(a) Consider the sequence $1, 4, 2, 3$. The greedy algorithm produces the rising trend $1, 4$, while the optimal solution is $1, 2, 3$.

(b) Let $OPT(j)$ be the length of the longest increasing subsequence on the set $P[j], P[j+1], ldots, P[n]$, including the element $P[j]$. Note that we can initialize $OPT(n) = 1$, and $OPT(1)$ is the length of the longest rising trend, as desired.

Now, consider a solution achieving $OPT(j)$. Its first element is $P[j]$, and its next element is $P[k]$ for some $k > j$ for which $P[k] > P[j]$. From k onward, it is simply the longest increasing subsequence that starts at $P[k]$; in other words, this part of the sequence has length $OPT(k)$, so including $P[j]$, the full sequence has length $1 + OPT(k)$. We have thus justified the following recurrence.

$$OPT(j) = 1 + \max_{k>j:P[k]>P[j]} OPT(k).$$

The values of OPT can be built up in order of decreasing j, in time $O(n-j)$ for iteration j, leading to a total running time of $O(n^2)$. The value we want is $OPT(1)$, and the subsequence itself can be found by tracing back through the array of OPT values.

[1] ex219.570.316