

資節筆記 1-4

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單元一：優先佇列 Priority Queue

□ 多種排序方式皆可實作優先佇列，但效能不同。

- selection sort $\rightarrow pgInsert() = O(1)$
 $\rightarrow pgDelete() = O(n)$ 未排序 \rightarrow 排序 (找出優先)
通常為最小或最大
 \uparrow
- insertion sort $\rightarrow pgInsert() = O(n)$ 邊插入邊排序
 $\rightarrow pgDelete() = O(1)$
- Tree sort = Binary Search Tree

$\rightarrow pgInsert() = O(\log n)$ 較上述二種方法來強

$\rightarrow pgDelete() = O(\log n)$ 效能更平均！

▲ worst case = $O(n)$ (But!)

最好理想狀態 \rightarrow 平衡樹。

↓
Heap!

$\rightarrow heapInsert() = O(\log n)$

$\rightarrow heapDelete() = O(\log n)$

\rightarrow average, worst case:
 $O(n \log n)$

□ Heap = $pgInsert() = O(\log n)$

(1) Insert the new element in the next bottom rightmost.

(2) Fix the heap property by calling ReheapUp.

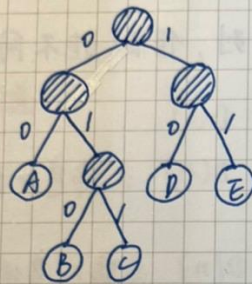
$pgDelete() = O(\log n)$

(1) copy the bottom rightmost element to the root.

(2) Delete the bottom rightmost node.

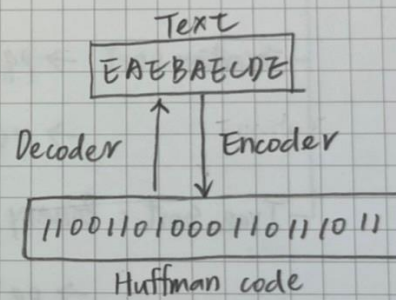
(3) Fix the heap property by calling ReheapDown.

□ Application: Huffman code. 霍夫曼編碼



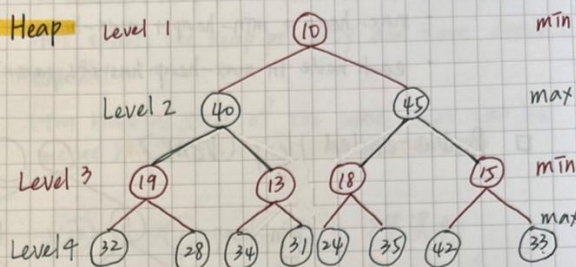
A: 00	D: 10
B: 010	E: 11
C: 011	

code



單元二：堆積變形 Variations of Heap

□ Min-max Heap level 1



補: grandparents of item[i]?

- Insert: if $(i-1)/2 > x$ grandparent = $(i-3)/4$;

1. Decide which level \rightarrow min or max

2. Check whether to swap with its parents.

• NO: ReheapUp from the current node.

• Yes: ReheapUp from its parents.

- Delete the smallest:

1. Replace the root with the last element.

2. Check whether to swap with its smaller child.

• NO: ReheapDown from the root (recursion)

• Yes: ReheapDown from the root (recursion)

- Delete the Largest:

1. Replace the maximum with the last element.

2. Check whether to swap with its larger child.

• NO: ReheapDown from the current node.

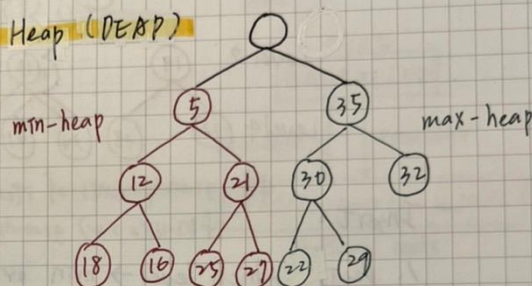
• Yes: ReheapDown from the current node.

□ Main idea in Min-max Heap

- Three 4-way tree

- max-heap + min-heap + max-heap
- each node in max-heap has its parent in min-heap.

□ Double-ended Heap (DEAP)



- Insert =

1. Examine the corresponding node = $Left < Right$
2. ReheapUp if necessary (recursion)

- Delete the smallest, largest =

1. Replace the root of $\begin{matrix} \text{min} \\ \text{max} \end{matrix}$ -heap with the last element.
2. ReheapDown if necessary // 若走到樹葉, 要檢查對應節點
3. Examine the corresponding nodes = $Left < Right$

□ Main idea in DEAP

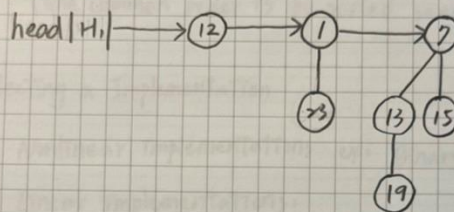
- Two heaps

- Pseudo root + min-heap + max-heap
- Each node in max-heap corresponds to one in min-heap

□ Binomial Heap $O(\log n)$

- Definition: A binomial heap is a collection of binomial trees that satisfy the heap property and have distinct orders.

