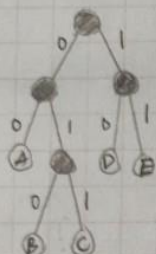


優點:



② Encoder

A: 60 D: 10
B: 010 E: 11
C: 011

Text:

EAEBAE CDEA

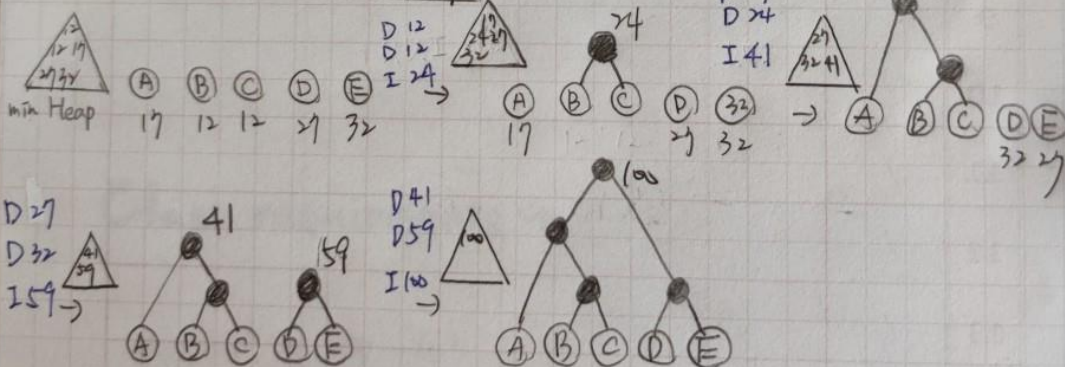
1. Average code length is minimized.
2. 出現 Frequency 高, 代碼長度短, 反之較長 \rightarrow heap
3. a priority queue can be used to build the binary tree

1166 1181066 11611 101160

② When use heap?

Character	A	B	C	D
Frequency	17	12	12	27

特別：此 binary tree 從樹葉開始做！



※ Heap 是為了做出 \uparrow binary tree 所需要使用的資料結構
[資料結構]

My Questions

Problems & Difficulties needing exploration

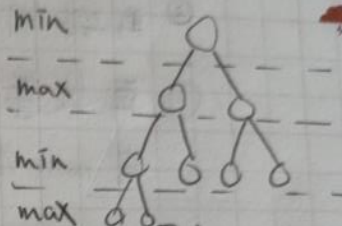
Double-ended Priority Queues [同時可知 Max, Min]

- Min-Max heap
- Double-end heap (Deap)

My learning
weather report



Min-Max heap



Insert: ① 檢查此節點所在 (Min or Max 層)

$$\text{level} = (\text{int}(\log_2(i+1)) \% 2) \rightarrow \begin{matrix} 0 & \text{Min 層} \\ 1 & \text{Max 層} \end{matrix}$$

② 與其父節點比較大小

若此節點在 Max 層, 而父節點大於此節點, swap

My Opinions

Thoughts, inspirations, and suggestions

$$\text{father} = (i-1)/2$$

③ 與其祖父節點比較大小

$$\text{if}((i-1)/2 > 2) \quad \text{grandfather} = ((i-3))/4$$

Delete: 1. Delete smallest (刪root) / Largest (看刪左)

① 與其兩 child 比較, 是否要換

② 在往下跟子祖比較

$$\text{grandchildren} = (i*4) + j$$

$$j = 3, 4, 5, 6$$

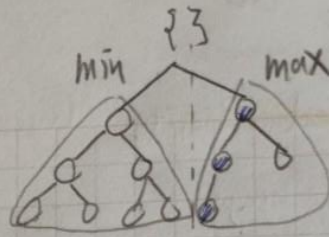
在對的時間, 做對的事,
以表明對人的重視。

《培基文教》

My Notes

Important Concepts worth keeping

Heap



Insert: ① 先找此 node 在 Min or Max 上

$$\text{middle} = \text{int}(\log_2(i+1))/2$$

② 再與相對應節點比較

③ 再依序往上檢查至 smallest or Largest,
若 node 有與其父節點 大 or 小就 swap.

