# Large-Scale Machine Learning

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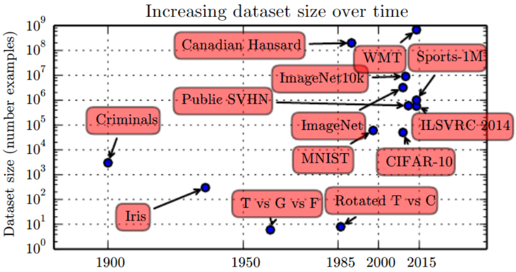
Machine Learning

# Outline

1. When ML Meets Big Data
2. Representation Learning
3. Curse of Dimensionality
4. Trade-Offs in Large-Scale Learning
5. SGD-Based Optimization

# Outline

# The Big Data Era

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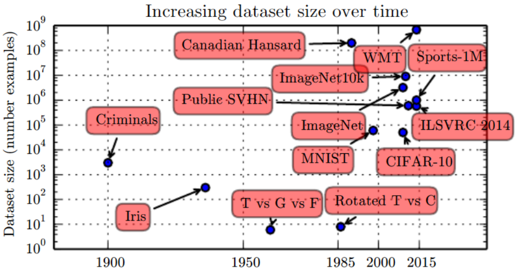


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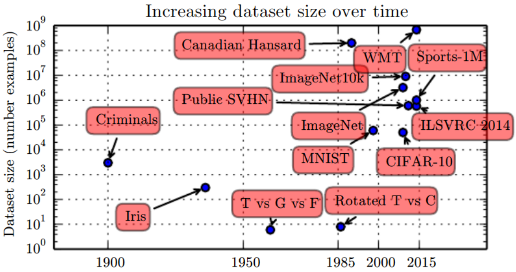


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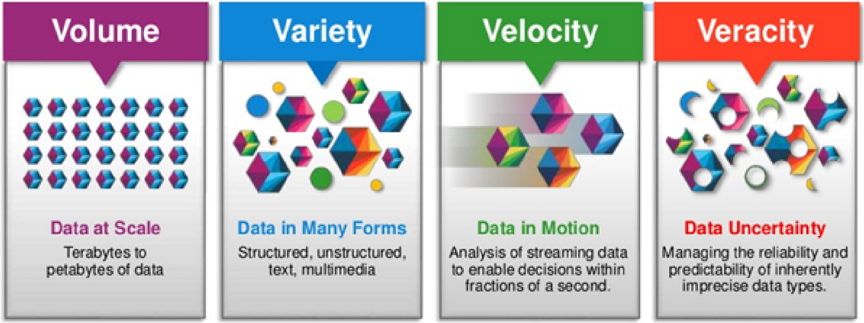
- ***Large-scale machine learning*** techniques solve problems by leveraging the posteriori knowledge learned from the big data

# Characteristics of Big Data



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- Volume
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- Velocity
  - Online learning

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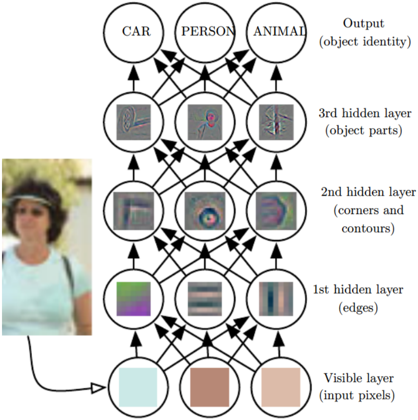
# The Rise of Deep Learning

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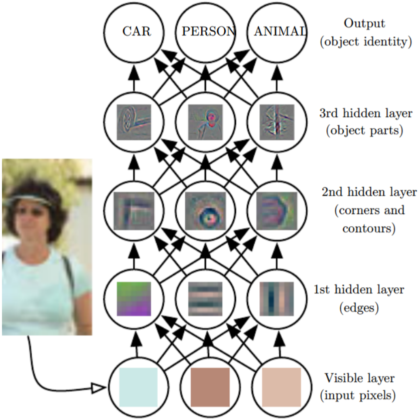


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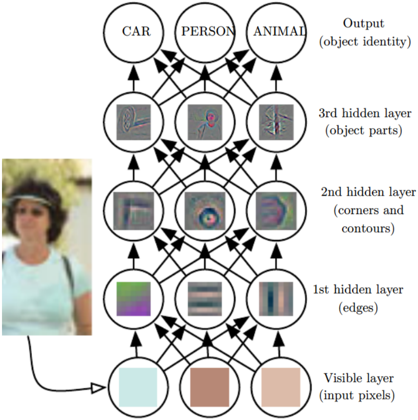


