# Deliverable 2

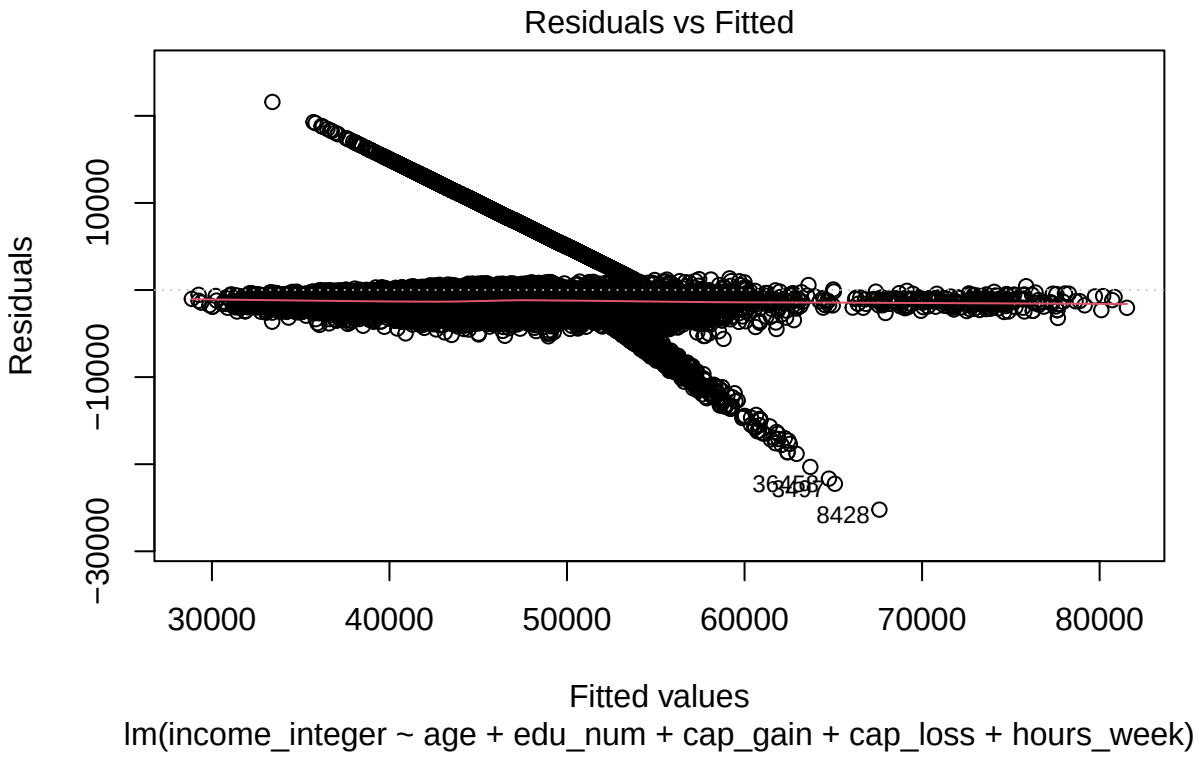
2025-05-01

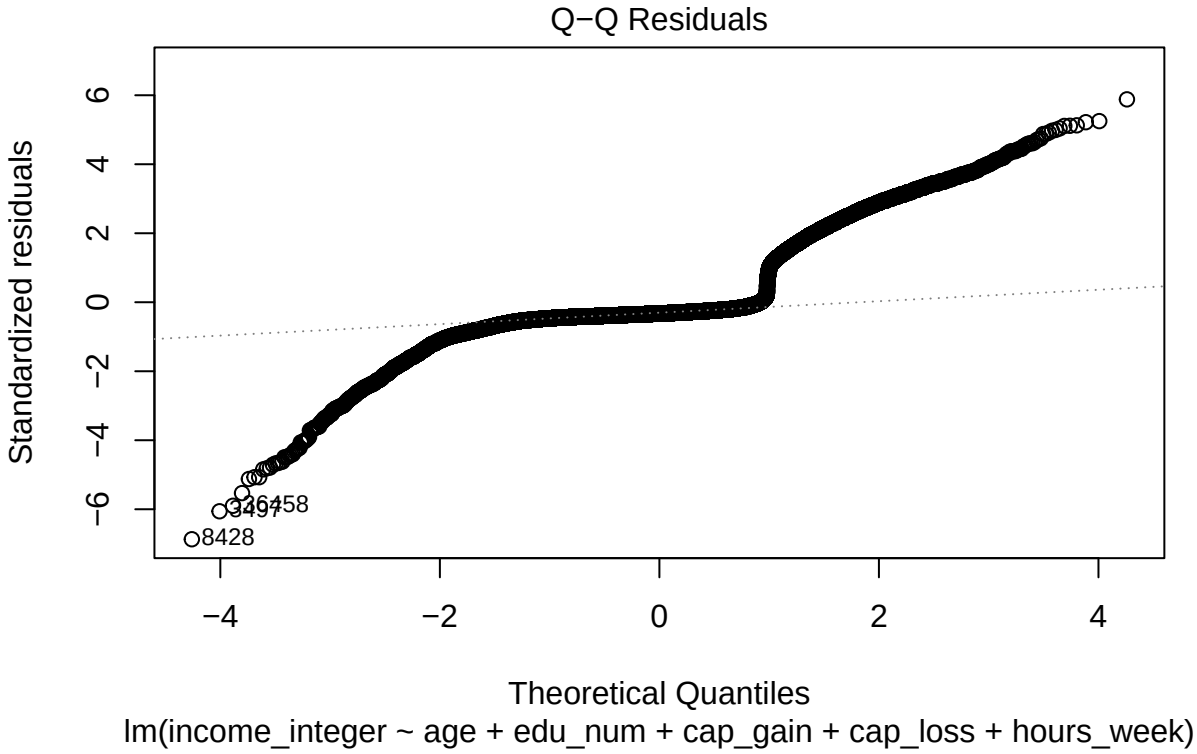
```
setwd("~/Escritorio/ADEI/D2")
dd <- read.csv("adult_def.csv")
```

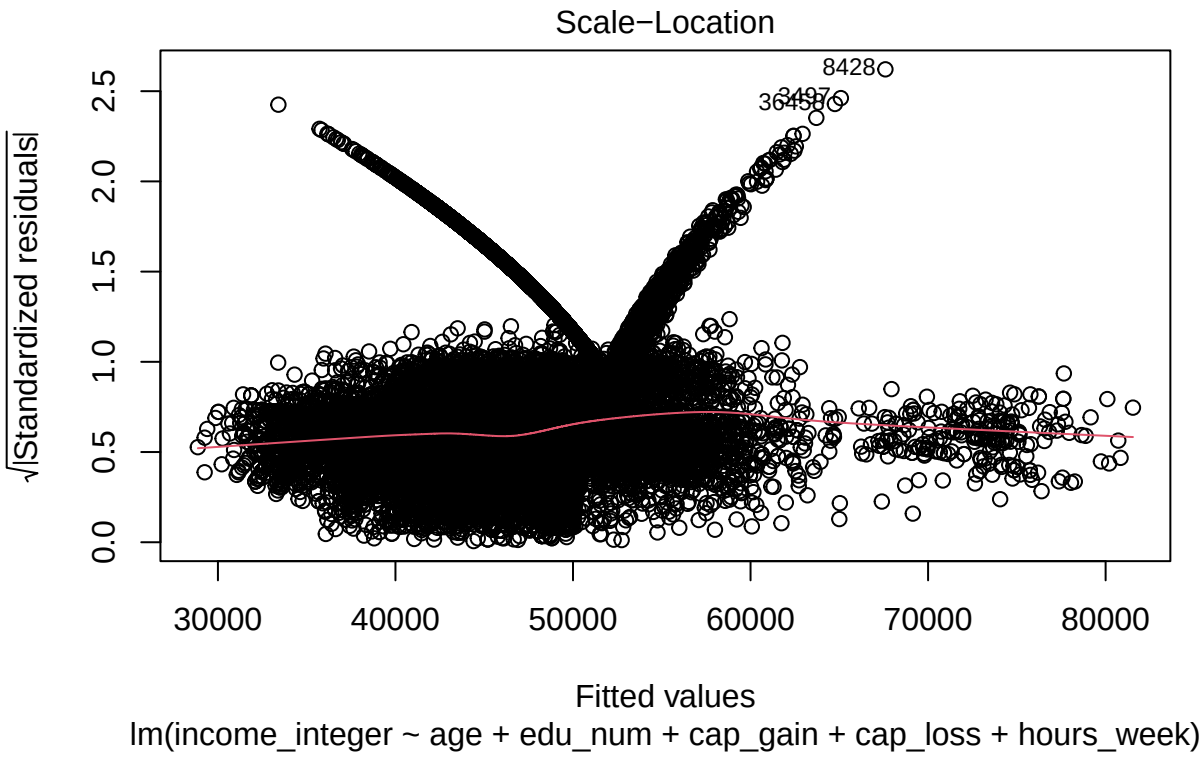
```
initial_model <- lm(income_integer ~ age + edu_num + cap_gain + cap_loss + hours_week, data = dd)
summary(initial_model)
```

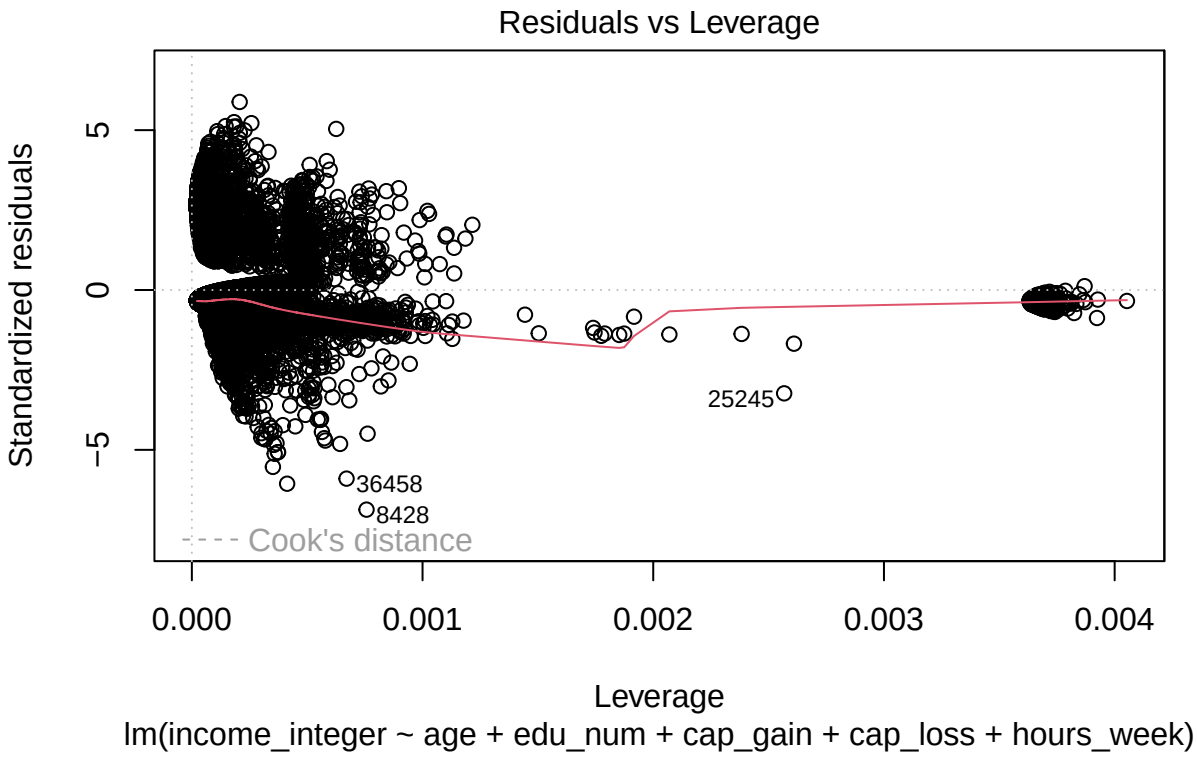
```
##
## Call:
## lm(formula = income_integer ~ age + edu_num + cap_gain + cap_loss +
##     hours_week, data = dd)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -25216.0  -1522.3  -1201.8   -699.4  21594.2
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 2.091e+04  9.221e+01  226.74   <2e-16 ***
## age         2.317e+02  1.220e+00  189.87   <2e-16 ***
## edu_num     1.144e+03  6.595e+00  173.42   <2e-16 ***
## cap_gain    2.003e-01  2.260e-03   88.60   <2e-16 ***
## cap_loss    7.848e-01  4.151e-02   18.91   <2e-16 ***
## hours_week  9.924e+01  1.362e+00   72.87   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3672 on 48836 degrees of freedom
## Multiple R-squared:  0.6683, Adjusted R-squared:  0.6682
## F-statistic: 1.968e+04 on 5 and 48836 DF,  p-value: < 2.2e-16
```

```
plot(initial_model)
```

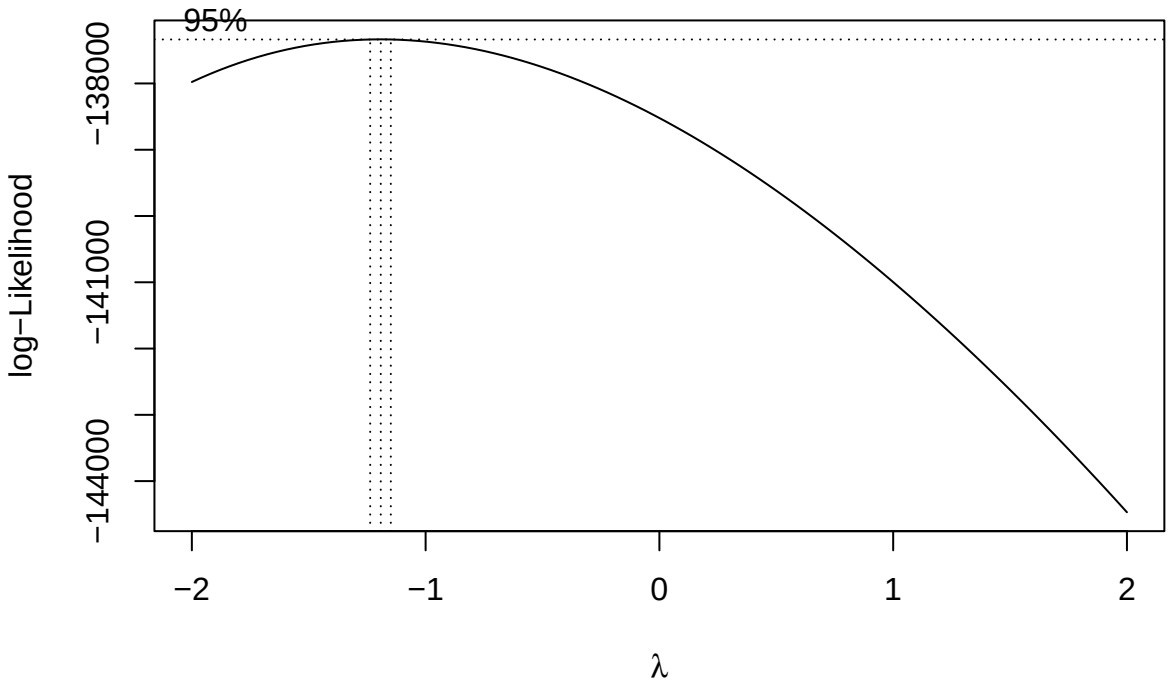

Residuals vs Fitted
Residuals
10000
−10000
−30000
30000
40000
50000
60000
70000
80000
8428
Fitted values
lm(income_integer ~ age + edu_num + cap_gain + cap_loss + hours_week)


Q−Q Residuals
Standardized residuals
6
4
2
0
−2
−4
−6
26458
3497
8428
−4
−2
0
2
4
Theoretical Quantiles
lm(income_integer ~ age + edu_num + cap_gain + cap_loss + hours_week)


Scale−Location
8428
3497
36458
√|Standardized residuals|
0.0
0.5
1.0
1.5
2.0
2.5
30000
40000
50000
60000
70000
80000
Fitted values
lm(income_integer ~ age + edu_num + cap_gain + cap_loss + hours_week)




```
#target variable transformation, so the normality assumption is met
boxcox(income_integer ~ age + edu_num + cap_gain + cap_loss + hours_week, data = dd)
```

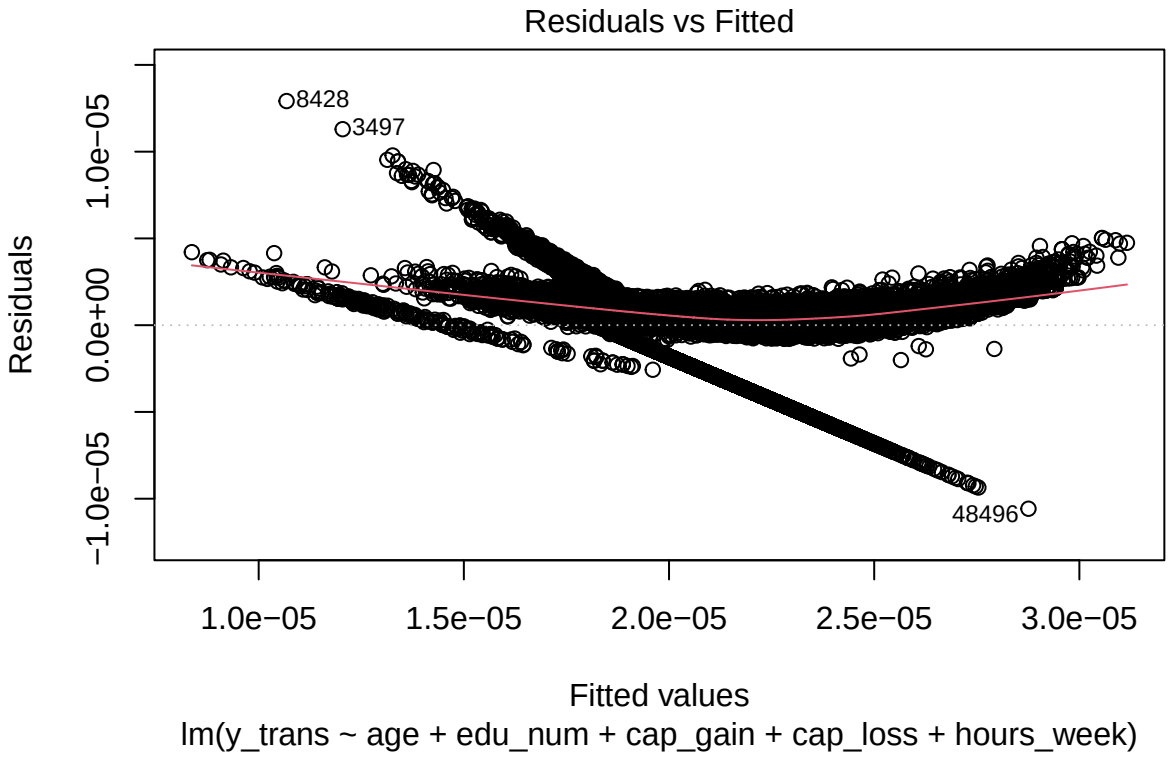


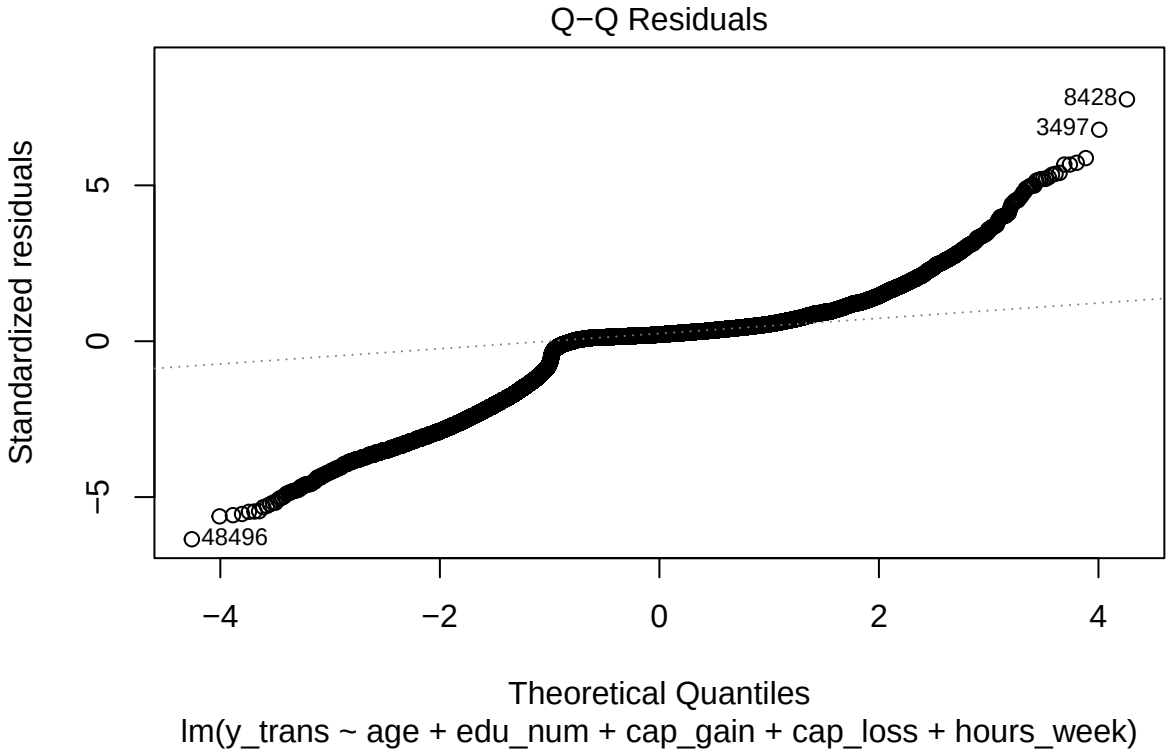

```
#given lambda is aproximatedly -1 we do the inverse transformation
y_trans <- 1 / dd$income_integer
transformed_model <- lm(y_trans ~ age + edu_num + cap_gain + cap_loss + hours_week, data = dd)
summary(transformed_model)
```

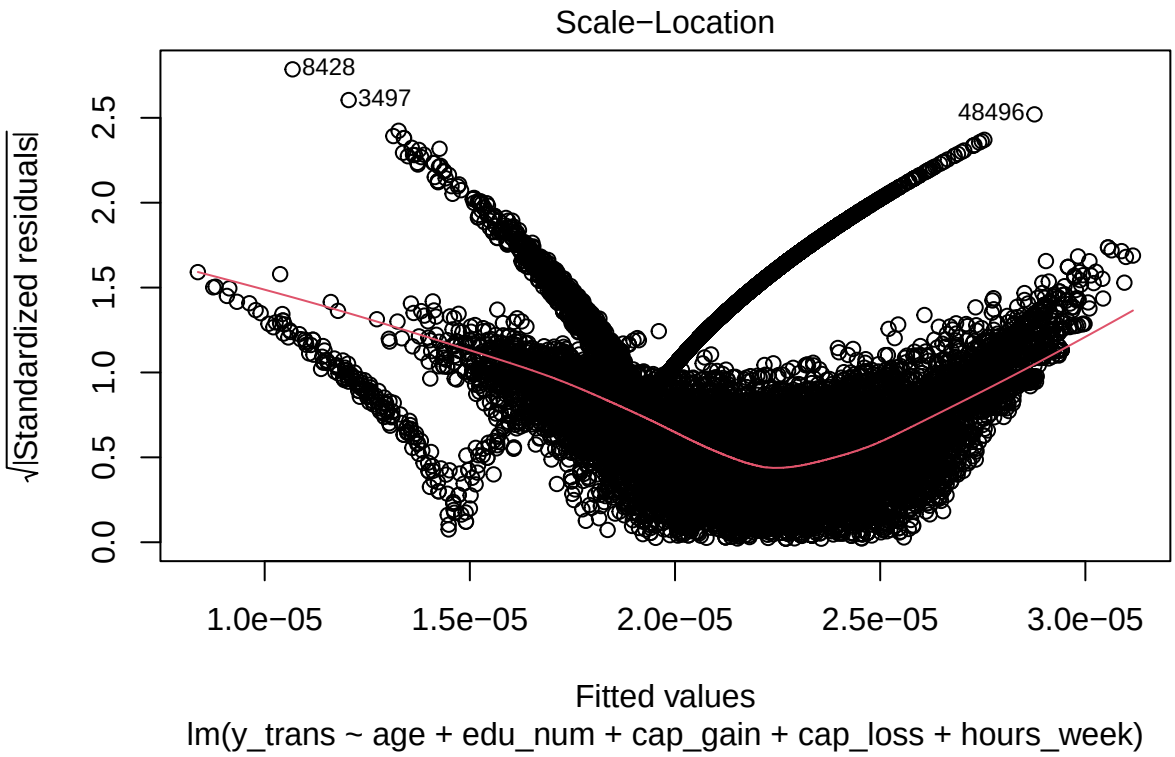
```
##
## Call:
## lm(formula = y_trans ~ age + edu_num + cap_gain + cap_loss +
##     hours_week, data = dd)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -1.058e-05  1.373e-07  3.552e-07  6.859e-07  1.292e-05
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.536e-05  4.182e-08  845.57   <2e-16 ***
## age         -1.221e-07  5.533e-10 -220.74   <2e-16 ***
## edu_num     -6.086e-07  2.991e-09 -203.48   <2e-16 ***
## cap_gain    -5.503e-11  1.025e-12  -53.68   <2e-16 ***
## cap_loss    -2.612e-10  1.883e-11  -13.88   <2e-16 ***
## hours_week  -5.219e-08  6.176e-10  -84.51   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.665e-06 on 48836 degrees of freedom
```