Delete: ① 走至樹葉

② 與對應節點比較

③ 若交換,再往上依序檢查要不要換

Forest (union) of Heaps

- Binomial heap = 項式堆積
- Fibonacci heap

Binomial tree of order k

The root has k children

Merge by two binomial tree of order $k-1$

Number of nodes: 2^k

Tree height = $k+1 \Rightarrow O(\log n)$

$C_i^k = \frac{k!}{i!(k-i)!}$ node at level i , for $i=0, \dots, k$

$\hookrightarrow 13 = (1101)_2$
 $= 2^3 + 2^2 + 0 + 2^0$
not \rightarrow

Example:

Binomial tree of order 4 (B_4)?

level 0: $C_0^4 = 1$

level 1: $C_1^4 = 4$

level 2: $C_2^4 = 6$

level 3: $C_3^4 = 4$

level 4: $C_4^4 = 1$

① 5 levels

② level 2 = 4 nodes

③ total $2^4 = 16$ nodes

不要為明天憂慮，
因為明天自有明天的憂慮，
一天的難處一天當就夠了。

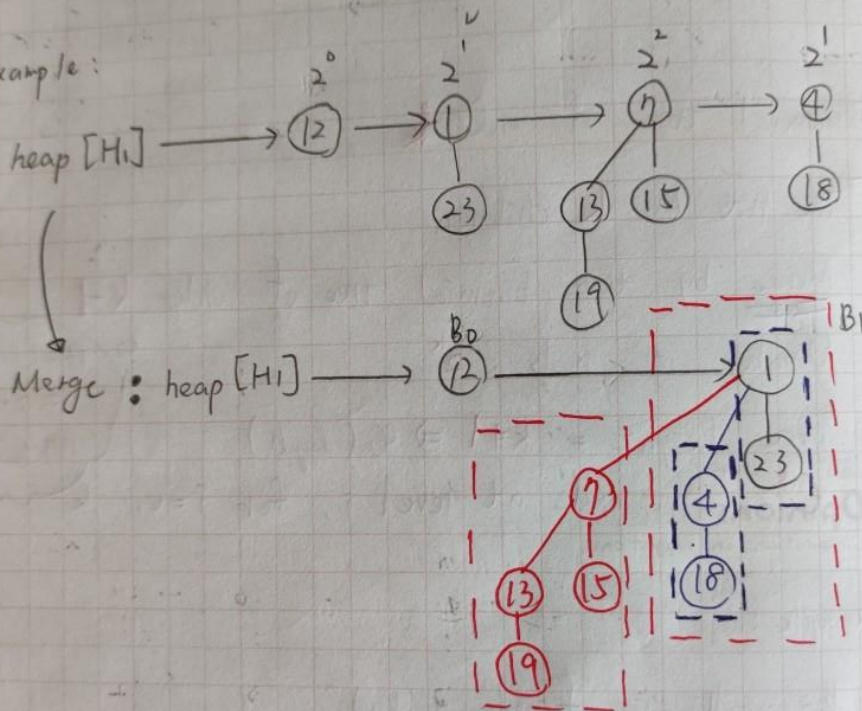
《新約聖經》

密碼
cipher key

Merge

- ① 每個 root 都是 1 個 min heap
- ② Two binomial tree of the same order can be Merged ✓

Example:



Example:

What does a binomial heap of "26" node looks like?

