

Let  $Y$  denote the number of steps in which your net profit is positive. Then  $Y = Y_1 + Y_2 + \cdots + Y_n$ , where  $Y_k = 1$  if your net profit is positive at step  $k$ , and 0 otherwise.

Now, consider a particular step  $k$ .  $Y_k = 1$  if and only if you have had more than  $k/2$  steps in which your profit increased. Since the expected number of steps in which your profit increased is  $k/3$ , we can apply the Chernoff bound (4.1) with  $\mu = k/3$  and  $1 + \delta = 3/2$  to conclude that  $EY_k$  is bounded by

$$\left[ \frac{e^{1/2}}{(3/2)^{(3/2)}} \right]^{(k/3)} < (.97)^k.$$

Thus,

$$EY = \sum_{k=1}^n EY_k < \sum_{k=1}^n (.97)^k < \frac{1}{1 - (.97)} < 34,$$

which is a constant independent of  $n$ .

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<sup>1</sup>ex251.139.906