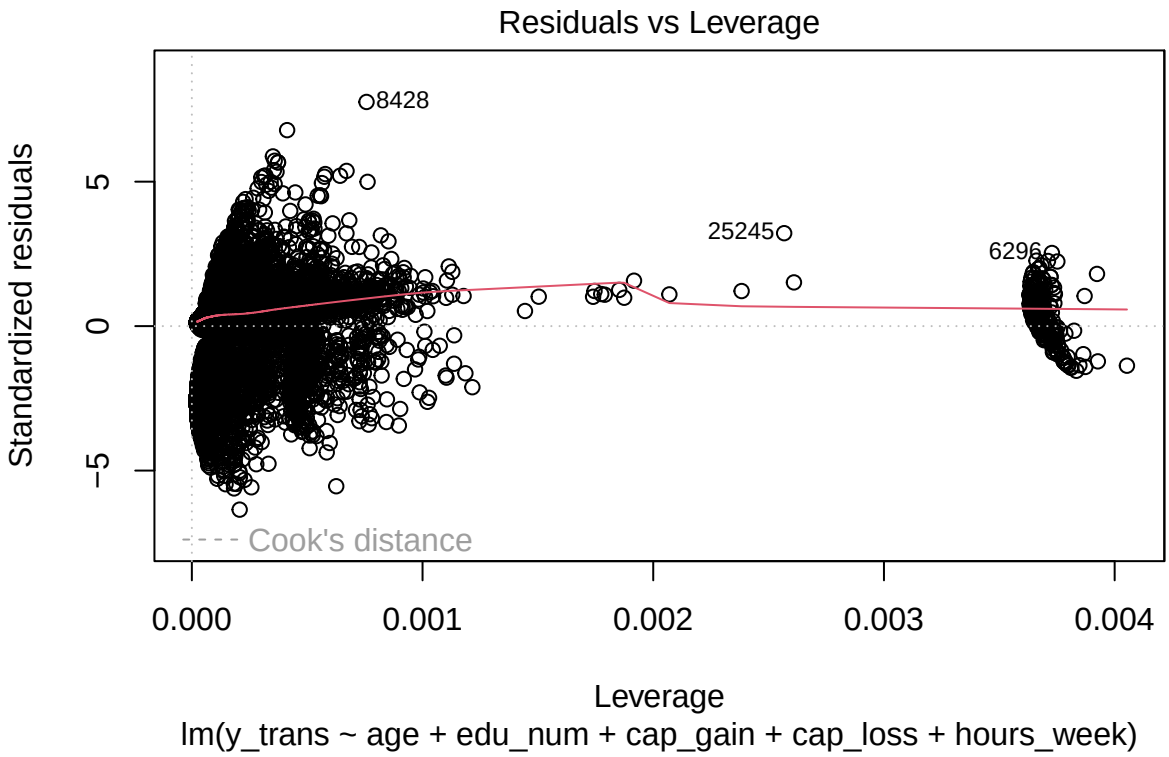
```
## Multiple R-squared:  0.7109, Adjusted R-squared:  0.7109
## F-statistic: 2.402e+04 on 5 and 48836 DF,  p-value: < 2.2e-16
```

```
plot(transformed_model) #we cannot accept the basic hypothesis yet
```

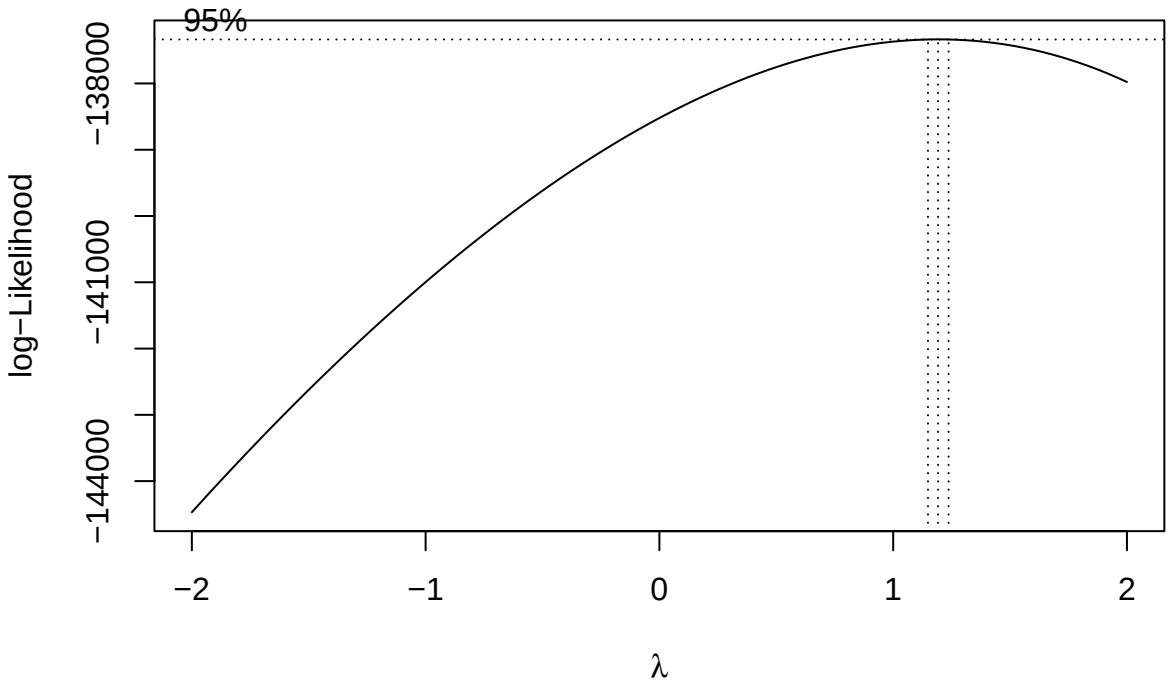



Q−Q Residuals
Standardized residuals
5
0
−5
8428
3497
48496
−4
−2
0
2
4
Theoretical Quantiles
lm(y_trans ~ age + edu_num + cap_gain + cap_loss + hours_week)


Scale−Location
8428
3497
48496
√|Standardized residuals|
0.0
0.5
1.0
1.5
2.0
2.5
1.0e−05
1.5e−05
2.0e−05
2.5e−05
3.0e−05
Fitted values
lm(y_trans ~ age + edu_num + cap_gain + cap_loss + hours_week)




```
boxcox(y_trans ~ age + edu_num + cap_gain + cap_loss + hours_week, data = dd) # lambda is now around 1 
```




```
#As seen before, the basic hypothesis cannot be accepted, we need to perform transformation on the regr
boxTidwell(y_trans ~ age + edu_num + hours_week, data = dd)
```

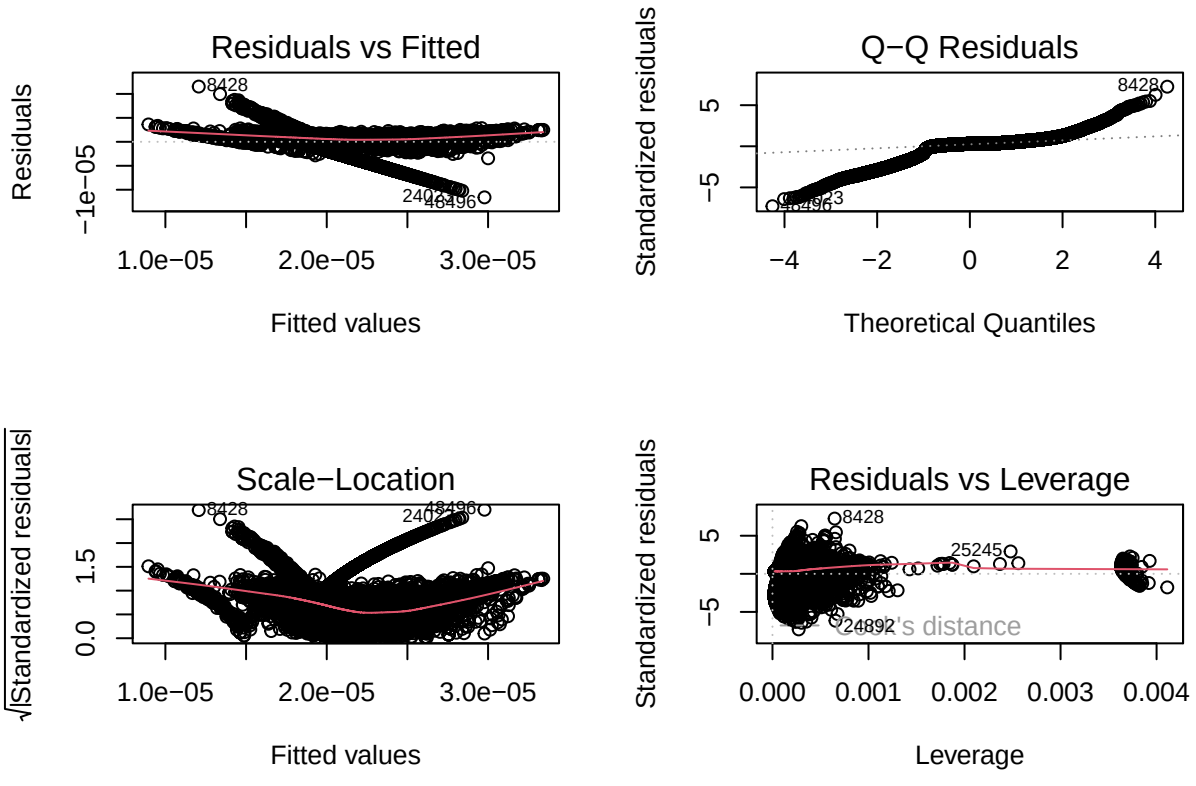
```
##             MLE of lambda Score Statistic (t)  Pr(>|t|)
## age              -0.69324             75.0355 < 2.2e-16 ***
## edu_num           0.38487             40.7224 < 2.2e-16 ***
## hours_week        0.85116              4.6881 2.764e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## iterations =  4
##
## Score test for null hypothesis that all lambdas = 1:
## F = 2566.4, df = 3 and 48835, Pr(>F) = < 2.2e-16
```

```
dd$agebt <- sqrt(dd$age)
edu_num_bt <- sqrt(dd$edu_num)
btmodel <- lm(y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week, data = dd)
summary(btmodel)
```

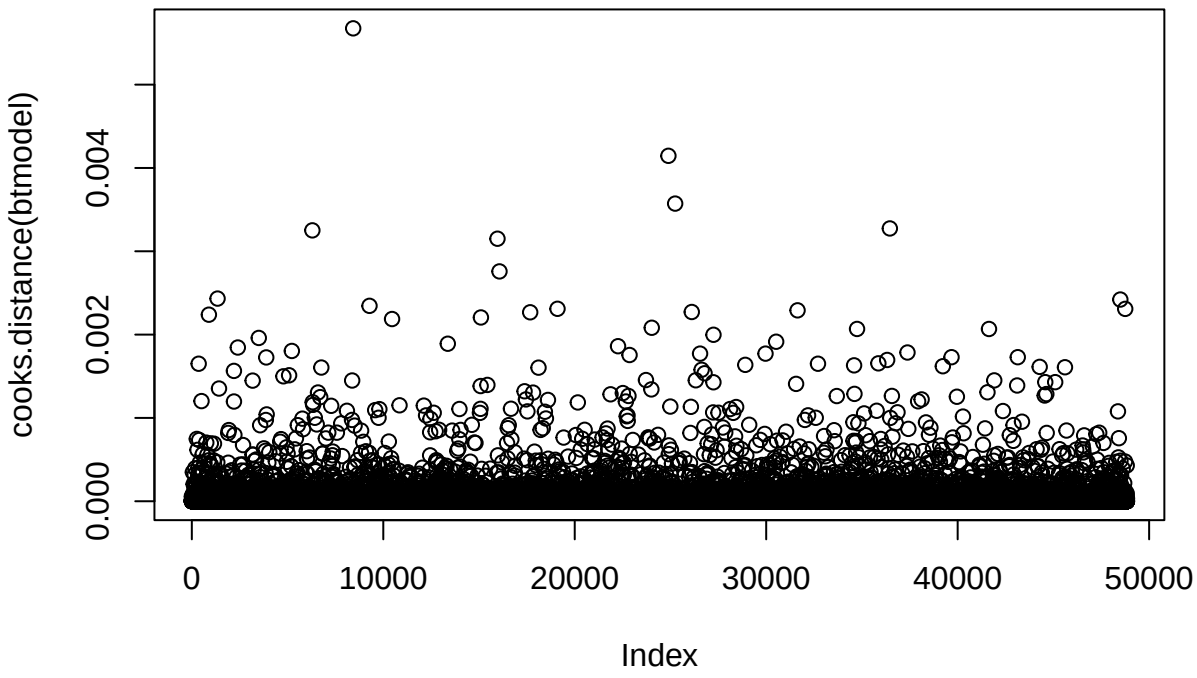
```
##
## Call:
## lm(formula = y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss +
##     hours_week, data = dd)
##
```

```
## Residuals:
##        Min         1Q     Median         3Q        Max
## -1.160e-05  1.249e-07  4.788e-07  6.404e-07  1.153e-05
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.542e-05  6.717e-08  676.15   <2e-16 ***
## agebt       -1.606e-06  6.642e-09 -241.80   <2e-16 ***
## edu_num_bt  -3.585e-06  1.679e-08 -213.48   <2e-16 ***
## cap_gain    -5.684e-11  9.779e-13  -58.12   <2e-16 ***
## cap_loss    -2.708e-10  1.797e-11  -15.07   <2e-16 ***
## hours_week  -4.752e-08  5.911e-10  -80.38   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.591e-06 on 48836 degrees of freedom
## Multiple R-squared:  0.7363, Adjusted R-squared:  0.7362
## F-statistic: 2.727e+04 on 5 and 48836 DF,  p-value: < 2.2e-16
```

```
par(mfrow=c(2,2))
plot(btmodel)
```




```
#Check cook's distance
plot(cooks.distance(btmodel))
```




```
#Try adding polynomial terms
age2 <- dd$agebt^2
hours_week2 <- dd$hours_week^2
model_poly <- lm(y_trans ~ agebt + age2 + edu_num + cap_gain + cap_loss + hours_week + hours_week2 , da

#comparing model performance
summary(model_poly)
```

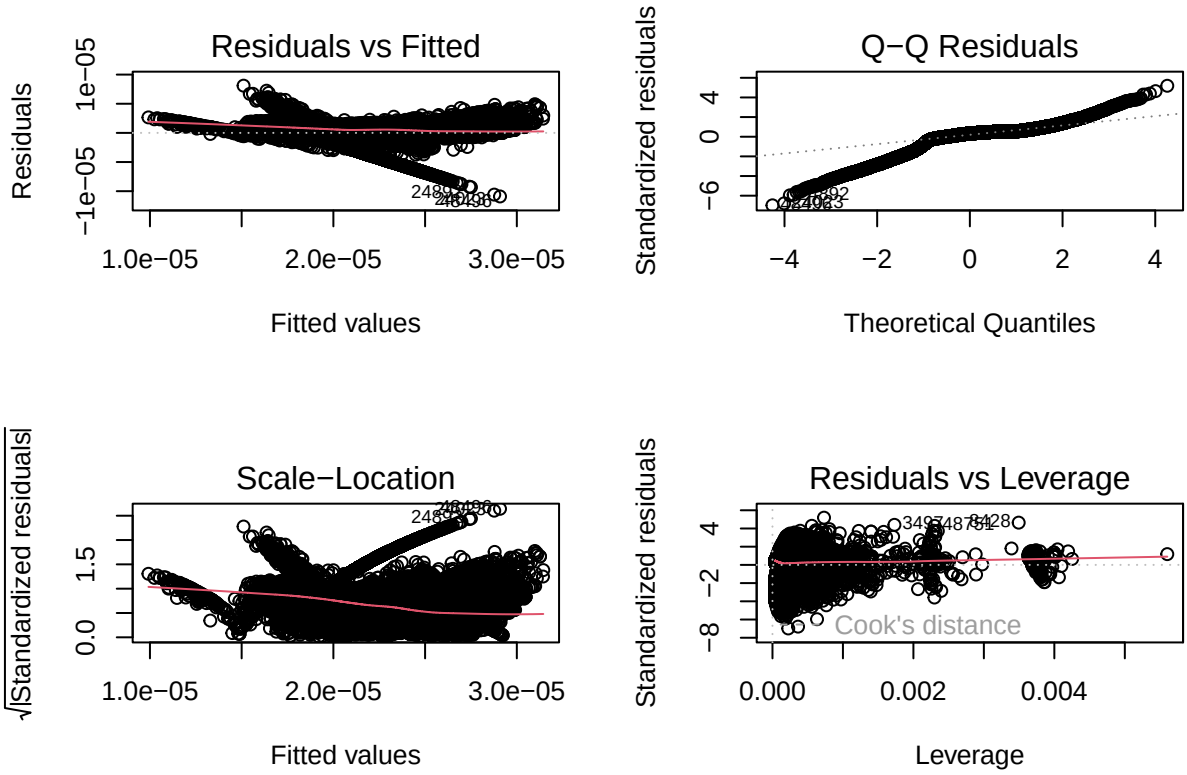
```
##
## Call:
## lm(formula = y_trans ~ agebt + age2 + edu_num + cap_gain + cap_loss +
##     hours_week + hours_week2, data = dd)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -1.090e-05 -1.907e-07  5.084e-07  8.102e-07  8.100e-06
##
## Coefficients:
##               Estimate Std. Error  t value Pr(>|t|)
## (Intercept)  5.161e-05  2.063e-07  250.184   <2e-16 ***
## agebt       -5.548e-06  7.237e-08  -76.663   <2e-16 ***
## age2         3.172e-07  5.759e-09   55.070   <2e-16 ***
## edu_num     -5.729e-07  2.841e-09 -201.635   <2e-16 ***
## cap_gain    -5.647e-11  9.619e-13  -58.706   <2e-16 ***
## cap_loss    -2.568e-10  1.766e-11  -14.540   <2e-16 ***
## hours_week  -5.224e-08  1.946e-09  -26.843   <2e-16 ***
```

```
## hours_week2  2.073e-10  2.140e-11    9.687   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.562e-06 on 48834 degrees of freedom
## Multiple R-squared:  0.7456, Adjusted R-squared:  0.7456
## F-statistic: 2.045e+04 on 7 and 48834 DF,  p-value: < 2.2e-16
```