$$26 = (11010)_2 = 2^4 + 2^3 + 0 + 2^1 + 0 \quad (3 \text{ root})$$

$$200 = (11001000)_2 = 2^7 + 2^6 + 2^3 \quad (3 \text{ root})$$

$$\begin{array}{r} 2 \overline{) 200} \quad -0 \\ \underline{100} \\ 2 \overline{) 100} \quad -0 \\ \underline{50} \\ 2 \overline{) 50} \quad -0 \\ \underline{25} \\ 2 \overline{) 25} \quad -1 \\ \underline{12} \\ 2 \overline{) 12} \quad -0 \\ \underline{6} \\ 2 \overline{) 6} \quad -0 \\ \underline{3} \\ 2 \overline{) 3} \quad -1 \\ \underline{1} \end{array}$$

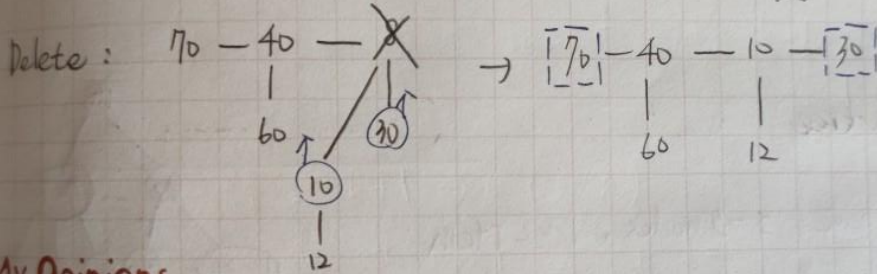
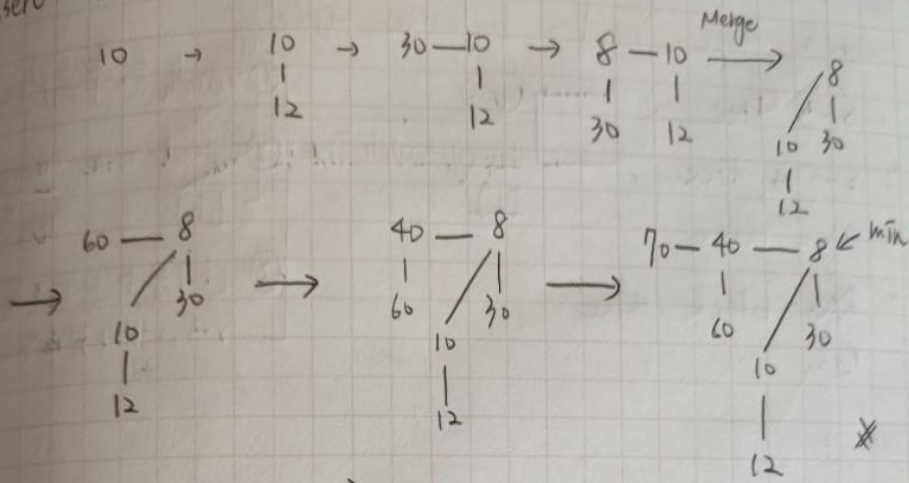
So in everything, do to others what you would have them do to you...

(New Testament)

My Questions

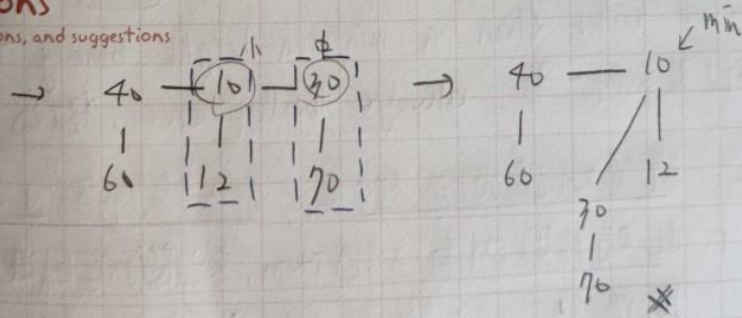
Problem & Difficulties needing exploration