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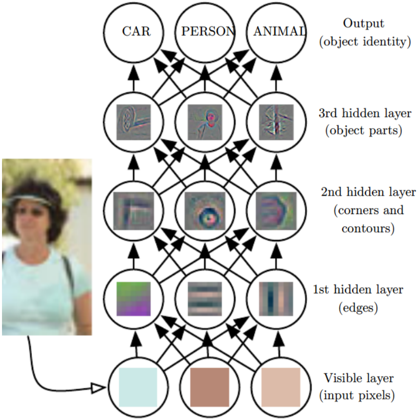


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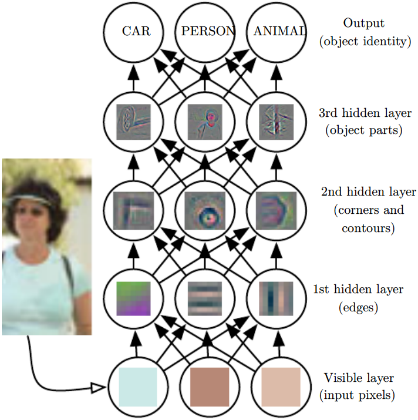


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- Supports online & transfer learning

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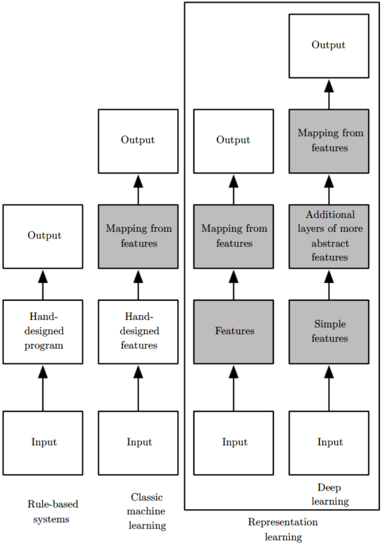
- For simple (linear) $f$, there are specialized large-scale ML techniques (e.g., LIBLINEAR [4]) that are much more efficient
  - E.g., text classification

# Outline

# Representation Learning

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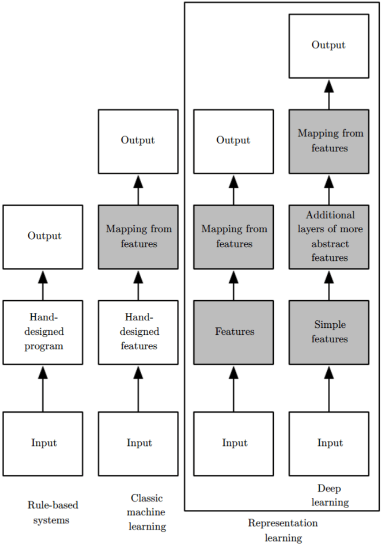


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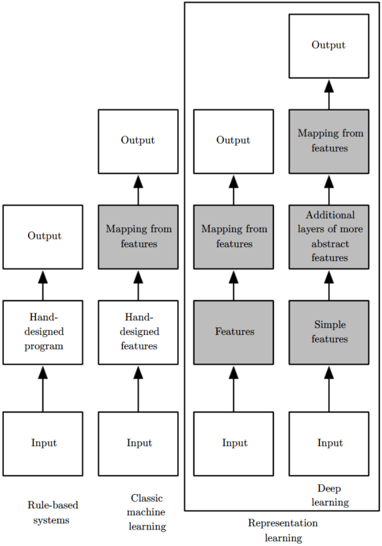


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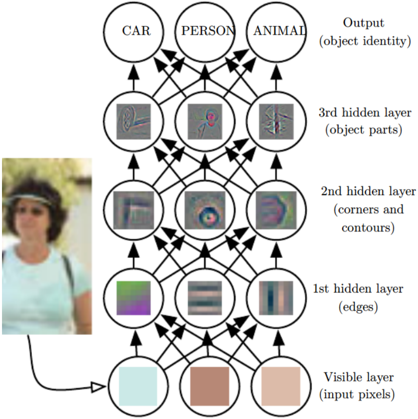
- In deep learning, features/presentations are ***distributed***

# Distributed Representations of Data

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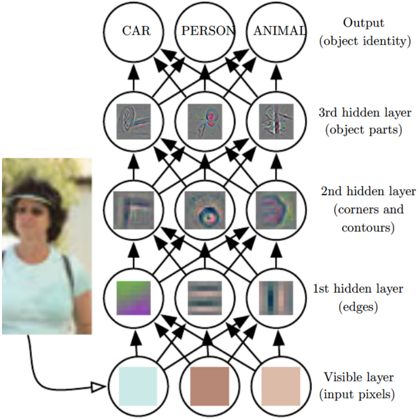


