

$\Rightarrow \binom{8}{2}\left(\frac{1}{6}\right)^{2}\left(\frac{5}{6}\right)^{6} = \cancel{0.009303} \quad 0.260484$

$X = 3$

$\binom{8}{3}\left(\frac{1}{6}\right)^{3}\left(\frac{5}{6}\right)^{5} = \cancel{0.0018605} \quad 0.104188$

$1 - [0.37208 + 0.260484 + 0.104188 + 0.23257 + 0.02605]$

$\therefore = 0.0046\cancel{28}$

6. Solution.

$P(X = x) = \binom{n}{x} p^x q^{n-x}$

$p = 95/100 = 0.95$

$q = 0.05$

$n = 12$

$P(7) = \binom{12}{7}(0.95)^7(0.05)^5 = 0.0001728$

$P(8) = \binom{12}{8}(0.95)^8(0.05)^4 = 0.002052$

$P(9) = \binom{12}{9}(0.95)^9(0.05)^3 = 0.017332$

$P(10) = \binom{12}{10}(0.95)^{10}(0.05)^2 = 0.098792$

6. $P(11) = \binom{12}{11}(0.95)^{11}(0.05)^{1} = 0.341280$

$P(12) = \binom{12}{12}(0.95)^{12}(0.05)^{0} = 0.540360$

$\cancel{x}$ $[P(7) + P(8) + P(9) + P(10) + P(11) + P(12)]$

$0.0001728 + 0.002052 + 0.017332 + 0.098792 + 0.034128 +$
0.540360

$\therefore = 0.9999888$

7. Solution.

$p = 1/8 \qquad q = 7/8 \qquad n = 5$

a) No faulty products.

$x = 0$

$\binom{5}{0}(1/8)^{0}(7/8)^{8} = 0.51291$

$\therefore = 0.51291$

b) $x = \cancel{4}\ 1$

$\binom{5}{1}(1/8)^{1}(7/8)^{7} = 0.36636$

$\therefore = 0.36636$

c) $x = \cancel{2}$ Atleast 2

$1 - [P(0) + P(1)]$

$1 - [0.51291 + 0.36636]$

$1 - 0.87927$

$\therefore = 0.12073$

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7 d.) x = not more than 3

$x = 2$

$$\binom{5}{2}\left(\tfrac{1}{8}\right)^2\left(\tfrac{7}{8}\right)^6 = 0.104675$$

$$\binom{5}{3}\left(\tfrac{1}{8}\right)^3\left(\tfrac{7}{8}\right)^5 = 0.0149561$$

$\Rightarrow$ $0.51291 + 0.366364 + 0.104675 + 0.0149561$

$\therefore = 0.9989051$

10. Solution.

$$f(x) = \begin{cases} \dfrac{4x(4-x^2)}{m} & 0 \le x \le 3 \\ 0 \end{cases}$$

$$\int_0^3 \frac{4x(9-x^2)}{m}\,dx = 1$$

$$\cancel{\tfrac{1}{m}\int_0^3 (36x - 4x^3}$$

$$\tfrac{1}{m}\int_0^3 (36x - 4x^3)\,dx = 1$$

$$\tfrac{1}{m}\left[\frac{36x^2}{2} - \frac{4x^4}{4}\right]_0^3 = 1$$

$$\left[\tfrac{1}{m}[162 - 81] - \tfrac{1}{m}[0-0]\right] = 1$$

$$\tfrac{1}{m}[162 - 81] = 1$$

$$\frac{81}{m} = \frac{1}{1}$$

$m = 81$

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10. $\cancel{\int_0^3 \frac{4x(9-x^2)}{81}}$

$$f(x) = \begin{cases} \frac{4x(9-x^2)}{81} & , 0 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

$$\mu = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_0^3 \frac{x \cdot 4x(9-x^2)}{81} = \int_0^3 \frac{4x^2(9-x^2)\,dx}{81}$$

