

## Ch6 佇列 Queue

- Queue —
1. ADT Queue Operation
  2. Simple Queue Application
    - Reading a String of Characters
    - Recognize a Palindrome
  3. Implementations of ADT Queue
  4. Application of Queues — Simulation

### Ob-1 佇列的運作

1. New item enter at the back, or rear, of the queue  
Item leave from the front of the queue  
First-in, First-out (FIFO)

2. ex: `aQueue.createQueue()`  
`aQueue.enqueue(5)`  
`aQueue.enqueue(2)`  
`aQueue.enqueue(7)`  
`aQueue.getFront(queueFront)`  
`aQueue.dequeue(queueFront)`  
`aQueue.dequeue(queueFront)`

5  
52  
527 ← back  
527 (queueFront is 5)  
27 (queueFront is 5)  
7 (queueFront is 2)

3. ex:  $247 = (2 \times 10 + 4) \times 10 + 7$

```
do {  
    aQueue.dequeue(ch);  
} while (ch is blank) // 空白, 忽略  
n = 0;  
done = FALSE;  
while (!done and ch is digit) {  
    n = n * 10 + integer represented by ch;  
    if (aQueue.isEmpty())  
        done = TRUE;  
    else  
        aQueue.dequeue(ch)  
}
```

### Ob-2 佇列辨識迴文

1. Stack reverse the order of occurrences  
Queue preserves the order of occurrences
2. Insert character into both a queue and a stack  
Compare the characters at the front of the queue and the top of the stack

```

3. isPal(in str: string): boolean
   aQueue.createQueue();
   aStack.createStack();
   for (the next character ch in str) {
       aQueue.enqueue(ch);
       aStack.push(ch);
   }
   charEqual = true;
   while (!aQueue.isEmpty() && charEqual) {
       aQueue.dequeue(front);
       aStack.pop(top);
       if (front != top)
           charEqual = FALSE;
   }

```

### 06-3 以指標實作佇列

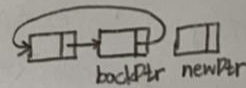
1. linear-linked list — front, back  
    Circular-linked list — back

2. 新增 enqueue — ① newPtr → next = NULL;  
                               backPtr → next = newPtr;  
                               backPtr = newPtr;  
                               ② frontPtr = newPtr;  
                               backPtr = newPtr;

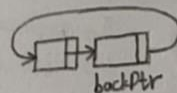
3. 刪除 dequeue — ① tempPtr = frontPtr;  
                               frontPtr = frontPtr → next;  
                               tempPtr → next = NULL;  
                               delete tempPtr;  
                               ② tempPtr = frontPtr;  
                               frontPtr = NULL;  
                               backPtr = NULL;  
                               tempPtr → next = NULL;  
                               delete tempPtr;

### 06-4 以指標實作環狀佇列

1. enqueue: newPtr → next = backPtr → next  
                   backPtr → next = newPtr

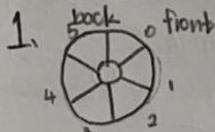


2. dequeue: QueueNode \*tempPtr = backPtr → next  
                   if (backPtr == backPtr → next)  
                       backPtr = NULL;  
                   else backPtr → next = tempPtr → next;  
                   tempPtr → next = NULL;  
                   delete tempPtr;



stack

## 06-5 陣列實現佇列



- ① Declare  $\text{MaxQueue} + 1$  for array item, see only  $\text{MaxQueue}$  for queue item
- ② Use a flag  $\text{isFull}$  to distinguish between full and empty

## 2. ADT

- ① statically — fixed-size queue can get full prevents the enqueue operation adding a item to queue, if array is full
  - ② dynamically or pointer — no size restriction on the queue
  - ③ point-based more efficient
- ADT list is simpler to write, save programming time

## 06-7 事件驅動模擬

1. simulated time advance to the time of next event using mathematical model base on statistics and probability
  2. Arrival events, Departure events
  3. keeps track of arrival and departure events that will occur but have not occurred yet
- contain at most one arrival event and one departure event

## 06-9 多佇列模擬

1. single teller / single queues
- Multiple teller / single queues
- Multiple tellers / Multiple queues

summary: 1. FIFO, first in first out

2. circular array  $\rightarrow$  problem of rightward drift

3. time-driven simulation, event-driven simulation

To implement an event-driven simulation, you maintain an event list that contains events that have not yet occurred

## 07-01 演算法的基本概念

1. Time efficiency, space efficiency
2. Specific implementation, computer, data
3. ex: Traverse a linked list of  $n$  nodes

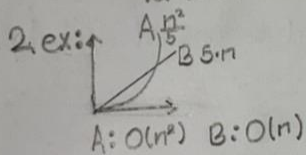
```
Node *cur = head;
while (cur != NULL) {
    cout << cur->item << endl;
    cur = cur->next;
}
```

