POLITECNICO
MILANO 1863

# Policy Gradient Algorithms for the Asset Allocation Problem

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# Animal Spirits

# The Sound of Silence

# Plan

1. Basics of Reinforcement Learning

2. Policy Gradient Algorithms

3. Asset Allocation with Transaction Costs

4. Conclusions

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# The Reinforcement Learning Framework

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Environment

$\mathcal{P}$, $\mathcal{R}$

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Agent

$\pi$

Environment

$\mathcal{P}, \mathcal{R}$

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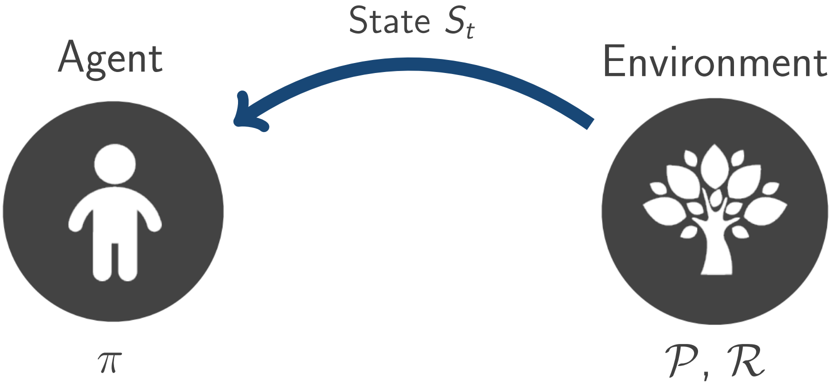

State $S_t$

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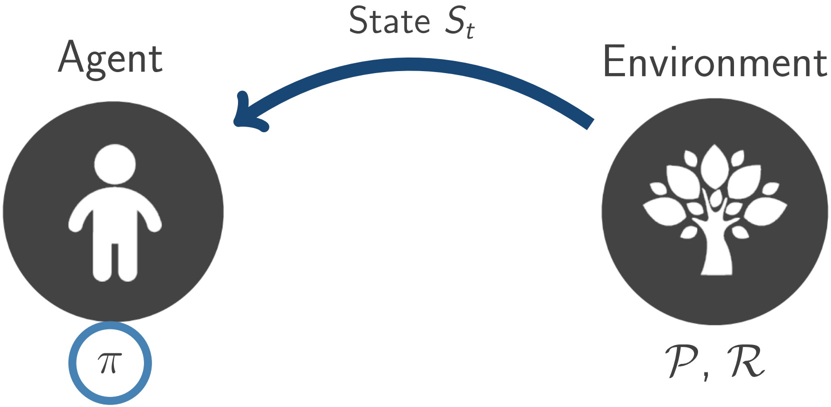

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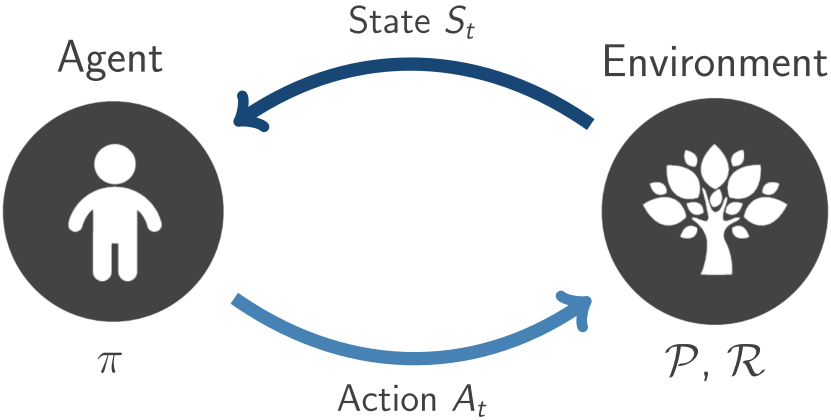

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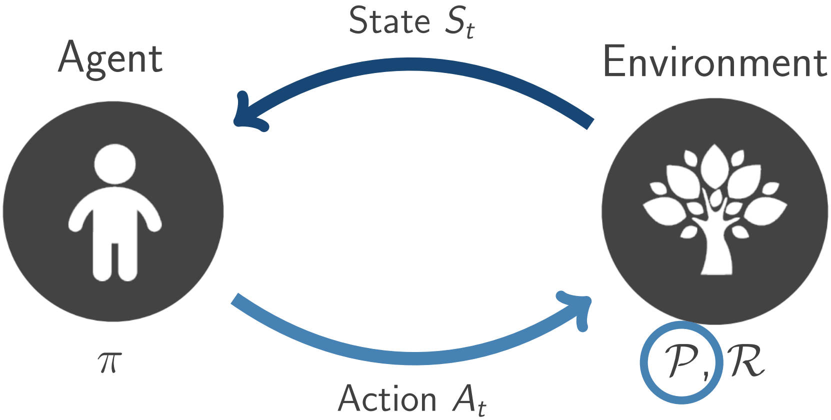

