Temperature dependence of Johnson Noise

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OBJECTIVES:

1. Calibrating the Thermal probe by the 'Two-point calibration' method.
2. Controlling the temperature of the copper base in the range of 77K - 300K.
3. Measuring Johnson Noise of a resistor as a function of temperature.

THEORY: Johnson noise is characterised in terms of mean square voltage as $\mathbf{\langle v_J^2(t)\rangle = 4k_B RT\Delta f}$. We verified this relation for a given resistor within the appropriate bandwidth(Δf).

We used a thermal probe to measure the temperatures. The probe comes with a 'trans-diode' or 'diode-connected transistor' as the electrical transducer for temperature measurement. We used the I-V response of the ideal p-n diode operating at the current of 10μA. It gives voltage in terms of temperature as $\mathbf{\Delta V(i=10\mu A,\ T) = d_1 - d_2T - d_3\,T\,Log(T)}$. We get the value of d_1, and d_2 by measuring the voltage at two known temperature points- room condition(298 K) and liquid Nitrogen boiling point(77K). The value of d_3 is supplied in the manual itself.

To control the temperature of the probe, we managed the power input to the heater by allowing a certain amount of current to pass through it. We get the desired temperature by setting a thermal equilibrium between heat delivered by the heater and heat dissipation to the surrounding atmosphere and liquid nitrogen.

PROCEDURE: We stabilized the system at four different temperatures by the method discussed above. For each fixed temperature we took the reading for all three remote[1] resistors $R_A=10\Omega$, $R_B=10k\Omega$ and $R_C=100k\Omega$. We did the same for local resistors located inside LLE[2].

OBSERVATION AND CALCULATIONS :

Table-1 (Calibrating thermal probe)

Experimental condition and their associated temperature	Voltage measured (Volt)
Temperature of the room = 298 K	0.431 V
Liquid nitrogen boiling point = 77 K	0.966 V

Putting these value in equation [$\mathbf{\Delta V(i=10\mu A,T)=d_1 - d_2T - d_3\ T\ Log(T)}$] [3] gives

$$\mathbf{\Delta V(i=10\mu A,T) = 1.10 - (8.15 \times 10^{-5})T - 4.05\times 10^{-4}\,T\,Log(T).}$$

Using this equation we get the temperature-voltage table as below. From the following table, we can infer the temperature by looking at the corresponding voltage. The voltmeter connected to the temperature module of LLE record the voltage.

[1] Remote resistor are embedded inside thermal probe and there are directly connected to temperature varying cooper base.
[2] LLE refers to low level electronic device and resistors inside them are always near ambient temperature
[3] Value of d_3=0.431mV is given in manual to make 'two point calibration' method more precise.

voltage(volt)	Temperature(Kelvin)	voltage(volt)	Temperature(Kelvin)
0.966	77	0.523	260
0.922	100	0.465	280
0.874	120	0.441	290
0.825	140	0.427	300
0.775	160	0.386	320
0.723	180	0.305	340
0.671	200	0.258	360
0.618	220	0.184	380
0.564	240	0.113	400

TABLE-2(variation of johnson noise as temperature)

Bandwidth range: 1 kHz - 33 kHz. Measured Amplifier-noise[4] $\langle V_N^2(t)\rangle$ = [$2.50\times10^{-12}V^2$]

We calculated the effect of the **remote resistors** R_A, R_B and R_C

The voltage of DMM correspond to $\langle V_{sq}(t)\rangle$ which is related to johnson noise as

$\boldsymbol{\langle V_{sq}(t)\rangle = \{(G_1G_2)^2/10V\}\langle V_J^2(t) + V_N^2(t)\rangle}$; where G_1= 600 and G_2 managed using HLE[5]

For resistor $R_A=10\Omega$

Temperature (K)	77	126	212	298
$\langle V_{sq}(t)\rangle$ (mV^2)	4.11	4.18	4.30	4.58

For resistor $R_B=10k\Omega$

Temperature (K)	77	126	212	298
$\langle V_{sq}(t)\rangle$ (mV^2)	9.50	9.76	10.01	10.31

For resistor $R_B=100k\Omega$

Temperature (K)	77	126	212	298
$\langle V_{sq}(t)\rangle$ (mV^2)	13.72	13.89	14.29	14.75

[4]Amplifier noise only depends on bandwidth of operation and we are keeping the same bandwidth while taking all the readings.

