

We build the following flow network. There is a node v_i for each patient i , a node w_j for each hospital j , and an edge (v_i, w_j) of capacity 1 if patient i is within a half hour drive of hospital j . We then connect a super-source s to each of the patient nodes by an edge of capacity 1, and we connect each of the hospital nodes to a super-sink t by an edge of capacity $\lceil n/k \rceil$.

We claim that there is a feasible way to send all patients to hospitals if and only if there is an s - t flow of value n . If there is a feasible way to send patients, then we send one unit of flow from s to t along each of the paths s, v_i, w_j, t , where patient i is sent to hospital j . This does not violate the capacity conditions, in particular on the edges (w_j, t) , due to the load constraints. Conversely, if there is a flow of value n , then there is one with integer values. We send patient i to hospital j if the edge (v_i, w_j) carries one unit of flow, and we observe that the capacity condition ensures that no hospital is overloaded.

The running time is the time required to solve a max-flow problem on a graph with $O(n + k)$ nodes and $O(nk)$ edges.

¹ex297.369.93