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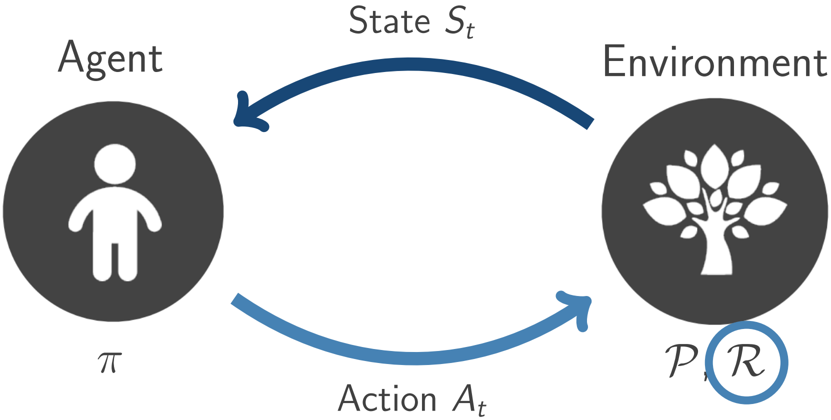

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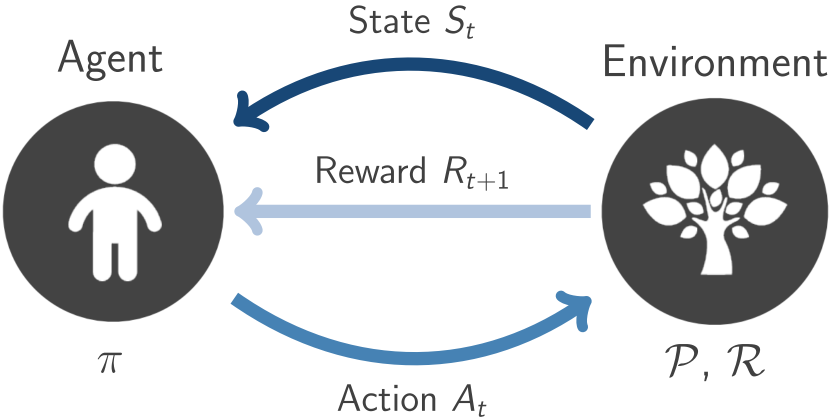

# Mathematical Formulation

**State-value function**

$$V_\pi(s) = \mathbb{E}_\pi\left[\sum_{t=0}^{\infty} \gamma^t R_{t+1} \middle| S_0 = s\right]$$

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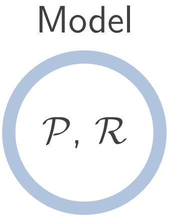
**Optimal policy**

$$\pi_* \text{ s.t. } V_{\pi_*}(s) = V_*(s),\ \forall s \in \mathbb{S}$$

# Taxonomy of RL Algorithms



Model

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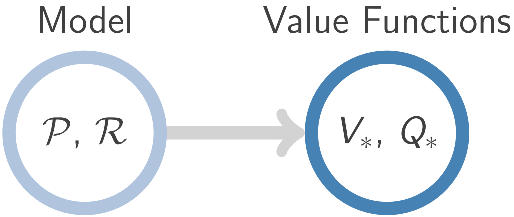

Model

Value Functions

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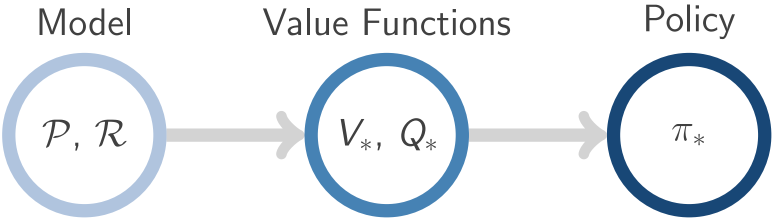

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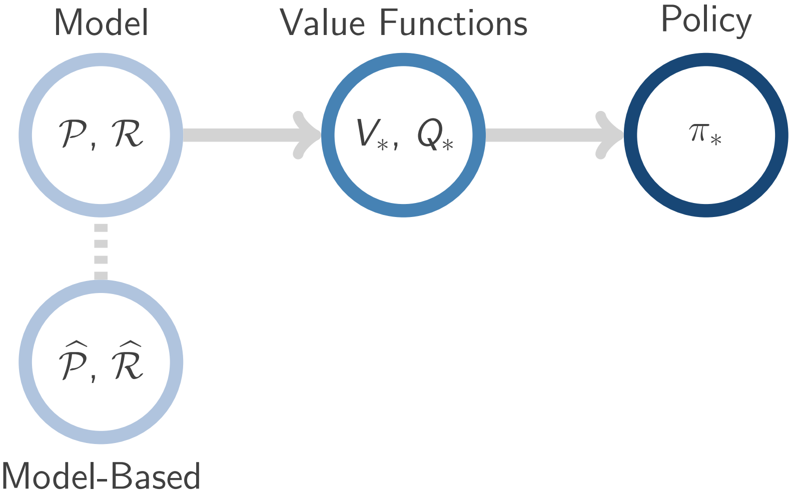

