Quantum Mechanics

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Chapter 1

The wave function

1.1 The Schrodinger equation

Imagine a particle of mass m, constrained to move along the x-axis, subjet to some to specifies force $F(x,t)$. The program of *classical* mechanics is to determine the position of the particle at any give time: $x(t)$. Once we know that, we can figure out the velocity ($v = \mathrm{d}x/\mathrm{d}t$), the momentum ($p = mv$), the kinetic energy ($T = (1/2)mv^2$), or any other dynamical variable of interest. And how do we go about determining $x(t)$? We apply Newton's second law: $F = ma$. (For *conservative* systems–the only kins we shall consider, and, fortunately, the only kins that *occur* at the microscopic level–the force can be expresed as the derivative of a potential energy function [1], $F = -\partial V/\partial x$, and Newton's law reads $m\mathrm{d}^2x/\mathrm{d}t^2 = -\partial V/\partial x$) This, together with appropiate initial conditions (typically the position and velocity at $t = 0$), determines $x(t)$.

Quantum mechanics approaches this same problem quite differently. In this case what we're loocking for ir the particle's **wave function**, $\Psi(x,t)$, and we get it by solving the **Schrodinger equaation**:

$$\boxed{i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi} \tag{1.1}$$

Schrodinger equation

This equation plays a role logically analogous to Newton's second law: Given suitable initial conditions (typically, $\Psi(x,0)$), the Schrodignes equation determines $\Psi(x,t)$ for all future time, just as, in classical mechanics, Newton's law determines $x(t)$ for all future time.

1.2 The statistical interpretation

What exactly is this "wave function"? Born's **statistical interpretation** of the wave function, says that $|\Psi(x,t)|^2$ gives the **probability** of finding the particle at point x, at time t, or more precisely

$$\boxed{\int_a^b |\Psi(x,t)|^2 \mathrm{d}x = \text{probability of finding the particle between } a \text{ and } b\text{, at time } b} \tag{1.2}$$

Statistical interpretation

[1]Magnetic forces are an exception, but let's not worry about them just yet. By the eay, we shall assume for the moment that the motion is nonrelativistic ($v << c$)

Probability is the *area* inder the graph of $|\Psi(x,t)|^2$

Yhe statistical interpretation introduces a kind of **indeterminacy** into quantum mechanics, for even if you know evertything the theory has to tell you about the particle /to wit: its wave function, still you cannot predict with certainty the outcome of a simple experiment to measure its position– all quantum mechanics has to offer is *statistical* information about the *possible* results.

We say that the wave function **collapses**, upon measurement, to spike some point. There are, then, two entirely distinct kinds of physical processes: "ordinary" oenes, in which the wave function evolves in a leisurely fashion inder the Schrodinger equation, and "measurements", in which Ψ suddenly and discontinously collapses [2].

1.3 Probability

I will give some definitions. The total number is

$$N = \sum_{j=0}^{\infty} N(j) \tag{1.3}$$

The probability of getting j is

$$P(j) = \frac{N(j)}{N} \tag{1.4}$$

In particular, the sum of all the probabilities is 1:

$$\sum_{j=0}^{\infty} P(j) = 0 \tag{1.5}$$

In general, the average value of some *function* of j es given by

$$\boxed{< f(j) >= \sum_{j=0}^{\infty} f(j)P(j)} \tag{1.6}$$

Average value of a function

In quantum mechanics the average is usually the quantity of interest; in that context it has come to be called the **expectation value**.

Now we need a numerial measure of the amount of "spread" in a distribution, with respect to the aveerge. The most obvious way to do this would be to find out how far each individual deviates from de average,

$$\Delta j = j- < j > \tag{1.7}$$

and compute the average of Δj. Trouble is, of course, that you get zero, by the nature of the avergae, Δj is as often negative as positive. To avoid this irritating problem you might decide

[2]The role of measurement in quantum mechanics is so critical and so bizarre that you may well be wondering what precisely constitutes a measurement. Does it have to do with the interaction between a miscroscopic (quantum) system and a mecorscopic (classical) measuring apparatus (as Bohr insisted), or is it intervention of a conscious "oberver" (as Wigner proposed)? For the moment let's take the naive view: A measurement is the kind of thing that a scientist does in the laboratory, with rules, stopwatches, and so on

to average de *absolute value* of Δj. But absolute values are nasty to work with; instead, we get around the sign problem by *squaring* before averaging:

$$\sigma^2 \equiv < (\Delta j)^2 > \tag{1.8}$$

Variance

This quantity is known as the **variance** of the distribution; σ itsel is called the **standard deviation**.

There is a useful little theorem pn variances:

$$\sigma = \sqrt{< j^2 > - < j >^2} \tag{1.9}$$

1.4 Continous variables