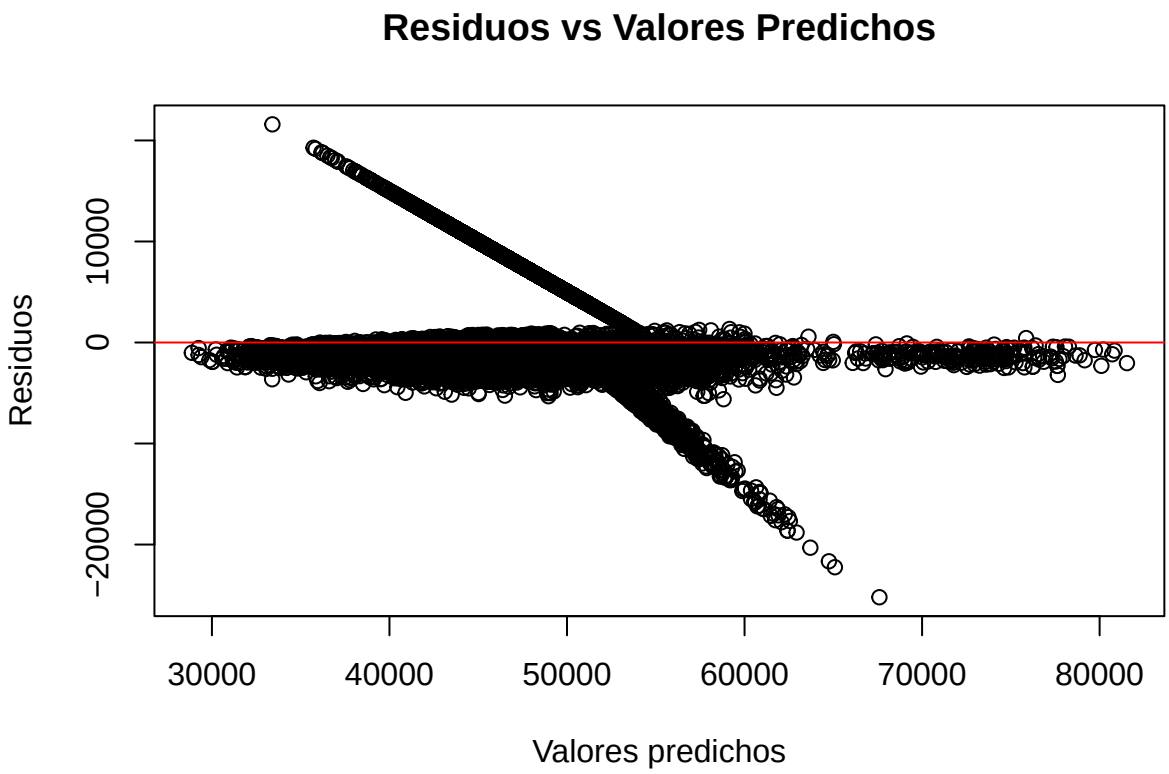
```
anova(initial_model, model_poly)
```

```
## Warning in anova.lmlist(object, ...): models with response '"y_trans"' removed
## because response differs from model 1

## Analysis of Variance Table
##
## Response: income_integer
##               Df     Sum Sq    Mean Sq  F value    Pr(>F)
## age            1 5.9443e+11 5.9443e+11 44080.01 < 2.2e-16 ***
## edu_num        1 5.3884e+11 5.3884e+11 39958.37 < 2.2e-16 ***
## cap_gain       1 1.1520e+11 1.1520e+11  8542.91 < 2.2e-16 ***
## cap_loss       1 6.5529e+09 6.5529e+09   485.93 < 2.2e-16 ***
## hours_week     1 7.1603e+10 7.1603e+10  5309.76 < 2.2e-16 ***
## Residuals  48836 6.5856e+11 1.3485e+07
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

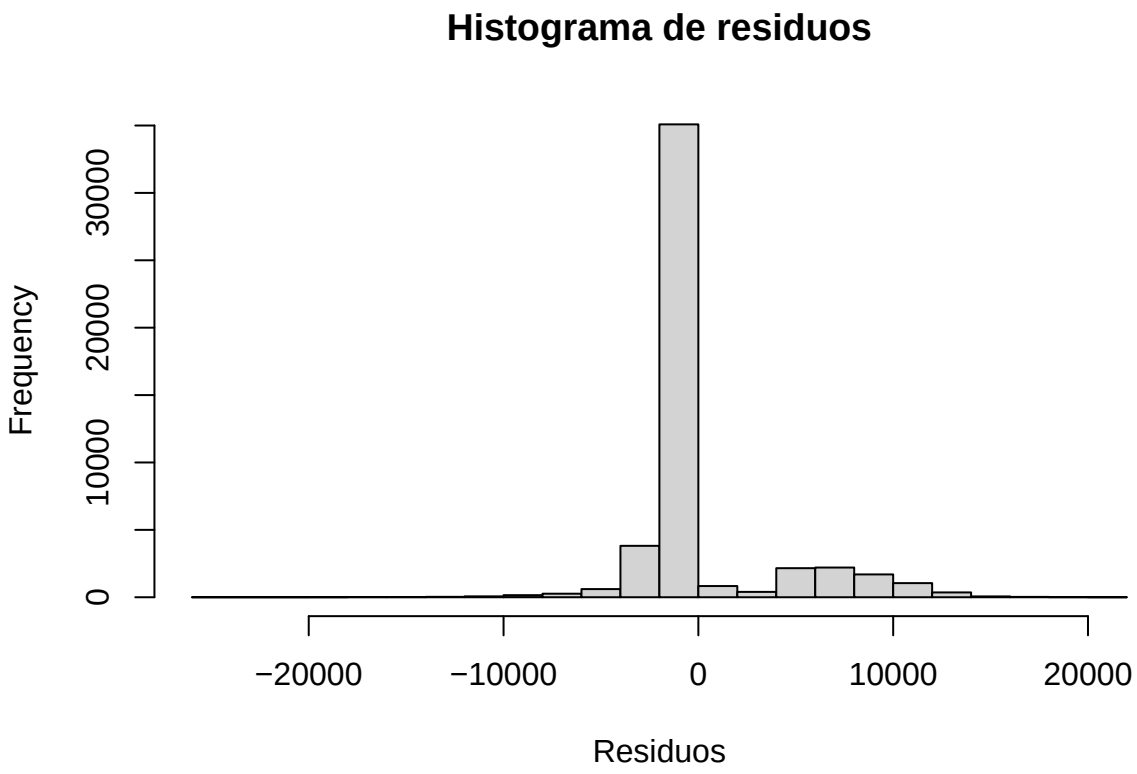
```
par(mfrow=c(2,2))
plot(model_poly)
```



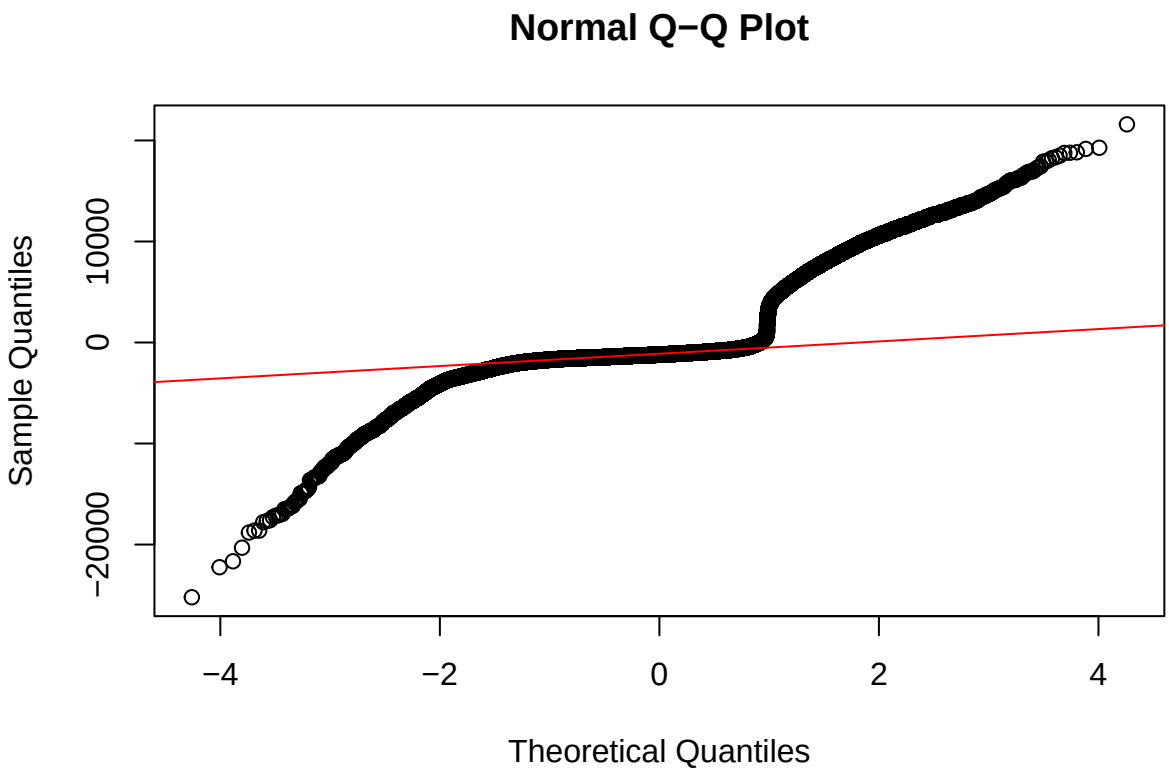

```r
# Análisis de residuos (modelo original)
plot(initial_model$fitted.values, resid(initial_model),
     xlab = "Valores predichos", ylab = "Residuos",
     main = "Residuos vs Valores Predichos")
abline(h = 0, col = "red")
```



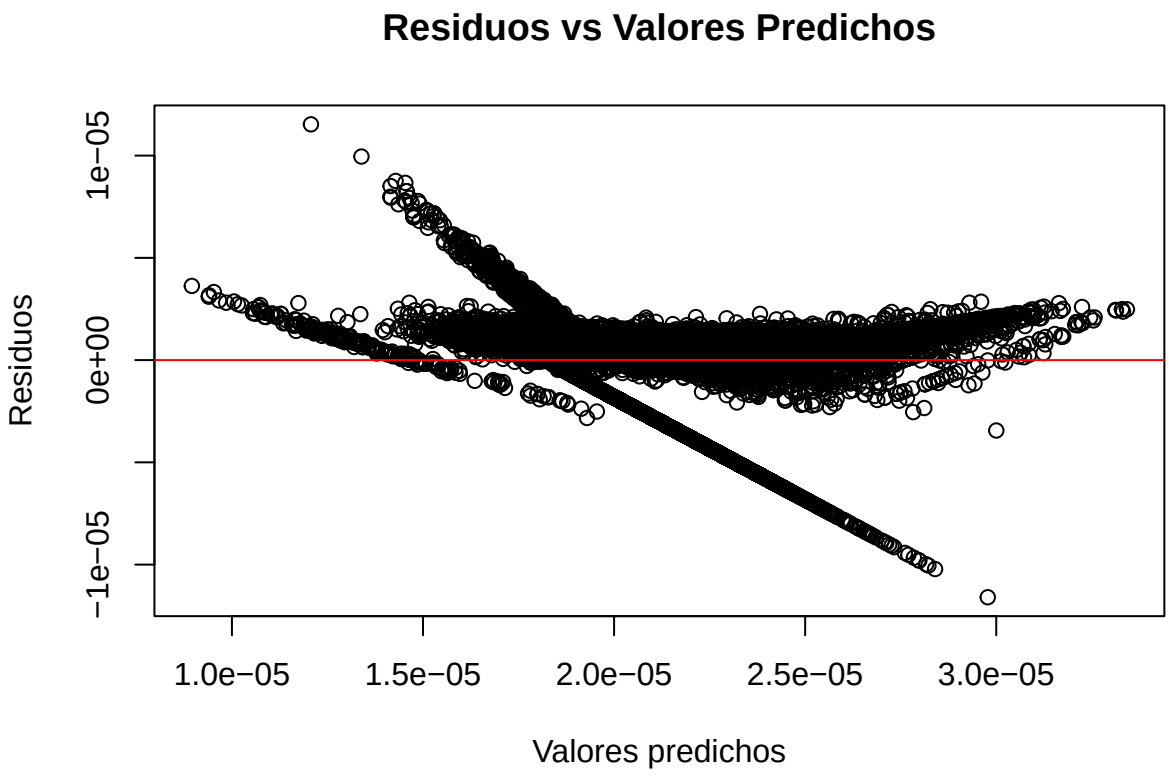

```r
hist(resid(initial_model), main = "Histograma de residuos", xlab = "Residuos")
```



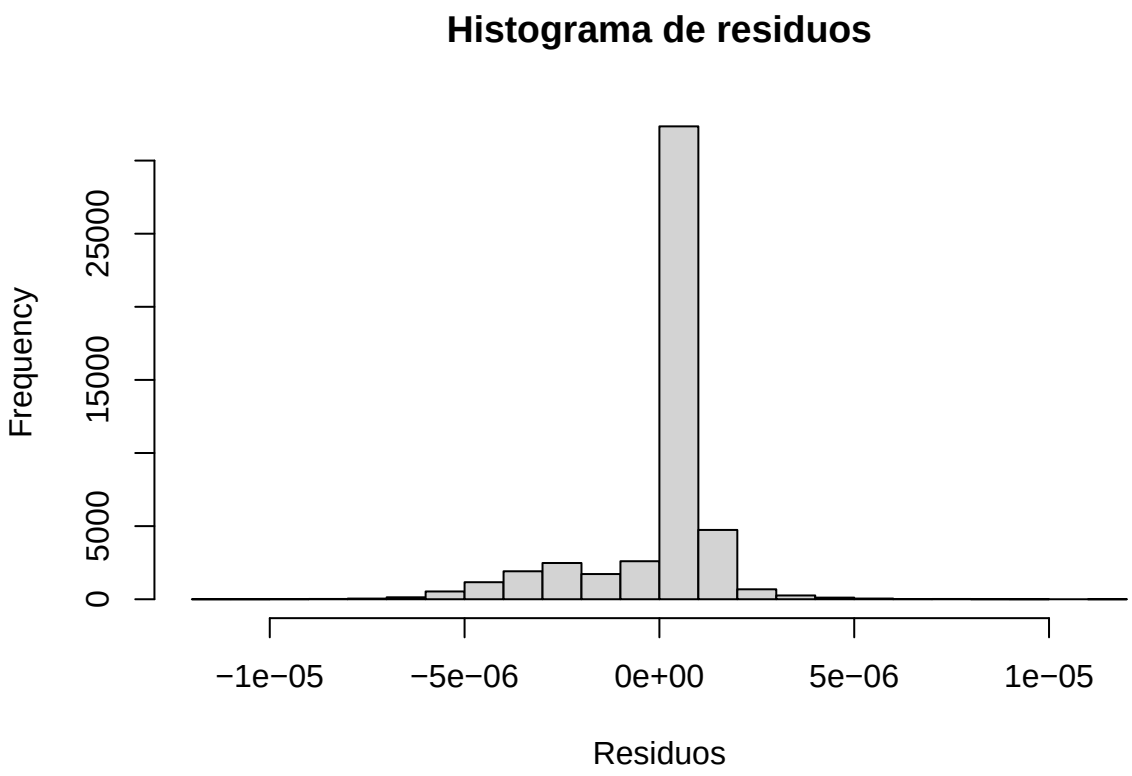

```
qqnorm(resid(initial_model))
qqline(resid(initial_model), col = "red")
```



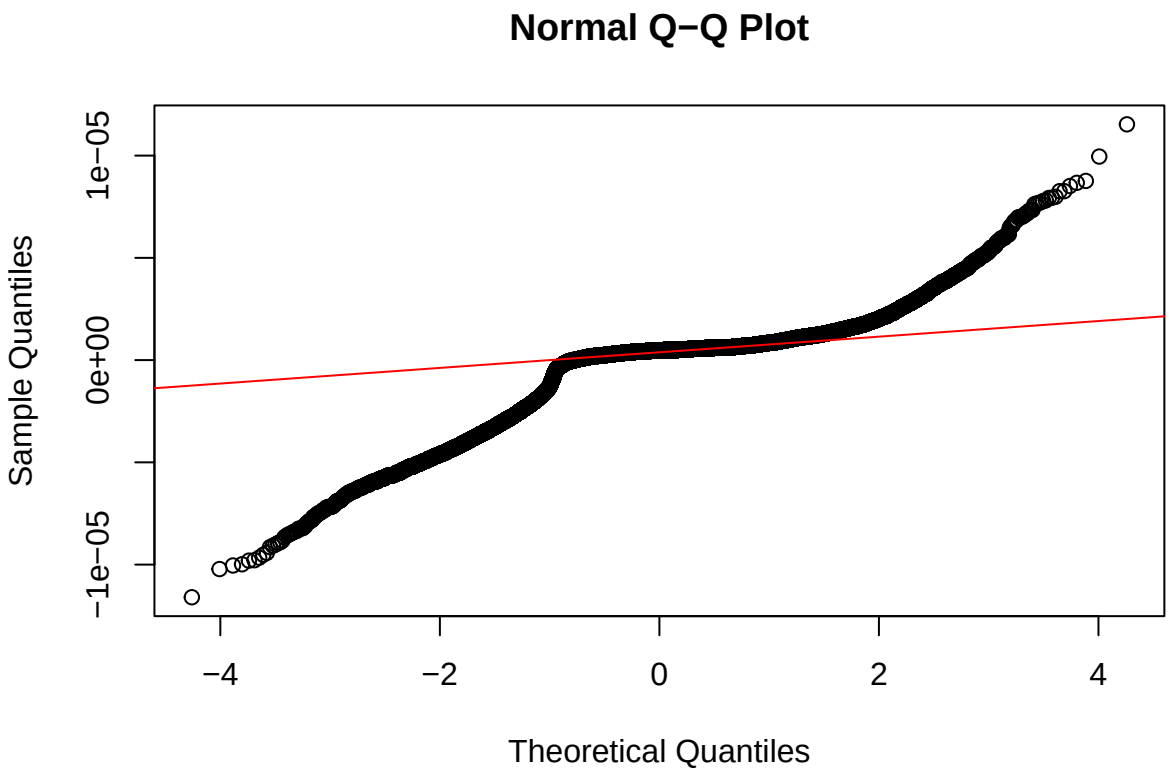

```r
# Análisis de residuos (bt_model transformado)
plot(btmodel$fitted.values, resid(btmodel),
     xlab = "Valores predichos", ylab = "Residuos",
     main = "Residuos vs Valores Predichos")
abline(h = 0, col = "red")
```




```r
hist(resid(btmodel), main = "Histograma de residuos", xlab = "Residuos")
```




```
qqnorm(resid(btmodel))
qqline(resid(btmodel), col = "red")
```




```
# Multicolinealidad (initial_model original)
vif(initial_model)
```

```
##        age    edu_num   cap_gain   cap_loss hours_week
##   1.013606   1.041277   1.027722   1.013638   1.031481
```

```
# Multicolinealidad (bt_model transformado)
vif(btmodel)
```

```
##      agebt edu_num_bt   cap_gain   cap_loss hours_week
##   1.020432   1.034825   1.024972   1.012790   1.035586
```

```
#Incorporating Factors
#Add Occupation
modelo_occ <- update(btmodel, . ~ . + occupation)
anova(btmodel, modelo_occ)  # p < 2.2e-16 ***
```

```
## Analysis of Variance Table
##
## Model 1: y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week
## Model 2: y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week +
##     occupation
##   Res.Df        RSS Df  Sum of Sq      F    Pr(>F)
## 1  48836 1.2357e-07
```

```
## 2  48823 1.2040e-07 13 3.1647e-09 98.712 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# Add estado civil (7 categories)
modelo_marital <- update(modelo_occ, . ~ . + marital)
anova(modelo_occ, modelo_marital)  # p = < 2.2e-16
```

```
## Analysis of Variance Table
##
## Model 1: y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week +
##     occupation
## Model 2: y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week +
##     occupation + marital
##   Res.Df        RSS Df  Sum of Sq      F    Pr(>F)
## 1  48823 1.2040e-07
## 2  48819 1.0817e-07  4 1.2237e-08 1380.7 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# Add género (2 categories)
modelo_gender <- update(modelo_marital, . ~ . + sex)
anova(modelo_marital, modelo_gender)  # p = 0.009058 ***
```

```
## Analysis of Variance Table
##
## Model 1: y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week +
##     occupation + marital
## Model 2: y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week +
##     occupation + marital + sex
##   Res.Df        RSS Df  Sum of Sq      F   Pr(>F)
## 1  48819 1.0817e-07
## 2  48818 1.0815e-07  1 1.5091e-11 6.8119 0.009058 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# Add clase trabajadora (9 categories)
modelo_workclass <- update(modelo_gender, . ~ . + workclass)
anova(modelo_gender, modelo_workclass)  # p = < 2.2e-16
```

```
## Analysis of Variance Table
##
## Model 1: y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week +
##     occupation + marital + sex
## Model 2: y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week +
##     occupation + marital + sex + workclass
##   Res.Df        RSS Df  Sum of Sq     F    Pr(>F)
## 1  48818 1.0815e-07
## 2  48812 1.0787e-07  6 2.8323e-10 21.36 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# Add relación familiar (6 categories)
modelo_relat <- update(modelo_workclass, . ~ . + relationship)
anova(modelo_workclass, modelo_relat)  # p = < 2.2e-16
```

```
## Analysis of Variance Table
##
## Model 1: y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week +
##     occupation + marital + sex + workclass
## Model 2: y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week +
##     occupation + marital + sex + workclass + relationship
##   Res.Df        RSS Df  Sum of Sq      F    Pr(>F)
## 1  48812 1.0787e-07
## 2  48807 1.0664e-07  5 1.2266e-09 112.28 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# Add raza (5 categories)
modelo_race <- update(modelo_relat, . ~ . + race)
anova(modelo_relat, modelo_race)  # p = 0.0008009
```

```
## Analysis of Variance Table
##
## Model 1: y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week +
##     occupation + marital + sex + workclass + relationship
## Model 2: y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week +
##     occupation + marital + sex + workclass + relationship + race
##   Res.Df        RSS Df  Sum of Sq      F    Pr(>F)
## 1  48807 1.0664e-07
## 2  48803 1.0660e-07  4 4.1418e-11 4.7404 0.0008009 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# Add país origen (42 categorías)
modelo_country <- update(modelo_race, . ~ . + native_country)
anova(modelo_race, modelo_country)  # p = < 2.2e-16
```

```
## Analysis of Variance Table
##
## Model 1: y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week +
##     occupation + marital + sex + workclass + relationship + race
## Model 2: y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week +
##     occupation + marital + sex + workclass + relationship + race +
##     native_country
##   Res.Df        RSS Df  Sum of Sq      F    Pr(>F)
## 1  48803 1.0660e-07
## 2  48802 1.0644e-07  1 1.5805e-10 72.462 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
#Add income (2 categorias)
modelo_income <- update(modelo_country,. ~ . + income)
anova(modelo_country,modelo_income)
```

```
## Analysis of Variance Table
##
## Model 1: y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week +
##     occupation + marital + sex + workclass + relationship + race +
##     native_country
## Model 2: y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week +
##     occupation + marital + sex + workclass + relationship + race +
##     native_country + income
##   Res.Df        RSS Df  Sum of Sq     F    Pr(>F)
## 1  48802 1.0644e-07
## 2  48801 4.9257e-08  1 5.7187e-08 56657 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
#Model with all significant variables including categorical variables
catmodel <- lm(y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week + occupation + marital +
stepmodel <- stepAIC(catmodel, direction = "back")
```

```
## Start:  AIC=-1349059
## y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week +
##     occupation + marital + sex + workclass + relationship + race +
##     native_country + income
##
##                  Df  Sum of Sq        RSS      AIC
## <none>                          4.9257e-08 -1349059
## - sex             1 4.8000e-11 4.9305e-08 -1349013
## - cap_loss        1 8.4000e-11 4.9341e-08 -1348978
## - workclass       6 1.0700e-10 4.9364e-08 -1348965
## - race            4 1.0400e-10 4.9361e-08 -1348964
## - native_country  1 1.6400e-10 4.9420e-08 -1348899
## - marital         4 2.9400e-10 4.9551e-08 -1348777
## - relationship    5 9.7500e-10 5.0232e-08 -1348112
## - occupation     13 1.2400e-09 5.0497e-08 -1347871
## - cap_gain        1 1.9790e-09 5.1236e-08 -1347137
## - hours_week      1 4.8230e-09 5.4080e-08 -1344499
## - agebt           1 5.0523e-08 9.9780e-08 -1314583
## - edu_num_bt      1 5.1493e-08 1.0075e-07 -1314110
## - income          1 5.7187e-08 1.0644e-07 -1311425
```

```
vif(catmodel)
```

```
##                    GVIF Df GVIF^(1/(2*Df))
## agebt          1.781237  1        1.334630
## edu_num_bt     1.429725  1        1.195711
## cap_gain       1.071123  1        1.034951
## cap_loss       1.030597  1        1.015183
## hours_week     1.221805  1        1.105353
## occupation     2.257130 13        1.031807
## marital       60.315146  4        1.669373
## sex            1.990125  1        1.410718
## workclass      1.441251  6        1.030928
## relationship  75.507075  5        1.540986
## race           1.283651  4        1.031706
```

```
## native_country  1.239624  1        1.113384
## income          1.549349  1        1.244728
```

```
summary(catmodel)
```

```
##
## Call:
## lm(formula = y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss +
##     hours_week + occupation + marital + sex + workclass + relationship +
##     race + native_country + income, data = dd)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -8.257e-06 -5.550e-07 -8.660e-08  3.838e-07  7.935e-06
##
## Coefficients:
##                              Estimate Std. Error  t value Pr(>|t|)
## (Intercept)                 4.073e-05  9.922e-08  410.476  < 2e-16 ***
## agebt                      -1.240e-06  5.542e-09 -223.731  < 2e-16 ***
## edu_num_bt                 -2.816e-06  1.247e-08 -225.869  < 2e-16 ***
## cap_gain                   -2.796e-11  6.314e-13  -44.280  < 2e-16 ***
## cap_loss                    1.046e-10  1.145e-11    9.131  < 2e-16 ***
## hours_week                 -2.803e-08  4.055e-10  -69.124  < 2e-16 ***
## occupationArmy              2.978e-07  2.611e-07    1.141 0.254025
## occupationCraftRep         -7.949e-08  2.017e-08   -3.940 8.14e-05 ***
## occupationExecMan           2.275e-07  1.960e-08   11.607  < 2e-16 ***
## occupationFarmFish          1.664e-07  3.137e-08    5.304 1.14e-07 ***
## occupationHandlCl           2.901e-07  2.697e-08   10.756  < 2e-16 ***
## occupationHouse             3.830e-07  6.655e-08    5.755 8.70e-09 ***
## occupationMachOp            8.582e-08  2.361e-08    3.634 0.000279 ***
## occupationOther             2.784e-07  2.008e-08   13.860  < 2e-16 ***
## occupationProf              3.612e-07  1.776e-08   20.340  < 2e-16 ***
## occupationProtServ         -1.413e-07  3.627e-08   -3.897 9.77e-05 ***
## occupationSales             1.443e-07  1.977e-08    7.297 2.99e-13 ***
## occupationTech             -1.148e-07  2.987e-08   -3.843 0.000122 ***
## occupationTrans            -8.449e-08  2.598e-08   -3.253 0.001145 **
## maritalMarried             -1.581e-07  5.642e-08   -2.803 0.005072 **
## maritalNevMarr              2.225e-07  1.719e-08   12.945  < 2e-16 ***
## maritalSep                  8.720e-08  2.516e-08    3.466 0.000529 ***
## maritalWidow               -1.863e-07  2.954e-08   -6.307 2.87e-10 ***
## sexMale                     9.425e-08  1.362e-08    6.919 4.61e-12 ***
## workclassLoc               -3.535e-08  3.269e-08   -1.081 0.279562
## workclassNoPay              2.314e-07  1.828e-07    1.266 0.205479
## workclassPriv              -3.407e-08  2.776e-08   -1.227 0.219771
## workclassSelfI              2.224e-07  3.698e-08    6.015 1.81e-09 ***
## workclassSelfN              1.276e-09  3.221e-08    0.040 0.968406
## workclassState              2.610e-08  3.522e-08    0.741 0.458720
## relationshipNot-in-family  -2.334e-07  5.621e-08   -4.153 3.29e-05 ***
## relationshipOther-relative  1.129e-07  5.518e-08    2.046 0.040760 *
## relationshipOwn-child       2.358e-07  5.614e-08    4.200 2.67e-05 ***
## relationshipUnmarried      -9.660e-08  5.829e-08   -1.657 0.097445 .
## relationshipWife           -2.354e-07  2.564e-08   -9.182  < 2e-16 ***
## raceAsian-Pac-Islander     -2.456e-07  5.447e-08   -4.509 6.53e-06 ***
## raceBlack                  -7.987e-08  4.871e-08   -1.640 0.101042
```