1 assignment  
n+1 comparisons  
n write  
n assignments

4. Algorithm's execution  $\rightarrow$  number of operations  
ex: Towers of Hanoi  $\rightarrow 2^n - 1$  moves

## 07-02 大O表示法

1. ex: for ( $a=1; a \leq n; a++$ )  $n$   
for ( $b=1; b \leq a; b++$ )  $1+2+\dots+n = n(n+1)/2$   
for ( $c=1; c \leq 5; c++$ )  $5$



## 07-03 演算法複雜度分析

1. Definition of the order of an algorithm

$\Rightarrow$  if constants  $k$  and  $n_0$  exist such that A requires no more than  $k \cdot f(n)$  time units to solve a problem of size  $n \geq n_0$

Algorithm A is order  $f(n)$  - denoted  $O(f(n))$

2. ex:  $25n^2 - 25n$ ,  $k=?$   $n_0=?$

$$k=3, n_0=n^2$$

$$\text{ex: } (n+1) * (c+a) + n * w, k=? n_0=?$$

$$k, n_0 = n$$

$$\text{ex: } 2^n - 1, k=? n_0=?$$

$$k=1, n_0=2^n$$



## 07-04 複雜度成長函數

1.  $O(n^3 + 3n)$  is  $O(n^3) \Rightarrow$  ignore low-order terms  
 $O(5f(n)) = O(f(n)) \Rightarrow$  ignore multiplicative constant in high-order term  
 $O(f(n)) + O(g(n)) = O(f(n) + g(n))$
2.  $O(1) < O(\log_2 n) < O(n) < O(n * \log_2 n) < O(n^2) < O(n^3) < O(2^n)$

## 07-05 以搜尋為例說明演算法效率

1. frequently, seldom-used but critical operations must be efficient
2. Worst-case analysis, Average-case analysis, Best-case analysis
3. ex: sequential search

worst case:  $O(n)$   
Average case:  $O(n)$   
Best case:  $O(1)$

ex: Binary search of a sorted array

worst case:  $O(\log_2 n)$   
Average case:  $O(\log_2 n)$   
Best case:  $O(1)$

## 07-06 循序搜尋的效率比較

1. Worst case	sorted	unsorted	found
Worst case	$O(n)$	$O(n)$	$O(n)$
Average case	$O(n)$	$O(n)$	$O(n)$
Best case	$O(1)$	$O(n)$	$O(1)$

n)-time

## 07-07 排序演算法的分析

### 1. Internal sort

⇒ Requires that the collection of the data fit entirely in the computer's main memory

### External sort

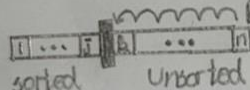
⇒ The collection of data will not fit in the computer's main memory all at once, but must reside in secondary storage

### 2. Stable sort vs. Unstable

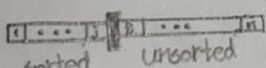
bubble  
insertion  
merge  
radix

quick  
heap

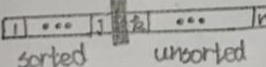
## 07-08 氣泡排序法

1.  Bubble up  
sorted unsorted

## 07-09 選擇排序法

1.  Swap the smallest among kth to nth numbers with kth number  
sorted unsorted

## 07-10 插入排序法

1.  Insert first element of unsorted section at the appropriate place in sorted section  
sorted unsorted

## 07-11 簡易排序方法比較

1. bubble sort	stable
selection sort	unstable
insertion sort	stable

### 07-12 氣泡排序法的複雜度分析

1. compare adjacent elements and exchange them if they are out of order  
Move the largest (smallest) to the end of the array  
Repeating this process eventually sorts the array into ascending (descending) order

2. worst case  $O(n^2)$

Best case  $O(n)$

### 07-13 選擇排序法的複雜度分析

1. Place the largest (smallest) item in correct place  
Place the next largest (smallest) item in its correct place, and so on

2. unstable

worst case  $O(n^2)$

Best case  $O(n^2)$

### 07-14 插入排序法的複雜度分析

1. Take the first item in the unsorted region and place it into its correct position in the sorted region

At each step, the sorted region grows by 1 and the unsorted shrinks by 1

2. worst case  $O(n^2)$

best case  $O(n)$

### 07-15 希爾排序

1. best case  $O(n)$

worst case  $O(n \log^2 n)$

## 07-16, 17, 18 合併演算法

### 1. a recursive sorting algorithm

Divide an array into halves

sort each half

Merge the sorted halves into one sorted array

Divide-and-conquer

2. worst case:  $O(n \log_2 n)$

Average case:  $O(n \log_2 n)$

3. advantage  $\rightarrow$  fast algorithm

disadvantage  $\rightarrow$  require second array as large as the original array

## 07-19, 20 快速演算法

### 1. divide-and-conquer algorithm

choose a pivot

partition the array about the pivot

items  $<$  pivot

items  $\geq$  pivot

pivot is now in correct sorted position

sort the left section

sort the right section

2. Average case:  $O(n \cdot \log_2 n)$

Worst case:  $O(n^2)$

Even if the worst case occurs, quicksort's performance is acceptable for moderately large arrays

