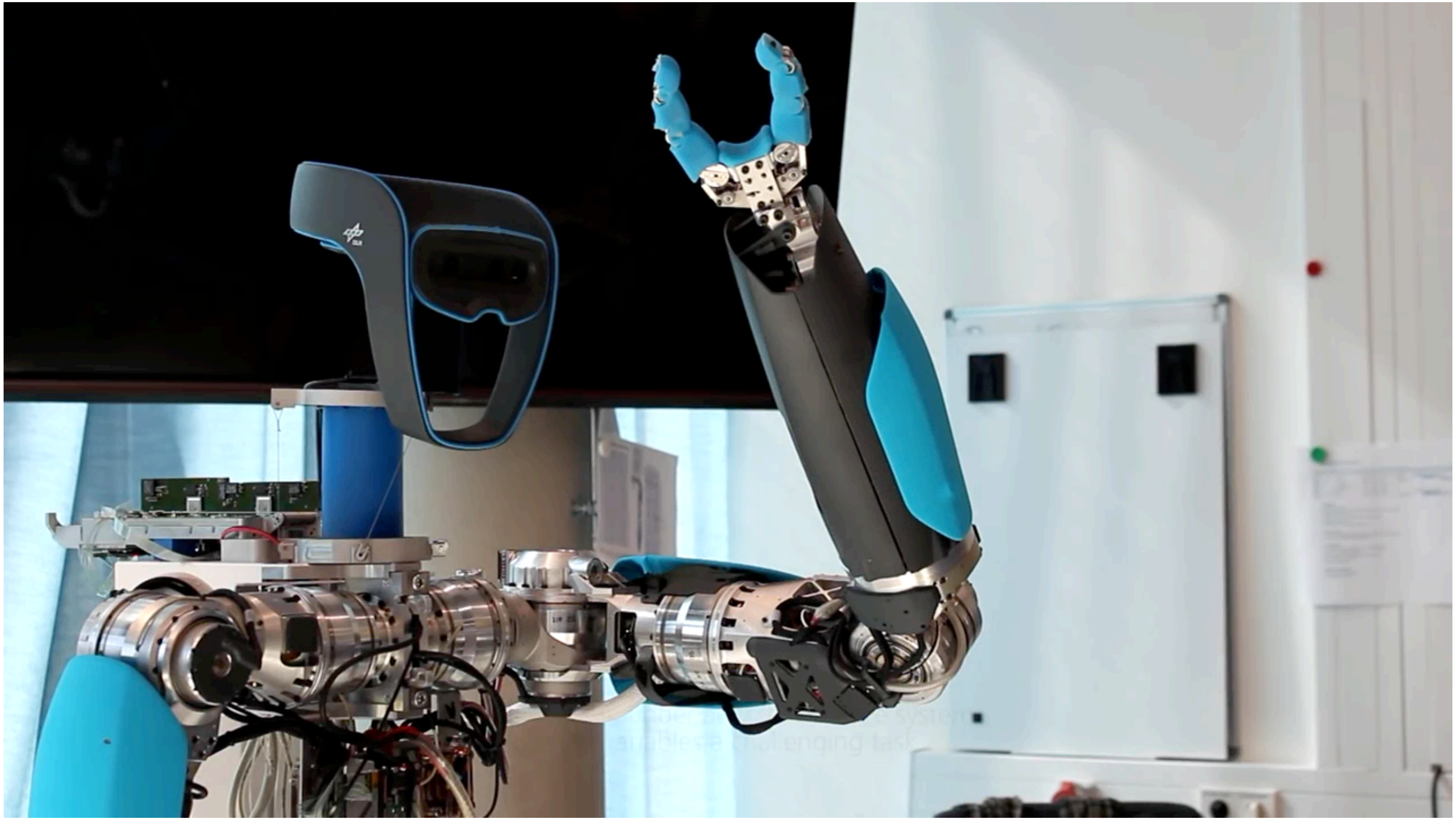
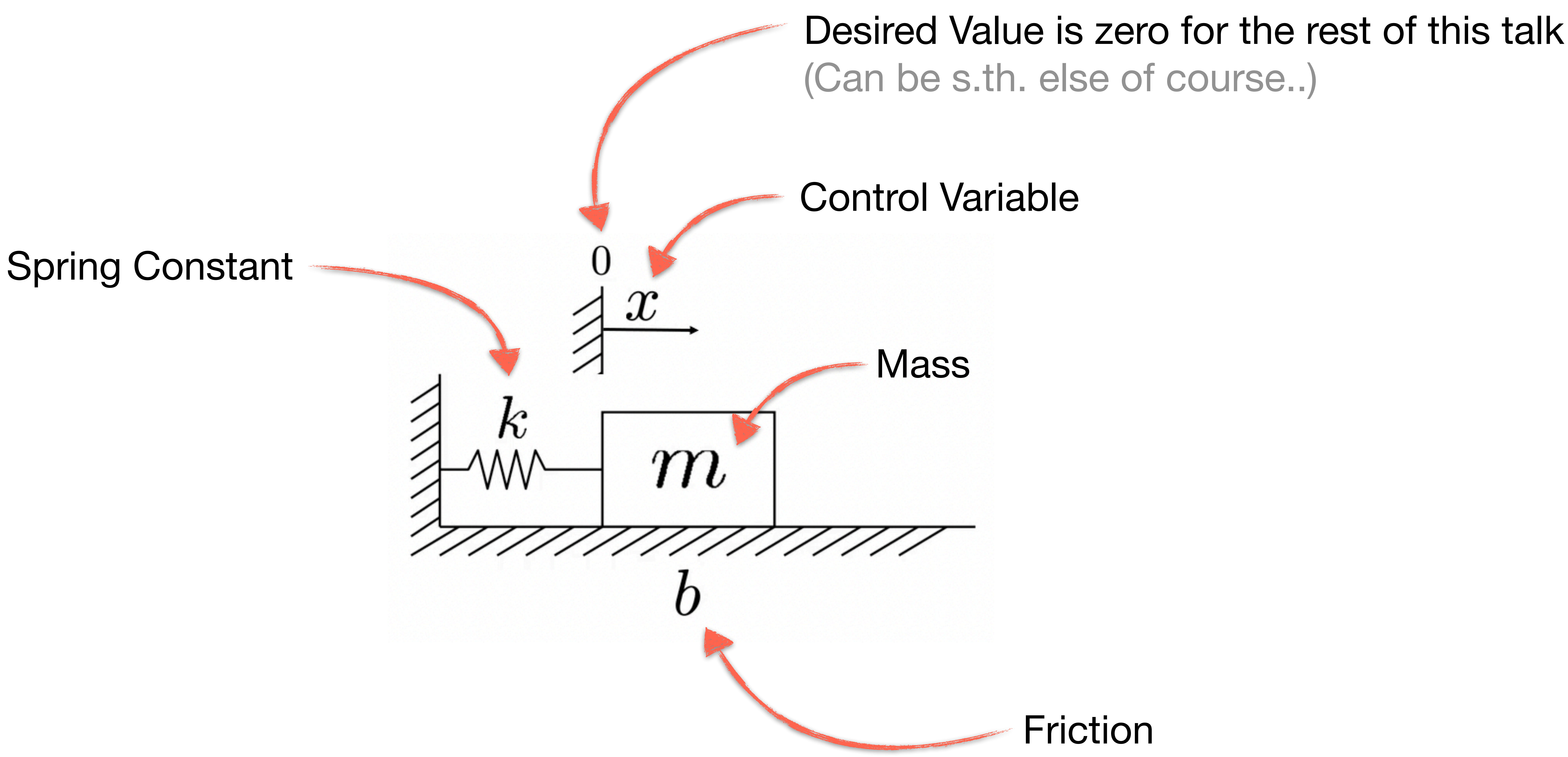
