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Mathematical Problems in Image Processing

Partial Differential Equations and the
Calculus of Variations

Second Edition



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Contents

Foreword	vn
Preface to the Second Edition	xi
Preface to the First Edition	xv
Guide to the Main Mathematical Concepts and Their Application	xxv
Notation and Symbols	xxvii
1 Introduction	1
1.1 The Image Society.	1
1.2 What Is a Digital Image?.	3
1.3 About Partial Differential Equations (PDEs).	5
1.4 Detailed Plan.	5
2 Mathematical Preliminaries	29
How to Read This Chapter.	29
2.1 The Direct Method in the Calculus of Variations.	30
2.1.1 Topologies on Banach Spaces.	30
2.1.2 Convexity and Lower Semicontinuity.	32
2.1.3 Relaxation.	37
2.1.4 About T-Convergence.	40
2.2 The Space of Functions of Bounded Variation.	42

2.2.1	Basic Definitions on Measures.	43
2.2.2	Definition of $BV(\dot{U})$.	45
2.2.3	Properties of $BV(\dot{U})$.	46
2.2.4	Convex Functions of Measures.	50
2.3	Viscosity Solutions in PDEs.	50
2.3.1	About the Eikonal Equation.	50
2.3.2	Definition of Viscosity Solutions.	52
2.3.3	About the Existence.	54
2.3.4	About the Uniqueness.	55
2.4	Elements of Differential Geometry: Curvature.	57
2.4.1	Parametrized Curves.	58
2.4.2	Curves as Isolevel of a Function u .	58
2.4.3	Images as Surfaces.	59
2.5	Other Classical Results Used in This Book.	60
2.5.1	Inequalities.	60
2.5.2	Calculus Facts.	62
2.5.3	About Convolution and Smoothing.	62
2.5.4	Uniform Convergence.	63
2.5.5	Dominated Convergence Theorem.	64
2.5.6	Well-Posed Problems.	64
3	Image Restoration	65
	How to Read This Chapter.	65
3.1	Image Degradation.	66
3.2	The Energy Method.	68
3.2.1	An Inverse Problem.	68
3.2.2	Regularization of the Problem.	69
3.2.3	Existence and Uniqueness of a Solution for the Minimization Problem.	72
3.2.4	Toward the Numerical Approximation.	76
	The Projection Approach.	76
	The Half-Quadratic Minimization Approach . . .	79
3.2.5	Some Invariances and the Role of A .	87
3.2.6	Some Remarks on the Nonconvex Case.	90
3.3	PDE-Based Methods.	94
3.3.1	Smoothing PDEs.	95
	The Heat Equation.	95
	Nonlinear Diffusion.	98
	The Alvarez-Guichard-Lions-Morel Scale Space Theory.	107
	Weickert's Approach.	113
	Surface Based Approaches.	117
3.3.2	Smoothing-Enhancing PDEs.	121
	The Perona and Malik Model.	121

	Regularization of the Perona and Malik Model:	
	Catté et al.	123
3.3.3	Enhancing PDEs.	128
	The Osher and Rudin Shock Filters.	128
	A Case Study: Construction of a Solution by the Method of Characteristics.	130
	Comments on the Shock-Filter Equation.	134
3.3.4	Neighborhood Filters, Nonlocal Means Algorithm, and PDEs.	137
	Neighborhood Filters.	138
	How to Suppress the Staircase Effect?.	143
	Nonlocal Means Filter (NL-Means).	146
4	The Segmentation Problem	149
	How to Read This Chapter.	149
4.1	Definition and Objectives.	150
4.2	The Mumford and Shah Functional.	153
	4.2.1 A Minimization Problem.	153
	4.2.2 The Mathematical Framework for the Existence of a Solution.	154
	4.2.3 Regularity of the Edge Set.	162
	4.2.4 Approximations of the Mumford and Shah Functional.	166
	4.2.5 Experimental Results.	171
4.3	Geodesic Active Contours and the Level-Set Method.	173
	4.3.1 The Kass-Witkin-Terzopoulos model.	173
	4.3.2 The Geodesic Active Contours Model.	175
	4.3.3 The Level-Set Method.	182
	4.3.4 The Reinitialization Equation.	194
	Characterization of the Distance Function	195
	Existence and Uniqueness.	198
	4.3.5 Experimental Results.	206
	4.3.6 About Some Recent Advances.	208
	Global Stopping Criterion.	208
	Toward More General Shape Representation	211
5	Other Challenging Applications	213
	How to Read This Chapter.	213
5.1	Reinventing Some Image Parts by Inpainting	215
	5.1.1 Introduction.	215
	5.1.2 Variational Models.	216
	The Masnou and Morel Approach.	216
	The Ballester et al. Approach.	218
	The Chan and Shen Total Variation Minimization Approach.	220

