We interpret the constraint (μ_i, μ_j, μ_k) to mean that we require one of the subsequences $\ldots, \mu_i, \ldots, \mu_j, \ldots, \mu_k, \ldots$ or $\ldots, \mu_k, \ldots, \mu_j, \ldots, \mu_i, \ldots$ to occur in the ordering of the markers. (One could also interpret it to mean that just the first of these subsequences occurs; this will affect the analysis below by a factor of 2.)

Suppose that we choose an order for the n markers uniformly at random. Let X_t denote the random variable whose value is 1 if the t^{th} constraint (μ_i, μ_j, μ_k) is satisfied, and 0 otherwise. The six possible subsequences of $\{\mu_i, \mu_j, \mu_k\}$ occur with equal probability, and two of them satisfy the constraint; thus $EX_t = \frac{1}{3}$. Hence if $X = \sum_t X_t$ gives the total number of constraints satisfied, we have $EX = \frac{1}{3}k$.

So if our random ordering satisfies a number of constraints that is at least the expectation, we have satisfied at least $\frac{1}{3}$ of all constraints, and hence at least $\frac{1}{3}$ of the maximum number of constraints that can be simultaneously satisfied.

We can extend this to construct an algorithm that *only* produces solutions within a factor of $\frac{1}{3}$ of optimal: We simply repeatedly generate random orderings until $\frac{1}{3}k$ of the constraints are satisfied. To bound the expected running time of this algorithm, we must give a lower bound on the probability p^+ that a single random ordering will satisfy at least the expected number of constraints; the expected running time will then be at most $1/p^+$ times the cost of a single iteration.

First note that k is at most n^3, and define $k' = \frac{1}{3}k$. Let k'' denote the greatest integer strictly less than k'. Let p_j denote the probability that we satisfy j of the constraints. Thus $p^+ = \sum_{j \geq k'} p_j$; we define $p^- = \sum_{j<k'} p_j = 1 - p^+$. Then we have

$$\begin{aligned}
k' &= \sum_j j p_j \\
&= \sum_{j<k'} j p_j + \sum_{j \geq k'} j p_j \\
&\leq \sum_{j<k'} k'' p_j + \sum_{j \geq k'} n^3 p_j \\
&= k''(1-p^+) + n^3 p^+
\end{aligned}$$

from which it follows that

$$(k'' + n^3)p^+ \geq k' - k'' \geq \frac{1}{3}.$$

Since $k'' \leq n^3$, we have $p^+ \geq \frac{1}{6n^3}$, and so we are done.

[1]ex449.507.100