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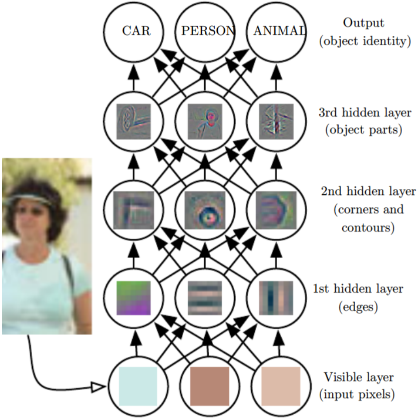


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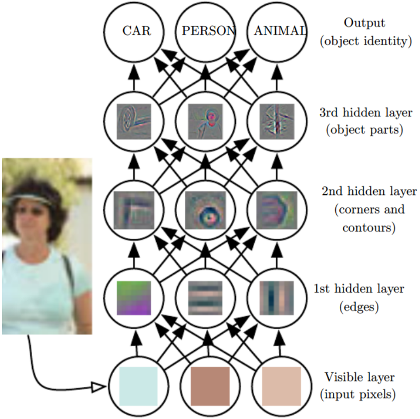


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- To be fed into the factors in the next (deeper) level
  - Face = 0.3 * 1 + 0.7 * 2

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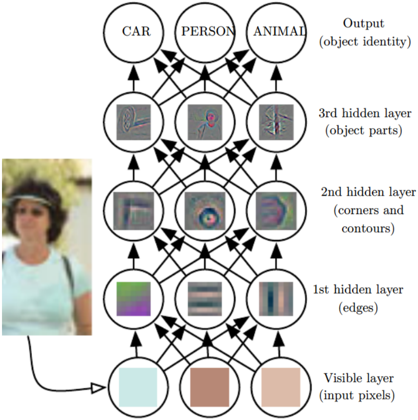


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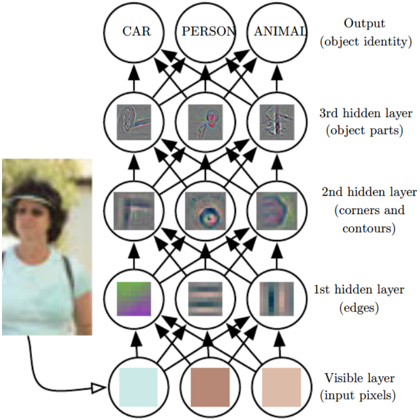


- Transfer learning: to reuse the knowledge learned from a task to help another task
- In deep learning, it's common to reuse the feature extraction network from one task in another
    - Weights may be further updated when training model in a new task

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# Curse of Dimensionality

- Most classic nonlinear ML models find $\theta$ by assuming function smoothness:

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$$\hat{y} = \sum_i \alpha_i y^{(i)} k(\boldsymbol{x}^{(i)},\boldsymbol{x}) + b, \text{ where } k(\boldsymbol{x}^{(i)},\boldsymbol{x}) = \exp(-\gamma\|\boldsymbol{x}^{(i)} - \boldsymbol{x}\|^2)$$

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- Suppose $f$ is smooth within a bin, we need ***exponentially more examples*** to get a good interpolation as $D$ increases

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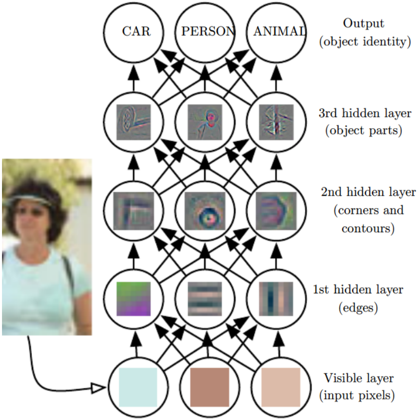


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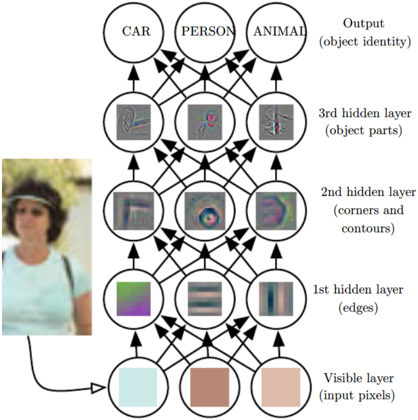