Insert: 10, 12, 30, 8, 60, 40, 70



My Opinions

Thoughts, inspirations, and suggestions



所以無論何事，
你們想要人怎樣待你們，
你們也要怎樣待人。
《新約聖經》

search Tree and balance tree

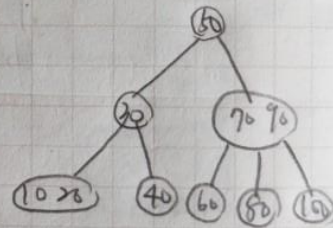
- 2-3 tree
- 2-3-4 tree
- AVL tree
- Red-black tree

V.S
binary search Tree
[b]

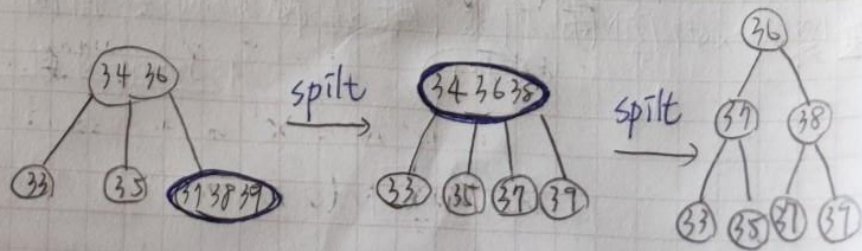
- ① search 效率高於 [b]
- ② need more storage than [b] 儲存空間
- ③ [b] 是由上往下長
2-3, 2-3-4 是由下往上長

2-3 tree

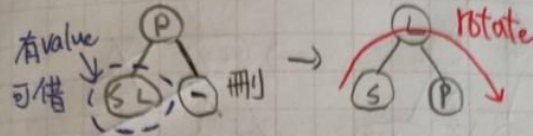
1. have 2~3 nodes, 1~2 item
2. never taller than a minimum-height binary tree $\lceil \log_2(n+1) \rceil$
3. 樹葉在同一層, always balance, 樹高一樣



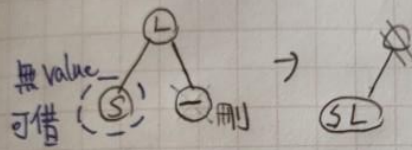
Insert: 新資料遞迴至樹葉, 加入該節點並 sorted, 若此時節點內有 3 個 item, 就依序往回進行 split()



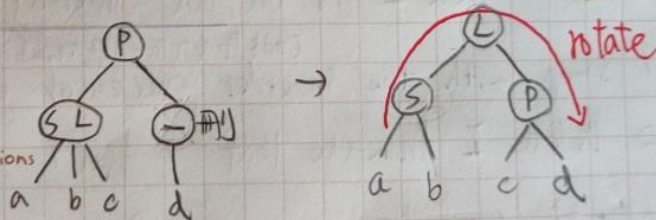
Delete : (a) Redistribute values → 當 Delete item 位於 "樹葉"
重新分配



(b) Merge into a leaf → " " " " " "



(c) Redistribute values and child → " 位於 "中間"
重新分配



My Opinions

Thoughts, inspirations, and suggestions

(d) Merge into internal node → 位於 "root"