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Model-Based

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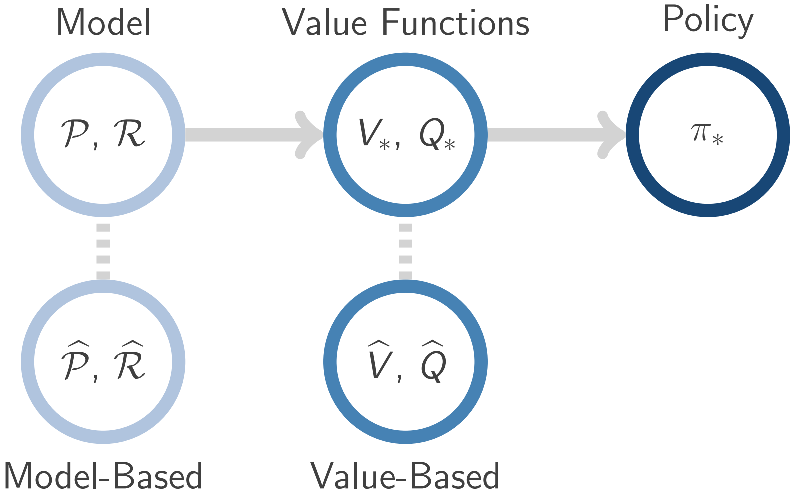

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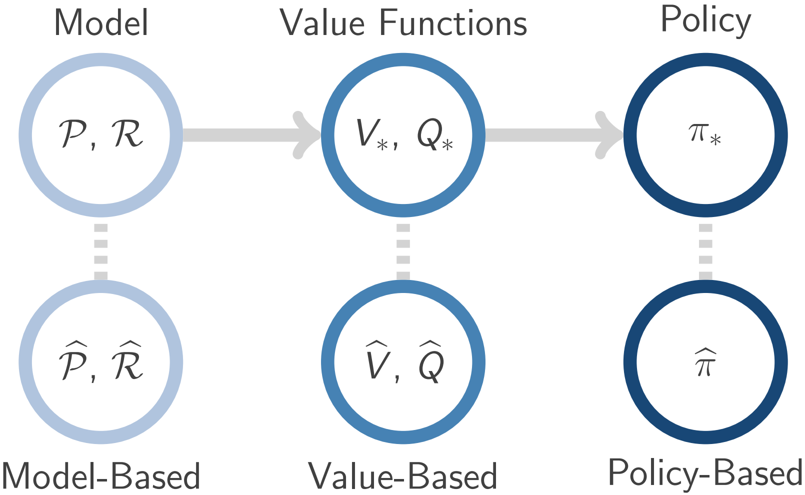

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3. Which is solved numerically via gradient descent

$$\theta_{k+1} = \theta_k + \alpha_k \, \nabla_\theta J\left(\theta_k\right)$$

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# The Keystone of Policy Gradient Algorithms

## Policy Gradient Theorem

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For an episodic environment, the policy gradient can be approximated via Monte-Carlo

$$\nabla_\theta J(\theta_k) \approx \frac{1}{M}\sum_{m=0}^{M}\sum_{u=0}^{T^{(m)}-1}\nabla_\theta \log \pi_{\theta_k}\left(s_u^{(m)}, a_u^{(m)}\right)\sum_{v\geq u}^{T^{(m)}-1}\gamma^{v-u} r_{v+1}^{(m)}$$

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However, this estimate is characterized by a large variance. Possible improvements:

1. Optimal baseline
2. Actor-critic methods
3. Natural gradient

# Policy Gradient with Parameter-Based Exploration (PGPE)

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More formally, the update scheme becomes

$$\xi_{k+1} = \xi_k + \alpha_k \nabla_\xi J(\xi_k)$$

where the policy gradient is given by

## Parameter-Based Policy Gradient Theorem

$$\nabla_\xi J(\xi) = \mathbb{E}_{\substack{S \sim d^\xi \\ \theta \sim p_\xi}} \left[ \nabla_\xi \log p_\xi(\theta) Q_\xi \left( S, F_\theta(S) \right) \right]$$

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# Problem Formulation

## Investor’s Goal

How to dynamically invest the available capital in a portfolio of different assets in order to maximize the expected total return or another relevant performance measure.