5.1.3	PDE-Based Approaches.	222
	The Bertalmio et al. Approach.	223
	The Chan and Shen Curvature-Driven Diffusion Approach	224
5.1.4	Discussion.	225
5.2	Decomposing an Image into Geometry and Texture	228
5.2.1	Introduction.	228
5.2.2	A Space for Modeling Oscillating Patterns	229
5.2.3	Meyer's Model.	232
5.2.4	An Algorithm to Solve Meyer's Model.	233
	Prior Numerical Contribution.	234
	The Aujol et al. Approach.	234
	Study of the Asymptotic Case.	241
	Back to Meyer's Model.	242
5.2.5	Experimental Results.	245
	Denoising Capabilities.	245
	Dealing With Texture.	248
5.2.6	About Some Recent Advances.	248
5.3	Sequence Analysis.	249
5.3.1	Introduction.	249
5.3.2	The Optical Flow: An Apparent Motion.	250
	The Optical Flow Constraint (OFC).	252
	Solving the Aperture Problem	253
	Overview of a Discontinuity-Preserving Variational Approach.	256
	Alternatives to the OFC.	260
5.3.3	Sequence Segmentation.	261
	Introduction.	261
	A Variational Formulation.	264
	Mathematical Study of the Time-Sampled Energy.	265
	Experiments.	269
5.3.4	Sequence Restoration.	271
	Principles of Video Inpainting.	276
	Total Variation (TV) Minimization Approach	277
	Motion Compensated (MC) Inpainting.	277
5.4	Image Classification.	281
5.4.1	Introduction.	281
5.4.2	A Level-Set Approach for Image Classification.	282
5.4.3	A Variational Model for Image Classification and Restoration.	290
5.5	Vector-Valued Images.	299
5.5.1	Introduction.	299
5.5.2	An Extended Notion of Gradient	300

5.5.3	The Energy Method.	300
5.5.4	PDE-Based Methods.	302
A	Introduction to Finite Difference Methods	307
	How to Read This Chapter.	307
A.1	Definitions and Theoretical Considerations Illustrated by the 1-D Parabolic Heat Equation.	308
A.1.1	Getting Started.	308
A.1.2	Convergence.	311
A.1.3	The Lax Theorem.	313
A.1.4	Consistency.	313
A.1.5	Stability.	315
A.2	Hyperbolic Equations.	320
A.3	Difference Schemes in Image Analysis.	329
A.3.1	Getting Started.	329
A.3.2	Image Restoration by Energy Minimization	333
A.3.3	Image Enhancement by the Osher and Rudin Shock Filters	336
A.3.4	Curve Evolution with the Level-Set Method	338
	Mean Curvature Motion.	339
	Constant Speed Evolution.	340
	The Pure Advection Equation.	341
	Image Segmentation by the Geodesic Active Contour Model.	342
B	Experiment Yourself!	343
	How to Read This Chapter.	343
B.1	The Cimg Library.	344
B.2	What Is Available Online?.	344
	References	349
	Index	373