步驟: 1. Delete I from the leaf

2. If the leaf now contain one item, you are done

3. else, choose one of the following operation to fix

(a) Redistribute values and child

(b) Merge into a leaf

(d) Merge into internal node

密碼
cipher key

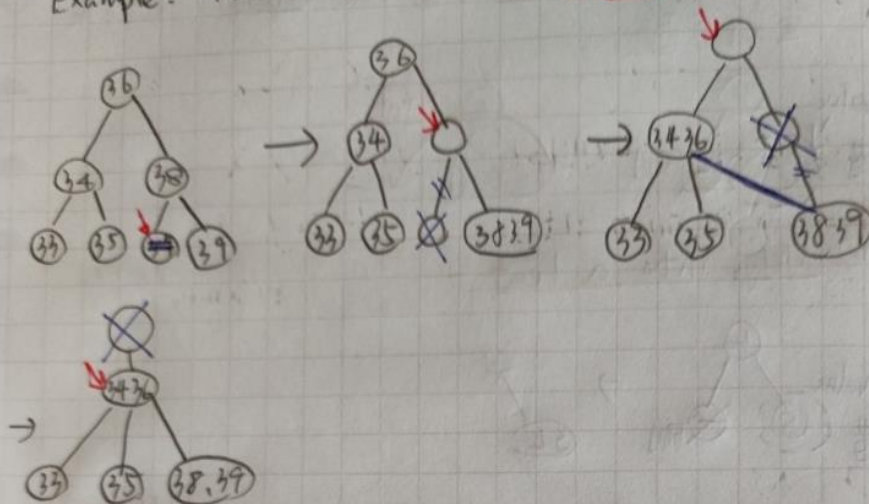
終始功：以終為始
— 史蒂芬·柯維

My Notes

Important Concepts worth keeping

Today: / /

Example: Delete item 37 [若此即點 item 為 empty 就等於 fix()]



② What if item I is on an internal (內部) node?

1. Swap with the in-order successor on leaf [排序的下一個 item]
2. Delete I from the leaf \rightarrow fix()

deleteItem(in ttree, in thekey) {

X = the tree node whose search key equals thekey;

if (X is not leaf)

Y = successor(X);

swapKey(X, Y);

Delete key from X;

if (X now has no item) fix(X);

10 "We recognize individual differences with respect to talents, character, capability, and background. We believe that full development of one's potential signifies success"

(CYCU's Education Philosophy)

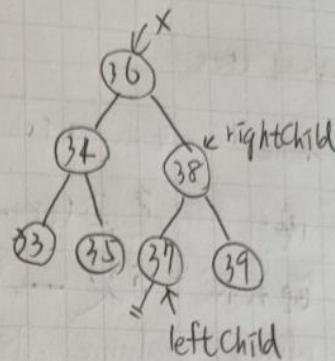
My Questions

Problems & Difficulties needing exploration

```

Ptr Successor (Ptr X) {
    rightChild = X -> right;
    leftChild = rightChild -> left;
    while (leftChild -> left != NULL)
        leftChild = leftChild -> left;
    return leftChild;
}

```



My learning
weather report



void fix (Ptr X)

if (X == root) remove the root;

else

p = parent of X;

if (the nearest sibling of X has two items)

Redistribute items among the sibling p and X;

if (X is not leaf)

Move appropriate child from sibling to X;

else // merge

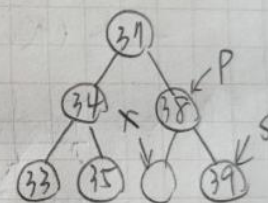
S = the nearest sibling of X;

Move appropriate item down from p to S

if (X is not leaf) Move X's child to S;

remove X;

if (p now has no item) fix(p);