## 07-21, 22, 23 基數演算法

### 1. 10 is the radix of the decimal system

Treats a key as a character string

Repeatedly assign the keys into group (buckets) according to the  $i$ th character

2. worst case  $O(n)$

best case  $O(n)$



summary: 1. quicksort, mergesort are recursive algorithms  
2. selection sort, bubble sort, insertion sort are  $O(n^2)$  algorithms

## Ch8 樹 tree

### 1. Terminology — ① composed of node and edges

② Parent-child  $\Rightarrow$  two nodes

Ancestor-descendant  $\Rightarrow$  among nodes

③ subtree — Any node and its descendant

④ general tree — one or more node  
A single node  $r$ , the root  
called subtrees of  $r$

⑤ parent, child, root  $\Rightarrow$  only in the tree with no parent  
subtree of node  $B \Rightarrow$  consists of a child of node  $B$  and  
the child descendants

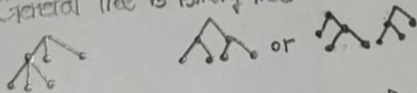
Leaf  $\Rightarrow$  no children, sibling  $\Rightarrow$  common parent

Ancestor node  $B \Rightarrow$  on the path from root to  $B$

Descendant node  $B \Rightarrow$  path from  $B$  to leaf

### 2. Binary tree — ① single node, the root ② left subtree of $r$ , right subtree of $r$

#### General Tree vs Binary Tree



### 3. Height of Tree — ① number of nodes along the longest path from root to leaf ② $T$ is empty, height is 0 ③ $T$ is not empty $\text{height}(T) = 1 + \max[\text{height}(T_L), \text{height}(T_R)]$

### 4. Full binary tree — ① node at levels $< h$ have two children each ② $T$ is empty, $T$ is full binary tree height 0 $T$ is not empty, root subtree both full binary tree of height $h-1$

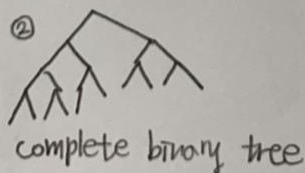
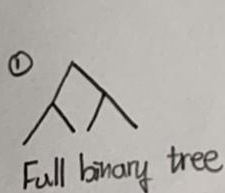
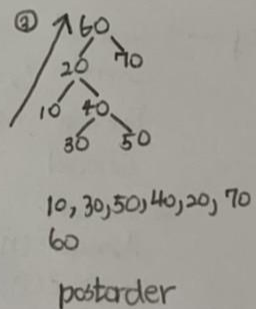
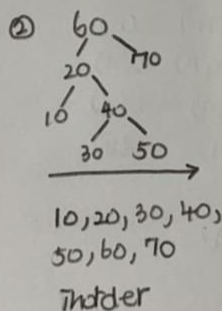
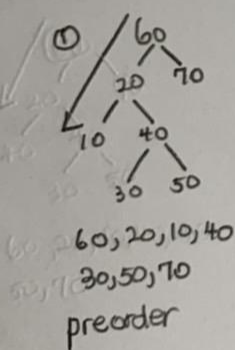
### 5. complete binary tree — ① full to level $h-1$ ② level is filled from left to right ③ level $< h-2$ , each two level $h-1$ , left

### 6. balanced binary tree — if the heights of any nodes two subtree differ by no more than 1

## (Full) complete (Balanced)

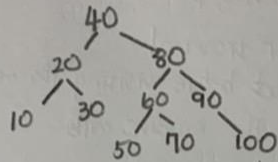
7. Representations of Binary tree —
- ① use an array of tree nodes
  - ② require creation of free list, keep track of available node
  - ③ pointer-based representation  
⇒ two pointers of nodes

8. Traversals of a binary tree —
- ① Preorder traversal  
visit root before visiting its subtrees  
Before the recursive calls
  - ② Inorder traversal  
visit root between visiting its subtrees  
Between the recursive calls
  - ③ Postorder traversal  
visit root after visiting its subtrees  
After the recursive calls

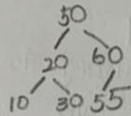


9. Binary search tree — ① has to following properties for each node  $n$   
 $n$ 's value is  $>$  all values in  $n$ 's left subtree  $T_L$   
 $n$ 's value is  $<$  all values in  $n$ 's right subtree  $T_R$   
 Both  $T_L, T_R$  are binary search tree

Insert:  
 ex: 40, 20, 10, 80, 90, 100, 30, 60, 50, 60



Delete:  
 ex: 55, 90, 70, 100, 40, 80



10. Efficiency — ① Retrieval  $O(\log n)$   $O(n)$   
 Insertion  $O(\log n)$   $O(n)$   
 Deletion  $O(\log n)$   $O(n)$   
 Traversal  $O(n)$   $O(n)$   
 ② tree sort  
 Average  $O(n \log n)$   
 worst  $O(n^2)$