# CORDIC-Based Trigonometric Functions

CH3 Computer Arithmetic
Programming Assignment

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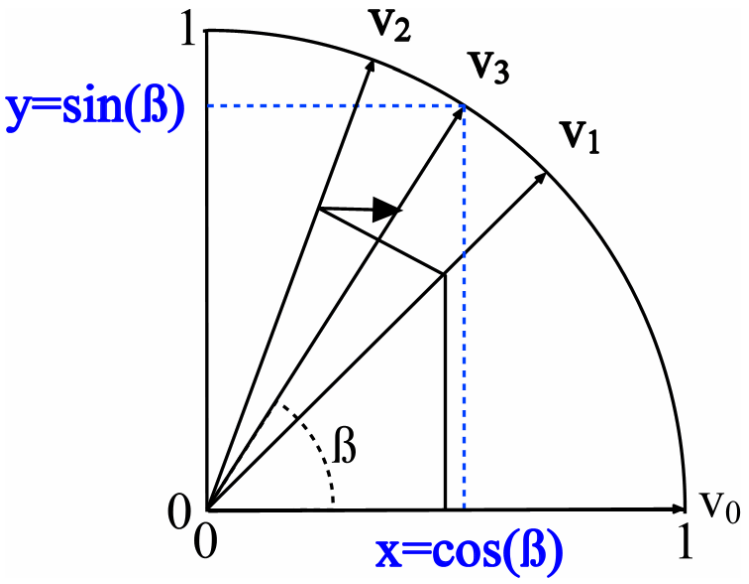
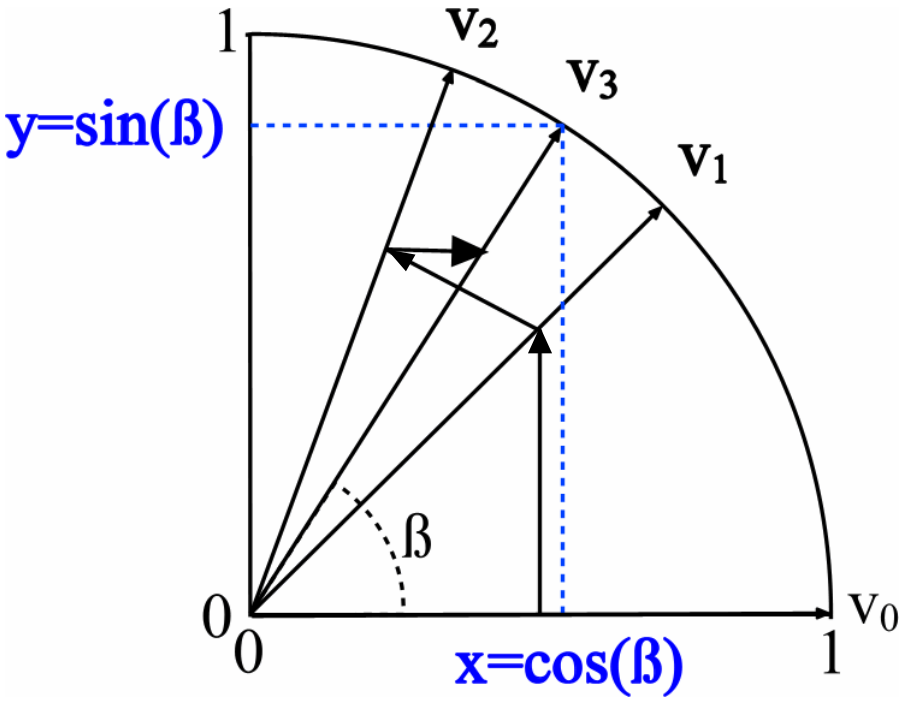


