We build the following flow network. There is a node v_i for each client i, a node w_j for each base station j, and an edge (v_i, w_j) of capacity 1 if client i is within range of base station j. We then connect a super-source s to each of the client nodes by an edge of capacity 1, and we connect each of the base station nodes to a super-sink t by an edge of capacity L.

We claim that there is a feasible way to connect all clients to base stations if and only if there is an s-t flow of value n. If there is a feasible connection, then we send one unit of flow from s to t along each of the paths s, v_i, w_j, t, where client i is connected to base station j. This does not violate the capacity conditions, in particular on the edges (w_j, t), due to the load constraints. Conversely, if there is a flow of value n, then there is one with integer values. We connect client i to base station j if the edge (v_i, w_j) carries one unit of flow, and we observe that the capacity condition ensures that no base station is overloaded.

The running is the time required to solve a max-flow problem on a graph with $O(n + k)$ nodes and $O(nk)$ edges.

[1] ex751.45.676