Manish Kumar (21044)
October 24, 2023

E0 230: Computational Methods of Optimization
Assignment 02

Q1. Trisection Method

(1). $r_1,\ s_1$ in terms of x_1, y_1: Let, $y > x$. Then:

$$r_1 = \frac{y_1 - x_1}{3} + x_1 = \frac{y_1 + 2x_1}{3}$$

$$s_1 = y_1 - \frac{y_1 - x_1}{3} = \frac{2y_1 + x_1}{3}$$

(2). Number of iteration in terms of x_1, y_1, ϵ:
Trisection process goes on like

$$(x_1, y_1) \to (x_2, y_2) \to ... \to (x_i, y_i) \to ... \to (x_n, y_n)$$

Key observation is $|x_2 - y_2| \leq \frac{2}{3}|x_1 - y_1|$ or, decrease in the search interval is atleast two-third.
Hence after k iterations the decrease in the search interval is:

$$|x_k - y_k| \leq \left(\frac{2}{3}\right)^{k-1} |x_1 - y_1|$$

We stop the iteration once this interval become smaller than tolerance error ϵ. Hence number of iteration k in terms of x_1, y_1, ϵ is:

$$\epsilon \leq \left(\frac{2}{3}\right)^{k-1} |x_1 - y_1|$$

Rearranging terms gives:

$$k \leq \log_{\frac{3}{2}} \left(\frac{|x_1 - y_1|}{\epsilon}\right)$$

(3). $g(\alpha)$ estimation by trisection method:

- Number of iterations: 24
- estimate of the minimizer α^*: 0.058814359
- estimate for $g(\alpha^*)$: 17.81381861

Q2. Damped Newton's method:

solution

(1). Show that $f(x)$ is convex.

Given, $f(x) = \sqrt{(1+2x^2)}$

Lemma 2.1: If f is C^2 function, then convexity $\iff$ $f''(x) \geq 0$.

Here, $f''(x) = \frac{2}{(1+2x^2)^{3/2}} > 0$. Thus, $f(x)$ is a convex function.

Proof of Lemma: This has been shown in the tutorial. We need to show one way implication, i.e. if $f''(x) \geq 0$ implies convexity.

(2) Show Newton's method diverges at $x_0 = 1$:

Using,

$$f'(x) = \frac{2x}{(1+2x^2)^{1/2}}$$

$$f''(x) = \frac{2}{(1+2x^2)^{3/2}}$$

Hence, update eqaution is:

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)} = x_k - x_k(1+2x_k^2) = -2x_k^3$$

$$x_{k+1} = -2x_k^3$$

for $x_0 = 1$ gives, $x_1 = -2$, $x_2 = 16$, ...

This sequence diverges if $x_0 = 1$ as per ratio test,

$$x_{k+1} = -2x_k^3 \implies |\frac{x_{k+1}}{x_k}| = 2x_k^2 > 1;\ if\ x_0 = 1$$

(3). What values of x_0 Newton's method converges:

As per ratio test, It definetly converges if:

$$|\frac{x_{k+1}}{x_k}| = 2x_k^2 < 1 \implies x_k < \frac{1}{2}$$

.

(4). Damped Newton's Method:

- Final point: $0.000000812 = 8.12 \times 10^{-7}$
- Numbers of iterations (or final k): 7

Q3. Nestrov's Accelarated Descent Method

solution

(1) Gradient Descent with $x_0 = [0, 0, 0, 0, 0]$ and $\alpha = 0.00004$ and $\epsilon = 0.0001$

- Number of iterations: 27
- final point x^*: [0.00687022, 0.00687022, 0.00687022, 0.00687022, 0.00687022]
- $f(x^*)$: 97.930285075

.

(2) Accelarated Descent with $x_0 = [0, 0, 0, 0, 0]$ and $\alpha = 0.00004$, $\theta = 0.142$ and $\epsilon = 0.0001$

- Number of iterations: 21
- final point x^*: [0.00687022, 0.00687022, 0.00687022, 0.00687022, 0.00687022]
- $f(x^*)$: 97.930285075

Q4. Conjugate Descent Method

solution

(1) Conjugate vectors:

- u_0**:** [1., 0., 0., 0., 0.]
- u_1**:** [-0.85, 1. , 0. , 0. , 0.]
- u_2**:** [-0.23943662 -0.71830986 1. 0. 0.]
- u_3**:** [-0.20935961 -0.62807882 -0.12561576 1. 0.]
- u_4**:** [-0.19212141 -0.57636422 -0.11527284 -0.08233775 1.]

.

(2) Convergence of Conjugate descent:

- x^*: [0.1575097 -0.52747091 -0.30549418 0.35321844 0.41208818]
- $f(x^*)$: -1.42438749
- Number of iterations: 5