

OS note 10927253 張易洋

2021-9-13 Basic of all fields in Computer Science

Data Structure

a way to store data (for easy use)

+

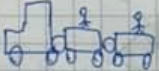
Algorithm 演算法


a finite set of instructions to complete a task

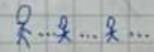
+

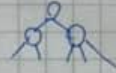
Coding

一些基本的資料結構

① 串列 List  串在一起

② 堆疊 Stack 

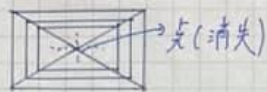
③ 佇列 Queue 

④ 樹 Tree 

# 递归 (recursion) v.s. 迴圈 (iteration)

↓  
优点: 简单明瞭、精简

缺点: 效率可能不好

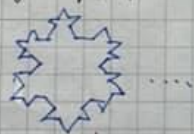


叙述: 很像两面镜子的画面, 把问题越来越小

— Linear recursion — Binary recursion

Work in:

- |                           |        |
|---------------------------|--------|
| □ Factorial               | 階乘     |
| □ Greatest Common Divisor | 最大公因数  |
| □ Search in Array         | 搜尋     |
| □ Fibonacci series        | 斐波那契数列 |
| □ Combinatorial numbers   | 组合数    |
| □ Towers of Hanoi         | 河内塔    |
| □ fractal                 | 碎形     |



• A binary search is recursive

→ Uses a divide and conquer strategy 分而治之  
↓  
分割 征服

(HW): 反向印字:

→ 字符串长度减1, 字元慢慢变少

```
void writeBackward (string s, int size)
{
    if (size > 0)
    {
        cout << s.substr(size-1, 1); // 输出最后一个字元
        writeBackward(s, size-1); // 递归呼叫
    }
}
```

Base case: 递归结束  
每次只取一个字符  
递归时, 印第一个字元

Box trace  
(箱型迴溯)

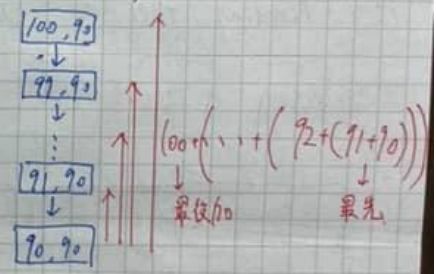
I  $S = \text{'cat'}$  size = 3  
 tac 输出最后一个字节  
 II  $S = \text{'cat'}$  size = 2  
 cat 输出第一个字节  
 III  $S = \text{'cat'}$  size = 1  
 tac 倒过来  
 IV  $S = \text{'cat'}$  size = 0

心得: I 较快, 其 't' 会先出来

Practice 1-1. Give two numbers  $a$  and  $b$ , where  $a > b$ . write a recursive function to compute the sum of all the integers from  $a$  to  $b$ , inclusively.

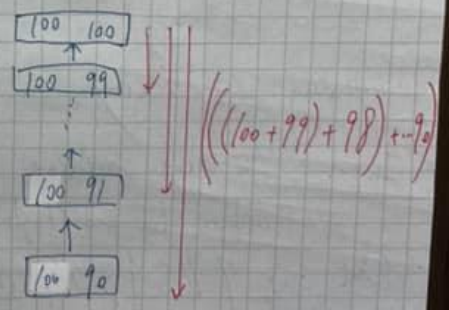
```

1. int sum(int a, int b)
{
  if (a > b)
    return sum(a-1, b) + a;
  else // a == b → 触底反弹
    return b;
}
  
```



```

2. int sum(int a, int b)
{
  if (b == a)
    return a;
  return sum(a, b+1) + b;
}
  
```

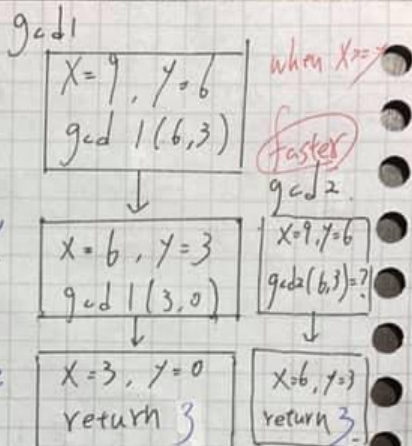




HW 找最大公因数

$$\begin{aligned} \text{gcd1}(x, y) &= x \quad \text{if } y=0 \\ &= \text{gcd1}(x, y \bmod x) \quad \text{if } y > x \\ &= \text{gcd1}(y, x \bmod y) \quad \text{otherwise } x, y \end{aligned}$$

$$\begin{aligned} \text{gcd2}(x, y) &= y \quad \text{if } x \bmod y = 0 \\ &= \text{gcd2}(y, x \bmod y) \quad \text{otherwise} \end{aligned}$$

