To do this, we apply the Cycle Property nine times. That is, we perform BFS until we find a cycle in the graph G, and then we delete the heaviest edge on this cycle. We have now reduced the number of edges in G by one, while keeping G connected, and (by the Cycle Property) not changing the identity of the minimum spanning tree. If we do this a total of nine times, we will have a connected graph H with $n-1$ edges and the same minimum spanning tree as G. But H is a tree, and so in fact it is the minimum spanning tree.

The running time of each iteration is $O(m+n)$ for the BFS and subsequent check of the cycle to find the heaviest edge; here $m \leq n+8$, so this is $O(n)$.

[1] ex258.711.547