```
## raceOther                   1.078e-07  6.862e-08    1.571 0.116123
## raceWhite                  -5.742e-10  4.674e-08   -0.012 0.990198
## native_countryUSA          -2.123e-07  1.668e-08  -12.728  < 2e-16 ***
## income>50K                 -3.157e-06  1.326e-08 -238.028  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.005e-06 on 48801 degrees of freedom
## Multiple R-squared:  0.8949, Adjusted R-squared:  0.8948
## F-statistic: 1.039e+04 on 40 and 48801 DF,  p-value: < 2.2e-16
```

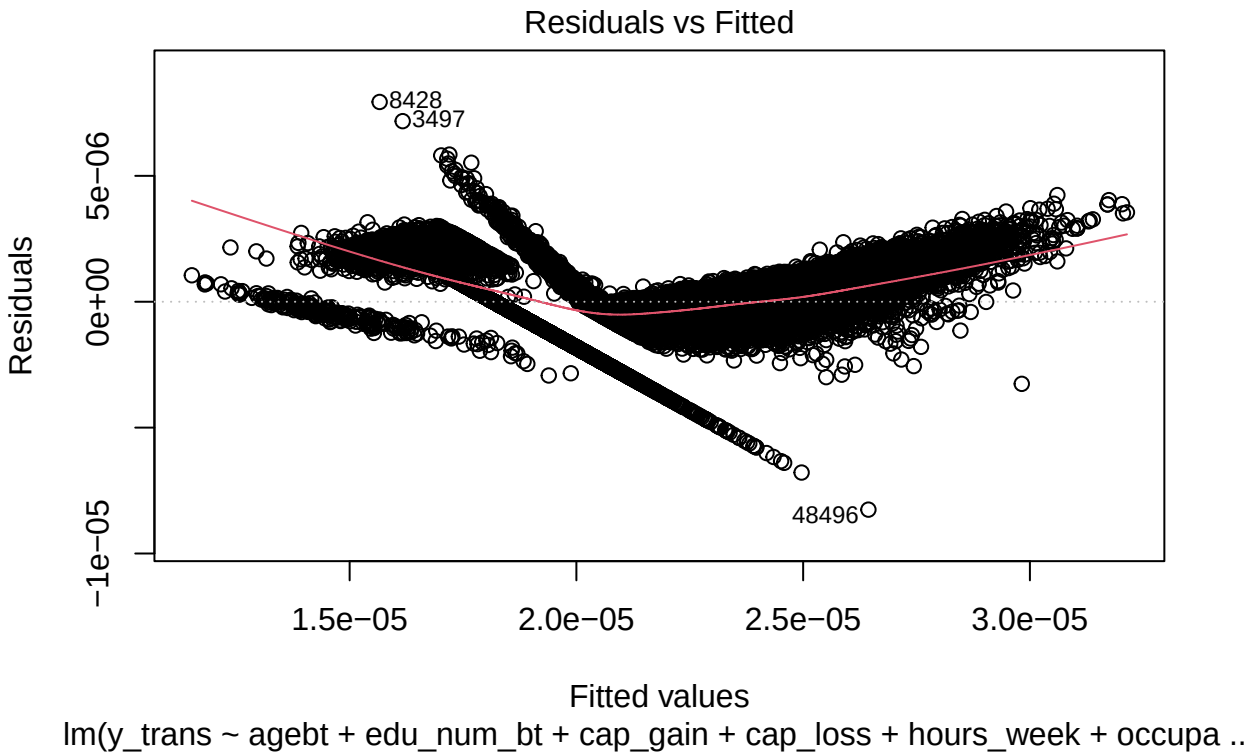
```
anova(catmodel)
```

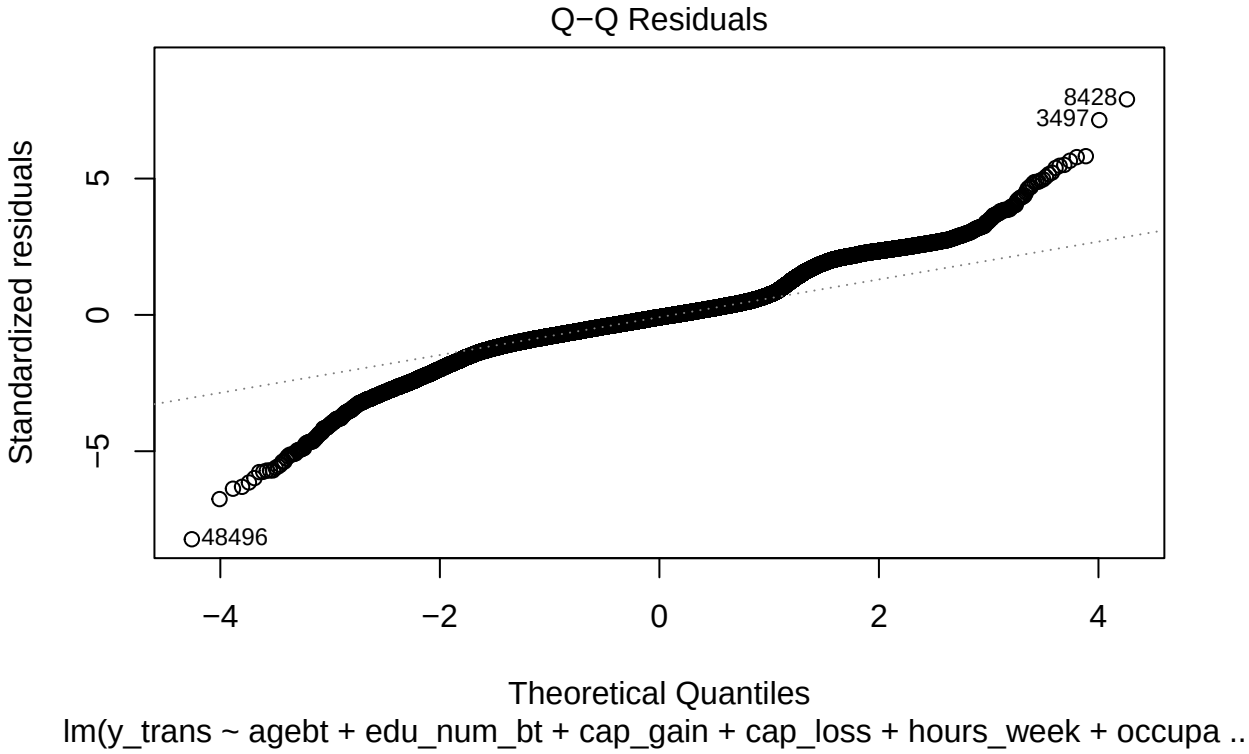
```
## Analysis of Variance Table
##
## Response: y_trans
##                   Df     Sum Sq    Mean Sq    F value    Pr(>F)
## agebt              1 1.7759e-07 1.7759e-07 175949.879 < 2.2e-16 ***
## edu_num_bt         1 1.4030e-07 1.4030e-07 138997.998 < 2.2e-16 ***
## cap_gain           1 9.8790e-09 9.8790e-09   9787.355 < 2.2e-16 ***
## cap_loss           1 8.6200e-10 8.6200e-10    853.794 < 2.2e-16 ***
## hours_week         1 1.6350e-08 1.6350e-08  16198.266 < 2.2e-16 ***
## occupation        13 3.1650e-09 2.4300e-10    241.185 < 2.2e-16 ***
## marital            4 1.2237e-08 3.0590e-09   3030.888 < 2.2e-16 ***
## sex                1 1.5000e-11 1.5000e-11     14.951 0.0001105 ***
## workclass          6 2.8300e-10 4.7000e-11     46.767 < 2.2e-16 ***
## relationship       5 1.2270e-09 2.4500e-10    243.055 < 2.2e-16 ***
## race               4 4.1000e-11 1.0000e-11     10.259 2.664e-08 ***
## native_country     1 1.5800e-10 1.5800e-10    156.585 < 2.2e-16 ***
## income             1 5.7187e-08 5.7187e-08  56657.237 < 2.2e-16 ***
## Residuals      48801 4.9257e-08 1.0000e-12
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

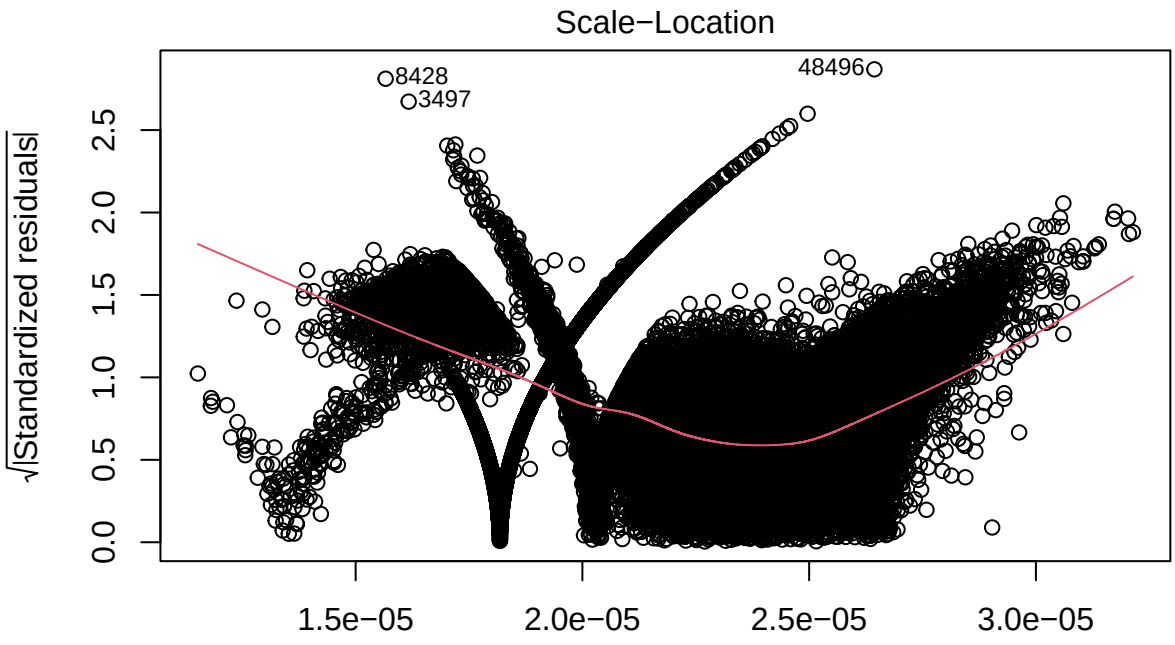
```
anova(transformed_model, catmodel)
```

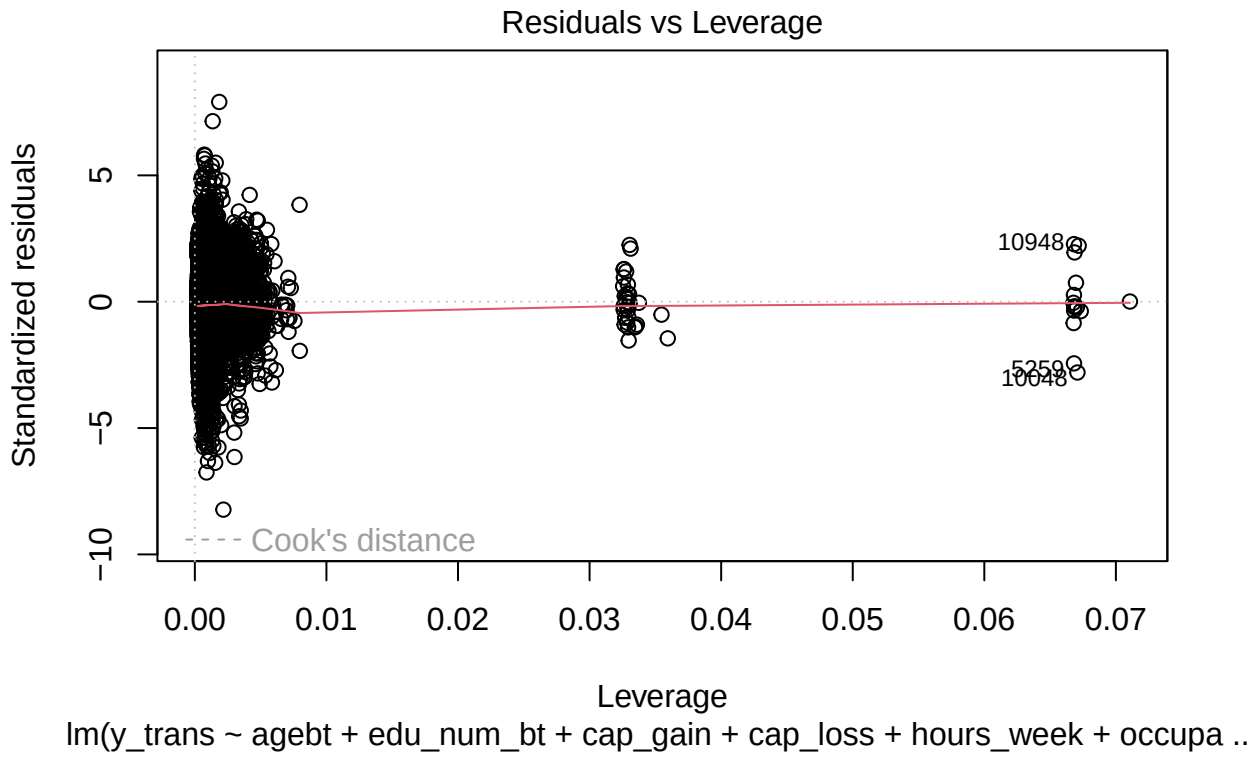
```
## Analysis of Variance Table
##
## Model 1: y_trans ~ age + edu_num + cap_gain + cap_loss + hours_week
## Model 2: y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week +
##     occupation + marital + sex + workclass + relationship + race +
##     native_country + income
##   Res.Df        RSS Df  Sum of Sq      F    Pr(>F)
## 1  48836 1.3544e-07
## 2  48801 4.9257e-08 35 8.6187e-08 2439.7 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
plot(catmodel)
```


Residuals vs Fitted
8428
3497
48496
Residuals
5e−06
0e+00
−1e−05
1.5e−05
2.0e−05
2.5e−05
3.0e−05
Fitted values
lm(y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week + occupa ..


Q−Q Residuals
Standardized residuals
5
0
−5
8428
3497
48496
−4
−2
0
2
4
Theoretical Quantiles
lm(y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week + occupa ..


Scale−Location
8428
3497
48496
√|Standardized residuals|
0.0
0.5
1.0
1.5
2.0
2.5
1.5e−05
2.0e−05
2.5e−05
3.0e−05
Fitted values
lm(y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week + occupa ..




```
full_model <- lm(y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week +
                   occupation + marital + sex + workclass + relationship + race +
                   native_country + income + sex:occupation + edu_num:marital, data = dd)

summary(full_model)
```

```
##
## Call:
## lm(formula = y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss +
##     hours_week + occupation + marital + sex + workclass + relationship +
##     race + native_country + income + sex:occupation + edu_num:marital,
##     data = dd)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -1.002e-05 -4.469e-07 -4.050e-08  3.931e-07  7.453e-06
##
## Coefficients: (1 not defined because of singularities)
##                               Estimate Std. Error  t value Pr(>|t|)
## (Intercept)                  4.568e-05  1.470e-07  310.629  < 2e-16 ***
## agebt                       -1.214e-06  4.992e-09 -243.222  < 2e-16 ***
## edu_num_bt                  -6.780e-06  7.403e-08  -91.586  < 2e-16 ***
## cap_gain                    -3.115e-11  5.636e-13  -55.274  < 2e-16 ***
## cap_loss                     7.709e-11  1.020e-11    7.554 4.29e-14 ***
## hours_week                  -2.671e-08  3.629e-10  -73.619  < 2e-16 ***
```

```
## occupationArmy              2.571e-07  2.330e-07    1.103 0.269883
## occupationCraftRep          2.561e-08  5.199e-08    0.493 0.622354
## occupationExecMan          -4.778e-10  2.620e-08   -0.018 0.985451
## occupationFarmFish          2.250e-07  9.332e-08    2.411 0.015901 *
## occupationHandlCl           1.423e-07  5.814e-08    2.447 0.014429 *
## occupationHouse             5.819e-09  6.167e-08    0.094 0.924830
## occupationMachOp            7.918e-08  3.514e-08    2.253 0.024253 *
## occupationOther             1.853e-07  2.285e-08    8.110 5.18e-16 ***
## occupationProf              1.060e-07  2.147e-08    4.935 8.04e-07 ***
## occupationProtServ          5.344e-08  8.266e-08    0.647 0.517935
## occupationSales             1.806e-07  2.520e-08    7.165 7.87e-13 ***
## occupationTech             -1.163e-07  4.053e-08   -2.870 0.004110 **
## occupationTrans             6.822e-09  8.084e-08    0.084 0.932739
## maritalMarried             -1.080e-06  7.278e-08  -14.837  < 2e-16 ***
## maritalNevMarr              2.752e-06  6.000e-08   45.858  < 2e-16 ***
## maritalSep                  4.225e-07  8.545e-08    4.944 7.67e-07 ***
## maritalWidow               -2.038e-06  9.467e-08  -21.525  < 2e-16 ***
## sexMale                    -6.636e-08  2.650e-08   -2.504 0.012280 *
## workclassLoc               -6.722e-08  2.925e-08   -2.298 0.021577 *
## workclassNoPay              2.392e-07  1.630e-07    1.467 0.142282
## workclassPriv              -2.480e-08  2.483e-08   -0.999 0.317789
## workclassSelfI              1.610e-07  3.311e-08    4.862 1.17e-06 ***
## workclassSelfN             -1.652e-08  2.882e-08   -0.573 0.566637
## workclassState             -2.388e-08  3.146e-08   -0.759 0.447916
## relationshipNot-in-family   5.333e-08  5.024e-08    1.062 0.288382
## relationshipOther-relative  1.832e-07  4.919e-08    3.724 0.000196 ***
## relationshipOwn-child       4.002e-07  5.007e-08    7.993 1.34e-15 ***
## relationshipUnmarried       7.771e-08  5.200e-08    1.494 0.135114
## relationshipWife           -2.564e-07  2.302e-08  -11.141  < 2e-16 ***
## raceAsian-Pac-Islander     -1.748e-07  4.853e-08   -3.602 0.000316 ***
## raceBlack                  -6.529e-08  4.340e-08   -1.504 0.132472
## raceOther                   9.531e-08  6.113e-08    1.559 0.118978
## raceWhite                   4.619e-09  4.164e-08    0.111 0.911681
## native_countryUSA          -1.757e-08  1.522e-08   -1.155 0.248276
## income>50K                 -3.407e-06  1.207e-08 -282.260  < 2e-16 ***
## occupationArmy:sexMale             NA         NA       NA       NA
## occupationCraftRep:sexMale  2.416e-08  5.729e-08    0.422 0.673259
## occupationExecMan:sexMale   2.295e-07  3.630e-08    6.323 2.60e-10 ***
## occupationFarmFish:sexMale -9.963e-08  9.850e-08   -1.012 0.311780
## occupationHandlCl:sexMale   1.408e-07  6.524e-08    2.158 0.030904 *
## occupationHouse:sexMale     4.951e-07  2.479e-07    1.997 0.045800 *
## occupationMachOp:sexMale    4.718e-08  4.499e-08    1.049 0.294285
## occupationOther:sexMale     2.715e-08  3.633e-08    0.747 0.454873
## occupationProf:sexMale      1.366e-07  3.228e-08    4.230 2.34e-05 ***
## occupationProtServ:sexMale -1.057e-07  9.032e-08   -1.170 0.242040
## occupationSales:sexMale    -8.991e-08  3.615e-08   -2.487 0.012889 *
## occupationTech:sexMale      4.481e-08  5.463e-08    0.820 0.412014
## occupationTrans:sexMale     6.095e-08  8.567e-08    0.711 0.476815
## maritalDiv:edu_num          6.989e-07  1.332e-08   52.464  < 2e-16 ***
## maritalMarried:edu_num      8.181e-07  1.285e-08   63.649  < 2e-16 ***
## maritalNevMarr:edu_num      4.515e-07  1.302e-08   34.670  < 2e-16 ***
## maritalSep:edu_num          6.647e-07  1.509e-08   44.037  < 2e-16 ***
## maritalWidow:edu_num        8.992e-07  1.584e-08   56.777  < 2e-16 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 8.948e-07 on 48784 degrees of freedom
## Multiple R-squared:  0.9166, Adjusted R-squared:  0.9165
## F-statistic:  9410 on 57 and 48784 DF,  p-value: < 2.2e-16
```

```
reduced_model <- step(full_model)
```

```
## Start:  AIC=-1360353
## y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week +
##     occupation + marital + sex + workclass + relationship + race +
##     native_country + income + sex:occupation + edu_num:marital
##
##                    Df  Sum of Sq        RSS      AIC
## - native_country    1 1.0000e-12 3.9062e-08 -1360353
## <none>                           3.9061e-08 -1360353
## - cap_loss          1 4.6000e-11 3.9107e-08 -1360298
## - workclass         6 6.1000e-11 3.9122e-08 -1360288
## - race              4 6.2000e-11 3.9123e-08 -1360283
## - occupation:sex   12 9.0000e-11 3.9151e-08 -1360265
## - relationship      5 5.9700e-10 3.9658e-08 -1359622
## - cap_gain          1 2.4460e-09 4.1507e-08 -1357388
## - hours_week        1 4.3400e-09 4.3401e-08 -1355209
## - edu_num_bt        1 6.7160e-09 4.5777e-08 -1352605
## - marital:edu_num   5 9.8820e-09 4.8943e-08 -1349347
## - agebt             1 4.7367e-08 8.6428e-08 -1321565
## - income            1 6.3792e-08 1.0285e-07 -1313067
##
## Step:  AIC=-1360353
## y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss + hours_week +
##     occupation + marital + sex + workclass + relationship + race +
##     income + occupation:sex + marital:edu_num
##
##                    Df  Sum of Sq        RSS      AIC
## <none>                           3.9062e-08 -1360353
## - cap_loss          1 4.6000e-11 3.9108e-08 -1360298
## - workclass         6 6.1000e-11 3.9124e-08 -1360289
## - race              4 6.2000e-11 3.9124e-08 -1360284
## - occupation:sex   12 9.0000e-11 3.9152e-08 -1360265
## - relationship      5 5.9600e-10 3.9658e-08 -1359624
## - cap_gain          1 2.4470e-09 4.1510e-08 -1357387
## - hours_week        1 4.3390e-09 4.3401e-08 -1355211
## - edu_num_bt        1 7.1180e-09 4.6180e-08 -1352180
## - marital:edu_num   5 1.0042e-08 4.9104e-08 -1349189
## - agebt             1 4.7414e-08 8.6476e-08 -1321540
## - income            1 6.3832e-08 1.0289e-07 -1313050
```