(two binomial trees of the same order can be merged)



筆算資料

$$7 = 2^0 + 2^1 + 2^2$$

- Insert $O(\log n)$

1. Insert into the linked list of the roots.

2. call merge function.

- Delete $O(\log n)$

1. Find the minimum from the linked list of the roots

2. Delete the root having the minimum.

3. Add its children into the linked list.

4. Call merge function

□ Binomial tree of order k (B_k)

- The root has k children

- Merged by two binomial trees of order $k-1$

- number of nodes $= 2^k$

- Tree height $= k+1 \rightarrow O(\log n)$

單元三：由下而上成長的平衡樹

□ The ADT table

- Our table assumes distinct search keys.
(Other tables could allow duplicate search keys.)
- The traverseTable operation visits table items in a specified order.
(One common order is by sorted search key.)

□ Selecting a Implementation

- Nonlinear Implementations ex: Binary search tree. ↓

- Linear Implementations:

	array based	pointer based
Unsorted	Insertion: $O(1)$ Deletion: $O(n)$ Retrieval: $O(n)$ Traversal: $O(n)$	Insertion: $O(1)$ Deletion: $O(n)$ Retrieval: $O(n)$ Traversal: $O(n)$
sorted	Insertion: $O(n)$ Deletion: $O(n)$ Retrieval: $O(\log n)$ Traversal: $O(n)$	Insertion: $O(n)$ Deletion: $O(n)$ Retrieval: $O(n)$ Traversal: $O(n)$

Insertion: $O(\log n)$
 Deletion: $O(\log n)$
 Retrieval: $O(\log n)$
 Traversal: $O(n)$

- 比較

- 線性：適用較少筆資料的情況。
- 非線性：通常為較好的選擇 (ex: balanced binary search tree).

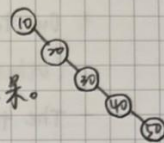
□ Binary Search Tree

- The efficiency tree is related to the tree height.

Maximum = n

Minimum = $\lceil \log_2(n+1) \rceil$

原因：新增和刪除的順序會影響到結果。



□ 2-3 Tree

- Are general trees, not binary trees.
 - Are never taller than a minimum-height binary tree.
(A 2-3 tree never has height greater than $\lceil \log_2(n+1) \rceil$)
- 2-3樹最糟糕的情況，是二元樹最好的情況。

- Insert = 由下往上.

1. 先找到要插入資料的位置(樹葉)

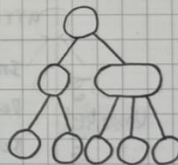
2. 新增資料

3. 如果樹葉沒有滿，則直接加入

4. 如果樹葉有3筆資料，則分離成2個節點

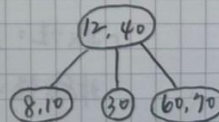
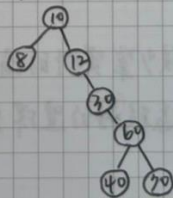
Case 1 → 若內部節點，則繼續分離 (recursion)

Case 2 → 若樹根，則分離成2節點(新樹根)，樹高增加



Practice: Answer (10, 12, 30, 8, 60, 40, 70)

Binary Search Tree v.s. 2-3 Tree



- Delete

1. 先找到要刪除資料的位置(樹葉)

2. 刪除資料

3. 若此樹葉只包含一筆資料, 則完成.

4. 若此樹葉內沒有資料了, 則

(a). 重新分配 = retain the tree structure

(b). 合併樹葉 = its parent has one less child

二者取決於兄弟節點有幾個 key (幾筆資料)

Q: 遇到要刪除內部節點, \rightarrow 和中序中的下一個樹葉節點交換
 \rightarrow 要刪的節點換到樹葉

□ 2-3-4 Tree

- Are general trees, not binary trees.

- Are never taller than a 2-3 tree

- Insert = 和 2-3 樹基本相同, 搜尋中遇到某節點包含 3 筆資料, 則先分離 (用空間換時間 \rightarrow 不用搬回上去)

• Splits occur only at the path from the root to a leaf.

• No upward recursion is needed.

- Delete = 和 2-3 樹基本相同, 搜尋中遇到某節點只有 1 筆資料, 則先合併。

• Transformations occur only at the path from the root to a leaf.

• No upward recursion is needed.

□ Search

- The efficiency depends on the tree height.
- All leaves are at the same level.