Human-Robot Interaction

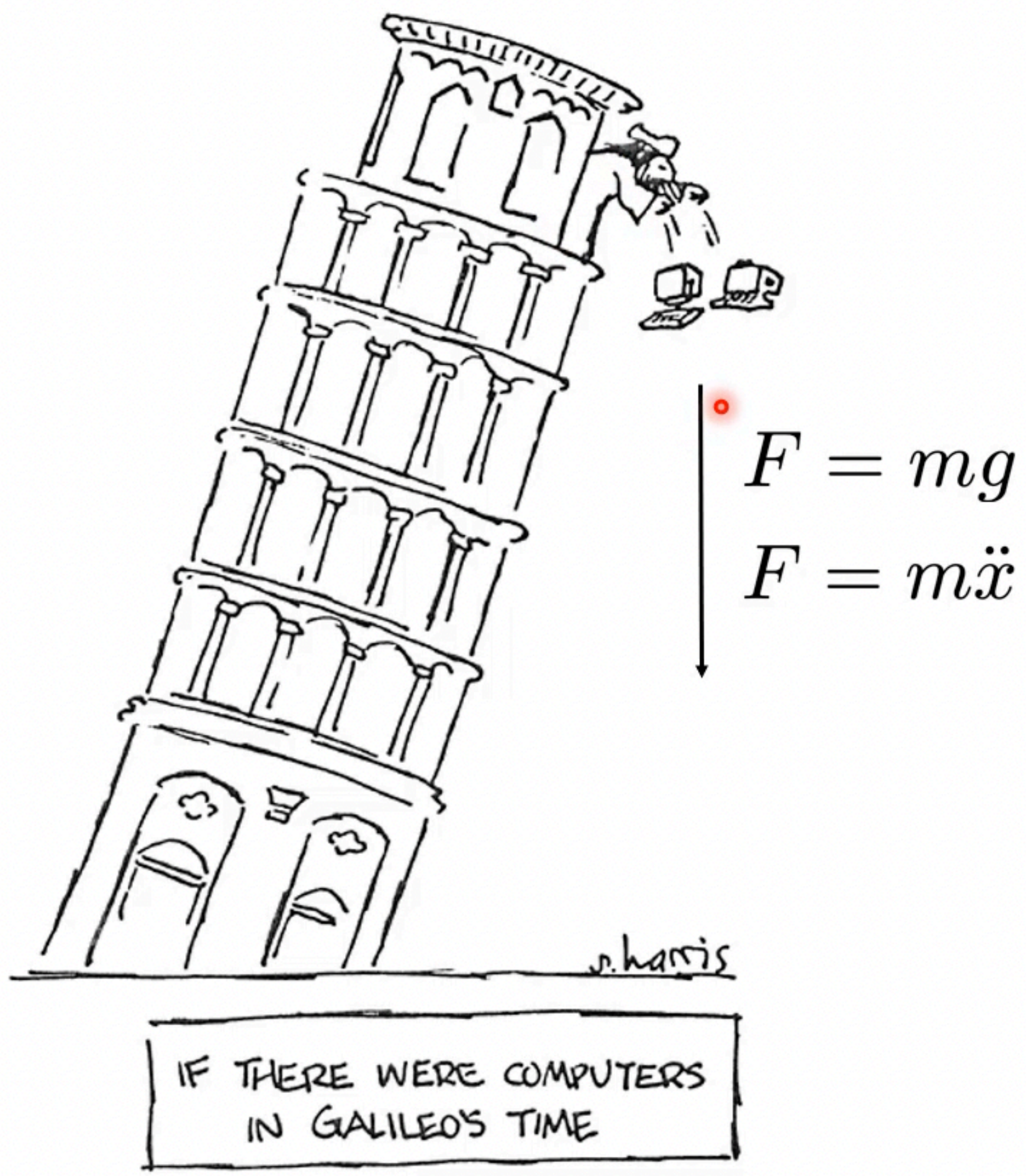
