One algorithm is the following.

```
For $i = 1, 2, \ldots, n$
  Receiver $j$ computes $\beta_{ij} = f(\beta_1^* \cdots \beta_{i-1}^*, \alpha_i^{\langle j \rangle})$.
  $\beta_i^*$ is set to the majority value of $\beta_{ij}$, for $j = 1, \ldots, k$.
  End for
Output $\beta^*$
```

We'll make sure to choose an odd value of k to prevent ties.

Let $X_{ij} = 1$ if $\alpha_i^{\langle j \rangle}$ was corrupted, and 0 otherwise. If a majority of the bits in $\{\alpha_i^{\langle j \rangle} : j = 1, 2, \ldots, k\}$ are corrupted, then $X_i = \sum_j X_{ij} > k/2$. Now, since each bit is corrupted with probability $\frac{1}{4}$, $\mu = \sum_j EX_{ij} = k/4$. Thus, by the Chernoff bound, we have

$$\begin{aligned}
\Pr[X_i > k/2] &= \Pr[X_i > 2\mu] \\
&< \left(\frac{e}{4}\right)^{k/4} \\
&\leq (.91)^k.
\end{aligned}$$

Now, if

$$k \geq 11 \ln n > \frac{\ln n - \ln .1}{\ln(1/.91)},$$

then

$$\Pr[X_i > k/2] < .1/n.$$

(So it is enough to choose k to be the smallest odd integer greater than $11 \ln n$.) Thus, by the union bound, the probability that *any* of the sets $\{\alpha_i^{\langle j \rangle} : j = 1, 2, \ldots, k\}$ have a majority of corruptions is at most .1.

Assuming that a majority of the bits in each of these sets are not corrupted, which happens with probability at least .9, one can prove by induction on i that all the bits in the reconstructed message β^* will be correct.

[1] ex482.918.336