Grouping Optimization Based Hybrid Beamforming for Multiuser MmWave Massive MIMO Systems

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Abstract—**In millimeter-wave massive multiple input multiple output multiuser systems, inter-user interference becomes a major factor limiting system capacity. The premise of increasing system capacity is to minimize inter-user interference on the basis of ensuring large receiving power. In response to this situation, this paper proposes a low complexity grouping optimization based hybrid beamforming (HBF) algorithm. Specifically, we group users according to user channel correlation and a correlation threshold. Users with strong correlation are grouped into a group. Then, with the goal of maximizing capacity, the low-dimensional exhaustive algorithm is used in each group to select the base station beamforming vector. Moreover, a greedy algorithm is adopted, i.e., the influence of the beamforming vectors of the previous groups is considered. Simulation results show that the system sum rate of the grouping optimization HBF algorithm is higher than that of the existing HBF algorithms.**

Keywords- multiple input multiple output; millimeter-wave; multiuser; interference;complexity;

I. INTRODUCTION

Millimeter-wave (mmWave) massive multiple input multiple output (MIMO) has been considered as a key candidate technology for 5G [1]. MmWave has the characteristic of short wavelength, which makes it possible to install a large number of antennas in limited space [2]. Large-scale antenna arrays provide high multiplexing gain to compensate for path loss during transmission [3]. More importantly, the mmWave massive MIMO system has a wide signal bandwidth and a high spectral efficiency, which can greatly increase the data transmission rate [4].

However, the implementation of mmWave massive MIMO systems has great difficulties. One of the challenges is that in a mmWave massive MIMO system, each antenna needs to be connected to a dedicated radio frequency (RF) chain [5]. The number of antennas in mmWave massive MIMO systems may be a hundred or several hundreds, which will bring huge hardware expend and power consumption. This problem can be solved by a hybrid beamforming (HBF) structure, which includes an analog beamforming (ABF) part and a digital beamforming part [6]. Some researches deal with HBF design. For example, [7] generalizes orthogonal matching pursuit algorithm, which iteratively finds the codebook vector with the strongest correlation with the rest of the channel. In [8], a greedy algorithm is proposed, in which only one element in the ABF matrix is optimized in each iteration.

For multiuser mmWave MIMO systems, there are also several HBF schemes. In [9], a beam steering algorithm is proposed, in which the transmitter selects the codebook vectors with the strongest correlation with the channel vectors as the ABF vectors. However, this method only maximizes the signal power and cannot effectively suppress inter-user interference. [10] proposes a HBF algorithm, which uses a greedy algorithm to select codebook vector with higher correlation with the channel vector as the candidate vector of the receiver, and calculate the capacity with the candidate vector. The codebook with the largest capacity is used as the ABF vector of the receiver. This method can mitigate inter-user interference. However, in [10], the algorithm only chooses the beamforming vector for one user at a time, and ignores some inter-user interference.

In this paper, a low complexity grouping optimization based HBF is proposed for multiuser mmWave massive MIMO systems. Specifically, we group users according to user channel correlation and a correlation threshold. Users with strong correlation are grouped into a group. The use of correlation threshold makes the grouping strict, and also selects a few users in each group. In order to reduce the computational complexity and make the algorithm feasible, n vectors (n greater than or equal to the number of users) which are highly correlated with the user channel are selected from the codebook to form a candidate codebook set for the BS. Then, starting from the first group, with the goal of maximizing capacity, the low-dimensional exhaustive algorithm is used to select the beamforming vector from the candidate codebook. Moreover, for the other groups, the selection of beamforming vector is based on the greedy algorithm. The main contributions of this paper are as follows. i) A strict grouping scheme that divides users with large channel correlations into groups is proposed, which make number of users small in each group. This makes it possible to use the exhaustive method. ii) The greedy algorithm that considers the impact of the users of former groups is proposed, which can effectively mitigate inter-user interference.

