- (a) Consider the sequence of weights 2, 3, 2. The greedy algorithm will pick the middle node, while the maximum weight independent set consists of the first and third.
- (b) Consider the sequence of weights 3, 1, 2, 3. The given algorithm will pick the first and third nodes, while the maximum weight independent set consists of the first and fourth.
- (c) Let  $S_i$  denote an independent set on  $\{v_1, \ldots, v_i\}$ , and let  $X_i$  denote its weight. Define  $X_0 = 0$  and note that  $X_1 = w_1$ . Now, for i > 1, either  $v_i$  belongs to  $S_i$  or it doesn't. In the first case, we know that  $v_{i-1}$  cannot belong to  $S_i$ , and so  $X_i = w_i + X_{i-2}$ . In the second case,  $X_i = X_{i-1}$ . Thus we have the recurrence

$$X_i = \max(X_{i-1}, w_i + X_{i-2}).$$

We thus can compute the values of  $X_i$ , in increasing order from i = 1 to n.  $X_n$  is the value we want, and we can compute  $S_n$  by tracing back through the computations of the max operator. Since we spend constant time per iteration, over n iterations, the total running time is O(n).

 $<sup>^{1}</sup>$ ex301.516.319