

This can be accomplished directly using a convolution. Define one vector to be $a = (q_1, q_2, \dots, q_n)$. Define the other vector to be $b = (n^{-2}, (n-1)^{-2}, \dots, 1/4, 1, 0, -1, -1/4, \dots, -n^{-2})$. Now, for each j , the convolution of a and b will contain an entry of the form

$$\sum_{i < j} \frac{q_i}{(j-i)^2} + \sum_{i > j} \frac{-q_i}{(j-i)^2}.$$

From this term, we simply multiply by Cq_j to get the desired net force F_j .

The convolution can be computed in $O(n \log n)$ time, and reconstructing the terms F_j takes an additional $O(n)$ time.

¹ex726.26.783