Notations: Letters are not bolded to indicate constants. Lowercase and uppercase letters in boldface represent vectors and matrices, respectively. For a matrix $\boldsymbol{A}$, $[\boldsymbol{A}]_{m,n}$ denotes the element in the m-th row and the n-th column. $[\boldsymbol{A}]_m$ represents the m-th column of $\boldsymbol{A}$. $\mathbf{A}^H$ denotes the conjugate transpose of $\mathbf{A}$. $\mathbb{E}[\bullet]$ is used to denote expectation. $\boldsymbol{I}_K$ is a $K \times K$ identity matrix. For a set, $Card(\bullet)$ denotes the number of elements in it. $\|\bullet\|_2$ denotes the Euclidean norm of a vector.

II. SYSTEM MODEL

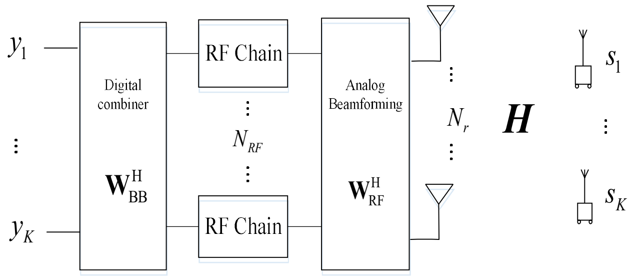

Fig.1. The architecture of a mmWave HBF system.

Consider multiple single antenna users communicating with a multi-antenna BS using mmWave channels. As shown in Fig. 1, in the system, there are K single antenna users. The BS is equipped with N_r antennas and N_{RF} RF chains. We consider the design of the BS as a receiver, and the receiver can be divided into an ABF part and a digital processing part. The use of HBF can greatly reduce the number of RF chains, that is $N_{\mathrm{RF}} << N_r$. Assuming K users simultaneously transmit signals to the BS. The signals received by the BS are first processed with the ABF matrix $\boldsymbol{W}_{\mathrm{RF}}^{\mathrm{H}}$ to reduce the dimension of the received signals, where $\boldsymbol{W}_{\mathrm{RF}} \in \mathbb{C}^{N_r \times N_{\mathrm{RF}}}$. Then, after analog to digital conversion, the reduced-dimensional signals are recovered by the digital processing matrix $\boldsymbol{W}_{\mathrm{BB}}^{\mathrm{H}}$, where $\boldsymbol{W}_{\mathrm{BB}} \in \mathbb{C}^{N_{\mathrm{RF}} \times K}$. In this paper, we set the number of RF chains as $N_{\mathrm{RF}} = K$. Since ABF is implemented by employing the analog phase shifters, we have the constraint $\left|\left[\mathbf{W}_{\mathrm{RF}}\right]_{n_r, n_{\mathrm{RF}}}\right| = 1/\sqrt{N_r}$, for $n_r = 1, \cdots, N_r, n_{\mathrm{RF}} = 1, \cdots, K$. Moreover, $\boldsymbol{s} \in \mathbb{C}^{K \times 1}$ is the signal vector, whose elements satisfy $\mathbb{E}\left[\boldsymbol{s}\boldsymbol{s}^{\mathrm{H}}\right] = \boldsymbol{I}_K$. The received signal vector at the BS is $\boldsymbol{x} = \sqrt{\rho}\boldsymbol{H}\boldsymbol{s} + \boldsymbol{n} \in \mathbb{C}^{N_r \times 1}$, where ρ is transmission power and $\boldsymbol{H} = \left[\boldsymbol{h}_1, \boldsymbol{h}_2, \cdots, \boldsymbol{h}_K\right] \in \mathbb{C}^{N_r \times K}$ denotes the channel matrix, and $\boldsymbol{h}_K \in \mathbb{C}^{N_r \times 1}$ is the channel vector of the k-th user. Moreover, $\boldsymbol{n} \in \mathbb{C}^{N_r \times 1}$ is the noise vector, whose elements obey $\mathcal{CN}\left(\boldsymbol{0}, \boldsymbol{I}_{N_r}\right)$. Then, the recovered signal vector can be expressed as