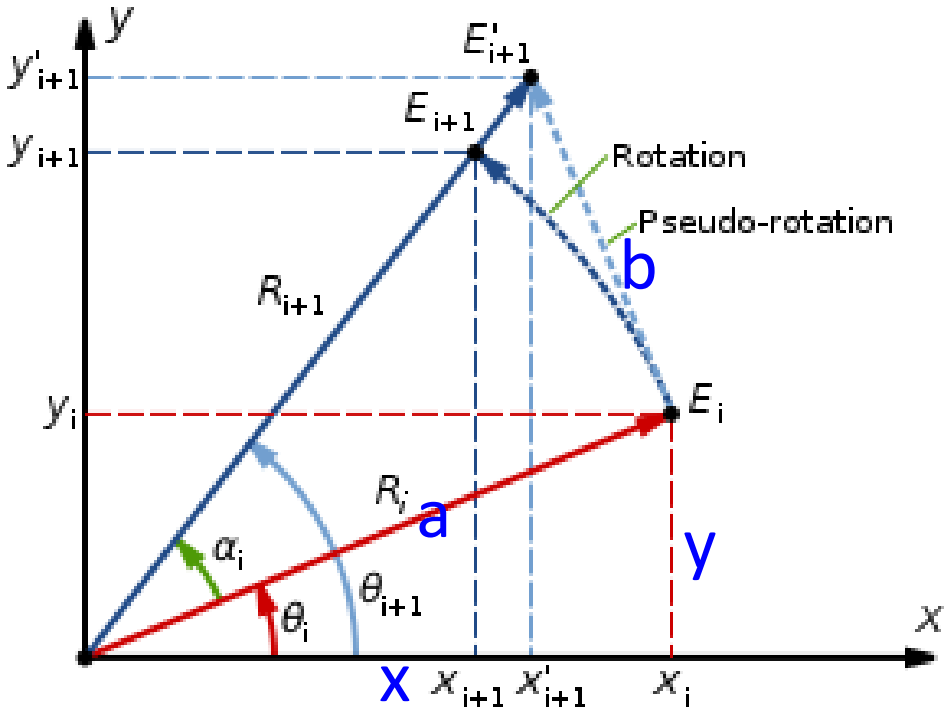
# Basic Concept

- CORDIC is a binary approximation algorithm to calculate sin(β) and cos(β) using only integer arithmetic



# Background

- (**x** + **y** $i$) × (**a** + **b** $i$) = (ax - by) + (bx + ay) $i$
  - Rotate the point by $\tan^{-1}(\frac{b}{a})$
  - Increase the distance by $\sqrt{\left(1+\left(\frac{b}{a}\right)^2\right)}$



# CORDIC

- a = 1
- b = $2^{-k}$
- Only shift and addition
  - (**x** + **y** $i$) × (**a** + **b** $i$)
    = (ax - by) + (bx + ay) $i$
    = (**x** - (y>>**k**)) + ((**x**>>k) + **y**)
- $\tan^{-1}$(b/a) is precomputed and stored in a table

| **a** | **b** | **$\tan^{-1}$(b/a)** |
|---|---|---|
| 1 | 1 | 45° |
| 1 | 0.5 | 26.5651…° |
| 1 | $2^{-2}$ | 14.0362…° |
| 1 | … | … |
| 1 | $2^{-(N-1)}$ | … |

# Software Implementation

```
x = initial_x;
y = 0;
N = # iterations, 20 (fixed);
th = the angle to compute;
for k = 0 … (N – 1)
   if th >= 0
     (x, y) *= (1, 2^-k);
     th = th - tan^-1(2^-k);
   else
     (x, y) *= (1, -2^-k);
     th = th + tan^-1(2^-k);

return x as cos(th)
   and y as sin(th);
```

| a | b | $\tan^{-1}(b/a)$ |
|---|---|---|
| 1 | 1 | 45° |
| 1 | 0.5 | 26.5651…° |
| 1 | $2^{-2}$ | 14.0362…° |
| 1 | … | … |
| 1 | $2^{-(N-1)}$ | … |

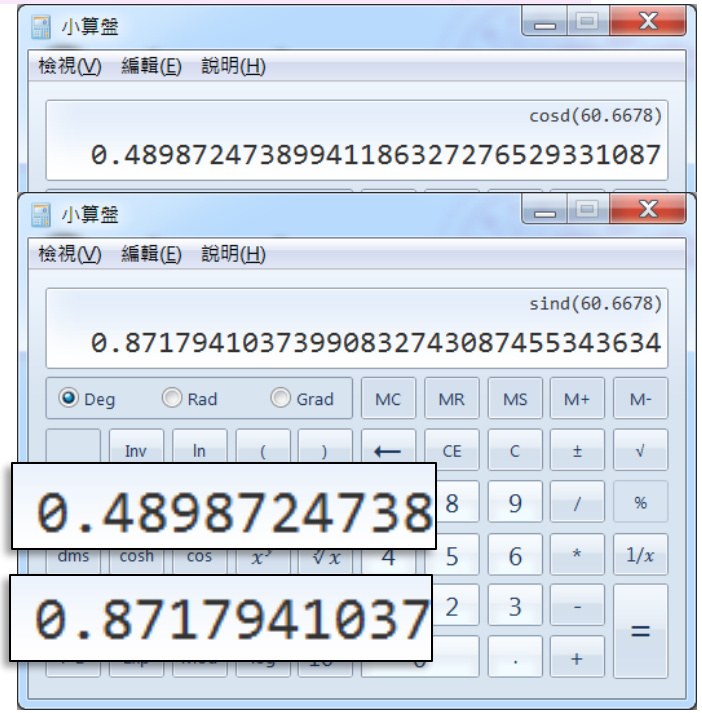
# Example Input and Output

- Input

```
5
606678
457006
-837565
-835975
395400
```

// number of inputs
// 60.6678° (scaled by 10,000)
// 45.7006° (scaled by 10,000)

