

If the minimum s - t cut has size $\leq k$, then we can reduce the flow to 0. Otherwise, let $f > k$ be the value of the maximum s - t flow. We identify a minimum s - t cut (A, B) , and delete k of the edges out of A . The resulting subgraph has a maximum flow value of at most $f - k$.

But we claim that for any set of edges F of size k , the subgraph $G' = (V, E - F)$ has an s - t flow of value at least $f - k$. Indeed, consider any cut (A, B) of G' . There are at least f edges out of A in G , and at most k have been deleted, so there are at least $f - k$ edges out of A in G' . Thus, the minimum cut in G' has value at least $f - k$, and so there is a flow of at least this value.

¹ex225.750.725