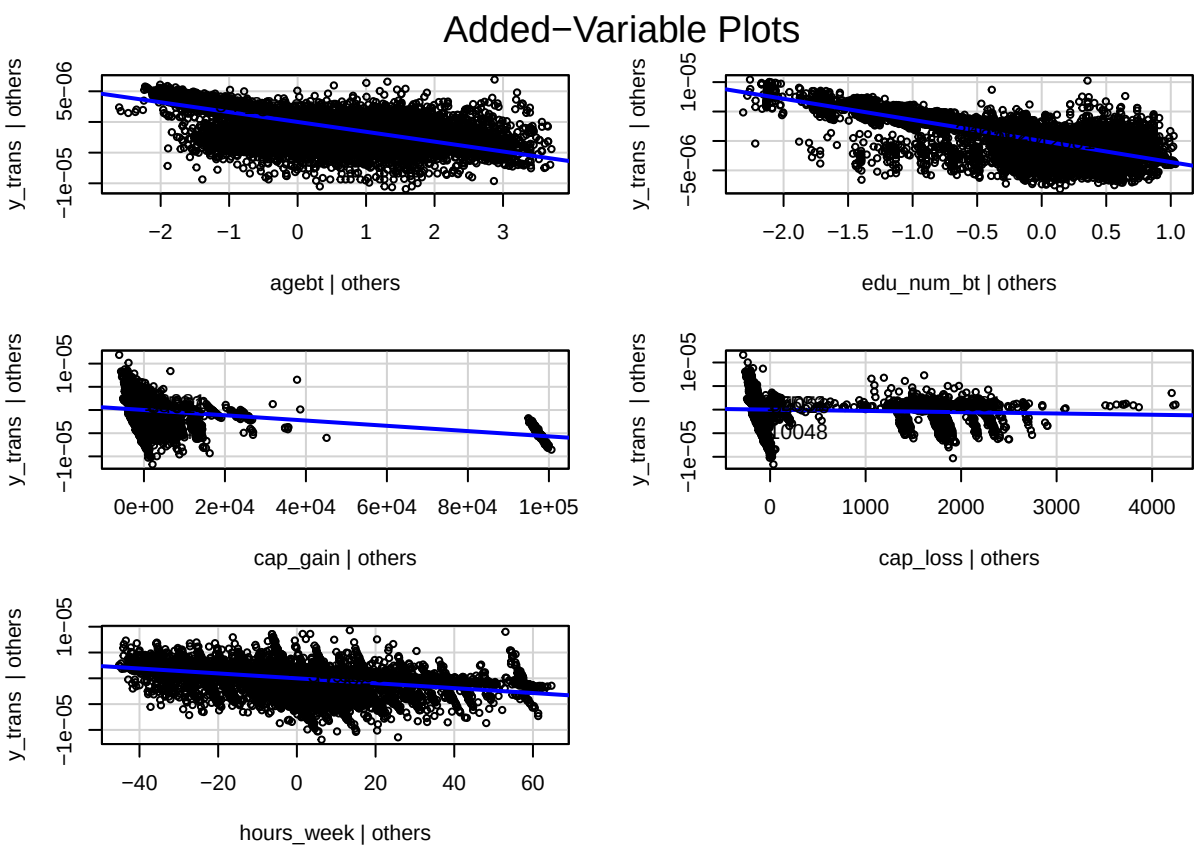
```
summary(reduced_model)
```

```
##
## Call:
## lm(formula = y_trans ~ agebt + edu_num_bt + cap_gain + cap_loss +
##     hours_week + occupation + marital + sex + workclass + relationship +
```

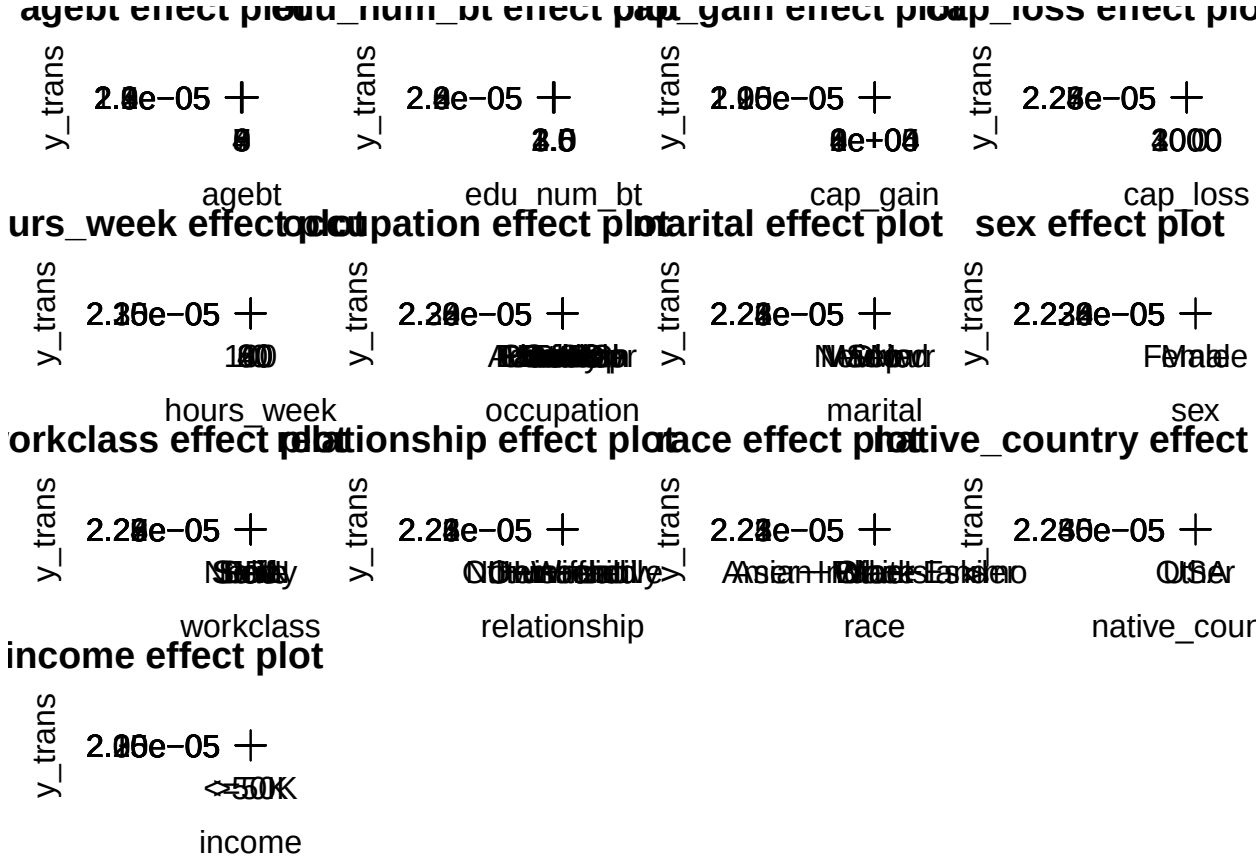
```
##     race + income + occupation:sex + marital:edu_num, data = dd)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -1.001e-05 -4.472e-07 -3.970e-08  3.933e-07  7.451e-06
##
## Coefficients: (1 not defined because of singularities)
##                               Estimate Std. Error  t value Pr(>|t|)
## (Intercept)                  4.569e-05  1.467e-07  311.344  < 2e-16 ***
## agebt                       -1.214e-06  4.990e-09 -243.342  < 2e-16 ***
## edu_num_bt                  -6.800e-06  7.212e-08  -94.282  < 2e-16 ***
## cap_gain                    -3.116e-11  5.636e-13  -55.285  < 2e-16 ***
## cap_loss                     7.701e-11  1.020e-11    7.546 4.56e-14 ***
## hours_week                  -2.671e-08  3.629e-10  -73.612  < 2e-16 ***
## occupationArmy               2.569e-07  2.330e-07    1.103 0.270244
## occupationCraftRep           2.589e-08  5.199e-08    0.498 0.618574
## occupationExecMan           -6.786e-10  2.620e-08   -0.026 0.979337
## occupationFarmFish           2.252e-07  9.332e-08    2.413 0.015819 *
## occupationHandlCl            1.426e-07  5.814e-08    2.453 0.014181 *
## occupationHouse              9.274e-09  6.160e-08    0.151 0.880324
## occupationMachOp             8.009e-08  3.513e-08    2.279 0.022646 *
## occupationOther              1.856e-07  2.285e-08    8.124 4.63e-16 ***
## occupationProf               1.058e-07  2.147e-08    4.929 8.31e-07 ***
## occupationProtServ           5.319e-08  8.266e-08    0.644 0.519880
## occupationSales              1.804e-07  2.520e-08    7.159 8.26e-13 ***
## occupationTech              -1.166e-07  4.053e-08   -2.876 0.004032 **
## occupationTrans              7.127e-09  8.084e-08    0.088 0.929743
## maritalMarried              -1.077e-06  7.275e-08  -14.809  < 2e-16 ***
## maritalNevMarr               2.753e-06  5.998e-08   45.909  < 2e-16 ***
## maritalSep                   4.263e-07  8.539e-08    4.992 5.99e-07 ***
## maritalWidow                -2.039e-06  9.467e-08  -21.535  < 2e-16 ***
## sexMale                     -6.591e-08  2.650e-08   -2.488 0.012862 *
## workclassLoc                -6.740e-08  2.925e-08   -2.304 0.021222 *
## workclassNoPay               2.383e-07  1.630e-07    1.462 0.143858
## workclassPriv               -2.444e-08  2.483e-08   -0.984 0.324941
## workclassSelfI               1.615e-07  3.310e-08    4.879 1.07e-06 ***
## workclassSelfN              -1.633e-08  2.882e-08   -0.567 0.571056
## workclassState              -2.421e-08  3.146e-08   -0.770 0.441579
## relationshipNot-in-family    5.415e-08  5.023e-08    1.078 0.280998
## relationshipOther-relative   1.858e-07  4.913e-08    3.782 0.000156 ***
## relationshipOwn-child        4.005e-07  5.007e-08    7.998 1.29e-15 ***
## relationshipUnmarried        7.898e-08  5.199e-08    1.519 0.128758
## relationshipWife            -2.559e-07  2.301e-08  -11.122  < 2e-16 ***
## raceAsian-Pac-Islander      -1.632e-07  4.747e-08   -3.438 0.000588 ***
## raceBlack                   -6.454e-08  4.339e-08   -1.487 0.136905
## raceOther                    1.029e-07  6.078e-08    1.693 0.090555 .
## raceWhite                    5.380e-09  4.164e-08    0.129 0.897189
## income>50K                  -3.407e-06  1.207e-08 -282.346  < 2e-16 ***
## occupationArmy:sexMale              NA         NA       NA       NA
## occupationCraftRep:sexMale   2.372e-08  5.729e-08    0.414 0.678841
## occupationExecMan:sexMale    2.293e-07  3.629e-08    6.317 2.70e-10 ***
## occupationFarmFish:sexMale -1.002e-07  9.850e-08   -1.017 0.309164
## occupationHandlCl:sexMale    1.404e-07  6.524e-08    2.152 0.031371 *
## occupationHouse:sexMale      4.917e-07  2.479e-07    1.984 0.047292 *
```

```
## occupationMachOp:sexMale    4.639e-08  4.498e-08    1.031 0.302404
## occupationOther:sexMale     2.757e-08  3.633e-08    0.759 0.447899
## occupationProf:sexMale      1.363e-07  3.228e-08    4.222 2.42e-05 ***
## occupationProtServ:sexMale -1.056e-07  9.032e-08   -1.169 0.242421
## occupationSales:sexMale    -9.026e-08  3.615e-08   -2.497 0.012542 *
## occupationTech:sexMale      4.475e-08  5.463e-08    0.819 0.412660
## occupationTrans:sexMale     5.997e-08  8.567e-08    0.700 0.483941
## maritalDiv:edu_num          7.021e-07  1.303e-08   53.867  < 2e-16 ***
## maritalMarried:edu_num      8.212e-07  1.258e-08   65.291  < 2e-16 ***
## maritalNevMarr:edu_num      4.546e-07  1.276e-08   35.628  < 2e-16 ***
## maritalSep:edu_num          6.677e-07  1.488e-08   44.875  < 2e-16 ***
## maritalWidow:edu_num        9.025e-07  1.558e-08   57.937  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 8.948e-07 on 48785 degrees of freedom
## Multiple R-squared:  0.9166, Adjusted R-squared:  0.9165
## F-statistic:  9578 on 56 and 48785 DF,  p-value: < 2.2e-16
```

```
#Possible Conclusio
avPlots(btmodel,id=list(method=hatvalues(catmodel),n=5))
```




```
plot(allEffects(catmodel))
```




```
income_bin <- ifelse(dd$income == ">50K", 1, 0)
income_bin <- as.factor(income_bin)
set.seed(123)
total_n     <- nrow(dd)
train_index <- sample(seq_len(total_n), size = floor(0.8 * total_n))
train_data  <- dd[train_index, ]
test_data   <- dd[-train_index, ]
test_data$income_bin  <- factor(ifelse(test_data$income  == ">50K", "1", "0"), levels = c("0","1"))
train_data$income_bin <- factor(ifelse(train_data$income == ">50K", "1", "0"), levels = c("0","1"))
```

```
## Build the Initial Logistic Regression Model
initial_model_b <- glm(income_bin ~ age + edu_num + cap_gain + cap_loss + hours_week, data = dd, family
```

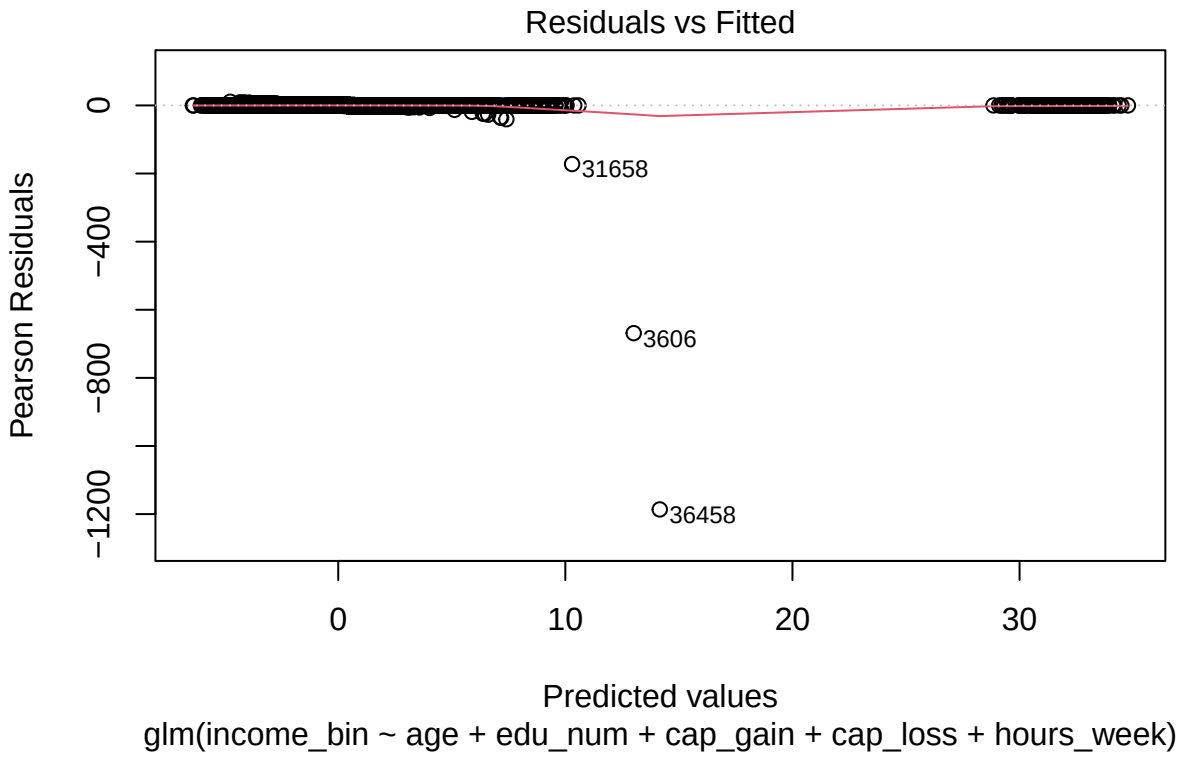
```
## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred
```

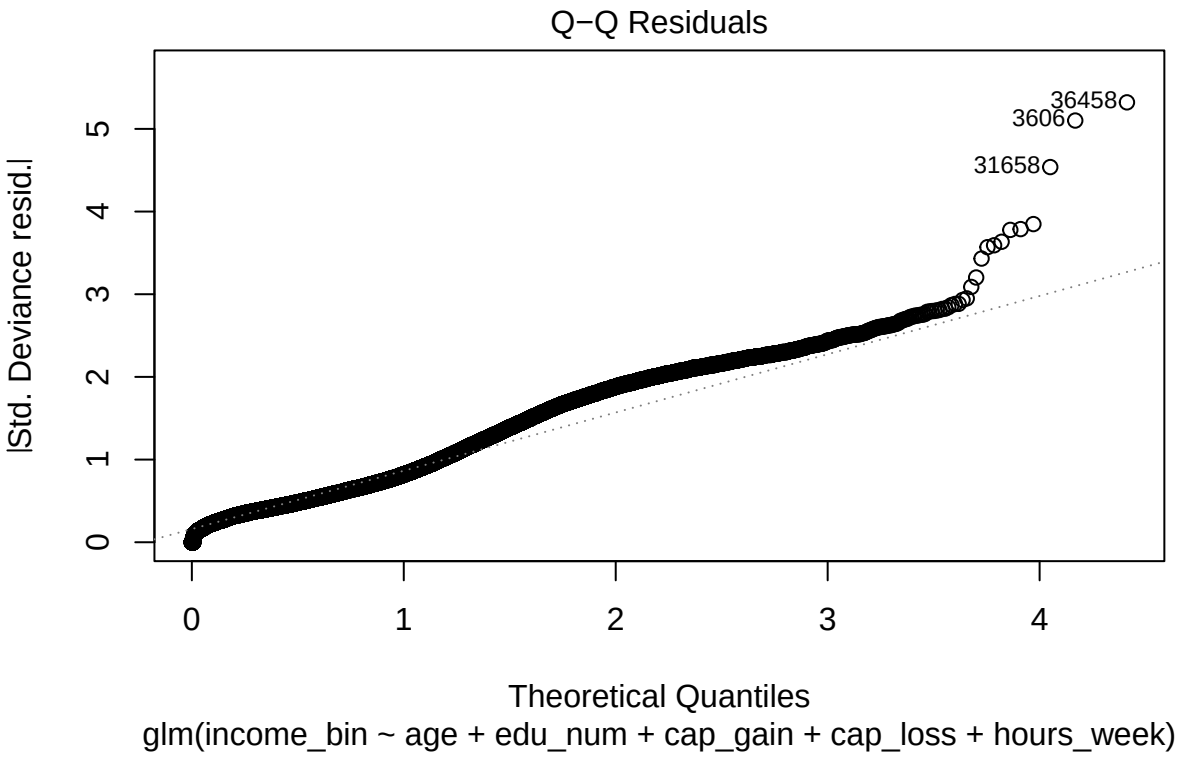
```
summary(initial_model_b)
```

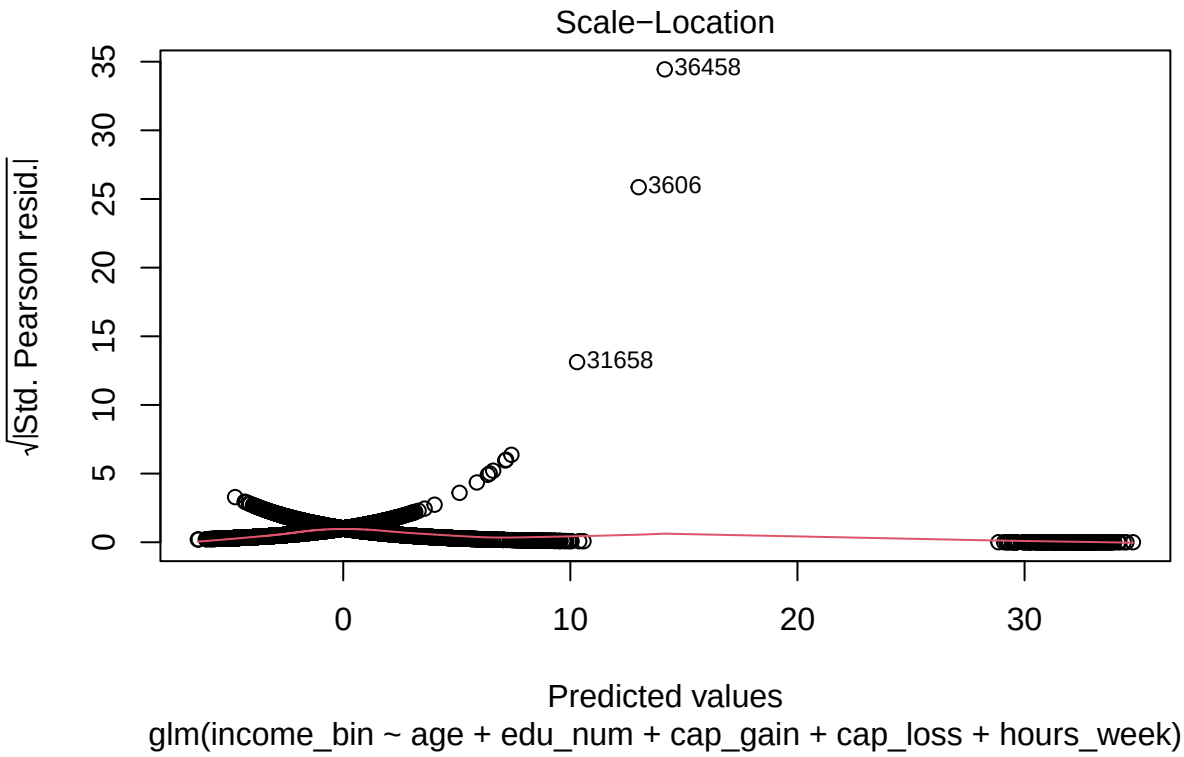
```
##
## Call:
## glm(formula = income_bin ~ age + edu_num + cap_gain + cap_loss +
##     hours_week, family = binomial, data = dd)
##
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
```