note: for simplicity,
input < 90°
&& input > -90°
&& input != 0°




- Output

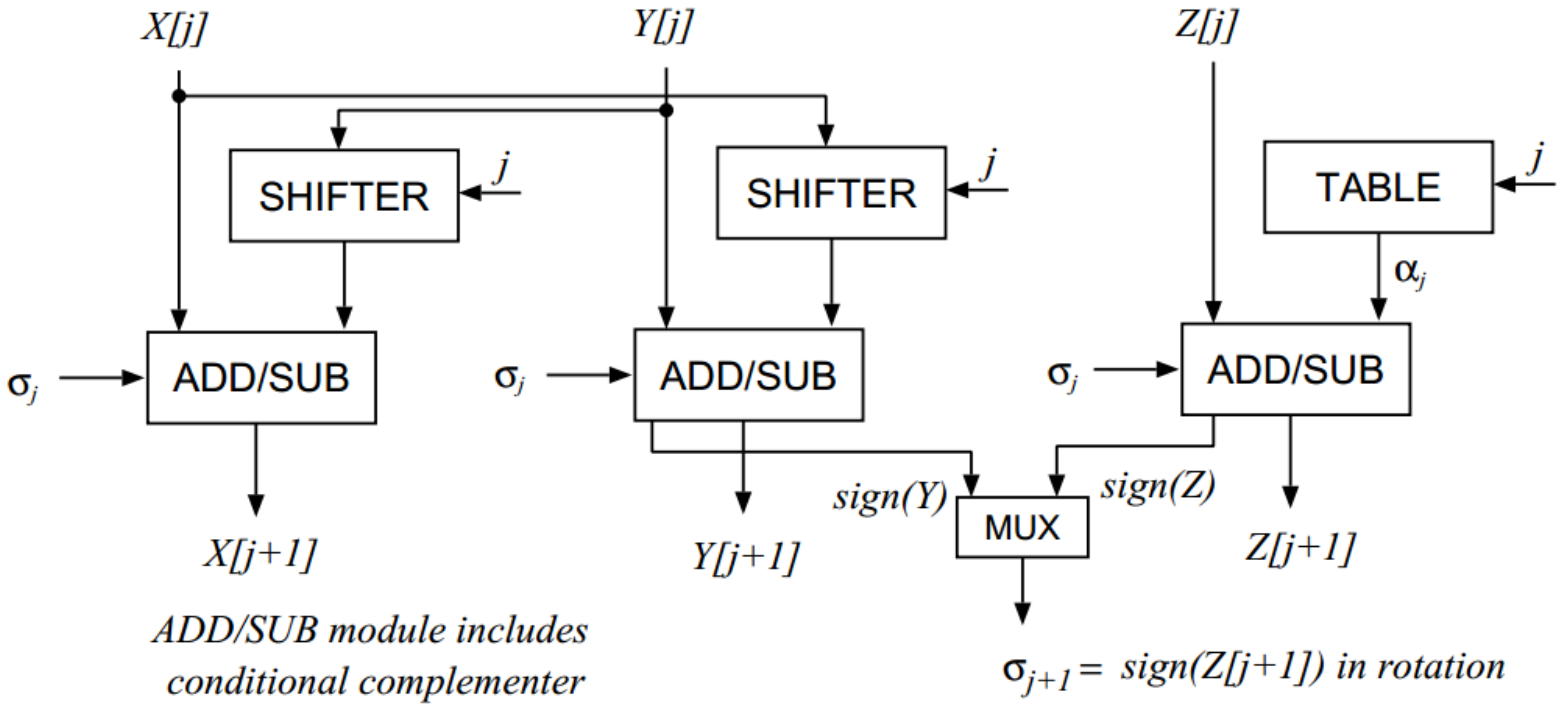
```
5
606678 4898750 8717926
457006 6984117 7156964
-837565 1087524 -9940686
-835975 1115127 -9937628
395400 7711808 6366160
```

60.6678°
cos(60.6678°) = 0.4898750 (scaled by 10,000,000)
sin(60.6678°) = 0.8717926 (scaled by 10,000,000)

# Parameters for this Assignment

| k | $\tan^{-1}(2^{-k})$ |
|---|---|
| 0 | 450000 |
| 1 | 265651 |
| 2 | 140362 |
| 3 | 71250 |
| 4 | 35763 |
| 5 | 17899 |
| 6 | 8952 |
| 7 | 4476 |
| 8 | 2238 |
| 9 | 1119 |

| k | $\tan^{-1}(2^{-k})$ |
|---|---|
| 10 | 560 |
| 11 | 280 |
| 12 | 140 |
| 13 | 70 |
| 14 | 35 |
| 15 | 17 |
| 16 | 9 |
| 17 | 4 |
| 18 | 2 |
| 19 | 1 |

initial_x = 6072529;

# Hardware (Just for Reference)