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**Rewards**: portfolio log-return with transaction costs

$$R_{t+1} = \log\left\{1 + \sum_{i=0}^{I}\left[\, a_t^i X_{t+1}^i \;-\; \delta_i \left|a_t^i - \widetilde{a}_t^i\right| \;-\; \delta_s \left(a_t^i\right)^- \,\right] - \;\delta_f \mathbf{1}_{a_t \neq \tilde{a}_{t-1}} \right\}$$

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**Actions**: Portfolio weights

$$\{a_t^i\}_{i=0}^I \quad \text{s.t.} \quad \sum_{i=0}^{I} a_t^i = 1 \quad \forall t \in \{0, 1, 2, \ldots\}$$

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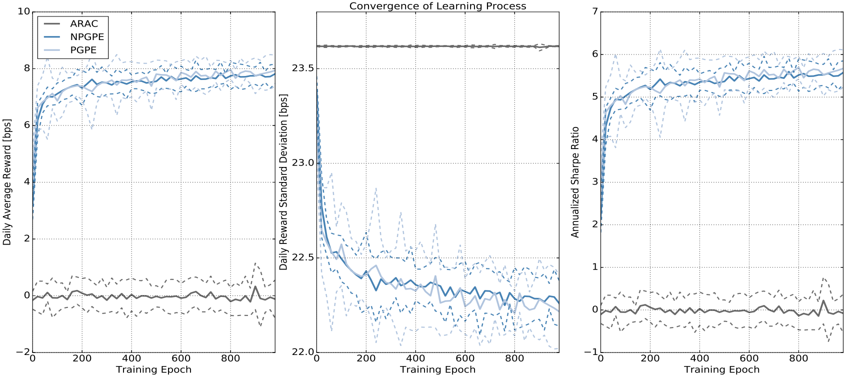
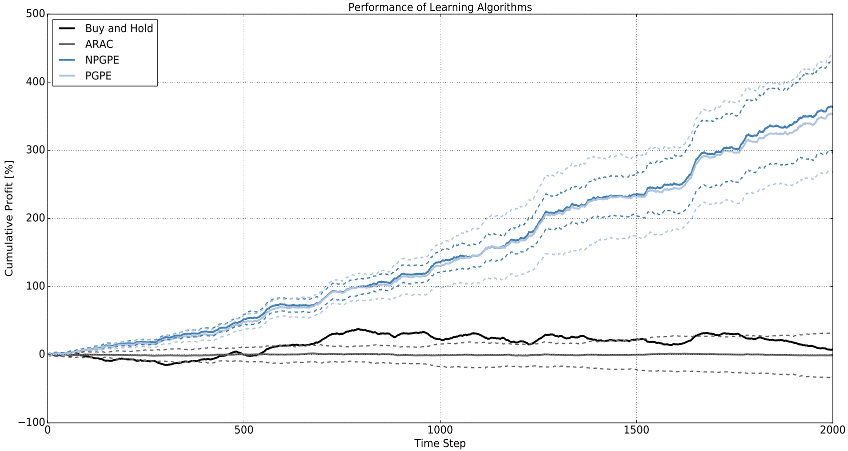
**States**: assets past returns and current allocation

$$S_t = \{X, X_t, X_{t-1}, \ldots, X_{t-P}, \tilde{a}_t\}$$

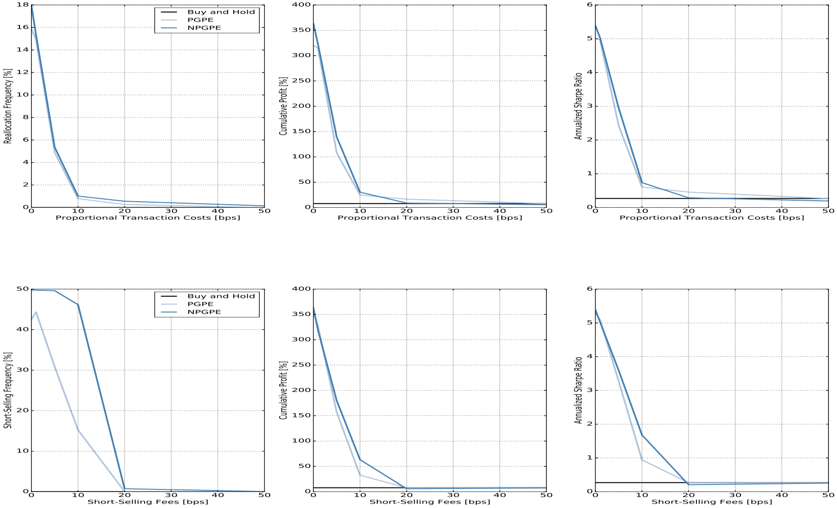
# Synthetic Asset: Convergence



# Synthetic Asset: Backtest Performance



# Synthetic Asset: Impact of Transaction Costs



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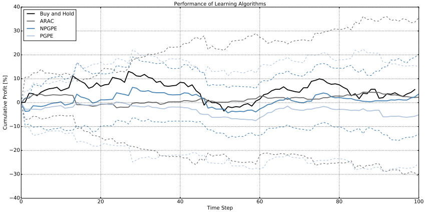
## Insuccess on Historical Data

Successfully applying these RL algorithms to historical data is much more challenging

1. Fail to converge
2. The strategies learned are not profitable



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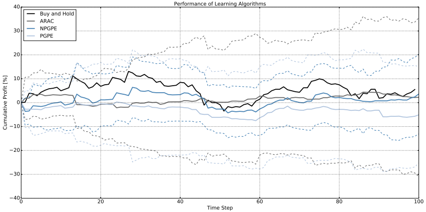

