Let (V, F) and (V, F') be distinct arborescences rooted at r. Consider the set of edges that are in one of F or F' but not the other; and over all such edges, let e be one whose distance to r in its arborescence is minimum. Suppose $e = (u, v) \in F'$. In (V, F), there is some other edge (w, v) entering v.

Now define $F'' = F - (w, v) + e$. We claim that (V, F'') is also an arborescence rooted at r. Clearly F'' has exactly one edge entering each node, so we just need to verify that there is an r-x path for every node x. For those x such that the r-x path in (V, F) does not use v, the same r-x path exists in F''. Now consider an x whose r-x path in (V, F) does use v. Let Q denote the r-u path in (V, F'), and let P denote the v-x path in (V, F). Note that all the edges of P belong to F'', since they all belong to F and (w, v) is not among them. But we also have $Q \subseteq F \cap F'$, since e was the closest edge to r that belonged to one of F or F' but not the other. Thus in particular, $(w, v) \notin Q$, and hence $Q \subseteq F''$. Hence the concatenated path $Q \cdot e \cdot P \subseteq F''$, and so there is an r-x path in (V, F'').

The arborescence (V, F'') has one more edge in common with (V, F') than (V, F) does. Performing a sequence of these operations, we can thereby transform (V, F) into (V, F') one edge at a time. But each of these operations changes the cost of the arborescence by at most 1 (since all edges have cost 0 or 1). So if we let (V, F) be a minimum-cost arborescence (of cost a) and we let (V, F') be a maximum-cost arborescence (of cost b), then if $a \leq k \leq b$, there must be an arborescence of cost exactly k.

Note: The proof above follows the strategy of "swapping" from the min-cost arborescence to the max-cost arborescence, changing the cost by at most one every time. The swapping strategy is a little complicated — choosing the highest edge that is not in both arborescences — but some complication of this type seems necessary. To see this, consider what goes wrong with the following, simpler, swapping rule: find any edge $e' = (u, v)$ that is in F' but not in F; find the edge $e = (w, v)$ that enters v in F; and update F to be $F - e + e'$ The problem is that the resulting structure may not be an arborescence. For example, suppose V consists of the four nodes $\{0, 1, 2, 3\}$ with the root at 0, $F = \{(0, 1), (1, 2), (2, 3)\}$, and $F' = \{(0, 3), (3, 1), (1, 2)\}$. Then if we find $(3, 1)$ in F' and update F to be $F - (0, 1) + (3, 1)$, we end up with $\{(1, 2), (2, 3), (3, 1)\}$, which is not an arborescence.

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