$$= \cancel{\frac{1}{81}\left[\frac{36x^2}{3} - 4x^5\right]}$$

$$= \frac{1}{81}\int_0^3 \left[36x^2 - 4x^4\right] dx$$

$$= \frac{1}{81}\left[\frac{36x^3}{3} - \frac{4x^5}{5}\right]_0^3$$

$$= \frac{1}{81}\left[324 - 194.4\right]$$

$\therefore$ Mean $= 1.6$

$$Var(x) = Ex^2 - [E(x)]^2$$

$$E x^2 = \int_a^b x^2 f(x)\, dx$$

$$= \frac{1}{81}\int_0^3 x^2 \cdot 4x(9-x^2)\, dx$$

$$= \frac{1}{81}\int_0^3 36x^3 - 4x^5\, dx$$

$$= \frac{1}{81}\left[9x^4 - \frac{4}{6}x^6\right]_0^3$$

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$= \frac{1}{81}\left[9(3)^4 - \frac{4}{6}(3)^6\right]$

$= \frac{1}{81}\left[729 - 486\right]$

$\Rightarrow 3$

$= 3 - (1.6)^2 = 0.44$

$\therefore$ Variance $= 0.44$

15. Solution.

$\lambda =$ 11 entries

$M = 0.5 \times 60 = 30$ minutes.

$P_0 = 1 - \frac{\lambda}{M}$

$= 1 - \frac{11}{30}$

$\therefore P_0 = \frac{19}{30}$

$P_3 = \left(\frac{\lambda}{M}\right)^n \left(1 - \frac{\lambda}{M}\right)$

$= \left(\frac{11}{30}\right)^3 \left(1 - \frac{11}{30}\right) = 0.031221$

$\therefore = 0.031221$

$L_s = \frac{\lambda}{M - \lambda}$

$= \frac{11}{30 - 11}$

$\therefore L_s = \frac{11}{19}$

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15. $L_q = \dfrac{\lambda^2}{M(M-\lambda)}$

$= \dfrac{11^2}{30(30-11)} = \dfrac{121}{570}$

$\therefore = \dfrac{121}{570}$

$W_s = \dfrac{1}{M-\lambda}$

$= \dfrac{1}{30-11} = 1/19$

$\therefore = 1/19$

$W_q = \dfrac{\lambda}{M(M-\lambda)}$

$= \dfrac{11}{30(30-11)}$

$\therefore = \dfrac{11}{570}$

12. Components of queuing system.

i) Arrival process.

- In this ~~pro~~ component, entities arrive to the system according to some arrival pattern. The arrivals are independent of preceding arrivals that is successive interarriv times are statistically independent and completely rando

ii) Queue Discipline.

- In this component, queue represents a certain number of customers waiting for service. ~~Therefore~~ Typically the customer being served is considered not to be in the queue.

iii) Service Mechanism.

- Service represents some activity that takes time and that the customers are waiting for. It may be

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12. a real service carried on persons or machines. Services takes a random time. Theoretical models are based on random distribution of service duration also called Service Pattern. Number of servers is another important parameter.

~~9-13. Solution.~~

9. Solution.

$$f(x)=\begin{cases} k & 50 \le x \le 60 \\ 0 & \text{otherwise} \end{cases}$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{50}^{60} k\,dx = [k x]_{50}^{60} = 1$$

$$60k - 50k = 1$$

$$\frac{10k}{10} = \frac{1}{10}$$

$$\therefore k = \frac{1}{10}$$

$$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_{50}^{60} x \cdot \frac{1}{10}\,dx$$

$$= \left[\frac{1}{20} x^2\right]_{50}^{60}$$

$$= \frac{1}{20}(60)^2 - \frac{1}{20}(50)^2 = 55$$

$$\therefore E(x) = 55$$

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9. Variance $(x) = \int_{50}^{60} x^2 f(x)\,dx - \mu^2$

$$= \int_{50}^{60} x^2 \cdot \frac{1}{10}\,dx$$

$$= \frac{1}{10}\int_{50}^{60} x^2\,dx$$

$$= \left[\frac{1}{30}x^3\right]_{50}^{60}$$

$$= \frac{1}{30}(60)^3 - \frac{1}{30}(50)^3$$

$$= 3033.33$$

$$\Rightarrow 3033.33 - 3025 = 8.33$$

$\therefore$ Variance $= 8.33$

9. $P(50 < x < 55)$

$$\int_{51}^{54} \frac{1}{10}\,dx$$

$$= \left[\frac{1}{10}x\right]_{51}^{54}$$

$$= \left[\frac{54}{10} - \frac{51}{10}\right]$$

$$\therefore = \frac{3}{10}$$

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11. Solution.