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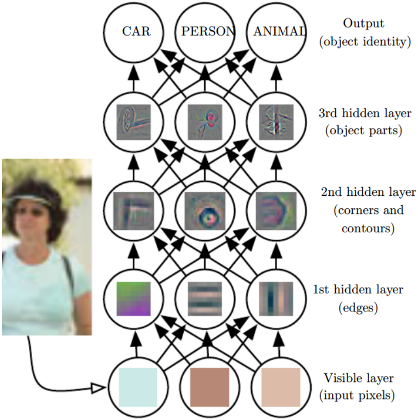


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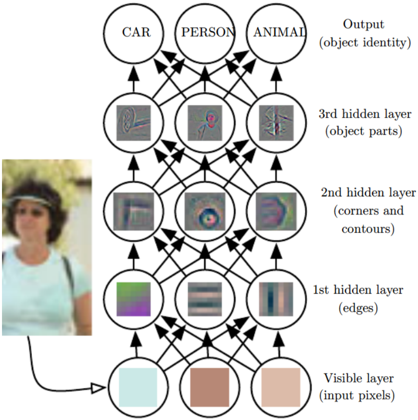


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- Exponential gains from depth counter the exponential challenges posed by the curse of dimensionality

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- But we only minimizes errors on limited examples in our objective, so we only have $f_N = \arg\min_{f\in\mathbb{F}} C_N[f]$
- The ***excess error*** $\mathscr{E} = C[f_N] - C[f^*]$:

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- ***Optimization error $\mathscr{E}_{opt}$***: reduced by
  1. Running optimization alg. longer (with smaller $\rho$)
  2. Choosing more efficient optimization alg.

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  - Mainly constrained by $N$
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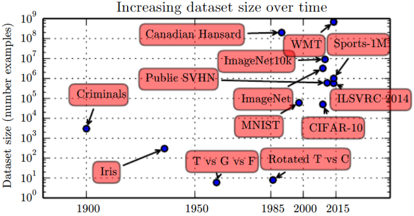
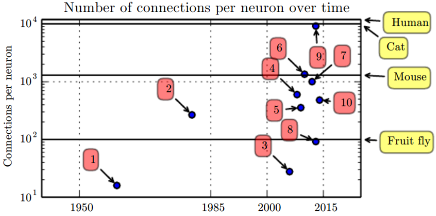
  - $N$ is large, so $\mathscr{E}_{\text{est}}$ can be reduced
  - ***Large model*** is preferred to reduce $\mathscr{E}_{\text{app}}$

# Big Data + Big Models




9. COTS HPC unsupervised convolutional network [3]
10. GoogleLeNet [6]

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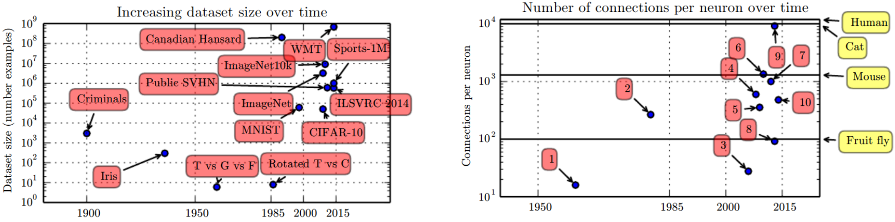


9. COTS HPC unsupervised convolutional network [3]
10. GoogleLeNet [6]

- With domain-specific architecture such as ***convolutional NNs*** (CNNs) and ***recurrent NNs*** (RNNs)

# Outline

# Stochastic Gradient Descent

## Gradient Descent (GD)

$\boldsymbol{w}^{(0)} \leftarrow$ a randon vector;
Repeat until convergence {
$\boldsymbol{w}^{(t+1)} \leftarrow \boldsymbol{w}^{(t)} - \eta \nabla_{\boldsymbol{w}} C_N(\boldsymbol{w}^{(t)}; \mathbb{X})$;
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$\boldsymbol{w}^{(0)} \leftarrow$ a randon vector;
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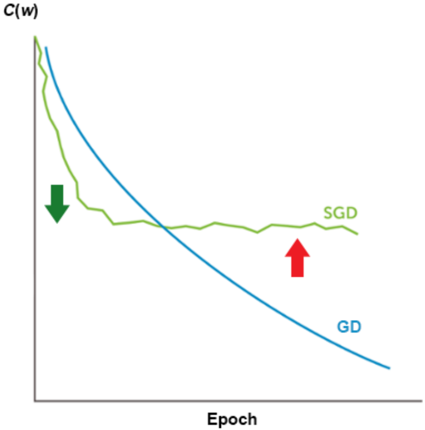
- Supports ***online*** training

