

In a Steiner tree  $T$  on  $X \cup Z \subseteq V$ ,  $|X| = k$ , we will refer to  $X$  as the *terminals* and  $Z$  as the *extra nodes*. We first claim that each extra node has degree at least 3 in  $T$ ; for if not, then the triangle inequality implies we can replace its two incident edges by an edge joining its two neighbors. Since the sum of the degrees in a  $t$ -node tree is  $2t - 2$ , every tree has at least as many leaves as it has nodes of degree greater than 2. Hence  $|Z| \leq k$ . It follows that if we compute the minimum spanning tree on all sets of the form  $X \cup Z$  with  $|Z| \leq k$ , the cheapest among these will be the minimum Steiner tree. There are at most  $\binom{n}{2k} = n^{O(k)}$  such sets to try, so the overall running time will be  $n^{O(k)}$ .