Tensores

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Capítulo 1

Tensor Algebra

1.1. Introduction

To work effectively in Newtonian theory, one reallyneeds the lenguage of vectors. This lenguage, first of all, is mire succint, since it summarized a set of three equations in one. Moreoveer, the formalism of vectors helps to solver cartain problems more readly, adn, most important of all, the language reveals structure and thereby offers insight. In exactly the same way, in relativity theory, one needs the language of tensors. Again, the language helps to summarize sets of equations succintly and to solve problems more readly, and i reveals structure in the equaions. This part is devoted to learning the formalism of tensors shich is a pre-condition for the rest.

The approach we adopt is to concentrate on the technique of tensors without taking into account the deeper geometrical significance behind the theory. We shall be concerned more with whay you do with tensors rather than what tensors actually are. There are two distinct approaches to the teaching of tensors: the abstract or index-free (coordinate-free) approach and the conventional approach based on indices. There has been a move in recent years in some quarters to introduce tensors from the stars using the more mdodern abstract approach (although some have subsequantly changed their mind and reverted to the conventional approach). The main advantage of this approach is that it offers deeper geometrical insight. However, it has two disadvantages. First of all, it requieres much more of a mathematical background, which in turn takes time to develop. The other disadvantage is that, for all its elegance, when one wants to do real calculation with tensors, as one frequently needs to, then recourse has to be made to indices. We shall adopt the more conventional index approach, because ir will prove faster and more practical. However, we advise those who wish to take their study of the subject further to look at the index-free approach at the first oppoertunity.

1.2. Manifolds and coordinates

*** Quizás se podría complementar algo más ***

We shall start by working with tensors defined in n dimensions since, and it is part of the power of the formalism, there is little extra effort involved. A tensor is an objet defined on a geometric entry called a (differential) **manifold**. We shall no define a manifold precisely because it would involve us too much of a digression. But, in simple terms, a manifold us simething which 'locally' looks like a bit of n-dimensional Euclidean space $\mathbb{R}^n$

We shall simply take an n-dimensional manifold M to be a set of points such that each point posseses a set of n **coordinates** $x^1, x^2, ..., x^n$, where each coordinate ranges over a subset of the reals, which may, in particular, range from $-\infty$ to $+\infty$. To start off with, we can think of these coordinates as corresponding to distances or angles in Euclidean space.

1.3. Curves and surfaces

Given a manifold, we shall be concerned with points in it and subsets of points which define curves and surfaces of diffeerent dimensions. We shall frequently define these curves and surfaces **parametrically**. Thus (in exactly the same way as is done in Euclidean 2- and 3-space), since a curve has one degree od freedom it dependes on one parameter and so we define a **curve** by the parametric equations

$$\boxed{x^a = x^a(u) \quad (a = 1, ..., n)} \tag{1.1}$$