# GD vs. SGD



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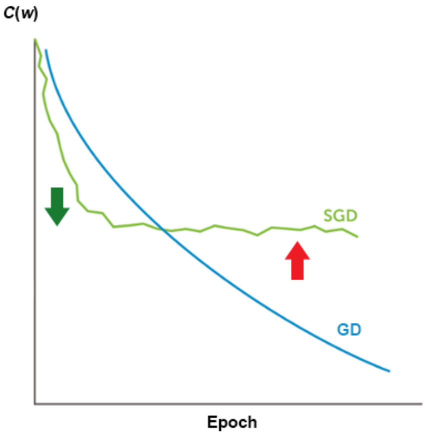
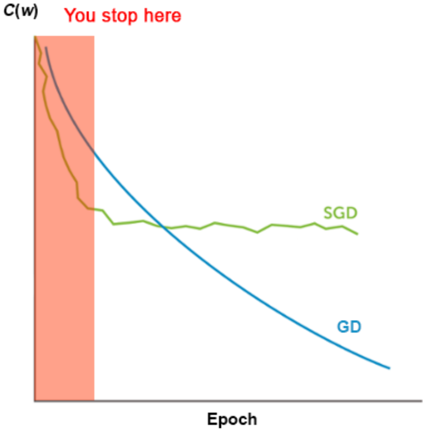



- Is SGD really a better algorithm?

# Yes, If You Have Big Data



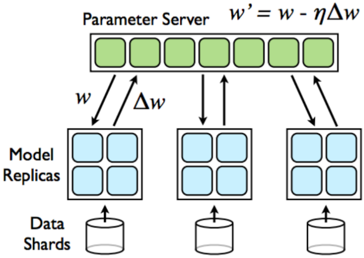

- Performance is limited by ***training time***

# Asymptotic Analysis [1]

| | GD | SGD |
|---|---|---|
| Time per iteration | $N$ | $1$ |
| #Iterations to opt. error $\rho$ | $\log \frac{1}{\rho}$ | $\frac{1}{\rho}$ |
| Time to opt. error $\rho$ | $N \log \frac{1}{\rho}$ | $\frac{1}{\rho}$ |
| Time to excess error $\varepsilon$ | $\frac{1}{\varepsilon^{1/\alpha}} \log \frac{1}{\varepsilon}$, where $\alpha \in [\frac{1}{2}, 1]$ | $\frac{1}{\varepsilon}$ |

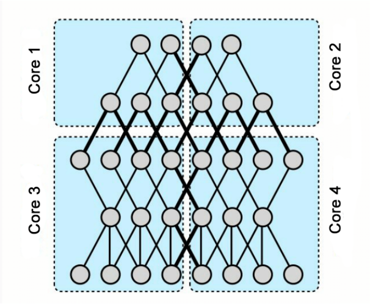
# Parallelizing SGD

## Data Parallelism




Every core trains the full model given partitioned data.

## Model Parallelism

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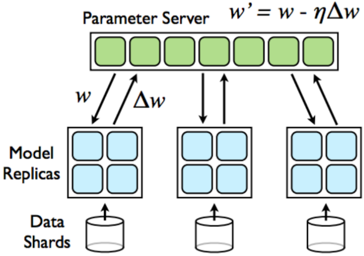


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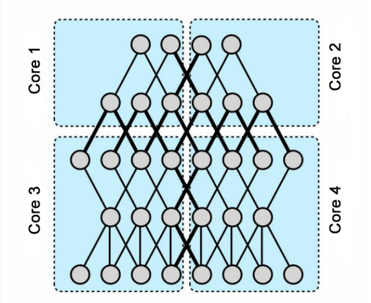


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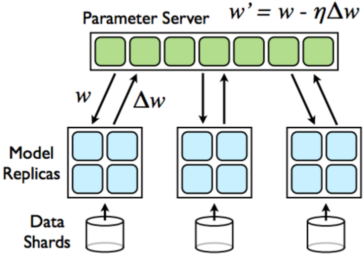


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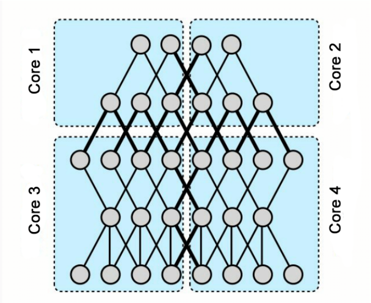


Every core train a partitioned model partition given full data.

- Normally, model parallelism exchange less parameters in a large NN and can support more cores
- The effectiveness depends on settings such as CPU speed, communication latency, and bandwidth, etc.

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