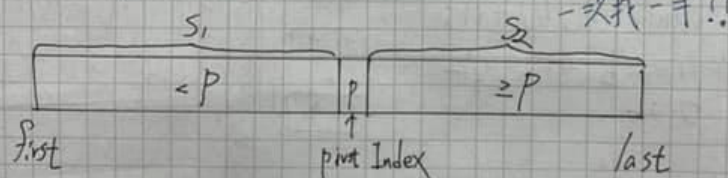


1-06. 分而治之的运算策略

Binary Search  $\rightarrow$  一分为二 (缩小问题)

1-07. Finding the  $k^{\text{th}}$  Smallest Item in an Array

Selecting a pivot item



Revering an Array

Input: An array anArray and nonnegative integer indices low and high. 0 1 2 3 4 5 6 7 8

Out: 8 7 6 5 4 3 2 1 0

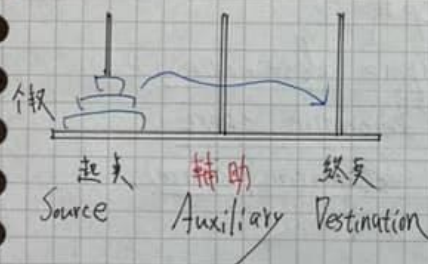
code: Algorithm ReverseArray (anArray, low, high) {  
if (low < high)

```

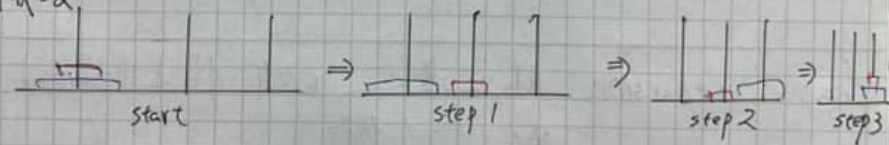
{ 交换位置 swap anArray[low] and anArray[high];
  ReverseArray(anArray, low+1, high-1);
  return; 递归调用
}

```

### Towers of Hanoi



个数=2



个数3



\* 把问题变小 递归

$$2^3 - 1 = 7 \Rightarrow 2^n - 1$$

$$2^4 - 1 = 15$$

3 + 1 + 3 + 1 + 7 = 15

3 + 1 + 3 + 1 + 7

心得: 河内塔可以想成 2倍 (n-1) 个盘子 + 1 次

Tower of Hanoi: Recursion:

void solveTowers (int <sup>个数</sup>count, char <sup>起始</sup>source, char <sup>终止</sup>destination, char <sup>辅助</sup>spare)

```
{
    if (count == 1)
    {
        cout << "Move top disk from pole" << source
            << " to pole" << destination << endl;
    } // if
    else
    {
        (n-1)个都移到辅助杆
        solveTowers (count-1, source, spare, destination);
        1最大盘移到终止
        solveTowers (1, source, destination, spare);
        (n-1)个从辅助杆移到终止
        solveTowers (count-1, spare, destination, source);
    } // else
}
```

Linear recursion 线性递归 (只有一条递归路径)

ex: 100 ~ 1 相加

Binary recursion = 元递归 (多次的递归呼叫)



## ● Multiplying Rabbits (Fibonacci)

- Base cases

- rabbit (2) rabbit (1)

□ Recursive Definition

$$\text{rabbit}(n) = 1, \text{ if } n \text{ is } 1 \text{ or } 2$$
$$= \text{rabbit}(n-1) + \text{rabbit}(n-2), \text{ if } n > 2$$

$$\text{Rabbit} * (7) = \text{Rabbit} * (6) + \text{Rabbit} * (5) + 1$$

● ★ 用空間換時間

## ~~✱~~ Better Fibonacci:

Use linear recursion instead

- Algorithm linear Fibonacci<sup>↑</sup>(k)

Input: A nonnegative integer  $k$

Output: Pair of Fibonacci numbers  $(F_k, F_{k-1})$

if  $k=1$  then

return (k, 0) // base cases:  $k=1 \rightarrow (F_1, F_0)$

else

else  
 $(i, j) = \text{linearFibonacci}(k-1) // (F_{k-1}, F_{k-2})$

return (i+j, i) // (F<sub>k</sub> = F<sub>k-1</sub> + F<sub>k-2</sub>, F<sub>k-1</sub>)

1-15 以遞迴求遊行隊伍排列數

## Organizing a Parade

Problem: How many ways can you organize a parade of length " $n$ "?

Subject: 樂隊不可緊跟樂隊

$$P(n) = F(n) + B(n)$$

$n$  個隊伍的排列數 =  $\underbrace{\text{事件=}}_{(1/2)} \text{花車殿後} + \underbrace{\text{事件=}}_{(1/2)} \text{樂隊殿後}$

$$F(n) = \underline{P(n-1)} \text{ 只要處理前 } (n-1) \text{ 個}$$

$$B(n) = \underline{F(n-1)} = P(n-2)$$

只能在下一個放花車，視同 "樂", "花", "..."  
( $n-2$ )

$\Rightarrow P(n) = P(n-1) + P(n-2)$ , as Fibonacci series.

