Let the contestants be numbered $1, \ldots, n$, and let s_i, b_i, r_i denote the swimming, biking, and running times of contestant i. Here is an algorithm to produce a schedule: arrange the contestants in order of decreasing $b_i + r_i$, and send them out in this order. We claim that this order minimizes the completion time.

We prove this by an exchange argument. Consider any optimal solution, and suppose it does not use this order. Then the optimal solution must contain two contestants i and j so that j is sent out directly after i, but $b_i + r_i < b_j + r_j$. We will call such a pair (i, j) an *inversion*. Consider the solution obtained by swapping the orders of i and j. In this swapped schedule, j completes earlier than he/she used to. Also, in the swapped schedule, i gets out of the pool when j previously got out of the pool; but since $b_i + r_i < b_j + r_j$, i finishes sooner in the swapped schedule than j finished in the previous schedule. Hence our swapped schedule does not have a greater completion time, and so it too is optimal.

Continuing in this way, we can eliminate all inversions without increasing the completion time. At the end of this process, we will have a schedule in the order produced by our algorithm, whose completion time is no greater than that of the original optimal order we considered. Thus the order produced by our algorithm must also be optimal.

[1] ex983.429.914