$$\boldsymbol{y} = \boldsymbol{W}_{\mathrm{BB}}^{\mathrm{H}}\boldsymbol{W}_{\mathrm{RF}}^{\mathrm{H}}\boldsymbol{x} = \sqrt{\rho}\boldsymbol{W}_{\mathrm{BB}}^{\mathrm{H}}\boldsymbol{W}_{\mathrm{RF}}^{\mathrm{H}}\boldsymbol{H}\boldsymbol{s} + \boldsymbol{W}_{\mathrm{BB}}^{\mathrm{H}}\boldsymbol{W}_{\mathrm{RF}}^{\mathrm{H}}\boldsymbol{n}, \quad (1)$$

Assume that the channel state information is known perfectly at the BS, which can be achieved by using the compressive sensing in [5]. Then, $\boldsymbol{W}_{\mathrm{BB}}$ can be designed with the minimum mean square error (MMSE) criterion as

$$\boldsymbol{W}_{\mathrm{BB}} = \tilde{\boldsymbol{H}}\left(\tilde{\boldsymbol{H}}^{\mathrm{H}}\tilde{\boldsymbol{H}} + \boldsymbol{I}_K\right)^{-1}, \quad (2)$$

where $\tilde{\boldsymbol{H}} = \boldsymbol{W}_{\mathrm{RF}}^{\mathrm{H}}\boldsymbol{H} \in \mathbb{C}^{K \times K}$.

Then, the system sum rate can be expressed as

$$R = \sum_{k=1}^{K} \log_2\left(1 + \eta_k\right), \quad (3)$$

where η_k is the signal-to-interference-plus-noise ratio (SINR) of the k-th user, and is expressed as

$$\eta_k = \frac{\left|\left[\boldsymbol{W}_{\mathrm{RF}}\boldsymbol{W}_{\mathrm{BB}}\right]_k^{\mathrm{H}} \boldsymbol{h}_k\right|^2}{\sum_{m \neq k}\left|\left[\boldsymbol{W}_{\mathrm{RF}}\boldsymbol{W}_{\mathrm{BB}}\right]_k^{\mathrm{H}} \boldsymbol{h}_m\right|^2 + \frac{1}{\rho}\left\|\left[\boldsymbol{W}_{\mathrm{RF}}\boldsymbol{W}_{\mathrm{BB}}\right]_k\right\|_2^2}. \quad (4)$$

A. Channel Model

We use the clustered channel model. For each user, there are N_{cl} scattering clusters, each of which contributes N_{ray} propagation paths. Thus, the discrete time narrowband channel vector of the k-th user can be expressed as

$$\boldsymbol{h}_k = \gamma \sum_{i,l} \alpha_{il}\mathbf{a}\left(\phi_{il}\right), \quad (5)$$

where $\gamma = \sqrt{N_r / N_{cl}N_{ray}}$, α_{il} is the complex gain of l-th ray in i-th scattering cluster, ϕ_{il} is the angle of arrival. $\mathbf{a}\left(\phi_{il}\right) \in \mathbb{C}^{N_r \times 1}$ is the antenna array response vector (AARV). In the paper, the BS uses a uniform linear array of N_r antennas to receive signals, then the AARV can be expressed as

$$\mathbf{a}_{\mathrm{ULA}}\left(\phi\right) = \frac{1}{\sqrt{N_r}}\left[1, e^{-jkd\sin(\phi)}, \cdots, e^{-j(N_r-1)kd\sin(\phi)}\right]^T, \quad (6)$$

where $k = 2\pi/\lambda$, λ is the carrier wavelength, and d is the distance between adjacent antennas in the array.