[5] HLE refers to high level electronics and it contains gain knobs to adjust appropriate G_2.

Johnson noise for **resistors inside LLE** device which is always at **ambient temperature**

R=10Ω

Temperature (K)	77	126	212	298
<V_{sq}(t)> (mV^2)	4.56	4.56	4.57	4.58

Slight deviation in voltage due to small internal heating of resistors (Joule heating).

R=10kΩ

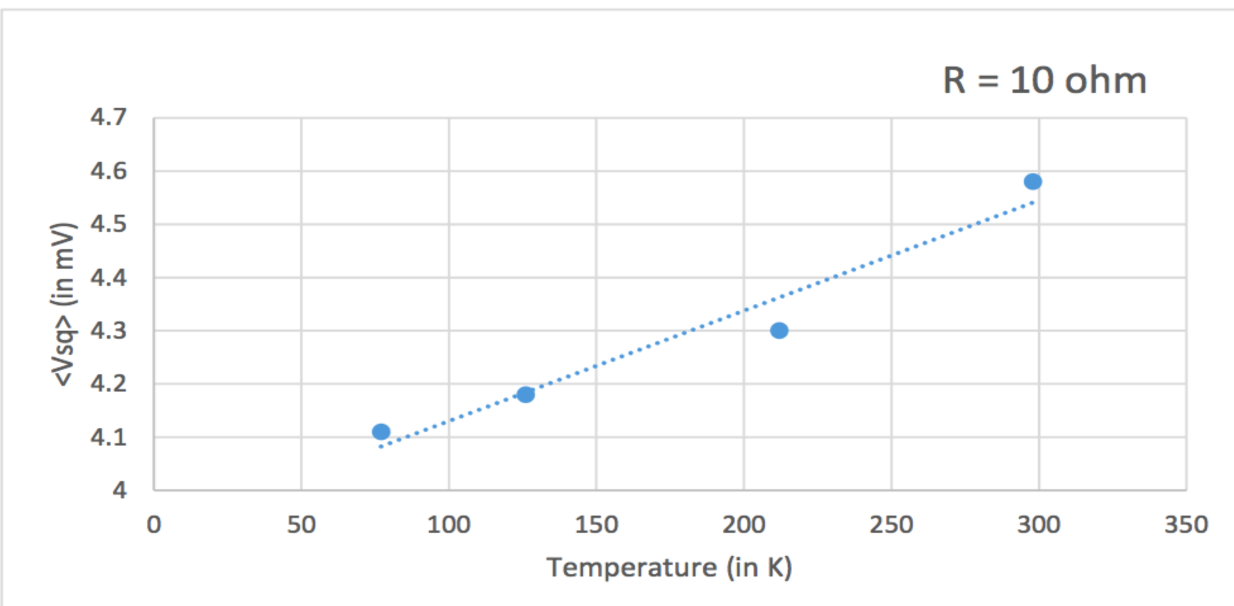
Temperature (K)	77	126	212	298
<V_{sq}(t)> (mV^2)	10.30	10.31	10.31	10.31