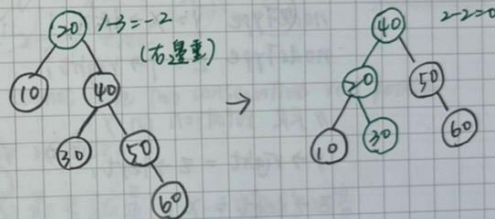
(2-3 tree, 2-3-4 tree)

二元樹無法控制樹高

單元四、由上而下成長的平衡樹

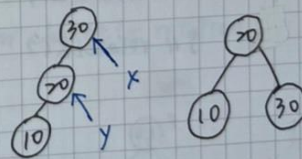
□ AVL tree

- A balanced search tree
- Maintains the tree height close to the minimum.
- Main idea: After each insertion or deletion
 1. Check whether the tree is still balanced.
 2. If the tree is unbalanced, rotate to restore the balance.
 - single rotation
 - double rotation



- LL Rotation = Pseudocode

```
nodeType rotateLL (nodeType x) {
    nodeType y = x->left;
    x->left = y->right;
    y->right = x;
    return y;
} // rotateLL()
```



- LR Rotation = Pseudocode

// first rotate left child with its right child = RR

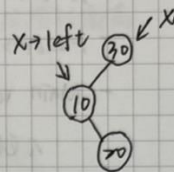
// then, rotate node x with its new left child: LL

```
nodeType rotateLR(nodeType x) {
```

```
    x->left = rotateRR(x->left);
```

```
    return rotateLL(x);
```

```
} // rotateLR
```



→ Version I.

```
nodeType rotateLR(nodeType x) {
```

```
    nodeType y = x->left;
```

```
    nodeType z = y->right;
```

```
    // RR rotation on y
```

```
    y->right = z->left;
```

```
    z->left = y;
```

```
    x->left = z;
```

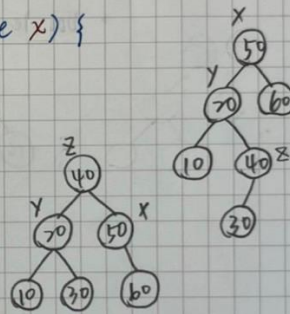
```
    // LL rotation on x
```

```
    x->left = z->right;
```

```
    z->right = x;
```

```
    return z;
```

```
} // rotateLR()
```



→ Version II.

```
nodeType rotateLR(nodeType x) {
```

```
    nodeType y = x->left;
```

```
    nodeType z = y->right;
```

```
    y->right = z->left;
```

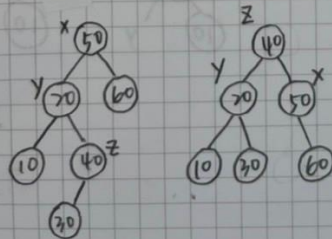
```
    x->left = z->right;
```

```
    z->right = x;
```

```
    z->left = y;
```

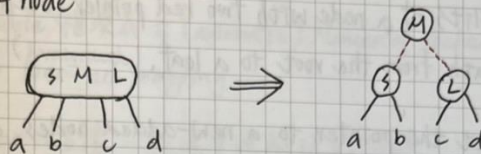
```
    return z;
```

```
} // rotateLR()
```

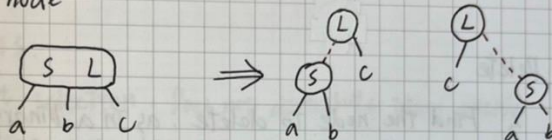


□ Red-black Tree

- 4 node



- 3 node



- Every external path has an equal number of black pointers.
⇒ Height of 2-3-4 tree

- External path cannot have two consecutive red pointer
⇒ Rotation to recover

- 紅黑樹樹高最多就是 2-3-4 樹的 2 倍

- 紅黑樹的搜尋有可能會比較慢, 但新增刪除會比較快

□ Splitting a 4-node

1. parent is a 2-node : Only change the colors.
2. parent is a 3-node : Color change (+ rotation)

□ Insertion =

1. Splits of a node with two red pointer occur only on the path from the root to a leaf, (downward)
2. Set the pointer to a new-added nodes as red.
3. Rotate if there are two consecutive red pointers.

□ Delete

1. Find the node to delete, as in a binary search tree
 - two children → swap with the In-order successor
 - only one child → pointed to by a black pointer
 - Leaf → pointed by a red or black pointer.
2. Replace the node of only one child with its child
3. Delete the leaf if the pointer to it is red
4. Recolor or rotate → Leaf pointed by a black pointer
black pointer → have a sibling

單元五、雜湊 Hashing

- Hash table 雜湊表 \cong (automatic) Manager of lockers
key \rightarrow location mapping.
- Collision 碰撞 \rightarrow 雜湊必須解決的問題
 - 不同的key 對應到同一個location
- Functions.
 - Digit selection: Does not distribute item evenly.
 - Folding: Involves the entire search key.
 - Modulo arithmetic 取餘數: 用質數 prime 可避免太多 collision.
 - Converting character strings: 轉換成數字.