B. Problem Formulation

In this paper, when we design HBF matrix, known by formula (2), $\boldsymbol{W}_{\mathrm{BB}}$ can be obtained from $\boldsymbol{W}_{\mathrm{RF}}$ and $\boldsymbol{H}$. Then, the design of HBF becomes the design of $\boldsymbol{W}_{\mathrm{RF}}$, and the object is to maximize the system sum rate, written as

$$\max_{\boldsymbol{W}_{\mathrm{RF}}} R \quad \text{s.t. } \left[\boldsymbol{W}_{\mathrm{RF}}\right]_k \in \mathcal{F}_{\mathrm{RF}}, k = 1, \cdots, K, \quad (7)$$

where $\mathcal{F}_{\mathrm{RF}}$ stands for the set of feasible ABF vectors and is written as

$$\mathcal{F}_{\mathrm{RF}} = \left\{\mathbf{a}\left(\phi_c\right) \middle| \phi_c = \frac{c\pi}{N_r}, c = -\frac{N_r}{2}, \cdots, \frac{N_r}{2} - 1\right\}. \quad (8)$$

For (8), the direct exhaustive method can obtain the optimal solution. However, its complexity is $N_r^{\ K}$. In mmWave massive MIMO systems, N_r is usually very large, e.g., $N_r = 128$, $K = 16$, and the complexity is too high for implementation. To solve this problem, we will propose a HBF algorithm based on the grouping optimization method, which will be described in detail in Section III.

III. GROUP OPTIMIZED HBF ALGORITHM

The existing HBF methods for multiuser mmWave massive MIMO systems still have shortcomings. The beam control method in [9] only maximizes signal power but does not suppress interference. Although [10] suppresses some interference, but the interference is not effectively mitigated. In this section, we propose a grouping optimization algorithm, which consists of two parts: i) Grouping users according to user channel correlation and the correlation threshold. ii) Using exhaustive searching in low-dimensional group and greedy search among groups.

A. Grouping Users

In multiuser mmWave MIMO systems, the k-th user is interfered by $K-1$ users. However, we do not know the interference strength between the k-th user and other users. If the interference is weak, these users can directly select the beamforming vector with the largest path gain. If the interference is strong, the beamforming vectors of these users should be selected together, and the selection of the beamforming vector is based on the capacity maximization.

We use the correlation between the user channel vector to measure the strength of the interference. To facilitate the comparison of the correlation, we use the matrix $\boldsymbol{X} \in \mathbb{R}^{K\times K}$ to represent the correlation, that is

$$[\boldsymbol{X}]_{i,l} = \begin{cases} \left|\boldsymbol{h}_i^{\mathrm{H}}\boldsymbol{h}_l\right|^2 & i \neq l, \\ 0 & i = l. \end{cases} \tag{9}$$

We know that the correlation between the i-th user and the l-th user is $[\boldsymbol{X}]_{i,l}$, and we have $[\boldsymbol{X}]_{i,l} = [\boldsymbol{X}]_{l,i}$. In order to avoid the occurrence of two maximum values in the subsequent calculation processes, we introduce another matrix $\boldsymbol{X}_{\mathrm{U}} \in \mathbb{R}^{K\times K}$, which consists of the upper triangular element of $\boldsymbol{X}$, and the remaining elements are all zeros. By extracting the index corresponding to the largest element from $\boldsymbol{X}_{\mathrm{U}}$, there is

$$(t_1, t_2) = \arg\max_{i,l} [\boldsymbol{X}_{\mathrm{U}}]_{i,l}, \tag{10}$$

At this time, we put user t_1 and user t_2 into one group, and use the set G to represent the users in the group, that is $G = \{t_1, t_2\}$. Assuming G has i elements, $i > 2, i \in \mathbb{Z}$,so $G = \{t_1, t_2, \cdots, t_i\}$. Then, we consider whether to add users to G. We define the user t_i correlation threshold e_{t_i} as the mean of the correlations between user t_i and other users, that is

$$e_{t_i} = \sum_{j=1}^{K} \frac{[\boldsymbol{X}]_{t_i,j}}{K}, \tag{11}$$

For user t_i, we select users whose correlation with user t_i is greater than e_{t_i} from the remaining $K-i$ users, which is represented by the set Γ, and form a set

$$P_{t_i} = \left\{ l \,\middle|\, [\boldsymbol{X}]_{t_i,l} > e_{t_i}, l \in \Gamma \right\}, \tag{12}$$