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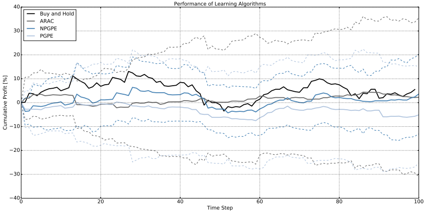

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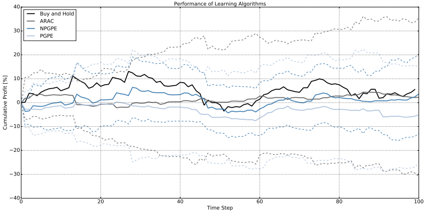

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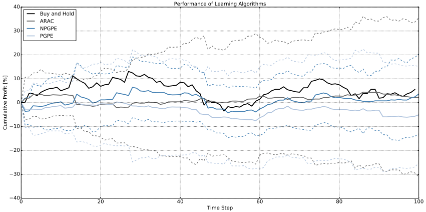

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4. **Non-stationarity of financial time-series**: a signal needs to be persistent

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# Conclusions

## What Has Been Done

1. In-depth bibliographical study of state-of-the-art policy gradient algorithms
2. Innovative contributions to the policy gradient literature
3. Applied these techniques to find a profitable long-short trading strategy

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## Research Directions

1. Improve the algorithms performance on historical data
2. Develop more complex features for the trading strategy
3. Combine policy gradient algorithms with state-of-the-art deep learning techniques
4. RL framework is versatile and can be applied to other financial decision problems

Thank you for your attention!

# References

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# Markov Decision Processes

## Reinforcement Learning

General class of algorithms that allow an agent to learn how to behave in a stochastic and possibly unknown environment by trial-and-error.

## Markov Decision Process (MDP)

stochastic dynamical system specified by $< \mathbb{S}, \mathbb{A}, \mathcal{P}, \mathcal{R}, \gamma >$

1. $(\mathbb{S}, \mathcal{S})$ is a measurable state space
2. $(\mathbb{A}, \mathcal{A})$ is a measurable action space
3. $\mathcal{P} : \mathbb{S} \times \mathbb{A} \times \mathcal{S} \to \mathbb{R}$ is a Markov transition kernel
4. $\mathcal{R} : \mathbb{S} \times \mathbb{A} \to \mathbb{R}$ is a reward function
5. $0 < \gamma < 1$ is the discount factor.

# Policy Gradient Theorem: Statement and Proof

## Policy Gradient Theorem

Let $\pi_\theta$ be a differentiable policy. For the gradient of the average reward is

$$\nabla_\theta \rho(\theta) = \mathbb{E}_{\substack{S\sim d^\theta \\ A\sim\pi_\theta}} \left[\nabla_\theta \log \pi_\theta(S,A) Q_\theta(S,A)\right]$$

where $d^\theta$ is the stationary distribution of the Markov chain induced by $\pi_\theta$.

**Proof**

$$\nabla_\theta V_\theta(s) = \nabla_\theta \int_{\mathbb{A}} \pi_\theta(s,a) Q_\theta(s,a) da = \int_{\mathbb{A}} \left[\nabla_\theta \pi_\theta(s,a) Q_\theta(s,a) + \pi_\theta(s,a) \nabla_\theta Q_\theta(s,a)\right] da$$

$$\nabla_\theta Q_\theta(s,a) = \nabla_\theta \left[\mathcal{R}(s,a) - \rho_\theta + \int_{\mathbb{S}} \mathcal{P}(s,a,s') V_\theta(s') ds'\right] = -\nabla_\theta \rho_\theta + \int_{\mathbb{S}} \mathcal{P}(s,a,s') \nabla_\theta V_\theta(s') ds'$$

$$\nabla_\theta V_\theta(s) = \int_{\mathbb{A}} \nabla_\theta \pi_\theta(s,a) Q_\theta(s,a) da - \nabla_\theta \rho_\theta + \int_{\mathbb{A}} \pi_\theta(s,a) \int_{\mathbb{S}} \mathcal{P}(s,a,s') \nabla_\theta V_\theta(s') ds'$$

$$\int_{\mathbb{S}} d^\theta(s) \int_{\mathbb{A}} \pi(s,a) \int_{\mathbb{S}} \mathcal{P}(s,a,s') \nabla_\theta V(s') ds' dads = \int_{\mathbb{S}} d^\theta(s) \nabla_\theta V_\theta(s) ds$$

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# Monte-Carlo Policy Gradient: Pseudocode

```
Input: Stochastic policy $\pi_\theta$, Initial parameters $\theta_0$, learning rate $\{\alpha_k\}$
Output: Approximation of the optimal policy $\pi_{\theta^*} \approx \pi_*$
repeat
    Sample $M$ trajectories $h^{(m)} = \{(s_t^{(m)}, a_t^{(m)}, r_{t+1}^{(m)})\}_{t=0}^{T^{(m)}}$ under policy $\pi_{\theta_k}$
    Approximate policy gradient
        $\nabla_\theta J(\theta_k) \approx \frac{1}{M} \sum_{m=0}^{M} \sum_{u=0}^{T^{(m)}-1} \nabla_\theta \log \pi_{\theta_k} \left( s_u^{(m)}, a_u^{(m)} \right) \sum_{v \geq u}^{T^{(m)}-1} \gamma^{v-u} r_{v+1}^{(m)}$
    Update parameters using gradient ascent $\theta_{k+1} = \theta_k + \alpha_k \nabla_\theta J(\theta_k)$
    $k \leftarrow k + 1$
until converged
```

