

This problem is very similar in flavor to the segmented least squares problem. We observe that the last line ends with word  $w_n$  and has to start with some word  $w_j$ ; breaking off words  $w_j, \dots, w_n$  we are left with a recursive sub-problem on  $w_1, \dots, w_{j-1}$ .

Thus, we define  $OPT[i]$  to be the value of the optimal solution on the set of words  $W_i = \{w_1, \dots, w_i\}$ . For any  $i \leq j$ , let  $S_{i,j}$  denote the slack of a line containing the words  $w_i, \dots, w_j$ ; as a notational device, we define  $S_{i,j} = \infty$  if these words exceed total length  $L$ . For each fixed  $i$ , we can compute all  $S_{i,j}$  in  $O(n)$  time by considering values of  $j$  in increasing order; thus, we can compute all  $S_{i,j}$  in  $O(n^2)$  time.

As noted above, the optimal solution must begin the last line somewhere (at word  $w_j$ ), and solve the sub-problem on the earlier lines optimally. We thus have the recurrence

$$OPT[n] = \min_{1 \leq j \leq n} S_{i,n}^2 + OPT[j-1],$$

and the line of words  $w_j, \dots, w_n$  is used in an optimum solution if and only if the minimum is obtained using index  $j$ .

Finally, we just need a loop to build up all these values:

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Compute all values  $S_{i,j}$  as described above.
Set  $OPT[0] = 0$ 
For  $k = 1, \dots, n$ 
     $OPT[k] = \min_{1 \leq j \leq k} (S_{j,k}^2 + OPT[j-1])$ 
Endfor
Return  $OPT[n]$ .

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As noted above, it takes  $O(n^2)$  time to compute all values  $S_{i,j}$ . Each iteration of the loop takes time  $O(n)$ , and there are  $O(n)$  iterations. Thus the total running time is  $O(n^2)$ .

By tracing back through the array  $OPT$ , we can recover the optimal sequence of line breaks that achieve the value  $OPT[n]$  in  $O(n)$  additional time.

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<sup>1</sup>ex771.275.715