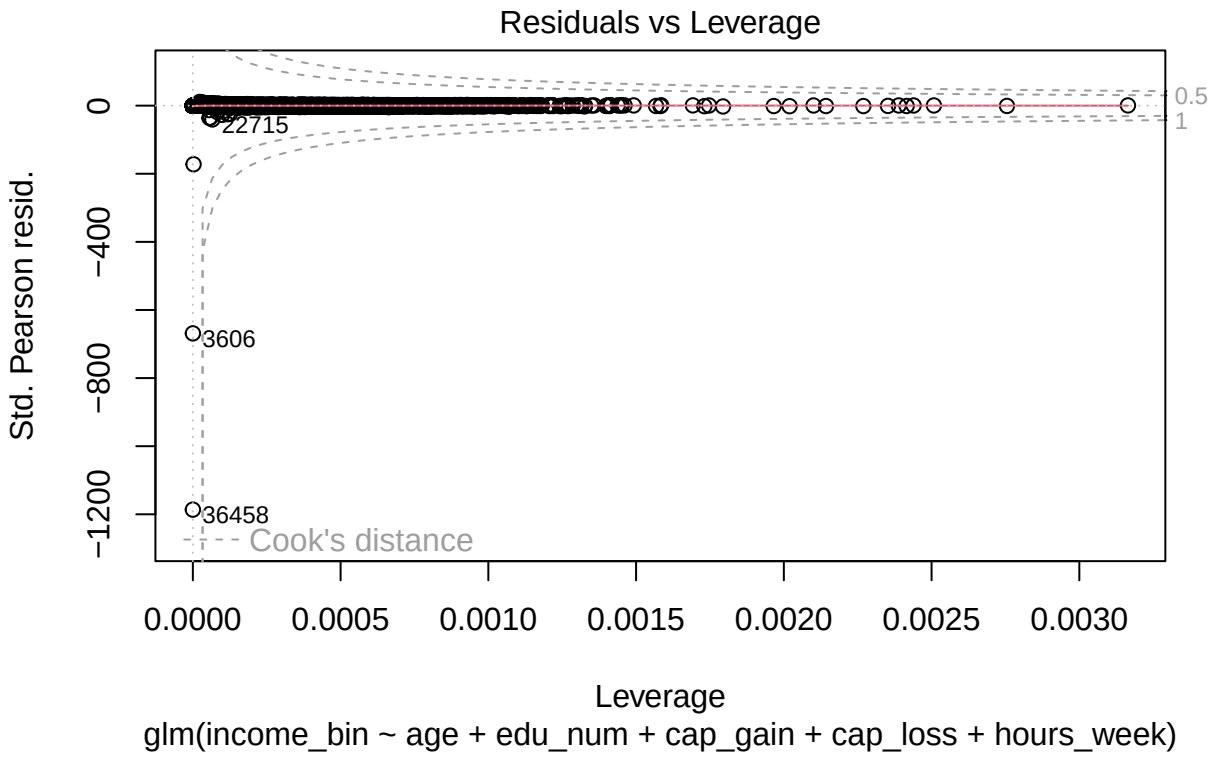
```
## (Intercept) -8.260e+00  9.371e-02  -88.14   <2e-16 ***
## age          4.220e-02  9.915e-04   42.57   <2e-16 ***
## edu_num      3.223e-01  5.556e-03   58.01   <2e-16 ***
## cap_gain     3.205e-04  7.985e-06   40.14   <2e-16 ***
## cap_loss     6.799e-04  2.634e-05   25.81   <2e-16 ***
## hours_week   4.012e-02  1.070e-03   37.51   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 53751  on 48841  degrees of freedom
## Residual deviance: 39775  on 48836  degrees of freedom
## AIC: 39787
##
## Number of Fisher Scoring iterations: 7
```

```
## Check Model Diagnostics
plot(initial_model_b) #the basic hypothesis are met, beware of Homoscedasticity
```

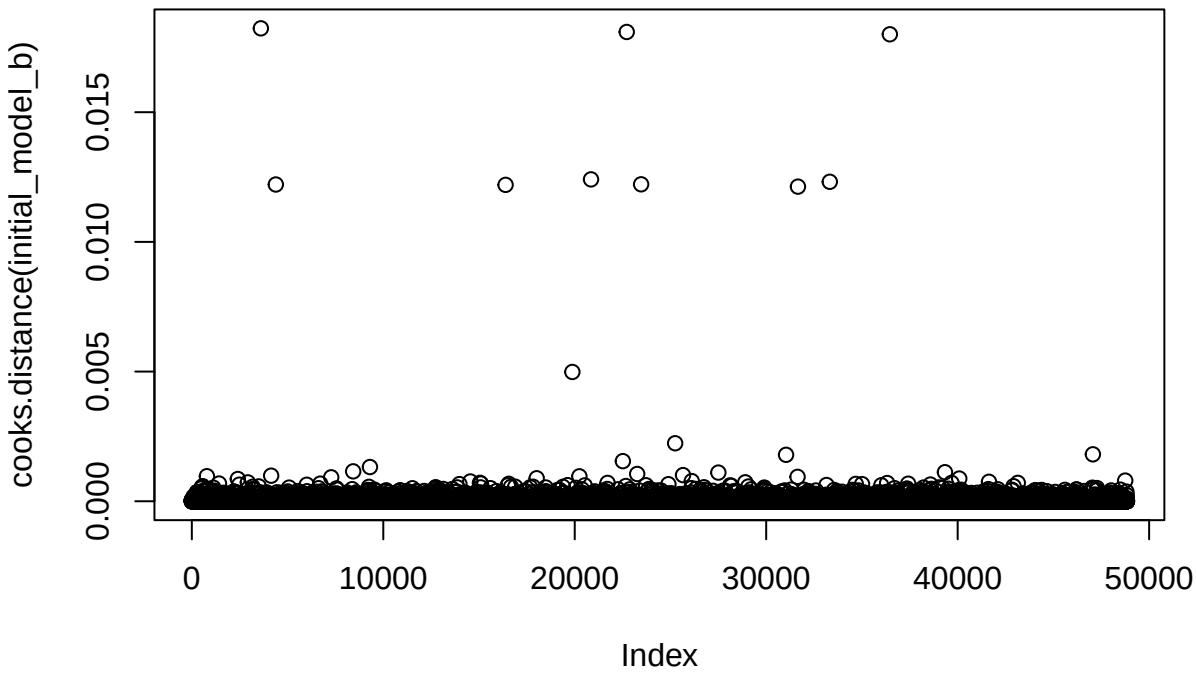



Q−Q Residuals
|Std. Deviance resid.|
0
1
2
3
4
5
36458
3606
31658
0
1
2
3
4
Theoretical Quantiles
glm(income_bin ~ age + edu_num + cap_gain + cap_loss + hours_week)


Scale−Location
√|Std. Pearson resid.|
0
5
10
15
20
25
30
35
36458
3606
31658
0
10
20
30
Predicted values
glm(income_bin ~ age + edu_num + cap_gain + cap_loss + hours_week)

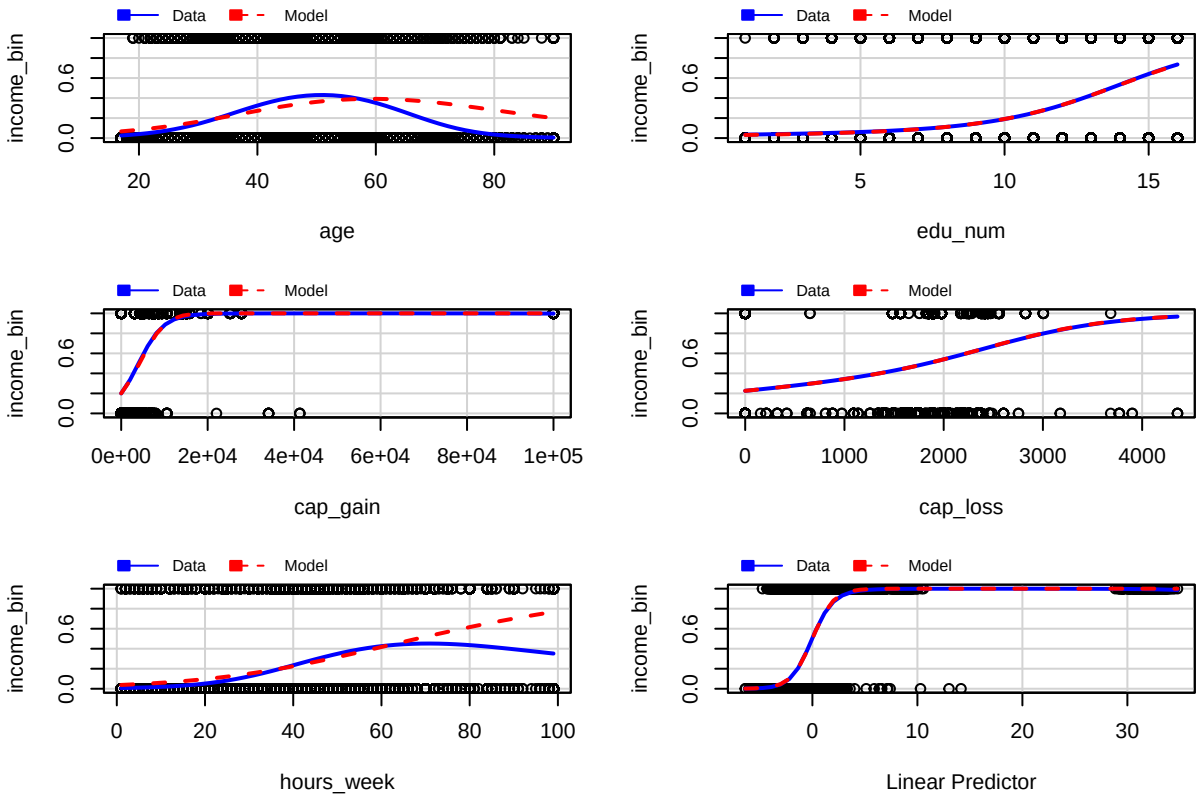



```
#Check cook's distance
plot(cooks.distance(initial_model_b)) #there are some influential observations that skew the data a lit
```



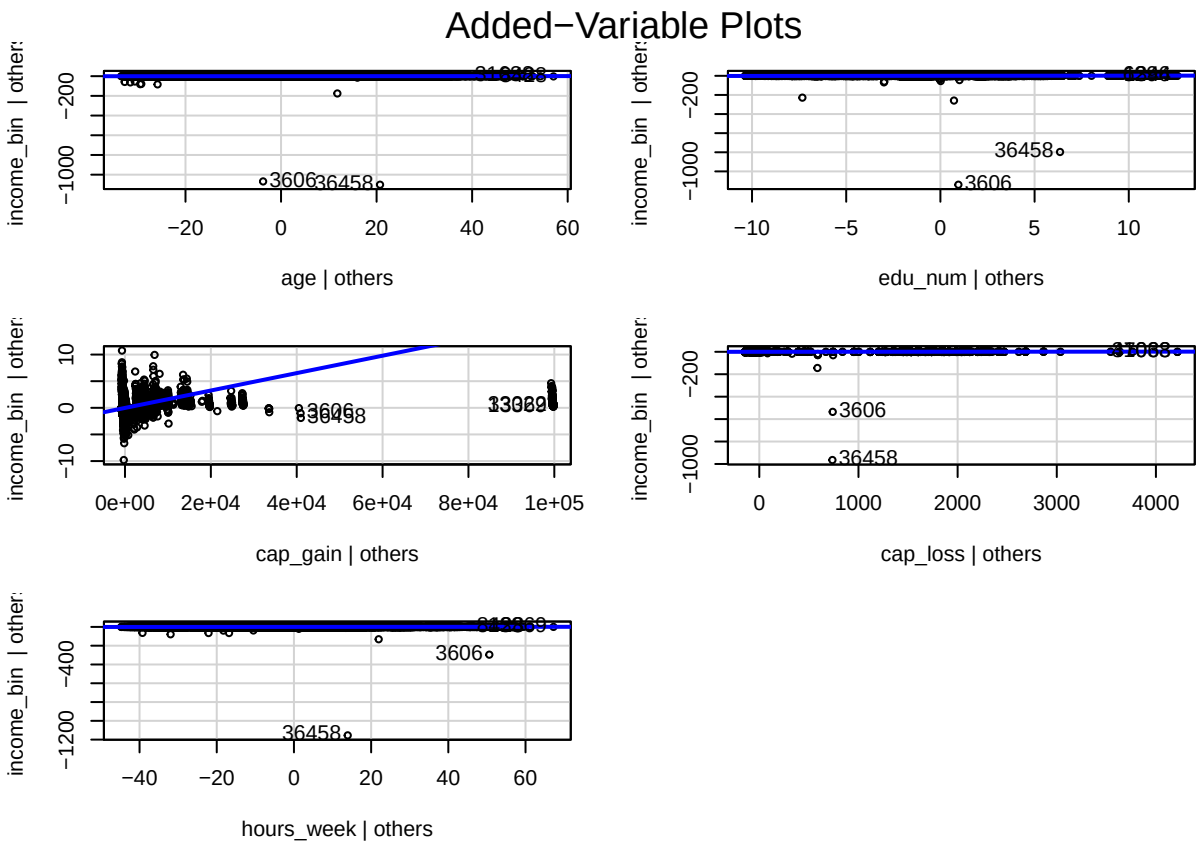

```
setwd("~/Escritorio/ADEI/D2")
df<- read.csv("adult_def.csv")

df$income <- ifelse(df$income == ">50K", 1, 0)

#GRAFICAS QUE APORTAN DE MÁS
marginalModelPlots(initial_model_b)
```

```
## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred
```

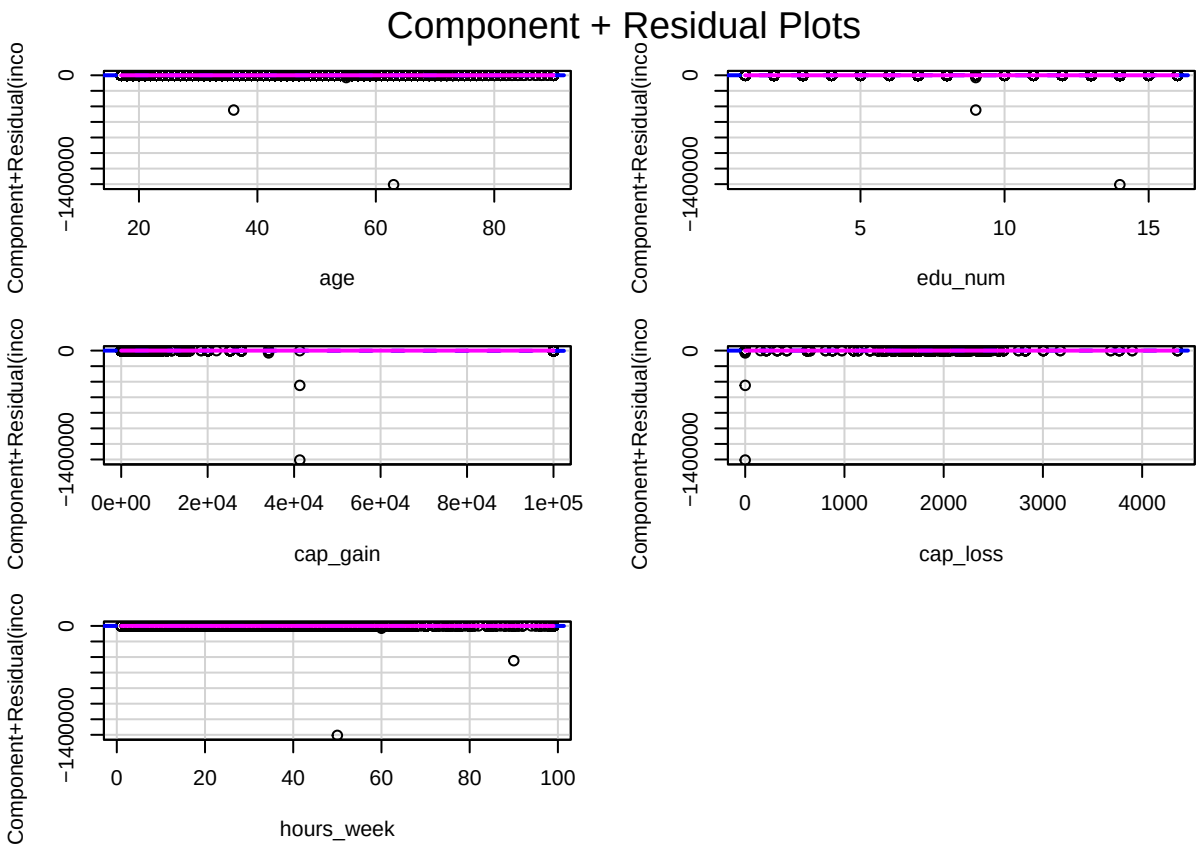



```
avPlots(initial_model_b)
```

```
## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred
## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred
## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred
## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred
```




```
crPlots(initial_model_b)
```

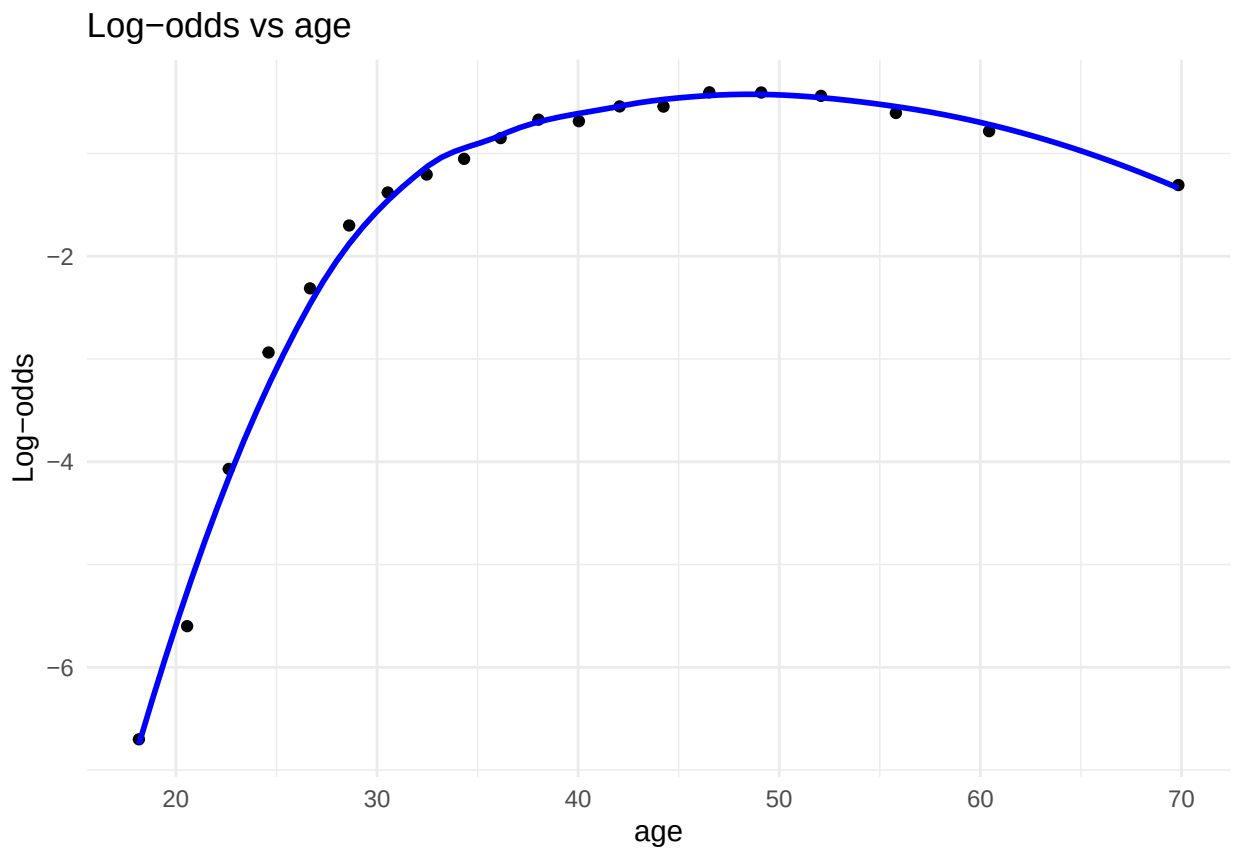



```
#PLOT LOGS

library(dplyr)
```

```
##
## Adjuntando el paquete: 'dplyr'

## The following object is masked from 'package:car':
##
##     recode

## The following object is masked from 'package:MASS':
##
##     select

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

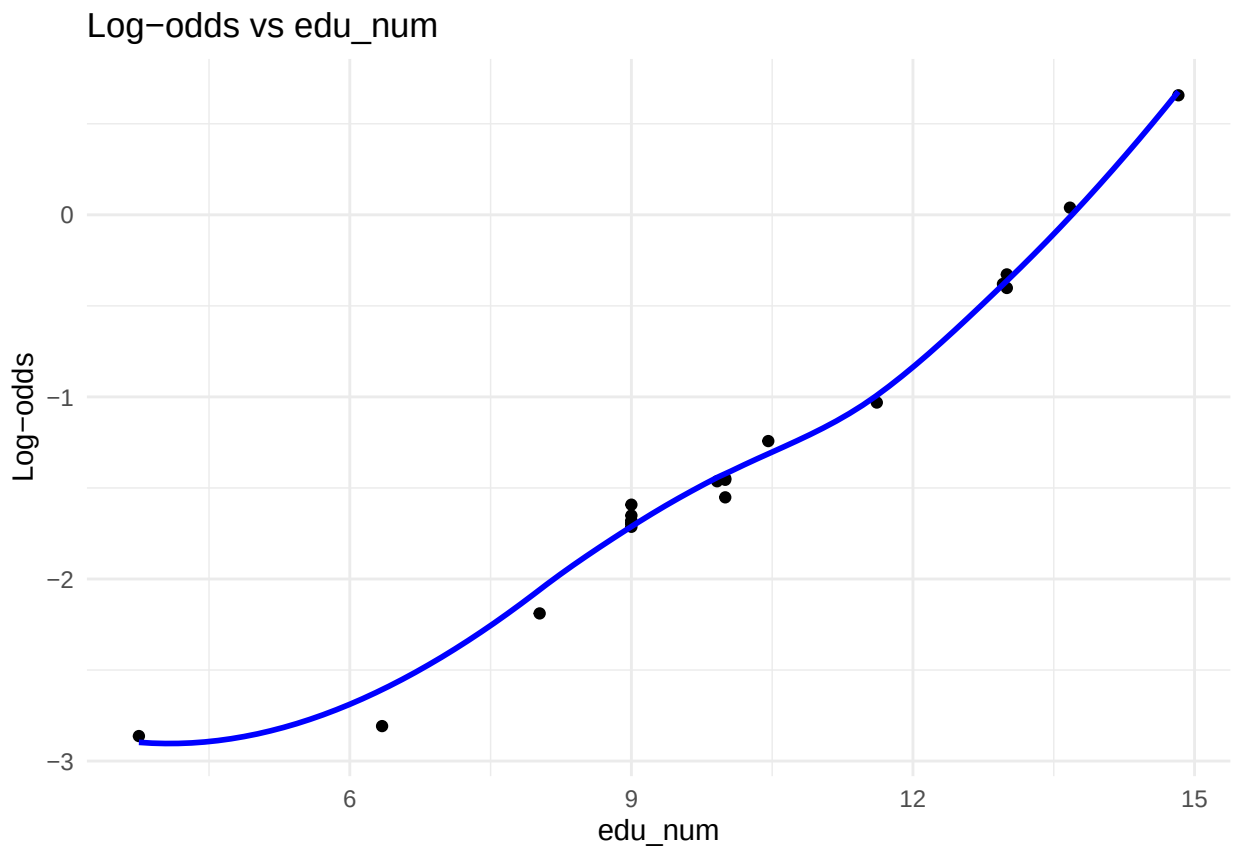
```r
library(ggplot2)

plot_log_odds <- function(df, var, response = "income") {
  df %>%
    mutate(bin = ntile(.data[[var]], 20)) %>%  # agrupa la variable numérica en 20 grupos
    group_by(bin) %>%
    summarise(
      x = mean(.data[[var]], na.rm = TRUE),
      p = mean(.data[[response]], na.rm = TRUE),
      logit = log(p / (1 - p))
    ) %>%
    ggplot(aes(x = x, y = logit)) +
    geom_point() +
    geom_smooth(method = "loess", se = FALSE, color = "blue") +
    labs(
      title = paste("Log-odds vs", var),
      x = var,
      y = "Log-odds"
    ) +
    theme_minimal()
}

plot_log_odds(df, "age")
```

```
## `geom_smooth()` using formula = 'y ~ x'
```



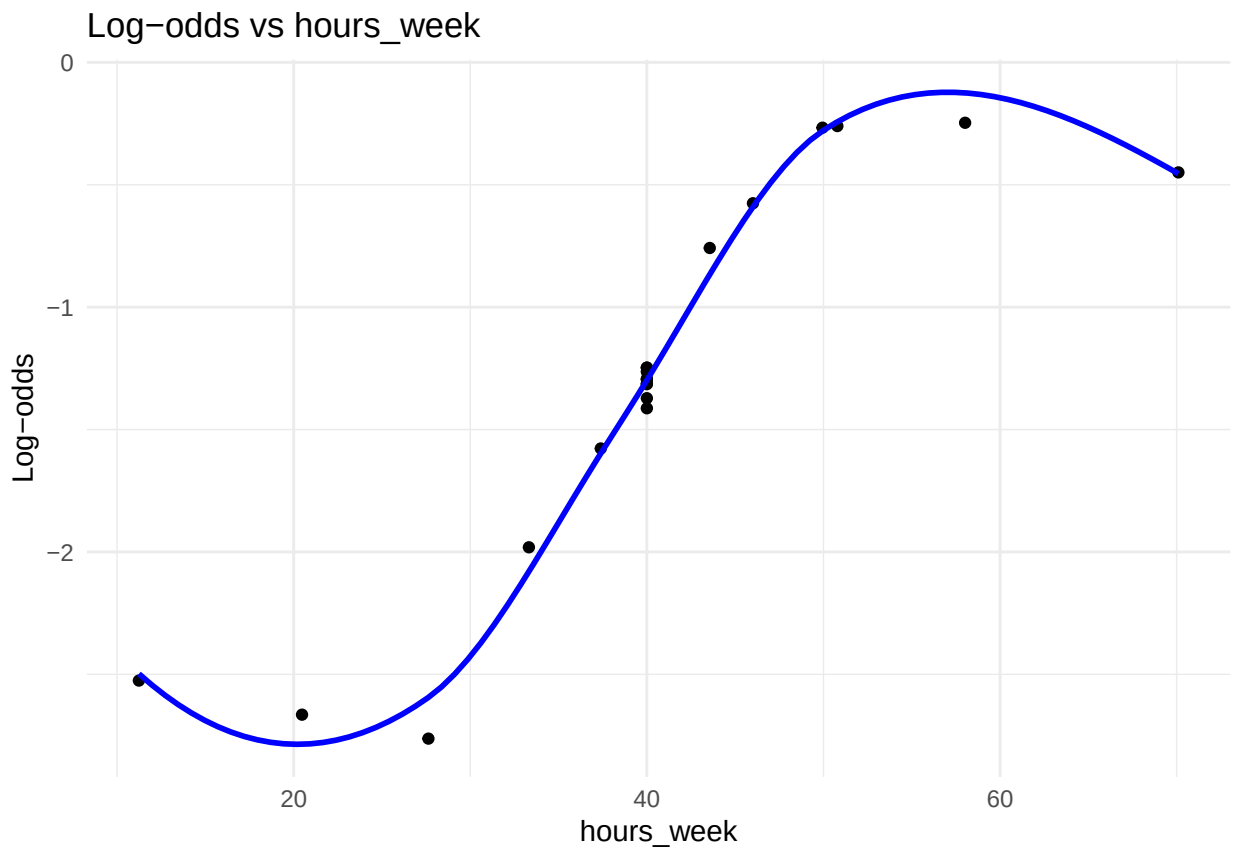
```
plot_log_odds(df, "edu_num")
```