Moreover, we have $\Omega_i = P_{t_1} \cap P_{t_2} \cap \cdots P_{t_i}$, where Ω_i indicates that candidate set of the $i+1$-th element of G. Now, we use the following method to determine whether we add the $i+1$-th element to G. If $Crad(\Omega_i) = 0$, it indicates that the group is completed, and there are only i elements in G. If $Crad(\Omega_i) = 1$, and the $i+1$-th element in G is the element in Ω_i. If $Crad(\Omega_i) > 1$, we assume $\Omega_i = \{k_1, k_2, \cdots, k_j\}$ and

$$q_k = [\boldsymbol{X}]_{t_1,k} + [\boldsymbol{X}]_{t_2,k} + \cdots + [\boldsymbol{X}]_{t_i,k}, k \in \Omega_i, \tag{13}$$

the $i+1$-th element of set G is

$$t_{i+1} = \arg\max_k q_k. \tag{14}$$

According to (11), $e_{t_{i+1}}$ can be updated. Then, by (12), $P_{t_{i+1}}$ can be updated. Moreover, we can update $\Omega_{i+1} = \Omega_i \cap P_{t_{i+1}}$, where Ω_{i+1} is the candidate set for the $i+2$-th element of G. Repeat the above process until the group is completed. According to the details, we know that the elements in the set G satisfy two conditions. i) High correlation between channels of the users. ii) A few number of elements.

Then, we remove the impact of grouped users before proceeding to the next group. Assume that the group is $G = \{t_1, t_2, \cdots, t_i\}$ and take element t_i as an example. We set the t_i-th row and the t_i-th column to zero vectors to remove the impact of user t_i. By using this all the elements in G, we can remove the impact of the grouped users in G. Then, we repeat (10)-(14) to group the remaining users until all users grouped.

In this process, we can see that the grouping condition is very strict, and there are a few elements in each group, which makes it suitable for employing low-dimensional exhaustive search in each group.

B. Low-dimensional Exhaustive and Greedy Algorithms

As mentioned above, in mmWave MIMO systems, directly using the exhaustive method to select the beamforming vector results into high complexity. With grouping, we make a few elements in each group. If the exhaustive search within the group is carried out at this time, the complexity is still high, as there are many beamforming vectors for select. However, the path gains of many paths are small and can be ignored in general. Therefore, we only need to select a part of beamforming vector that correspond to the paths with large gains for transmission. Here, we pick out the paths with large gains by correlating the user channel vector with the codebook，that is

$$\left[\boldsymbol{b}_k\right]_{1,n} = \left|\boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{F}_n\right|^2, \tag{15}$$

where $\boldsymbol{b}_k \in \mathbb{C}^{1\times N_r}$, and $\boldsymbol{F} \in \mathbb{C}^{N_r \times N_r}$ consists of the codebook set $\mathcal{F}_{\mathrm{RF}}$. Then, we extract the indices of the K maximum values in $\left[\boldsymbol{b}_k\right]_{1,n}, n = 1,2,\cdots,N_r$, and save them in the set C_k. Hence, the candidate codebook set of user k is

$$\mathcal{F}_k = \left\{ \mathbf{a}\left(\phi_c\right) \middle| \phi_c = \frac{\mathrm{c}\pi}{N_r}, \mathrm{c} \in C_k \right\}, \tag{16}$$