base case

$$\begin{cases} P(1) = 2 & \text{樂, 花} \end{cases}$$

$$P(2) = 3 \quad \text{樂花 or 花樂}$$

$$P(3) = 2 + 3$$

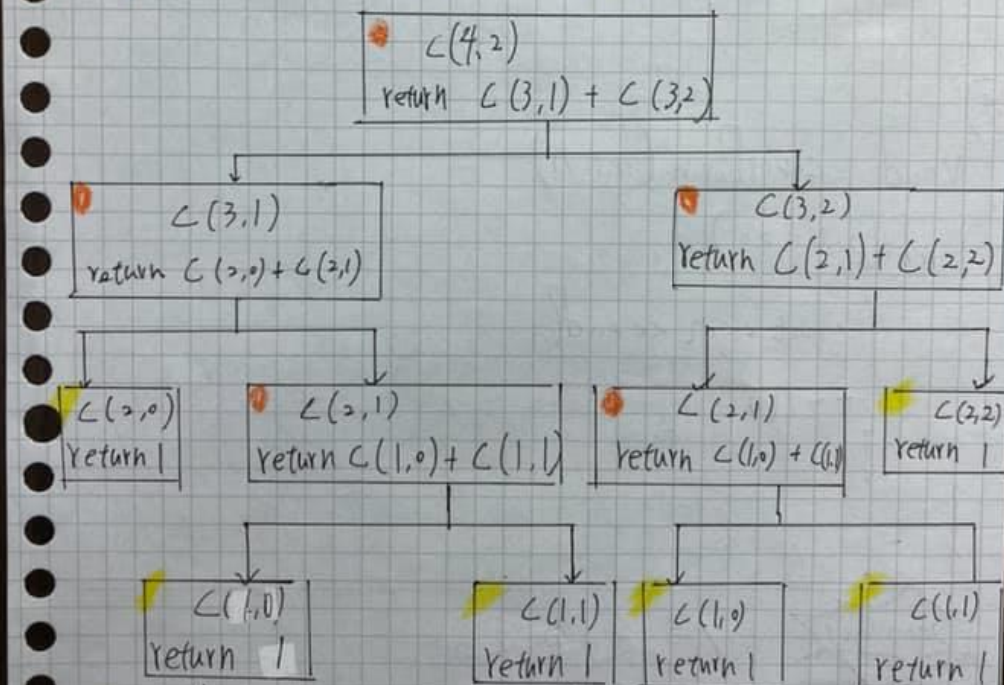


$C_n^k = ?$  例: 8行星

$C(n, k) = C(n-1, \underbrace{k-1}_{\text{必选地球}}) + C(n-1, \underbrace{k}_{\text{不选地球}})$

\* Base case 终止条件

- There is a group of everything  $C(k, k) = 1$
- There is a group of nothing  $C(n, 0) = 1$
- Special  $C(n, k) = 0$ , if  $n < k$



Leaf nodes : recursive calls to base cases 基例

Internal nodes : recursive calls to non-base cases 内部节点

\* Leaf - Internal = 1 数量差

可以把 tail Recursion 改成迴圈, 效果更好.  
(能)

例:

```
void countDown (int n)
{ if (n > 0)
  { cout << n << endl;
    countDown (n-1);
  }
}
```

改

```
void countDown (int n)
{ while (n > 0)
  { cout << n << endl;
    --n;
  }
}
```

Summary

1. 遞迴定義
2. 問題簡化
3. 終止條件
4. 保證終止

● ADT (抽象化). → Abstract Data Types

● ① Classes of object

● - Attribute : data members

● - Behavior : methods

● ② Principle of Object-Oriented Programming

● - Encapsulation (hide inner detail)

● • object combine data and operation

● - Inheritance (reused)

● • class can inherit properties from other classes.

● - Polymorphism

● • object can determine appropriate operations at execution time.

● ③ Operation contract

● - Purpose (What actions take place?)

● - Assumptions (What does the module assume?)

● - Input (What data is available to a module?)

● - Output (What effect does the module have on the data?)

● △ Begin the contract during analysis, finish during design

● △ Use to document code, particularly in header files.



## 物件導向

• class of object <sup>同類的物件</sup>

△ Attributes: data members (ex: 車)

△ Behaviors: methods (ex: 程序)

假設  
為一手車商



\* three characteristics

△ Encapsulation 封裝 → 相同東西, 更有效率

△ Inheritance 繼承 → (copy), 針對"不同"東西重寫

△ Polymorphism 多型

## 單元二. 連結串列 (link list)

\* 指標

不需移動資料

\* Variations

< Array → 需移動 >

└ circular linked lists 環狀

└ Doubly " 双向


\* 初始值為未定義, 不是 NULL

以免用到  
其它位置

\* pointer → \*P: X → P 指向 X

└ & : address → P: &X → P 得到 X 的地址

\* dangling reference illegal (access) 非法記憶體空間

ex:  → ? (不知道指向誰時)

## \* 动态(配置)数组 (Array dynamically)

→ int Arraysize = 50

double \* anArray = new double [arraysize];

(注) anArray[2] = \*(anArray + 2)

delete [] anArray ⇒ 全部删除  
(释放空间)