□ Collision Resolution

1. Open Address *hash table 最好用質數!!!

- key independent
- a. Linear probing: search the first location and the next.
 - Problem: 櫥位亂塞, 且當資料黏在一起, 要搜尋很多筆數
 - Delete: 刪掉某筆資料, 下次搜尋遇到空隔, 不代表不用繼續找
 - b. Quadratic probing: search the first location and continue at the increments of 1, 2, 3, ... (平方)

- key dependent
- Problem: 空間問題 \rightarrow overflow area (可能有空間, 但找不到)
 - 也有群聚問題, 但沒 Linear probing 嚴重

2. Double hashing: 用 2 個 hash function 來減少 collision.

- ($h_1 \neq h_2$ 且 $h_2(\text{key}) \neq 0$) \rightarrow step size
- table size v.s. step size 要互質, 才能用到每個空間
- 質數.

2. Restructuring the hash table = 允許多個 item 存在同一格.

a. Buckets

- Each location in the hash table is array.
- 若還有碰撞, 則前面三種方式擇一.

b. Separate chaining

- Each hash table location keep a linked list.
- The size of the hash table is dynamic.

c. Buckets + separate chaining.

- Memory management (把空掉的空間拼起來)
- Worst case: $O(n)$

□ Inefficient Operations on Hashing. 不適合用雜湊

- Traversal: sorted order
- NN Search: Find the smallest or largest item ex: Heap...
- Range Query: Find the item between two search key. ex: tree...

□ Application 1: Hash Join.

- Join is an important operation in database.
(在不同群的资料中, 找交集或特定资料)

□ Application 2: Count-Min Sketch

- 在资源有限的情況下, 紀錄下某些資料. (可能有一些錯誤).
- 應用: Hot IPs / users / files / words (熱門 → 超過某些門檻)

Python 不專業筆記

一、字串 string `phrase = "Hello Mr. White"`

- 相關函式
 - `phrase.lower()` # 字串變小寫
 - `phrase.upper()` # 字串變大寫
 - `phrase.islower()` # 判斷是否小寫
 - `phrase.index("H")` # H 所在 index
 - `phrase.replace("H", "h")` # 交換

• 相關知識 - 可直接表示 `phrase[0]` # 像陣列

二、讀入 `name = input("請輸入你的名字:")`

輸出 `print("哈囉" + name + "你今年" + age + "歲")`

三、列表 list (陣列)

`scores = [90, 70, 60, 50, 40]`

`friends = ["小黑", "小黃", "小綠"]`

`things = [90, "小黑", true]`

- 相關函式
 - `scores.extend(friends)` # `scores + friends`
 - `scores.append(30)` # `scores` 加一筆 30
 - `scores.insert(2, 50)` # 在第 2 格插入 50
 - `scores.remove(90)` # 刪除 90
 - `scores.clear()` # 清空 `scores`
 - `scores.pop` # 移除最後一位
 - `scores.sort()` # 排列
 - `scores.reverse()` # 反轉
 - `scores.count(60)` # 有幾筆 60

四. 元組 tuple scores = (90, 80, 70, 60, 50)

· 相關知識: 不可更改裡面的值, 不可新增, 刪除.

五. 函式 function

```
def hello(name, age)
    print("hello" + name + "你今年" + str(age) + "歲")
```

```
hello("小白", 87) # 呼叫函式
```

六. 判斷句

```
if score == 100:
    print("超棒")
elif score >= 60:
    print("普通")
else:
    print("不及格")
```

七. 字典 dictionary

```
dic = {0: "apple", 1: "banana", 2: "cat", 3: "dog"}
print(dic[3]) // output: dog.
```

八. for 迴圈

```
for letter in "小白你好"
    print(letter) // output = 小白你好
```


九、檔案讀取、寫入

```
open("檔案路徑", mode="開啟模式")
```

絕對路徑

相對路徑 以程式的位置做延伸

mode="r" 讀取

mode="w" 覆寫

mode="a" 在原先的資料後寫東西。

```
file = open("123.txt", mode="r")
```

```
print(file.read())
```

```
file.close()
```

十、類別 class & 物件 object

```
class Phone =
```

```
def __init__(self, os, number, waterproof) =
```

```
self.os = os
```

```
self.number = number.
```

```
self.waterproof = waterproof.
```

```
phone1 = phone("ios", 123, True)
```

十一、繼承 inheritance

```
from person import Person # 先引入模組
```

```
class Student(Person): # 寫法.
```

```
def __init__(self, name, age, school) =
```

```
≡
```

```
def print_school(self) =
```

```
≡
```