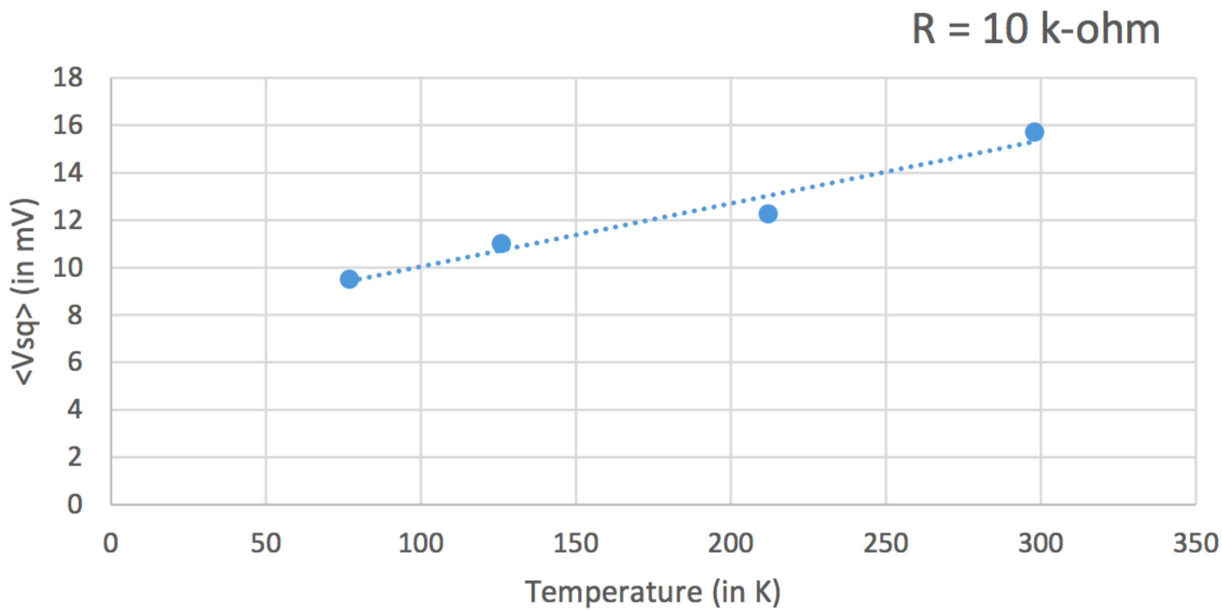
R=100kΩ

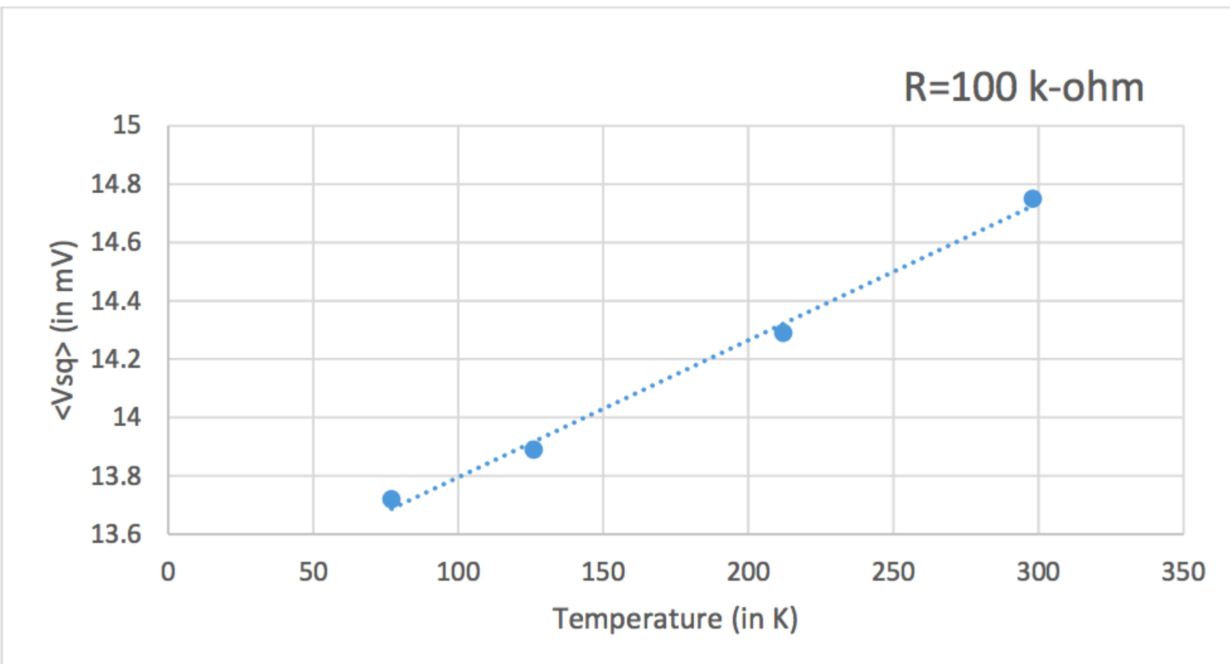
Temperature (K)	77	126	212	298
<V_{sq}(t)> (mV^2)	14.73	14.74	14.74	14.74

Result: Above three observation table shows that resistor inside LLE are at same temperature hence no change in their Johnson noise.

