Let X be a random variable equal to the number of times that b^* is updated. We write $X = X_1 + X_2 + \cdots + X_n$, where $X_i = 1$ if the i^{th} bid in order causes b^* to be updated, and $X_i = 0$ otherwise.

So $X_i = 1$ if and only if, focusing just on the sequence of the first i bids, the largest one comes at the end. But the largest value among the first i bids is equally likely to be anywhere, and hence $EX_i = 1/i$.

Alternately, the number of permutations in which the number at position i is larger than any of the numbers before it can be computed as follows. We can choose the first i numbers in $\binom{n}{i}$ ways, put the largest in position i, order the remainder in $(i-1)!$ ways, and order the subsequent $(n-i)$ numbers in $(n-i)!$ ways. Multiplying this together, we have $\binom{n}{i}(i-1)!(n-i)! = n!/i$. Dividing by $n!$, we get $EX_i = 1/i$.

Now, by linearity of expectation, we have $EX = \sum_{i=1}^{n} EX_i = \sum_{i=1}^{n} 1/i = H_n = \Theta(\log n)$.

[1] ex547.67.324