```
## `geom_smooth()` using formula = 'y ~ x'
```



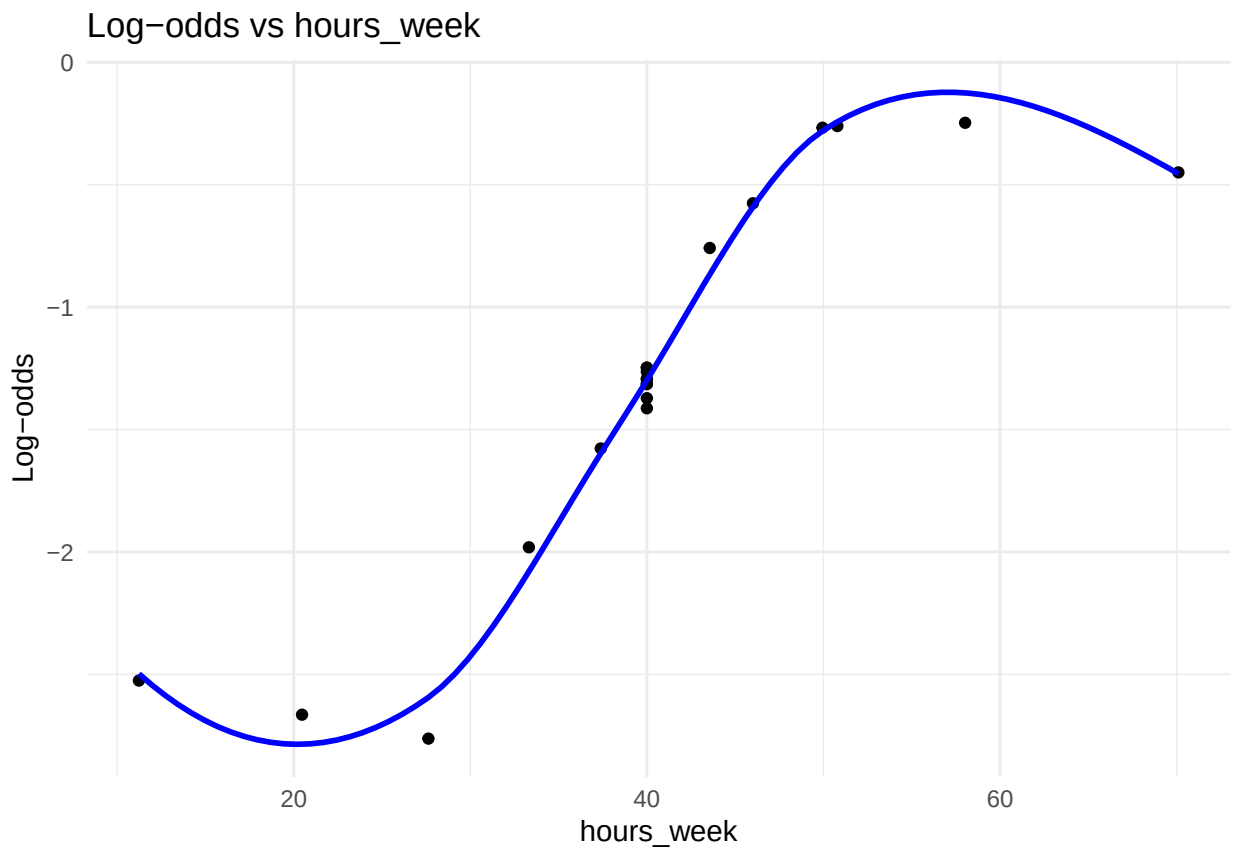

```
plot_log_odds(df, "hours_week")
```

```
## `geom_smooth()` using formula = 'y ~ x'
```

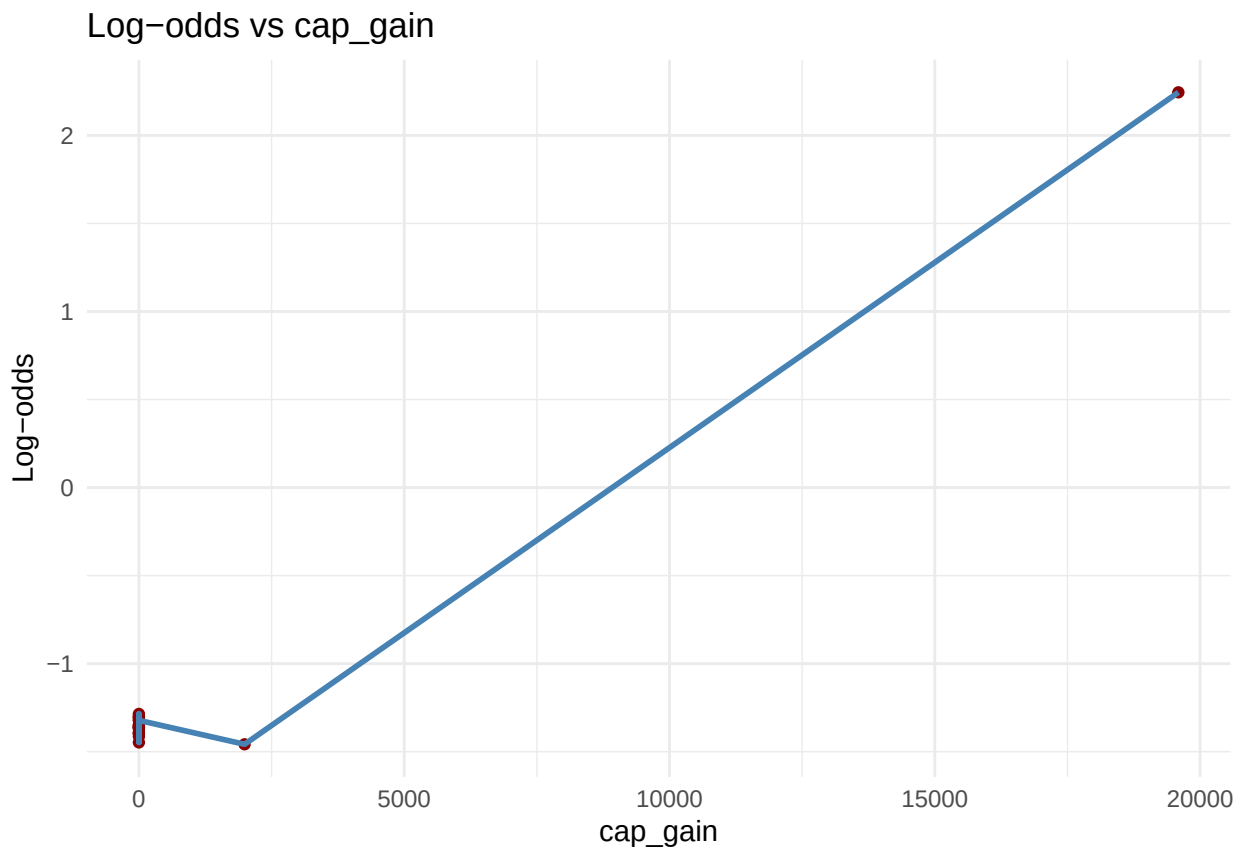



```
library(splines)
df$hours_week_spline <- ns(df$hours_week, df = 4)
plot_log_odds(df, "hours_week")
```

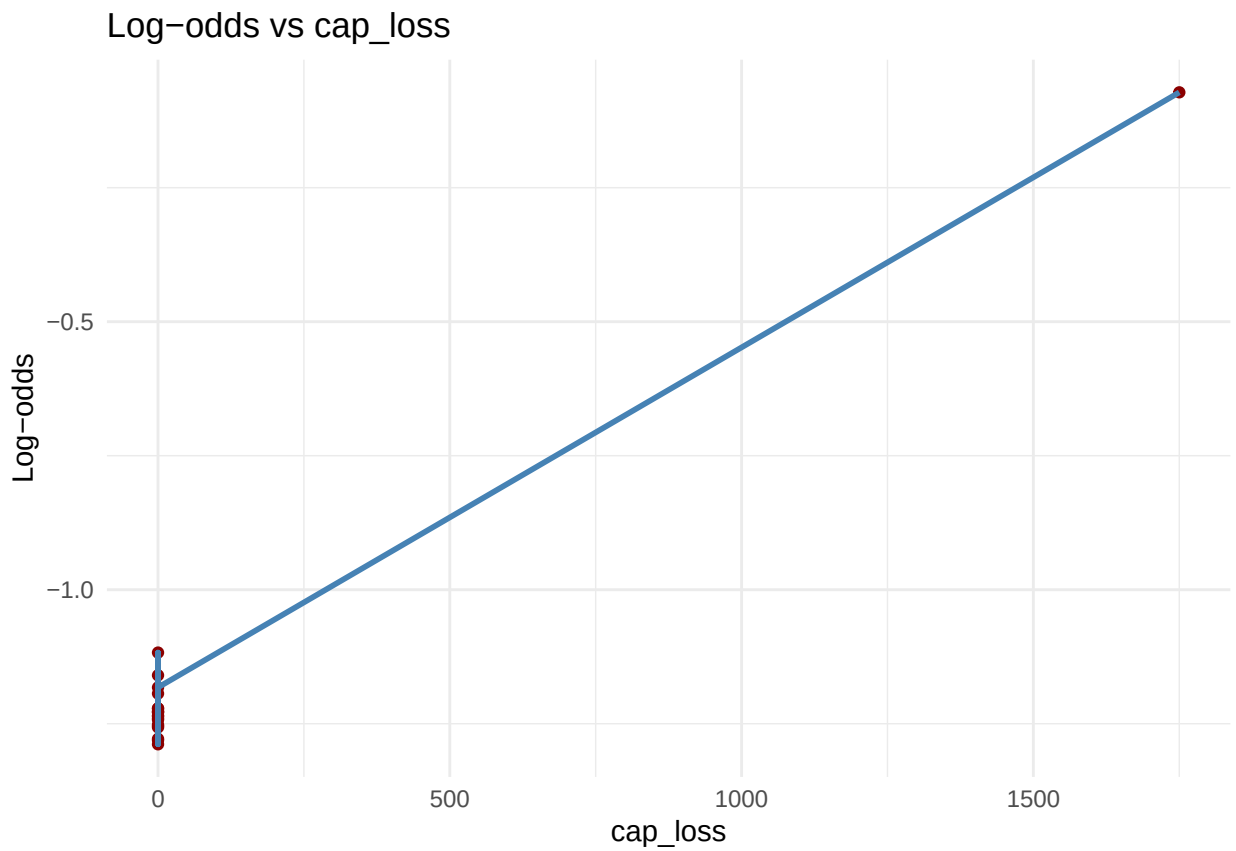
```
## `geom_smooth()` using formula = 'y ~ x'
```



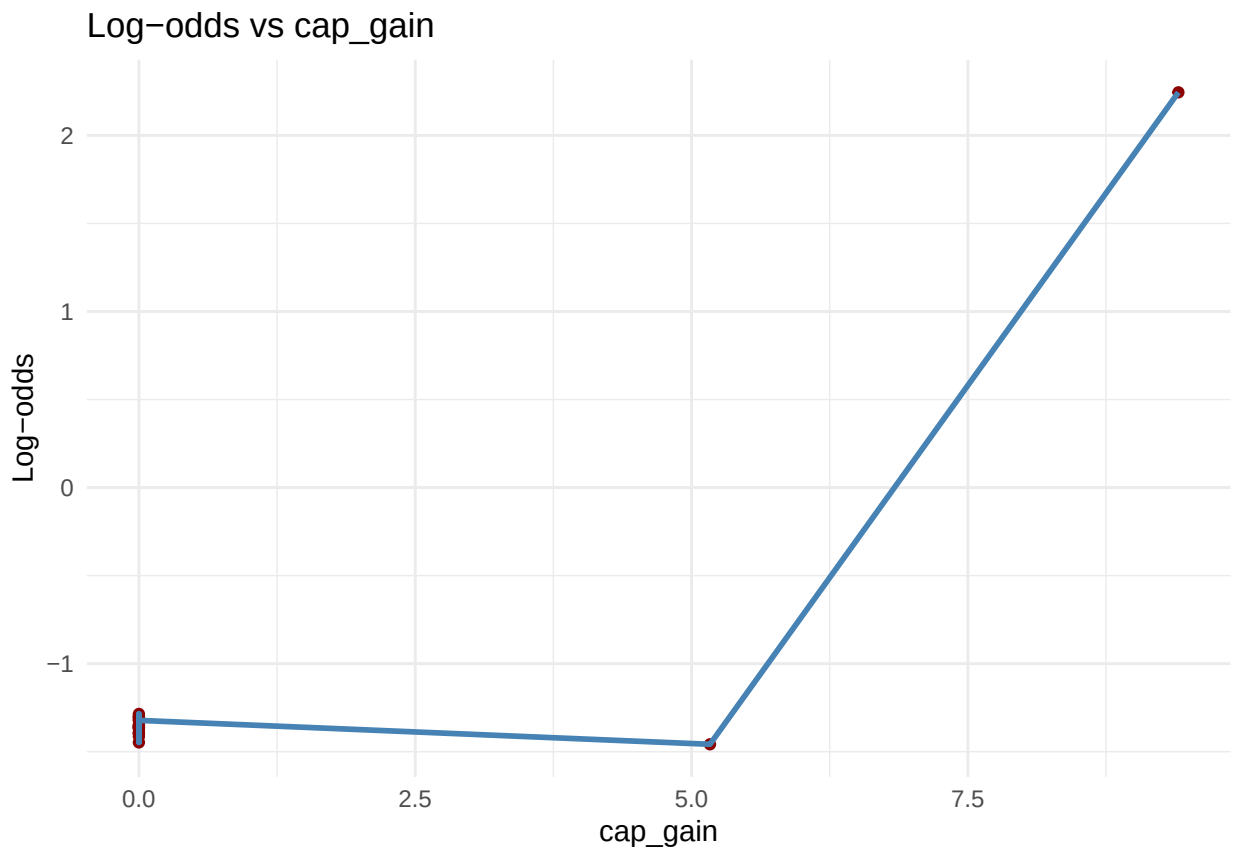

```r
# Función robusta para log-odds con suavizado controlado

plot_log_odds <- function(df, var, response = "income") {
  df %>%
    mutate(bin = ntile(.data[[var]], 20)) %>%
    group_by(bin) %>%
    summarise(
      x = mean(.data[[var]], na.rm = TRUE),
      p = mean(.data[[response]], na.rm = TRUE),
      p = ifelse(p == 0, 0.001, ifelse(p == 1, 0.999, p)),  # evita log(0) y log(1)
      logit = log(p / (1 - p))
    ) %>%
    ggplot(aes(x = x, y = logit)) +
    geom_point(color = "darkred") +
    geom_line(color = "steelblue", linewidth = 1) +  # sin loess
    labs(
      title = paste("Log-odds vs", var),
      x = var,
      y = "Log-odds"
    ) +
    theme_minimal()
}

# Ejecutar para cap_gain y cap_loss
plot_log_odds(df, "cap_gain")
```



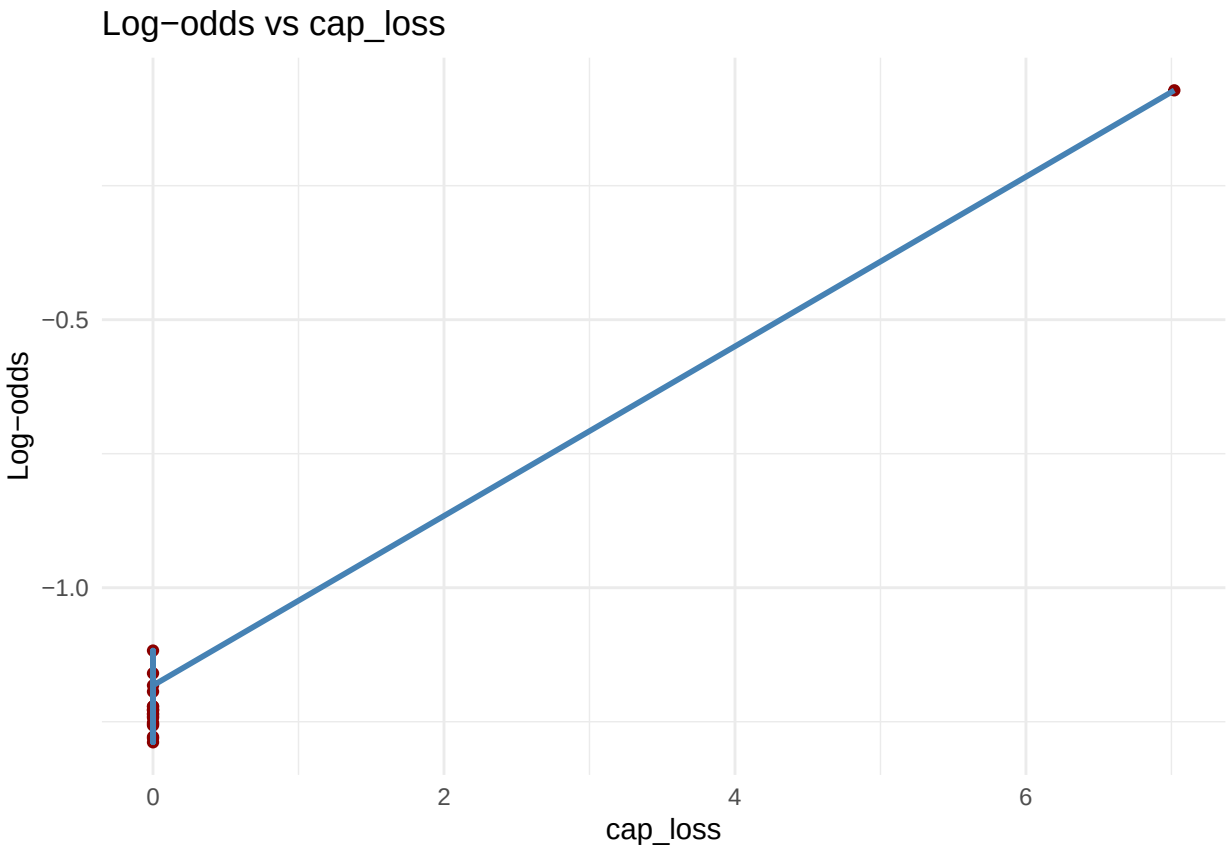

```
plot_log_odds(df, "cap_loss")
```




```
dd$cap_gain <- log1p(df$cap_gain)
dd$cap_loss <- log1p(df$cap_loss)

df$cap_gain <- log1p(df$cap_gain)
df$cap_loss <- log1p(df$cap_loss)

plot_log_odds(df, "cap_gain")
```




```
plot_log_odds(df, "cap_loss")
```




### Add Categorical Variables Step by Step

```
##Add_workclass_and_test_with_Chi_squared
initial_model_b <- glm(income_bin ~ age + edu_num + cap_gain + cap_loss + hours_week, data = dd, family
model_workclass <- update(initial_model_b, . ~ . + workclass)
anova(initial_model_b, model_workclass, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: income_bin ~ age + edu_num + cap_gain + cap_loss + hours_week
## Model 2: income_bin ~ age + edu_num + cap_gain + cap_loss + hours_week +
##     workclass
##   Resid. Df Resid. Dev Df Deviance  Pr(>Chi)
## 1     48836      40923
## 2     48830      40674  6   249.47 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##Add_marital_and_test_with_Chi_squared
model_marital <- update(model_workclass, . ~ . + marital)
anova(model_workclass, model_marital, test = "Chisq")
```

```
## Analysis of Deviance Table
```

```
##
## Model 1: income_bin ~ age + edu_num + cap_gain + cap_loss + hours_week +
##     workclass
## Model 2: income_bin ~ age + edu_num + cap_gain + cap_loss + hours_week +
##     workclass + marital
##   Resid. Df Resid. Dev Df Deviance  Pr(>Chi)
## 1     48830      40674
## 2     48826      33698  4   6976.1 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##Add_occupation_and_test_with_Chi_squared
model_occupation <- update(model_marital, . ~ . + occupation)
anova(model_marital, model_occupation, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: income_bin ~ age + edu_num + cap_gain + cap_loss + hours_week +
##     workclass + marital
## Model 2: income_bin ~ age + edu_num + cap_gain + cap_loss + hours_week +
##     workclass + marital + occupation
##   Resid. Df Resid. Dev Df Deviance  Pr(>Chi)
## 1     48826      33698
## 2     48813      32834 13   864.28 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##Add_relationship_and_test_with_Chi_squared
model_relationship <- update(model_occupation, . ~ . + relationship)
anova(model_occupation, model_relationship, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: income_bin ~ age + edu_num + cap_gain + cap_loss + hours_week +
##     workclass + marital + occupation
## Model 2: income_bin ~ age + edu_num + cap_gain + cap_loss + hours_week +
##     workclass + marital + occupation + relationship
##   Resid. Df Resid. Dev Df Deviance  Pr(>Chi)
## 1     48813      32834
## 2     48808      32584  5   249.44 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##Add_race_and_test_with_Chi_squared
model_race <- update(model_relationship, . ~ . + race)
anova(model_relationship, model_race, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: income_bin ~ age + edu_num + cap_gain + cap_loss + hours_week +
##     workclass + marital + occupation + relationship
## Model 2: income_bin ~ age + edu_num + cap_gain + cap_loss + hours_week +
```