密碼
cipher key

我們了解人人各承不同之稟賦，
其性格、能力與環境各異，
故充分發揮個人潛力就是成功。

《中原大學教育理念》

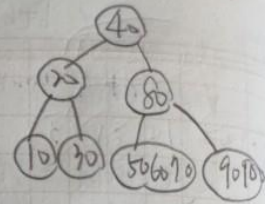
My Notes

Important Concepts worth keeping

Today: / /

2-3-4 tree

1. have ~4 nodes (child nodes), 1~3 item



2. 優化不用遞迴 (比 2-3 tree 好), 往下只要遇到 3 個 item 的

節點就先分裂 — 節省新增時間; 但樹高變高, search 會變慢

3. always balance, 樹高一樣

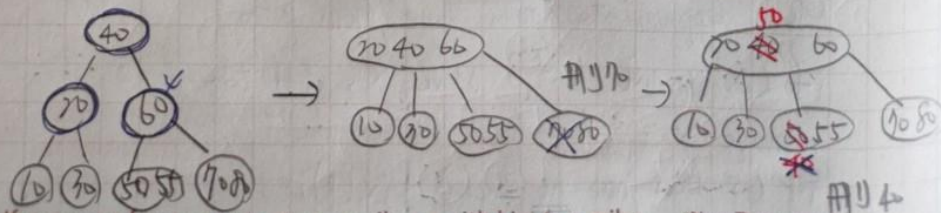
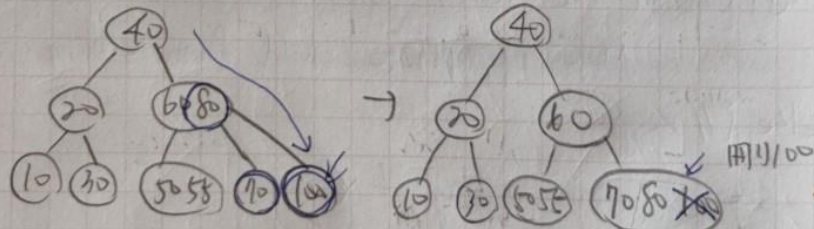
Insert: Splits occur only at the path from the root to leaf, finally at leaf add item (downward)

(No upward recursion is needed!)

Delete: ^{Merge} Transformation occur only at the path from root to leaf (downward) → 此節點只有 1 個 item 就 Merge

(No upward recursion is needed!)

Ex: Delete 100, 70, 40



AVL tree

1. 樹高可能差 1, balance

- Insert:
- ① +2, 0, +1 rotate LL()
 - ② +2, -1 rotate LR()
 - ③ -2, 0, -1 rotate RR()
 - ④ -2, +1 rotate RL()

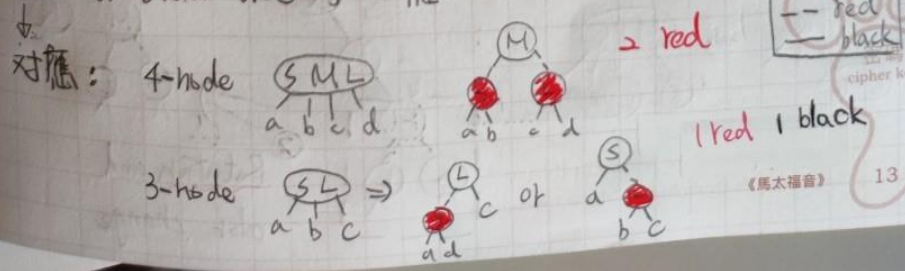
Delete: 此節點, ^{if} 只有一子節點, 將其往上提
^{else} 有 2 子節點, 找 in-order successor 往上提
 再往回依序檢查是否平衡, 不平衡則旋轉!

My Opinions

Thoughts, inspirations, and suggestions

Red-black tree (一種 binary search tree)

- 1. has the advantage of a 2-3-4 tree, without the storage overhead
- 2. notation like AVL tree [為 3 balance]
- 3. Red-black tree 可對應至少一個 2-3-4 tree

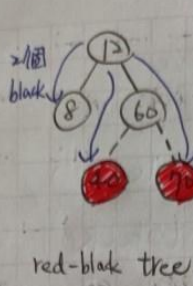
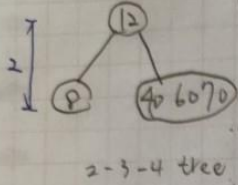


My Notes

Important Concepts worth keeping

Today: / /

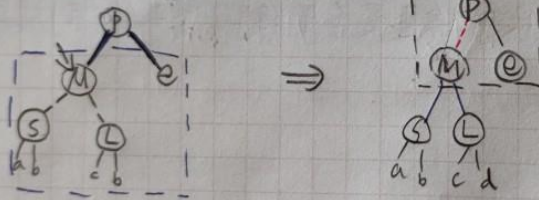
對應性質:



