

The statement is false and the following is a counterexample to it. Let us be given a number  $b > 1$  (we will without loss of generality assume that it is an integer, otherwise we will round it up). We consider the following graph. It has  $2(b + 1) + 2$  vertices: source  $s$  sink  $t$ , and vertices  $u_1, u_2, \dots, u_{b+1}$  that have an edge coming from the source and vertices  $v_1, v_2, \dots, v_{b+1}$  that have an edge going into the sink. There is also an edge from  $u_i$  to  $v_i$  and from  $v_i$  to  $u_{i+1}$ . All the edge capacities are 1.

Now assume that the first augmenting path was the path  $s \rightarrow u_1 \rightarrow v_1 \rightarrow u_2 \rightarrow v_2 \rightarrow \dots u_{b+1} \rightarrow v_{b+1} \rightarrow t$ . Then since all the backward edges are deleted from the residual graph according to the super-fast algorithm, the residual graph would contain no path from  $s$  to  $t$ , and therefore our final flow would equal 1. But there is a flow of value  $b + 1$  by using the horizontal edges (that is  $u_i \rightarrow v_i$ ). Therefore we failed to reach within  $b$  of the optimum.

Notice that for different  $b$ 's we would be considering different graphs, but we are allowed to do this, since the problem asks whether there exists a *universal*  $b$  that is independent of the choice of the flow graph  $G$ .

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<sup>1</sup>ex70.281.132