

Let's suppose that  $s$  has  $n$  characters total. To make things easier to think about, let's consider the repetition  $x'$  of  $x$  consisting of exactly  $n$  characters, and the repetition  $y'$  of  $y$  consisting of exactly  $n$  characters. Our problem can be phrased as: is  $s$  an interleaving of  $x'$  and  $y'$ ? The advantage of working with these elongated strings is that we don't need to "wrap around" and consider multiple periods of  $x'$  and  $y'$  — each is already as long as  $s$ .

Let  $s[j]$  denote the  $j^{\text{th}}$  character of  $s$ , and let  $s[1 : j]$  denote the first  $j$  characters of  $s$ . We define the analogous notation for  $x'$  and  $y'$ . We know that if  $s$  is an interleaving of  $x'$  and  $y'$ , then its last character comes from either  $x'$  or  $y'$ . Removing this character (wherever it is), we get a smaller recursive problem on  $s[1 : n - 1]$  and prefixes of  $x'$  and  $y'$ .

Thus, we consider sub-problems defined by prefixes of  $x'$  and  $y'$ . Let  $M[i, j] = \text{yes}$  if  $s[1 : i + j]$  is an interleaving of  $x'[1 : i]$  and  $y'[1 : j]$ . If there is such an interleaving, then the final character is either  $x'[i]$  or  $y'[j]$ , and so we have the following basic recurrence:

$$M[i, j] = \text{yes} \text{ if and only if } M[i-1, j] = \text{yes} \text{ and } s[i+j] = x'[i], \text{ or } M[i, j-1] = \text{yes} \text{ and } s[i+j] = y'[j].$$

We can build these up via the following loop.

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M[0,0] = yes
For k = 1, 2, ..., n
  For all pairs (i, j) so that i + j = k
    If M[i-1, j] = yes and s[i+j] = x'[i] then
      M[i, j] = yes
    Else if M[i, j-1] = yes and s[i+j] = y'[j] then
      M[i, j] = yes
    Else
      M[i, j] = no
  Endfor
Endfor
Return "yes" if and only there is some pair (i, j) with i + j = n
so that M[i, j] = yes.

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There are  $O(n^2)$  values  $M[i, j]$  to build up, and each takes constant time to fill in from the results on previous sub-problems; thus the total running time is  $O(n^2)$ .

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<sup>1</sup>ex357.417.692