

Let $OPT(j)$ denote the minimum cost of a solution on servers 1 through j , *given* that we place a copy of the file at server j . We want to search over the possible places to put the highest copy of the file before j ; say in the optimal solution this at position i . Then the cost for all servers up to i is $OPT(i)$ (since we behave optimally up to i), and the cost for servers $i + 1, \dots, j$ is the sum of the access costs for $i + 1$ through j , which is $0 + 1 + \dots + (j - i - 1) = \binom{j-i}{2}$. We also pay c_j to place the server at j .

In the optimal solution, we should choose the best of these solutions over all i . Thus we have

$$OPT(j) = c_j + \min_{0 \leq i < j} (OPT(i) + \binom{j-i}{2}),$$

with the initializations $OPT(0) = 0$ and $\binom{1}{2} = 0$. The values of OPT can be built up in order of increasing j , in time $O(j)$ for iteration j , leading to a total running time of $O(n^2)$. The value we want is $OPT(n)$, and the configuration can be found by tracing back through the array of OPT values.

¹ex25.372.49