Next, we take the u-th group as an example to analyze the process of obtaining the ABF vector using the low-dimensional exhaustive method in the group. Suppose there are P elements in the first $\mathrm{u}-1$ groups, and their ABF matrix is $\boldsymbol{W}_{\mathrm{RF,u-1}} \in \mathbb{C}^{N_r \times P}$. We denote the set that includes the indices of the users in u-th group as G and we ssume there are L elements in G. Then, the ABF matrix for the u-th group can be expressed as

$$\boldsymbol{W}_{\mathrm{RF,u}} = \left[\boldsymbol{W}_{\mathrm{RF,u-1}}, \mathbf{a}\left(\theta_1\right), \mathbf{a}\left(\theta_2\right), \cdots, \mathbf{a}\left(\theta_L\right)\right], \tag{17}$$

where $\boldsymbol{W}_{\mathrm{RF,u}} \in \mathbb{C}^{N_r \times E}$, $E = P + L$, and $\mathbf{a}\left(\theta_l\right) \in \mathcal{F}_{G_l}$, $G = \left\{G_l \mid l = 1,2,\cdots,L\right\}$. We know from equation (16), $\mathcal{F}_{G_1}, \mathcal{F}_{G_l}, \cdots, \mathcal{F}_{G_L}$ all have K elements. Thus, $\boldsymbol{W}_{\mathrm{RF,u}}$ has a total of K^L realizations. According to the MMSE criterion, the corresponding $\boldsymbol{W}_{\mathrm{BB,u}} \in \mathbb{C}^{E \times E}$ is obtained as

$$\boldsymbol{W}_{\mathrm{BB,u}} = \tilde{\tilde{\boldsymbol{H}}}\left(\tilde{\tilde{\boldsymbol{H}}}^{\mathrm{H}} \tilde{\tilde{\boldsymbol{H}}} + \boldsymbol{I}_E\right)^{-1}, \tag{18}$$

where $\tilde{\tilde{\boldsymbol{H}}} = \boldsymbol{W}_{\mathrm{RF,u}}^{\mathrm{H}} \boldsymbol{H}_{\mathrm{u}} \in \mathbb{C}^{E \times E}$, $\boldsymbol{H}_{\mathrm{u}} = \left[\boldsymbol{H}_{\mathrm{u-1}}, \boldsymbol{h}_{G_1}, \cdots, \boldsymbol{h}_{G_L}\right]$, and $\boldsymbol{H}_{\mathrm{u-1}}$ is the channel matrix of the former $\mathrm{u}-1$ groups. According to formula (3), the system capacity R_{u} of the u-th group can be obtained. Due to $\boldsymbol{W}_{\mathrm{RF,u}}$ has K^L realizations, R_{u} also has K^L kinds of values. Our goal is to design ABF matrix to maximize capacity, and the optimum beamforming matrix is

$$\boldsymbol{W}_{\mathrm{RF,u}}^{*} = \underset{\boldsymbol{W}_{\mathrm{RF,u}}}{\arg\max}\, R_{\mathrm{u}}. \tag{19}$$

Then, the above process is repeated until the calculation of the ABF matrix of all the groups is completed.

IV. SIMULATIONS

In this part, we use the clustered channel model to simulate and analyze the proposed algorithm from four aspects: signal-to-noise ratio (SNR), number of users, number of BS antennas, and number of scattered clusters in space. For comparison, we also analyze the full digital algorithm, the greedy algorithm in [10], and the beam control algorithm in [9]. For the full digital algorithm, each antenna is connected to one RF chain, and the zero forcing criterion is employed. The simulation parameters are as follows. φ_{il} is uniformly distributed in $\left[0, 2\pi\right]$, the antenna distance is $d = 0.5\lambda$, the number of the BS antennas is $N_r = 64$, the number of the users is $K = 8$. For any user, $N_{cl} = 8$, $N_{ray} = 10$, the SNR is $10\log_{10}\rho = 10$ dB.

In Fig. 2, sum rates versus the SNR is shown. It can be seen that the full digital algorithm has the highest system sum rate. With high SNR, the performance of the proposed algorithm is far superior to the beam control algorithm proposed in [9] and the greedy algorithm proposed in [10]. Especially, when SNR$=20\mathrm{dB}$, the system sum rate of the proposed algorithm differs from that of the greedy algorithm in [10] by $20\mathrm{bit/s/Hz}$.

