

$$f_1(u) = u^\beta, f_2(u) = \exp(-\alpha u)$$

$$\text{med}(D) - a > 0, a < \text{med}(D) < b$$

$$Q_a = f_1(\text{med}(D) - a)f_2(\text{med}(D) - a)$$

$$Q_b = f_1(b - \text{med}(D))f_2(b - \text{med}(D))$$

Defining optimal bounds separately:

$$\frac{\partial Q_a}{\partial a} = \frac{\partial}{\partial a}((\text{med}(D) - a)^\beta \exp(-\alpha(\text{med}(D) - a))) =$$

$$= -\beta(\text{med}(D) - a)^{\beta-1} \exp(-\alpha(\text{med}(D) - a)) + \alpha(\text{med}(D) - a)^\beta \exp(-\alpha(\text{med}(D) - a))$$

$$\frac{\partial Q_a}{\partial a} = 0, \Rightarrow -\beta + \alpha(\text{med}(D) - a) = 0$$

$$a_{opt} = \text{med}(D) - \frac{\beta}{\alpha}$$

$$\frac{\partial Q_b}{\partial b} = \frac{\partial}{\partial b}((b - \text{med}(D))^\beta \exp(-\alpha(b - \text{med}(D)))) =$$

$$= \beta(b - \text{med}(D))^{\beta-1} \exp(-\alpha(b - \text{med}(D))) - \alpha(b - \text{med}(D))^\beta \exp(-\alpha(b - \text{med}(D)))$$

$$\frac{\partial Q_b}{\partial b} = 0, \Rightarrow \beta - \alpha(b - \text{med}(D)) = 0$$

$$b_{opt} = \text{med}(D) + \frac{\beta}{\alpha}$$