```
##     workclass + marital + occupation + relationship + race
##   Resid. Df Resid. Dev Df Deviance  Pr(>Chi)
## 1     48808      32584
## 2     48804      32560  4   24.477 6.407e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##Add_sex_and_test_with_Chi_squared
model_sex <- update(model_race, . ~ . + sex)
anova(model_race, model_sex, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: income_bin ~ age + edu_num + cap_gain + cap_loss + hours_week +
##     workclass + marital + occupation + relationship + race
## Model 2: income_bin ~ age + edu_num + cap_gain + cap_loss + hours_week +
##     workclass + marital + occupation + relationship + race +
##     sex
##   Resid. Df Resid. Dev Df Deviance  Pr(>Chi)
## 1     48804      32560
## 2     48803      32426  1   133.44 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##Add_native_country_and_test_with_Chi_squared
model_country <- update(model_sex, . ~ . + native_country)
anova(model_sex, model_country, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: income_bin ~ age + edu_num + cap_gain + cap_loss + hours_week +
##     workclass + marital + occupation + relationship + race +
##     sex
## Model 2: income_bin ~ age + edu_num + cap_gain + cap_loss + hours_week +
##     workclass + marital + occupation + relationship + race +
##     sex + native_country
##   Resid. Df Resid. Dev Df Deviance  Pr(>Chi)
## 1     48803      32426
## 2     48802      32414  1   11.969 0.0005409 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
## Define the Final Model
final_model <- model_country
```

```
## Perform Stepwise Selection and Final Diagnostics
stepmodel <- stepAIC(final_model, direction = "back")
```

```
## Start:  AIC=32494.25
## income_bin ~ age + edu_num + cap_gain + cap_loss + hours_week +
##     workclass + marital + occupation + relationship + race +
##     sex + native_country
```

```
##
##                  Df Deviance   AIC
## <none>                32414 32494
## - native_country  1    32426 32504
## - race            4    32435 32507
## - sex             1    32549 32627
## - marital         4    32582 32654
## - workclass       6    32588 32656
## - age             1    32747 32825
## - relationship    5    32786 32856
## - cap_loss        1    32815 32893
## - hours_week      1    33050 33128
## - occupation     13    33239 33293
## - cap_gain        1    34131 34209
## - edu_num         1    34540 34618
```

```
vif(final_model)
```

```
##                     GVIF Df GVIF^(1/(2*Df))
## age             1.238318  1        1.112797
## edu_num         1.398393  1        1.182537
## cap_gain        1.029813  1        1.014797
## cap_loss        1.012070  1        1.006017
## hours_week      1.143801  1        1.069486
## workclass       1.489055  6        1.033735
## marital        48.550369  4        1.624703
## occupation      2.175083 13        1.030338
## relationship  106.507884  5        1.594917
## race            1.306566  4        1.033990
## sex             2.799541  1        1.673183
## native_country  1.255919  1        1.120678
```

```
summary(final_model)
```

```
##
## Call:
## glm(formula = income_bin ~ age + edu_num + cap_gain + cap_loss +
##     hours_week + workclass + marital + occupation + relationship +
##     race + sex + native_country, family = binomial, data = dd)
##
## Coefficients:
##                            Estimate Std. Error z value Pr(>|z|)
## (Intercept)               -9.185238   0.315764 -29.089  < 2e-16 ***
## age                        0.023163   0.001274  18.179  < 2e-16 ***
## edu_num                    0.307182   0.007142  43.009  < 2e-16 ***
## cap_gain                   0.206803   0.005260  39.317  < 2e-16 ***
## cap_loss                   0.151105   0.007593  19.901  < 2e-16 ***
## hours_week                 0.031309   0.001261  24.826  < 2e-16 ***
## workclassLoc              -0.591447   0.089280  -6.625 3.48e-11 ***
## workclassNoPay            -1.583736   0.791564  -2.001 0.045417 *
## workclassPriv             -0.544920   0.074448  -7.319 2.49e-13 ***
## workclassSelfI            -0.302041   0.097359  -3.102 0.001920 **
## workclassSelfN            -0.983363   0.087041 -11.298  < 2e-16 ***
```

```
## workclassState             -0.766828   0.098430  -7.791 6.67e-15 ***
## maritalMarried              2.190810   0.206831  10.592  < 2e-16 ***
## maritalNevMarr             -0.409496   0.067439  -6.072 1.26e-09 ***
## maritalSep                 -0.009917   0.106140  -0.093 0.925563
## maritalWidow                0.044763   0.117298   0.382 0.702747
## occupationArmy              0.498904   0.768296   0.649 0.516103
## occupationCraftRep          0.090487   0.063525   1.424 0.154319
## occupationExecMan           0.747849   0.060774  12.305  < 2e-16 ***
## occupationFarmFish         -0.974205   0.111772  -8.716  < 2e-16 ***
## occupationHandlCl          -0.652362   0.111965  -5.826 5.66e-09 ***
## occupationHouse            -1.687714   0.636758  -2.650 0.008038 **
## occupationMachOp           -0.292128   0.081693  -3.576 0.000349 ***
## occupationOther            -0.868541   0.094083  -9.232  < 2e-16 ***
## occupationProf              0.279347   0.059624   4.685 2.80e-06 ***
## occupationProtServ          0.446458   0.100970   4.422 9.79e-06 ***
## occupationSales             0.255818   0.065328   3.916 9.01e-05 ***
## occupationTech              0.526447   0.087529   6.015 1.80e-09 ***
## occupationTrans            -0.073437   0.079115  -0.928 0.353291
## relationshipNot-in-family   0.521617   0.204963   2.545 0.010930 *
## relationshipOther-relative -0.477064   0.190936  -2.499 0.012470 *
## relationshipOwn-child      -0.591879   0.202634  -2.921 0.003490 **
## relationshipUnmarried       0.294691   0.217736   1.353 0.175916
## relationshipWife            1.095140   0.079246  13.820  < 2e-16 ***
## raceAsian-Pac-Islander      0.506635   0.192251   2.635 0.008407 **
## raceBlack                   0.295384   0.179551   1.645 0.099944 .
## raceOther                   0.319483   0.257435   1.241 0.214597
## raceWhite                   0.493374   0.171068   2.884 0.003926 **
## sexMale                     0.686837   0.059889  11.468  < 2e-16 ***
## native_countryUSA           0.191995   0.055848   3.438 0.000586 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 53751  on 48841  degrees of freedom
## Residual deviance: 32414  on 48802  degrees of freedom
## AIC: 32494
##
## Number of Fisher Scoring iterations: 7
```

```
anova(final_model, test="LR")
```

```
## Analysis of Deviance Table
##
## Model: binomial, link: logit
##
## Response: income_bin
##
## Terms added sequentially (first to last)
##
##
##                Df Deviance Resid. Df Resid. Dev  Pr(>Chi)
## NULL                           48841      53751
## age             1   2537.0     48840      51214 < 2.2e-16 ***
```

```
## edu_num         1   5891.7     48839      45322 < 2.2e-16 ***
## cap_gain        1   2142.0     48838      43180 < 2.2e-16 ***
## cap_loss        1    660.7     48837      42519 < 2.2e-16 ***
## hours_week      1   1596.0     48836      40923 < 2.2e-16 ***
## workclass       6    249.5     48830      40674 < 2.2e-16 ***
## marital         4   6976.1     48826      33698 < 2.2e-16 ***
## occupation     13    864.3     48813      32834 < 2.2e-16 ***
## relationship    5    249.4     48808      32584 < 2.2e-16 ***
## race            4     24.5     48804      32560 6.407e-05 ***
## sex             1    133.4     48803      32426 < 2.2e-16 ***
## native_country  1     12.0     48802      32414 0.0005409 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

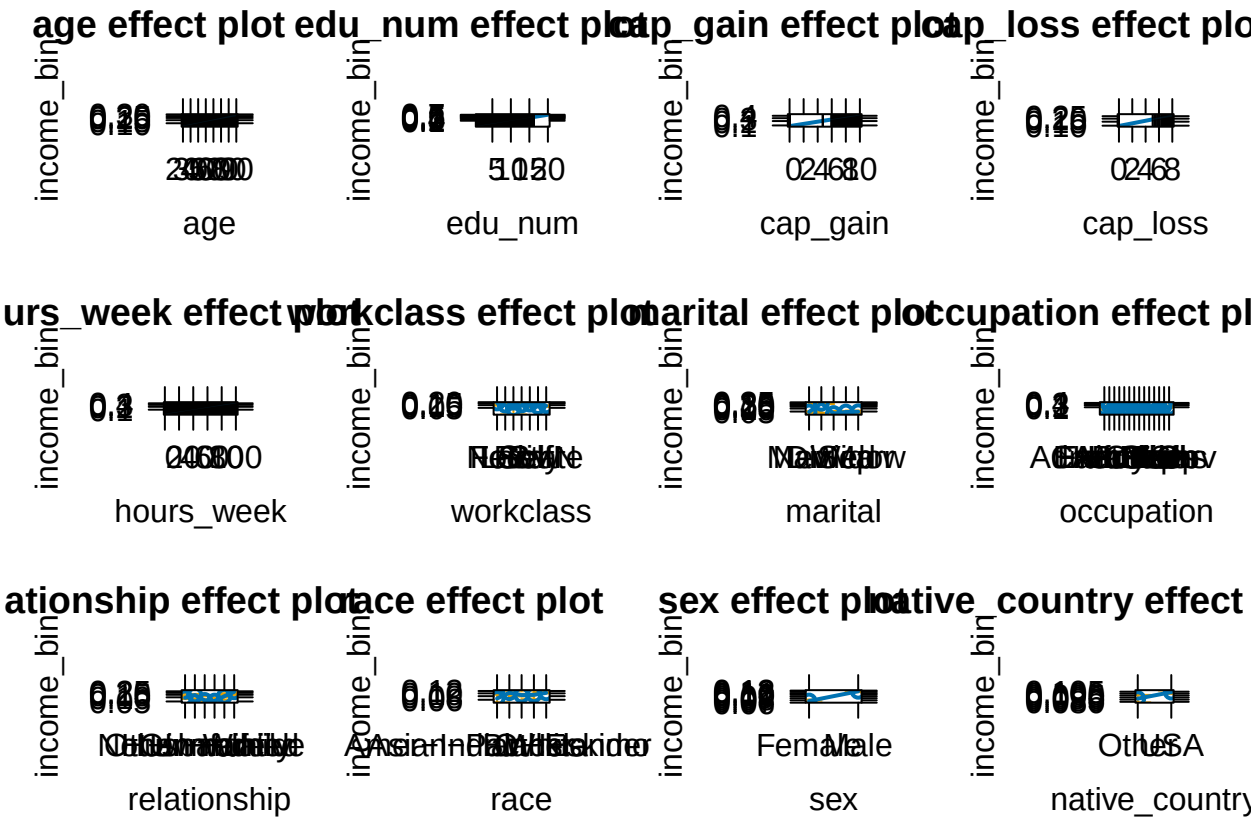
```
anova(initial_model_b, final_model)
```

```
## Analysis of Deviance Table
##
## Model 1: income_bin ~ age + edu_num + cap_gain + cap_loss + hours_week
## Model 2: income_bin ~ age + edu_num + cap_gain + cap_loss + hours_week +
##     workclass + marital + occupation + relationship + race +
##     sex + native_country
##   Resid. Df Resid. Dev Df Deviance  Pr(>Chi)
## 1     48836      40923
## 2     48802      32414 34   8509.2 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
AIC(initial_model_b, final_model)
```

```
##                 df      AIC
## initial_model_b  6 40935.40
## final_model     40 32494.25
```

```
plot(allEffects(final_model))
```




```
##Final Model Diagnostics and Test-Sample Evaluation
##Predictions on test set (probabilities)
probs_test <- predict(final_model, newdata = test_data, type = "response")
```

```
##Convert probabilities to class labels (threshold = 0.5)
preds_test <- factor(ifelse(probs_test > 0.5, "1", "0"), levels=c("0","1"))
```

```
##Confusion matrix and derived metrics
conf_tab   <- table(Pred=preds_test, True=test_data$income_bin)
TP <- conf_tab["1","1"]; TN <- conf_tab["0","0"]
FP <- conf_tab["1","0"]; FN <- conf_tab["0","1"]
accuracy    <- (TP + TN) / sum(conf_tab)
sensitivity <- TP / (TP + FN)
specificity <- TN / (TN + FP)
cat("Accuracy=", round(accuracy,3), " Sensitivity=", round(sensitivity,3),
    " Specificity=", round(specificity,3), "\n")
```

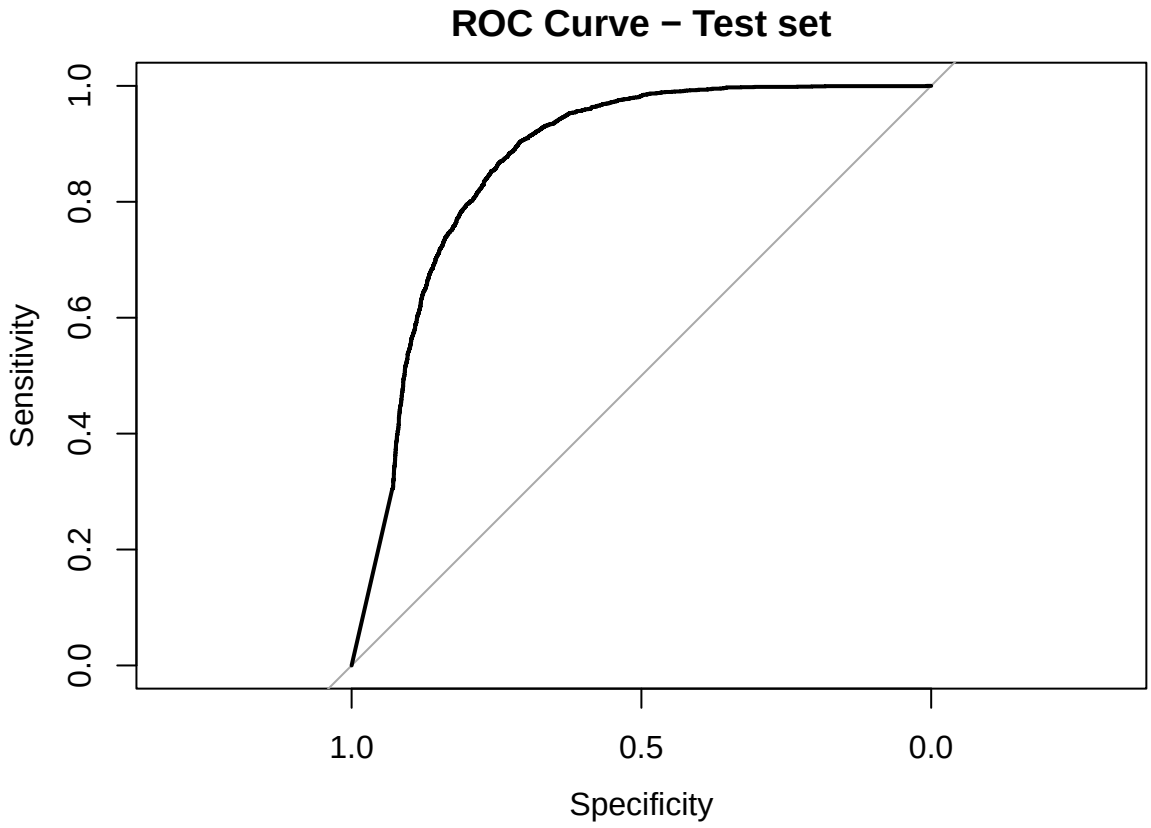
```
## Accuracy= 0.822  Sensitivity= 0.631  Specificity= 0.88
```

```
##ROC curve and AUC
test_roc <- roc(test_data$income_bin, probs_test)
```

```
## Setting levels: control = 0, case = 1
```

```
## Setting direction: controls < cases
```

```
plot(test_roc, main="ROC Curve - Test set")
```



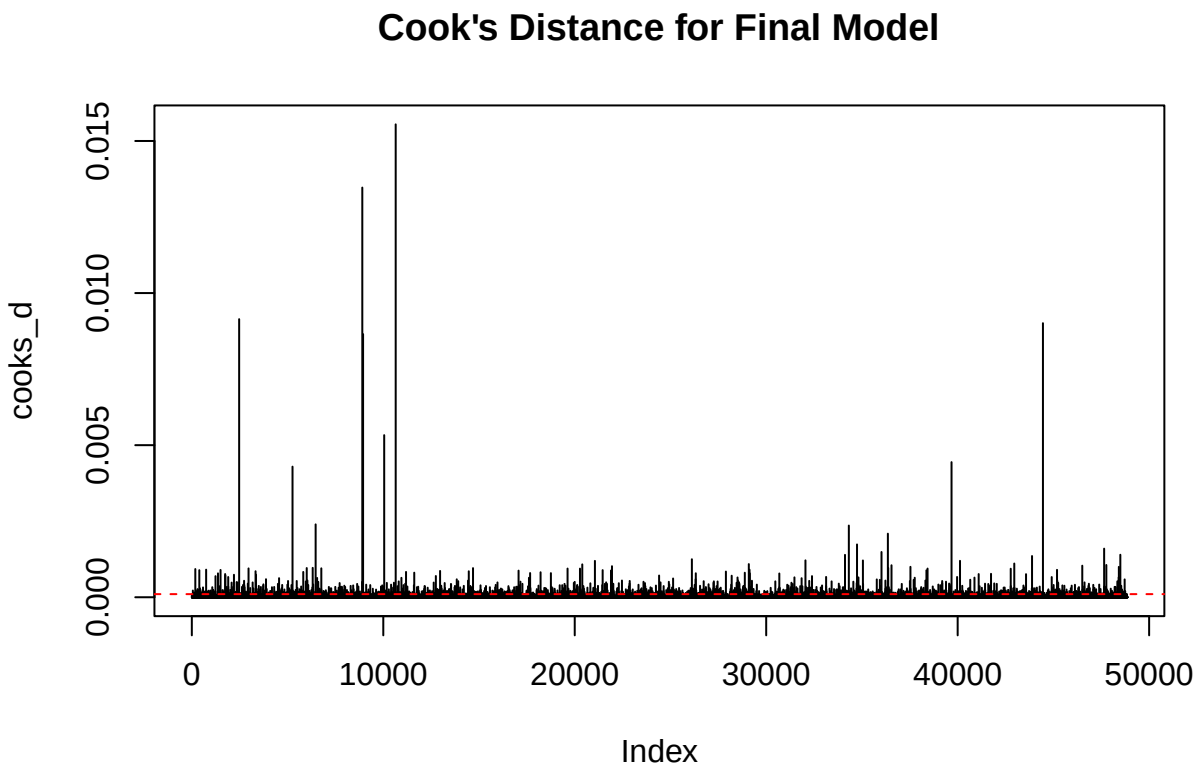

```
cat("AUC=", round(auc(test_roc),3), "\n")
```

```
## AUC= 0.869
```

```
##Influential observations (Cook's distance on training data)
cooks_d <- cooks.distance(final_model)
p <- length(coef(final_model))
threshold <- 4 / (nrow(train_data) - p)
inf_index <- which(cooks_d > threshold)
cat("Cook's D threshold=", round(threshold,4), " Influential obs indices:", inf_index, "\n")
```

```
## Cook's D threshold= 1e-04  Influential obs indices: 54 89 120 129 183 192 200 237 257 282 297 299 31
```

```
plot(cooks_d, type="h", main="Cook's Distance for Final Model")
abline(h=threshold, col="red", lty=2)
```




```r
##Odds ratios and 95% confidence intervals
or_vals <- exp(coef(final_model))
ci_vals <- exp(confint(final_model))
```

```
## Waiting for profiling to be done...
```

```r
or_table <- data.frame(Predictor=names(or_vals), OR=or_vals,
                       CI_low=ci_vals[,1], CI_high=ci_vals[,2])
print(or_table)
```

```
##                                      Predictor          OR       CI_low
## (Intercept)                        (Intercept) 0.000102542 5.510776e-05
## age                                        age 1.023433359 1.020882e+00
## edu_num                                edu_num 1.359588819 1.340756e+00
## cap_gain                              cap_gain 1.229740633 1.217172e+00
## cap_loss                              cap_loss 1.163119299 1.145962e+00
## hours_week                          hours_week 1.031804749 1.029262e+00
## workclassLoc                      workclassLoc 0.553525753 4.646435e-01
## workclassNoPay                  workclassNoPay 0.205207086 3.080170e-02
## workclassPriv                    workclassPriv 0.579888229 5.012133e-01
## workclassSelfI                  workclassSelfI 0.739308099 6.109383e-01
## workclassSelfN                  workclassSelfN 0.374050961 3.153680e-01
## workclassState                  workclassState 0.464484037 3.829003e-01
## maritalMarried                  maritalMarried 8.942456335 5.936575e+00
## maritalNevMarr                  maritalNevMarr 0.663985144 5.818136e-01
```

```
## maritalSep                                maritalSep 0.990132491 8.019945e-01
## maritalWidow                            maritalWidow 1.045779559 8.282226e-01
## occupationArmy                        occupationArmy 1.646914617 3.283989e-01
## occupationCraftRep                  occupationCraftRep 1.094707468 9.667868e-01
## occupationExecMan                    occupationExecMan 2.112451356 1.875836e+00
## occupationFarmFish                  occupationFarmFish 0.377492382 3.025977e-01
## occupationHandlCl                    occupationHandlCl 0.520814128 4.169858e-01
## occupationHouse                        occupationHouse 0.184941894 4.191211e-02
## occupationMachOp                      occupationMachOp 0.746672903 6.358455e-01
## occupationOther                        occupationOther 0.419563363 3.482770e-01
## occupationProf                          occupationProf 1.322265846 1.176761e+00
## occupationProtServ                  occupationProtServ 1.562766676 1.281604e+00
## occupationSales                        occupationSales 1.291517369 1.136528e+00
## occupationTech                          occupationTech 1.692906616 1.425696e+00
## occupationTrans                        occupationTrans 0.929194698 7.955171e-01
## relationshipNot-in-family   relationshipNot-in-family 1.684748978 1.122353e+00
## relationshipOther-relative relationshipOther-relative 0.620602876 4.227833e-01
## relationshipOwn-child           relationshipOwn-child 0.553286667 3.691506e-01
## relationshipUnmarried           relationshipUnmarried 1.342711769 8.728326e-01
## relationshipWife                     relationshipWife 2.989601577 2.560047e+00
## raceAsian-Pac-Islander         raceAsian-Pac-Islander 1.659697191 1.145587e+00
## raceBlack                                   raceBlack 1.343642241 9.517552e-01
## raceOther                                   raceOther 1.376415498 8.291536e-01
## raceWhite                                   raceWhite 1.637832905 1.180522e+00
## sexMale                                       sexMale 1.987418557 1.767830e+00
## native_countryUSA                 native_countryUSA 1.211665032 1.086470e+00
##                                CI_high
## (Intercept)                1.901149e-04
## age                        1.025994e+00
## edu_num                    1.378824e+00
## cap_gain                   1.242531e+00
## cap_loss                   1.180584e+00
## hours_week                 1.034363e+00
## workclassLoc               6.593593e-01
## workclassNoPay             8.037472e-01
## workclassPriv              6.710864e-01
## workclassSelfI             8.948575e-01
## workclassSelfN             4.436171e-01
## workclassState             5.632062e-01
## maritalMarried             1.336616e+01
## maritalNevMarr             7.578886e-01
## maritalSep                 1.216044e+00
## maritalWidow               1.312046e+00
## occupationArmy             6.929674e+00
## occupationCraftRep         1.240176e+00
## occupationExecMan          2.380480e+00
## occupationFarmFish         4.690383e-01
## occupationHandlCl          6.468926e-01
## occupationHouse            5.466311e-01
## occupationMachOp           8.758865e-01
## occupationOther            5.036749e-01
## occupationProf             1.486621e+00
## occupationProtServ         1.904027e+00
## occupationSales            1.468259e+00
```

```
## occupationTech             2.009344e+00
## occupationTrans            1.084805e+00
## relationshipNot-in-family  2.508580e+00
## relationshipOther-relative 8.946108e-01
## relationshipOwn-child      8.174550e-01
## relationshipUnmarried      2.050794e+00
## relationshipWife           3.492727e+00
## raceAsian-Pac-Islander     2.435947e+00
## raceBlack                  1.925668e+00
## raceOther                  2.277536e+00
## raceWhite                  2.310525e+00
## sexMale                    2.235676e+00
## native_countryUSA          1.352389e+00
```