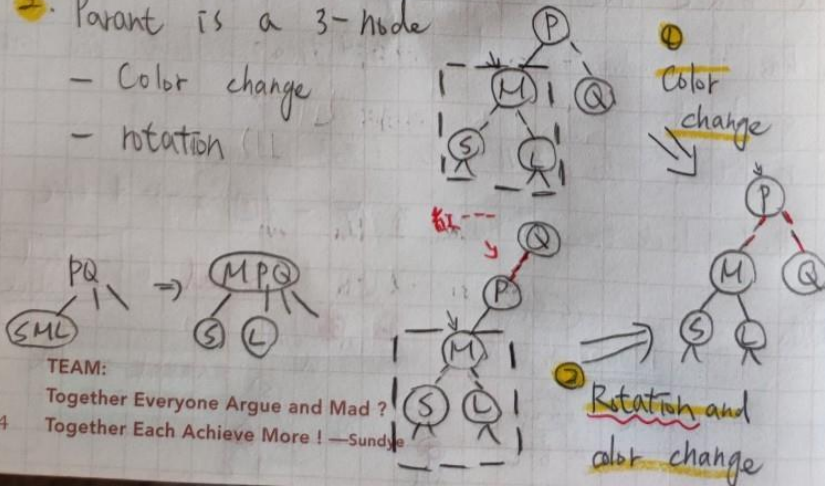
1. Every external path has an equal number of black pointers
2. black pointer 數量 = 2-3-4 tree 的高度

★ Split on red black tree

1. Parent is a 2-node
- Only change the colors



2. Parent is a 3-node
- Color change
- rotation



TEAM:

Together Everyone Argue and Mad?

Together Each Achieve More! —Sundye

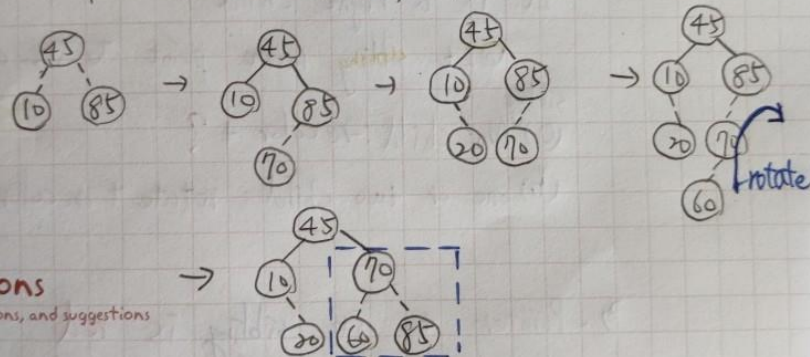
My Questions

Problems & Difficulties needing exploration

My learning
weather report

- Insert:
1. Splits of a node with two **red** pointers
(like the 4-node in 2-3-4 tree) occur only on path from the root to leaf (downward!)
 2. Set the pointer to a new-add node as **red**
 3. Rotate if there are two consecutive (連續) **red** pointer

Example: 45, 85, 10, 70, 20, 60



My Opinions

Thoughts, inspirations, and suggestions

- Delete:
- ① two children → swap with the in-order successor
 - ② one children → pointed to by a black pointer
child 指向
 - ③ leaf → ① pointed to by a **red** pointer
delete the leaf

- ② pointed to by a black pointer

★ - 一定 have sibling

三 ↓ 種

密碼
cipher key

團隊成效可好可壞，
運作之妙在於一心。

My Notes

Important Concepts worth keeping

Today: / /

1. Pointer to its parent is **red**

It is a right child

@ It sibling must point to its child by **red**

sibling

(a) No child: recolor

(b) one or two child: rotate + recolor

2. Pointer to its parent is **black**

It is a right child

@ It sibling must point to its child by **red**

sibling

(a) No child: recolor + ?

(b) one or two child: rotate + recolor

3. Pointer to its sibling is **red**

It is a right child

@ It sibling must have to **black** pointer

@ LL rotation + recolor

sibling

(a) No child: ok

(b) one child: rotate

(d) Two child: rotate + recolor