(a) For every node v_k that comes later than v_j, i.e. $k > j$, it has probability $\frac{1}{k-1}$ to link to v_j, since v_k chooses from the $k-1$ existing nodes with equal probabilities. For all the nodes coming before v_j, such probability is obviously zero.

So the expected number of incoming links to node v_j is

$$\begin{aligned}
\sum_{k=j+1}^{n} \frac{1}{k-1} &= \sum_{k=1}^{n-1} \frac{1}{k} - \sum_{k=1}^{j-1} \frac{1}{k} \\
&= H(n-1) - H(k-1) \\
&= \Theta(\ln n) - \Theta(\ln k) \\
&= \Theta(\ln \frac{n}{k})
\end{aligned}$$

(b) Consider a node v_j, every node v_k with $k > j$ has probability $1 - \frac{1}{k-1}$ not to link to v_j. So if we have random variable X_j s.t.

$$X_j = \begin{cases} 1 & \text{node } v_j \text{ has no in-coming links} \\ 0 & \text{otherwise} \end{cases}$$

then

$$\begin{aligned}
Exp[X_j] &= Pr[\text{no nodes links to } v_j] \\
&= \prod_{k=j+1}^{n} \left(1 - \frac{1}{k-1}\right) \\
&= \frac{j-1}{j} \cdot \frac{j}{j+1} \cdot \frac{j+1}{j+2} \cdots \frac{n-2}{n-1} \\
&= \frac{j-1}{n-1}
\end{aligned}$$

Therefore, by linearity of expectations, we get the expected number of nodes without in-coming links

$$\sum_{j=1}^{n} Exp[X_j] = \sum_{j=1}^{n} \frac{j-1}{n-1} = \frac{1}{n-1} \sum_{j=1}^{n} (j-1) = \frac{1}{n-1} \cdot \frac{n(n-1)}{2} = \frac{n}{2}$$

[1] ex976.627.720