Graph-A







Graph-C(above)

Slope calculation and % deviation of experimental result with the theory.

Experimental value of Slope calculation by *best fit line* of the graph

(i) For R=10Ω(Graph-A)

Slope = 0.00142 mV^2K^{-1}

(ii) For R=10kΩ(Graph-B)

Slope = 0.00281 mV^2K^{-1}

(iii) For R= 100kΩ(Graph-C)

Slope = 0.00466 mV^2K^{-1}

Theoretical values of each sllopes using the relation:

$\langle V_{sq}(t)\rangle = \{(G_1G_2)^2/10V\}\langle V_J^2(t) + V_N^2(t)\rangle$ and,

$\langle V_J^2(t) + V_N^2(t)\rangle=(4k_BR\Delta f)T$ which implies

Slop = **$[\{(G_1G_2)^2/10V\}(4k_BR\Delta f)]$** and y intercept = **$V_N^2(t)$.**

We are taking k_B=1.3×10^{-23} m^{2}kgs^{-2}K^{-1} and Δf = 32 kHz(controlled by HLE filter knobs)

For R= 10Ω

Taking G_1= 1000 G_2=1500

Slope = 0.0018 mV^2K^{-1}

For R=10kΩ

Taking G_1 =600 G_2=1000

Slope = 0.00241 mV^2K^{-1}

For R=100KΩ

Taking G_1=600 G_2=300

Slope= 0.0059 mV^2K^{-1}

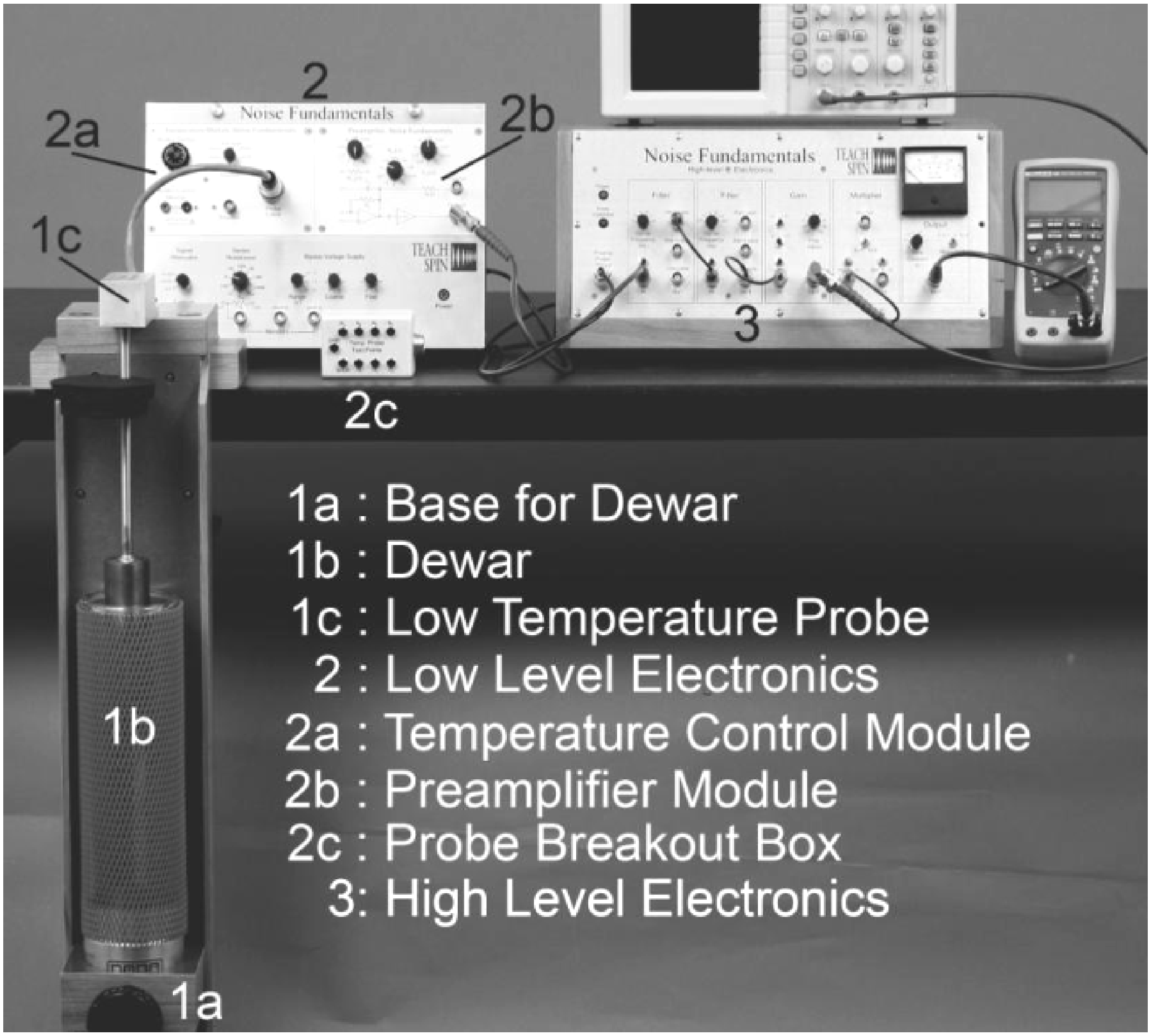
	R_A=10Ω	R_B=10kΩ	R_C=100KΩ
Experimental slope	0.00142 mV^2K^{-1}	0.00281 mV^2K^{-1}	0.00466 mV^2K^{-1}
Theoretical slope	0.00181 mV^2K^{-1}	0.00241 mV^2K^{-1}	0.00592 mV^2K^{-1}
%Error in Slope	21.5%	16.5%	20.3%