# Episodic PGPE Algorithm: Pseudocode

```
Input: Controller $F_\theta$, hyper-distribution $p_\xi$, initial guess $\xi_0$, learning rate $\{\alpha_k\}$
Output: Approximation of the optimal policy $F_{\xi^*} \approx \pi_*$
repeat
    for $m = 1, \dots, M$ do
        Sample controller parameters $\theta^{(m)} \sim p_{\xi_k}$
        Sample trajectory $h^{(m)} = \{(s_t^{(m)}, a_t^{(m)}, r_{t+1}^{(m)})\}_{t=0}^{T^{(m)}}$ under policy $F_{\theta^{(m)}}$
    end for
    Approximate policy gradient
        $$\nabla_\xi J(\xi_k) \approx \frac{1}{M} \sum_{m=1}^{M} \nabla_\xi \log p_\xi \left(\theta^{(m)}\right) \left[G\left(h^{(m)}\right) - b\right]$$
    Update hyperparameters using gradient ascent $\xi_{k+1} = \xi_k + \alpha_k \nabla_\xi J(\xi_k)$
    $k \leftarrow k + 1$
until converged
```

# Natural PGPE Algorithm: Pseudocode

**Input:** Controller $F_\theta$, hyper-distribution $p_\xi$, initial guess $\xi_0$, learning rate $\{\alpha_k\}$

**Output:** Approximation of the optimal policy $F_{\xi^*} \approx \pi_*$

```
repeat
    Observe current state $s_k$
    Draw $\zeta_k \sim \mathcal{N}(0, I_n)$
    Compute controller parameters $\theta_k = \mu_k + \Gamma^T \zeta_k$
    Perform action $a_k = F_{\theta_k}(s_k)$ and receive reward $r_{k+1}$
    Update average reward estimate $\widehat{\rho}_{k+1} = \widehat{\rho}_k + \alpha_k(r_{k+1} - \widehat{\rho}_k)$
    Compute natural policy gradients
        $$\widetilde{\nabla}_\mu \log p_{\xi_k}(\theta_k) = \theta_k - \mu_k \qquad \widetilde{\nabla}_\Gamma \log p_{\xi_k}(\theta_k) = \left(\operatorname{triu}(\zeta_k\zeta_k^T) - \frac{1}{2}\operatorname{diag}(\zeta_k\zeta_k^T) - \frac{1}{2}I\right)\Gamma$$
    Update eligibility trace $e_k = \lambda e_{k-1} + \nabla_\xi \log p_{\xi_k}(\theta_k)$
    Update hyper-parameters $\xi_{k+1} = \xi_k + \alpha_k(r_{k+1} - \widehat{\rho}_k)e_k$
    $k \leftarrow k + 1$
until converged
```

# Experiment on Synthetic Asset

The synthetic asset price is given by

$$Z_t = \exp\left(\frac{z_t}{\max_t z_t - \min_t z_t}\right)$$

where $\{z_t\}$ is a random walk with autoregressive trend $\{\beta_t\}$

$$\begin{aligned} z_t &= z_{t-1} + \beta_{t-1} + \kappa\epsilon_t \\ \beta_t &= \alpha\beta_{t-1} + \nu_t \end{aligned}$$

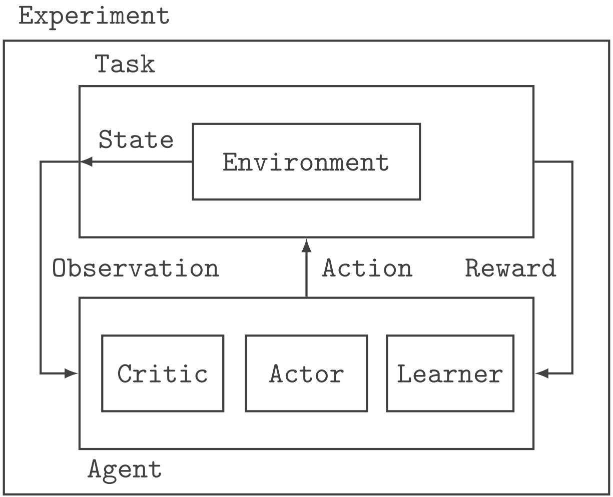
The policy used for the PGPE and the NPGPE algorithms is

