

The strategy is as follows. The seller watches the first  $n/2$  bids without accepting any of them. Let  $b^*$  be the highest bid among these. Then, in the final  $n/2$  bids, the seller accepts any bid that is larger than  $b^*$ . (If there is no such bid, the seller simply accepts the final bid.)

Let  $b_i$  denote the highest bid, and  $b_j$  denote the second highest bid. Let  $S$  denote the underlying sample space, consisting of all permutations of the bids (since they can arrive in any order.) So  $|S| = n!$ . Let  $E$  denote the event that  $b_j$  occurs among the first  $n/2$  bids, and  $b_i$  occurs among the final  $n/2$  bids.

What is  $|E|$ ? We can place  $b_j$  anywhere among the first  $n/2$  bids ( $n/2$  choices); then we can place  $b_i$  anywhere among the final  $n/2$  bids ( $n/2$  choices); and then we can order the remaining bids arbitrarily ( $(n-2)!$  choices). Thus  $|E| = \frac{1}{4}n^2(n-2)!$ , and so

$$P[E] = \frac{n^2(n-2)!}{4n!} = \frac{n}{4(n-1)} \geq \frac{1}{4}.$$

Finally, if event  $E$  happens, then the strategy will accept the highest bid; so the highest bid is accepted with probability at least  $1/4$ .

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<sup>1</sup>ex437.89.251