Conclusion of our experiment:

1. Johnson noise in resistor shows linear dependence with absolute temperature.

Special Note: We are unable to address the exact reason for relatively high deviations. We hypothesize it could be due to less data points on each graphs and some possible error committed in calibration of the temperature probe.

Acknowledgments: special thanks to the Liquid nitrogen plant (Southern Labs, IIT Kanpur) for providing liquid Nitrogen for the experiment.



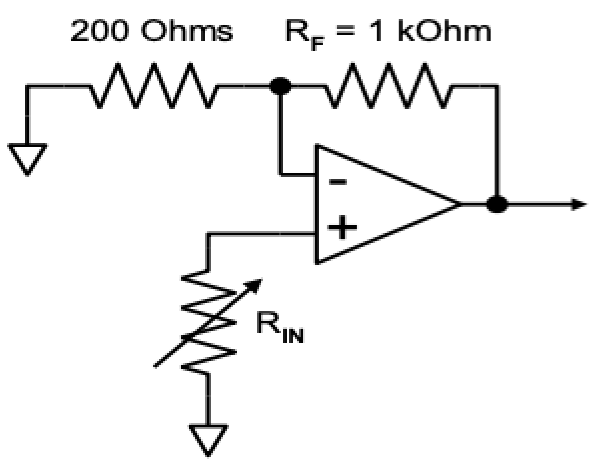
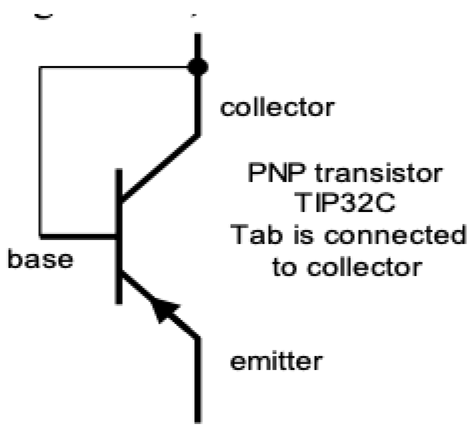


Fig 1.1a: Johnson noise preamplifier schematic



Schematic diagram for a pnp transistor in its transdiode configuration.