$$F_\theta(s) = \text{sign}(\theta \cdot s)$$

where

$$\theta \sim \mathcal{N}(\mu, \text{diag}(\sigma))$$

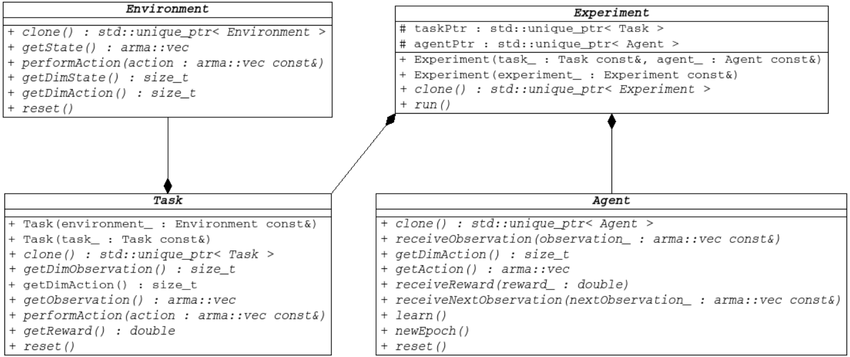
# PyBrain's Architecture for a RL Problem



# Agent-Environment Interaction in C++

## Adapting PyBrain's Architecture

1. Defined standard interfaces via pure abstract classes
2. Achieved modularity via polymorphic composition



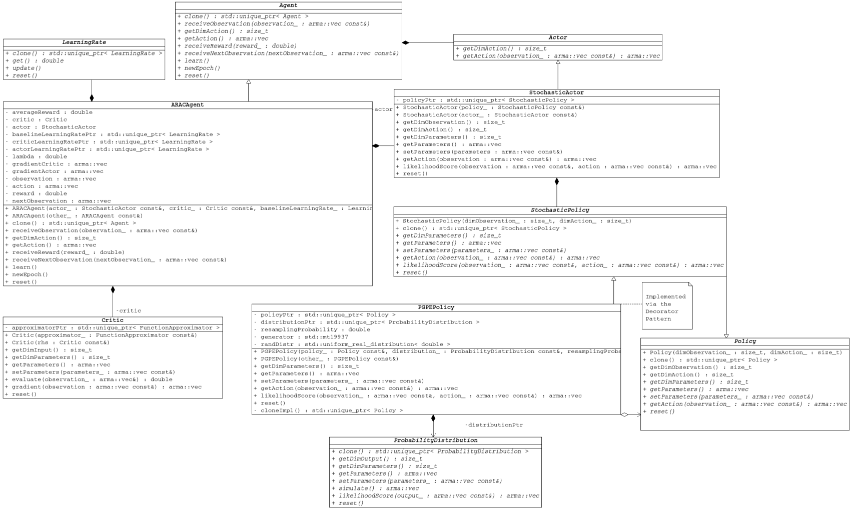
```
Environment
+ clone() : std::unique_ptr< Environment >
+ getState() : arma::vec
+ performAction(action : arma::vec const&)
+ getDimState() : size_t
+ getDimAction() : size_t
+ reset()
```

```
Experiment
# taskPtr : std::unique_ptr< Task >
# agentPtr : std::unique_ptr< Agent >
+ Experiment(task_ : Task const&, agent_ : Agent const&)
+ Experiment(experiment_ : Experiment const&)
+ clone() : std::unique_ptr< Experiment >
+ run()
```

```
Task
+ Task(environment_ : Environment const&)
+ Task(task_ : Task const&)
+ clone() : std::unique_ptr< Task >
+ getDimObservation() : size_t
+ getDimAction() : size_t
+ getObservation() : arma::vec
+ performAction(action : arma::vec const&)
+ getReward() : double
+ reset()
```

```
Agent
+ clone() : std::unique_ptr< Agent >
+ receiveObservation(observation_ : arma::vec const&)
+ getDimAction() : size_t
+ getAction() : arma::vec
+ receiveReward(reward_ : double)
+ receiveNextObservation(nextObservation_ : arma::vec const&)
+ learn()
+ newEpoch()
+ reset()
```