$$f(x) = \begin{cases} 7e^{-7x} & x > 0 \\ 0 & \end{cases}$$

a) $X > 2$

$$= \int_2^{\infty} 7e^{-7x}$$

$$= \left[\frac{7e^{-7x}}{-7} \right]_2^{\infty}$$

$$= \left[-e^{-\infty} + e^{-7(2)} \right]$$

$$= 0 + 8.315 \times 10^{-7}$$

$$\therefore = 8.315 \times 10^{-7}$$

b) $P(-3 < x \leq 4)$

$$\int_0^4 7e^{-7x} = \left[\frac{7e^{-7x}}{-7} \right]_1^4$$

$$= e^{-7(4)} + e^{-7(1)}$$

$$\therefore = 0.00091188$$

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8. Solution.

$P(x) = 1/4^x$

$x = 1$

$= 1/4$

$x = 2$

$= 1/4^2 = 1/16$

$E(x) = \frac{1}{4}(1) + \frac{1}{16}(2)$

$= \frac{1}{4} + \frac{2}{16} = 3/8$

$\therefore = 3/8$

14. Solution.

P_n

If $n = \infty$, then the sum of G.P is;

$\cancel{S_N \neq \lambda/\mu}$

$$\sum_{n=0}^{\infty} = \left(\frac{x}{\mu}\right)^n = \frac{1}{1-\left(\frac{\lambda}{\mu}\right)}$$

$$P_n = \frac{1}{1-\left(\frac{\lambda}{\mu}\right)} = P_n = \left(\frac{\lambda}{\mu}\right)^n\left(1-\frac{\lambda}{\mu}\right)$$

$$\therefore P_n = \left(\frac{\lambda}{\mu}\right)^n\left(1-\frac{\lambda}{\mu}\right)$$

L_s

$$L_s = E(n) = \sum_{n=0}^{\infty} n\left(\frac{\lambda}{\mu}\right)^n$$

$$\sum_{n=0}^{\infty} = 0\left(\frac{\lambda}{\mu}\right)^0 + 1\left(\frac{\lambda}{\mu}\right)^1$$

$$\therefore L_s = \frac{\lambda}{\mu - \lambda}$$

14. L_q

$$L_q = E(n-1) = \sum_{n=1}^{\infty} (n-1) P_n$$

$$= \sum_{n=1}^{\infty} n P_n - \sum_{n=1}^{\infty} P_n$$

$$= L_s - (1 - P_n)$$

$$L_q = L_s - (1 - P_0)$$

$$P_0 = 1 - \frac{\lambda}{\mu}$$

$$L_q = \frac{\lambda}{\mu - \lambda} - \left[1 - \left(1 - \frac{\lambda}{\mu}\right)\right]$$

$$\therefore L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

$$W_q = \frac{L_q}{\lambda}$$

$$= \frac{\lambda^2}{m(m-\lambda)} / \lambda$$

$$\therefore W_q = \frac{\lambda}{m(m-\lambda)}$$

$$W_s = \frac{L_s}{\lambda}$$

$$= \frac{\lambda}{\mu - \lambda} / \lambda$$

$$\therefore W_s = \frac{1}{\mu - \lambda}$$

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13\. Solution.

c.) a) Geometric distribution.

$$E(x) = \sum_{x=1}^{\infty} x f(x) = \sum_{x=1}^{\infty} x(1-p)^{x-1} p$$

$$= p\{1 + 2(1-p) + 3(1-p)^2 + 4(1+p)^3 + \cdots\}$$

$$= \frac{p}{[1-(1-p)]^2} = \frac{p}{p^2} = \frac{1}{p}$$

$$\therefore E(x) = \frac{1}{p}$$

Variance:

$$E(x^2) = \sum_{x=1}^{\infty} x^2 + (x) = \sum_{x=1}^{\infty} [x(x-1)+x] + (x)$$