In Fig. 3, sum rates versus the number of user is shown. As can be seen, the performance of the proposed algorithm is inferior to the full digital algorithm. The performance of the beam control algorithm in [9] is the worst. This is because that this approach cannot effectively mitigate interference.

In Fig. 4, sum rates versus the number of the BS antennas is shown. The sum rates of all the approaches increase with the increase of the number of the BS antennas. The proposed algorithm performs much better than the beam control algorithm proposed in [9] and the greedy algorithm proposed in [10].

In Fig. 5, sum rates versus the number of scattering clusters is shown. With the increase of the number of the scattering clusters, the inter-user interference is enhancing. Although the performance of the proposed algorithm slightly decreases, it has quickly stabilized. In contrast, the beam control algorithm proposed in [9] and the greedy algorithm proposed in [10] have a serious performance degradation. This also confirms the superiority of the proposed algorithm in reducing the inter-user interference .

V. CONCLUSIONS

In this paper, we propose a low-complexity HBF algorithm based on grouping optimization. We group users according to user channel correlation and the user correlation threshold. Users with strong channel correlations are grouped into a group. With the goal of maximizing capacity, a low-dimensional exhaustive search algorithm is used in the group and a greedy algorithm is used among the groups to select the beamforming vectors. The proposed approach considers the impact of the former groups of users and can effectively mitigate interference. Simulation results show that the sum rate of the proposed algorithm is higher than that of the existing HBF algorithms.

ACKNOWLEDGMENT

This research was supported by Project 61601152 supported by National Natural Science Foundation of China.

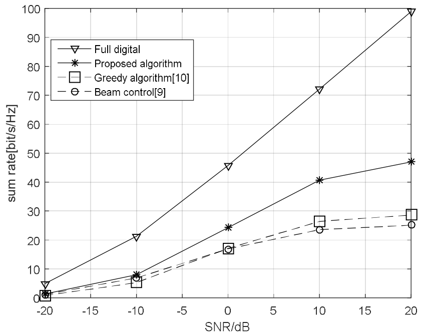

Fig.2. Sum rates versus the SNR.

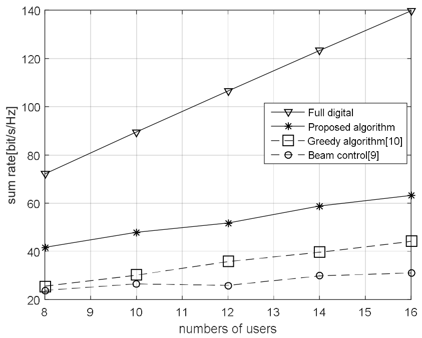

Fig.3. Sum rates versus the number of user.

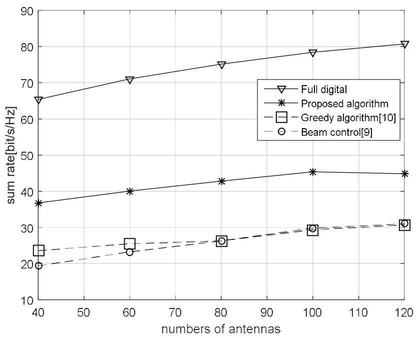

Fig.4. Sum rates versus the number of the BS antennas.

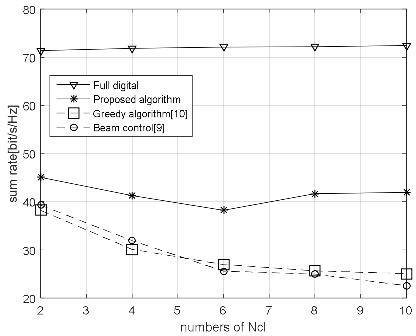

Fig.5. Sum rates versus the number of the scattering clusters.

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