# Agent’s Architecture in C++



```
Agent
+ clone() : std::unique_ptr< Agent >
+ receiveObservation(observation_ : arma::vec const&)
+ getDimAction() : size_t
+ getAction() : arma::vec
+ receiveReward(reward_ : double)
+ receiveNextObservation(nextObservation_ : arma::vec const&)
+ learn()
+ newEpoch()
+ reset()
```

```
LearningRate
+ clone() : std::unique_ptr< LearningRate >
+ get() : double
+ update()
+ reset()
```

```
Actor
+ getDimAction() : size_t
+ getAction(observation_ : arma::vec const&) : arma::vec
```

```
ARACAgent
- averageReward : double
- critic : Critic
- actor : StochasticActor
- baselineLearningRatePtr : std::unique_ptr< LearningRate >
- criticLearningRatePtr : std::unique_ptr< LearningRate >
- actorLearningRatePtr : std::unique_ptr< LearningRate >
- lambda : double
- gradientCritic : arma::vec
- gradientActor : arma::vec
- observation : arma::vec
- action : arma::vec
- reward : double
- nextObservation : arma::vec
+ ARACAgent(actor_ : StochasticActor const&, critic_ : Critic const&, baselineLearningRate_ : Learnin
+ ARACAgent(other_ : ARACAgent const&)
+ clone() : std::unique_ptr< Agent >
+ receiveObservation(observation_ : arma::vec const&)
+ getDimAction() : size_t
+ getAction() : arma::vec
+ receiveReward(reward_ : double)
+ receiveNextObservation(nextObservation_ : arma::vec const&)
+ learn()
+ newEpoch()
+ reset()
```

```
StochasticActor
- policyPtr : std::unique_ptr< StochasticPolicy >
+ StochasticActor(policy_ : StochasticPolicy const&)
+ StochasticActor(actor_ : StochasticActor const&)
+ getDimObservation() : size_t
+ getDimAction() : size_t
+ getDimParameters() : size_t
+ getParameters() : arma::vec
+ setParameters(parameters : arma::vec const&)
+ getAction(observation : arma::vec const&) : arma::vec
+ likelihoodScore(observation : arma::vec const&, action : arma::vec const&) : arma::vec
+ reset()
```

```
StochasticPolicy
+ StochasticPolicy(dimObservation_ : size_t, dimAction_ : size_t)
+ clone() : std::unique_ptr< StochasticPolicy >
+ getDimParameters() : size_t
+ getParameters() : arma::vec
+ setParameters(parameters_ : arma::vec const&)
+ getAction(observation_ : arma::vec const&) : arma::vec
+ likelihoodScore(observation_ : arma::vec const&, action_ : arma::vec const&) : arma::vec
+ reset()
```

```
Critic
- approximatorPtr : std::unique_ptr< FunctionApproximator >
+ Critic(approximator_ : FunctionApproximator const&)
+ Critic(rhs : Critic const&)
+ getDimInput() : size_t
+ getDimParameters() : size_t
+ getParameters() : arma::vec
+ setParameters(parameters_ : arma::vec const&)
+ evaluate(observation_ : arma::vec&) : double
+ gradient(observation : arma::vec const&) : arma::vec
+ reset()
```

```
PGPEPolicy
- policyPtr : std::unique_ptr< Policy >
- distributionPtr : std::unique_ptr< ProbabilityDistribution >
- resamplingProbability : double
- generator : std::mt19937
- randDistr : std::uniform_real_distribution< double >
+ PGPEPolicy(policy_ : Policy const&, distribution_ : ProbabilityDistribution const&, resamplingProba
+ PGPEPolicy(other_ : PGPEPolicy const&)
+ getDimParameters() : size_t
+ getParameters() : arma::vec
+ setParameters(parameters_ : arma::vec const&)
+ getAction(observation_ : arma::vec const&) : arma::vec
+ likelihoodScore(observation_ : arma::vec const&, action_ : arma::vec const&) : arma::vec
+ reset()
- cloneImpl() : std::unique_ptr< Policy >
```

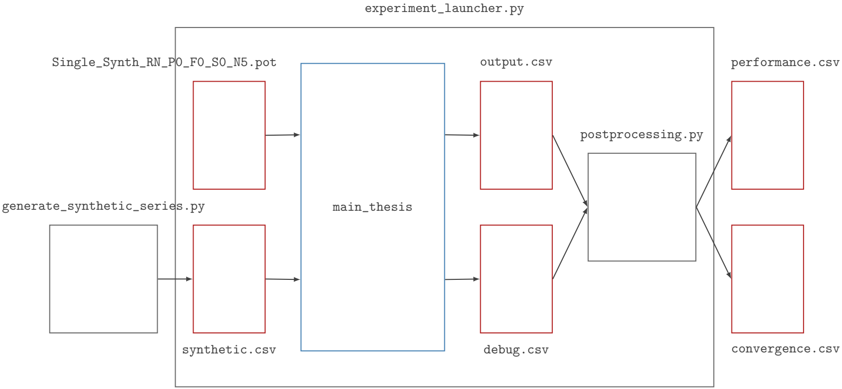
```
Policy
+ Policy(dimObservation_ : size_t, dimAction_ : size_t)
+ clone() : std::unique_ptr< Policy >
+ getDimObservation() : size_t
+ getDimAction() : size_t
+ getDimParameters() : size_t
+ getParameters() : arma::vec
+ setParameters(parameters_ : arma::vec const&)
+ getAction(observation_ : arma::vec const&) : arma::vec
+ reset()
```

```
ProbabilityDistribution
+ clone() : std::unique_ptr< ProbabilityDistribution >
+ getDimOutput() : size_t
+ getDimParameters() : size_t
+ getParameters() : arma::vec
+ setParameters(parameters_ : arma::vec const&)
+ simulate() : arma::vec
+ likelihoodScore(output_ : arma::vec const&) : arma::vec
+ reset()
```

# Execution Pipeline

`experiment_launcher.py`

1. Program execution is handled by a Python script
2. Responsible for analyzing the output of the C++ engine


experiment_launcher.py
Single_Synth_RN_P0_F0_S0_N5.pot
output.csv
performance.csv
postprocessing.py
generate_synthetic_series.py
main_thesis
synthetic.csv
debug.csv
convergence.csv