$$= \sum_{x=1}^{\infty} [x(x-1)+x](1-p)^{x-1} p$$

$$\sum_{x=1}^{\infty} [x(x-1)+x](1-p)^{x-1}.$$

$$p = p[2(1-p) + 6(1-p)^2 + 12(1-p)^3 + 20(1-p)^4 + \cdots]$$

$$= 2p(1-p)[1 + 3(1-p) + 6(1-p)^2 + 10(1-p)^3 + 15(1-p)^4 + \cdots]$$

$$= \frac{2p(1-p)}{[1-(1-p)]^3} = \frac{2p(1-p)}{p^3}$$

$$= \frac{2-2p}{p^2}$$

$$\sigma^2 = \frac{2-2p}{p^2} + \frac{1}{p} - \frac{1}{p^2} = \frac{1-p}{p^2} = \frac{q}{p^2}$$

$$\text{Variance} = \cancel{\frac{q}{q}} \quad \frac{q}{p^2}$$

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13. b) Beta distribution.

$$f(x:a\ \beta) = x^{a-1}\ (1-x)^{\beta-1} \quad 0 \le x \le 1,\ a>0\ \beta>0$$

$$E(x) = \int_0^1 x\ \frac{x^{a-1}\ (1-x)^{\beta-1}}{\beta\ (a,\beta)}$$

$$dx = \frac{1}{\beta(a,\beta)} \int_0^1 x x (1-x)^{\beta-1}\ dx$$

$$= \frac{\beta(a+1,\beta)}{\beta(a,\beta)} = \frac{\Gamma(a+1)\ \Gamma(\beta)}{\Gamma(a+\beta+1)} \div \frac{\Gamma(a)\ \Gamma\beta}{\Gamma(a)+(\beta)}$$

$$= \frac{\Gamma(a+1)}{\Gamma a+\beta+1)} \frac{\Gamma(a)+(\beta))}{\Gamma(a)} = \frac{a!}{(a+\beta)!},\ \frac{(a+\beta-1)!}{(a-1)}$$

$$= \frac{a(a-1)!}{(a+\beta)(a+\beta-1)!} \frac{(a+\beta-1)!}{(a-1)!} = \frac{a}{a+\beta}$$

$$\therefore E(x) = \frac{a}{a+\beta}$$

$$\text{Variance} = E(x^2) - [E(x)]^2 \quad dx = \frac{1}{\beta(a,\beta)} \int_0^1 x^{a+1} (1-x)^{\beta-1} dx$$

$$= \frac{\Gamma(a+2)}{\Gamma a+\beta+2} \quad \frac{\Gamma a+\beta}{\Gamma a}$$

$$= \frac{(a+1)'}{a+\beta+1} \frac{(a+\beta-1)'}{(a-1)'} = \frac{(a+1)a(a-1)!}{(a+\beta+1)!\ (a+\beta-1)!} \frac{(a+\beta-1)!}{(a-1)!}$$

$$E(x)^2 = \frac{a(a+1)!}{(a+\beta+1)(a+\beta)} = \frac{a(a+1)!}{(a+\beta+1)(a+\beta)} - \left[\frac{a}{a+\beta}\right]^2 = \frac{a}{(a+\beta+1)(a+\beta)^2}$$

$$\therefore \text{Variance} = \frac{a\beta}{(a+\beta+1)(a+\beta)^2}$$

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13 a) Bernoulli

$x \sim$ Bernouli if

$$x = \begin{cases} p^x q^{1-x} & x = 0, 1 \\ 0 & \text{elsewhere} \end{cases}$$

Probability of success

$q = 1 - p$

$$E(x) = \sum_{x=0}^{1} x p^x q^{1-x} = 0 p^0 q^{1-0} + 1 \times p^1 q^{1-1}$$

$\therefore E(x) = p$

Variance (x)

$= E(x^2) - [E(x)]^2$

$$E(x)^2 = \sum_{x=0} x^2 p^x q^{1-x}$$

$= 0^2 \times p^0 q^{1-0} + 1^2 * p^1 q^{1-1}$

$= 0 + p$

$= p$

$E(x^2) = p$

Variance $(x) = p - p^2 = p(1-p)$

$\therefore$ Variance$(x) = pq$