Spring-Mass-Damper System



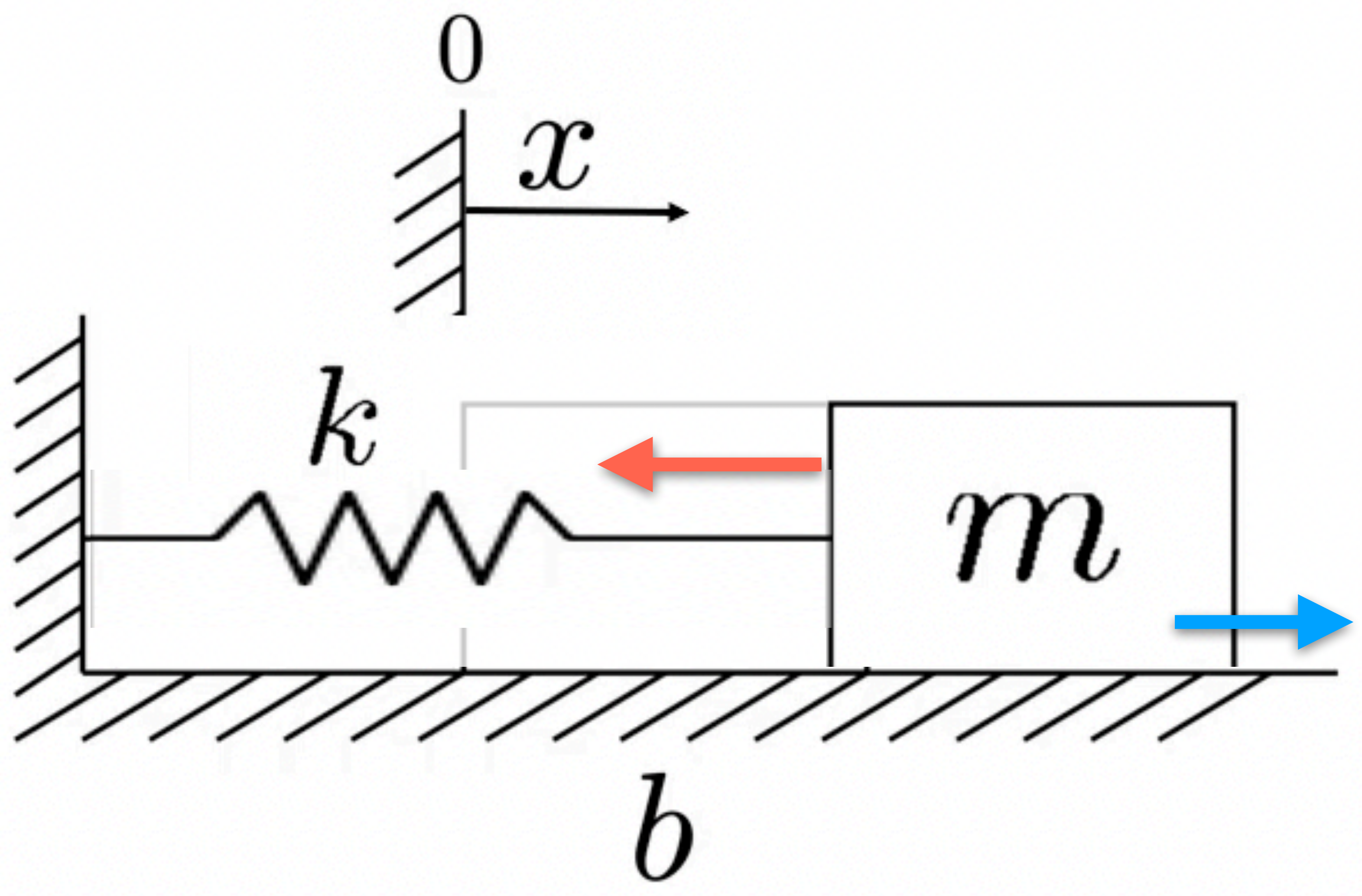
Spring-Mass-Damper System



Natural System



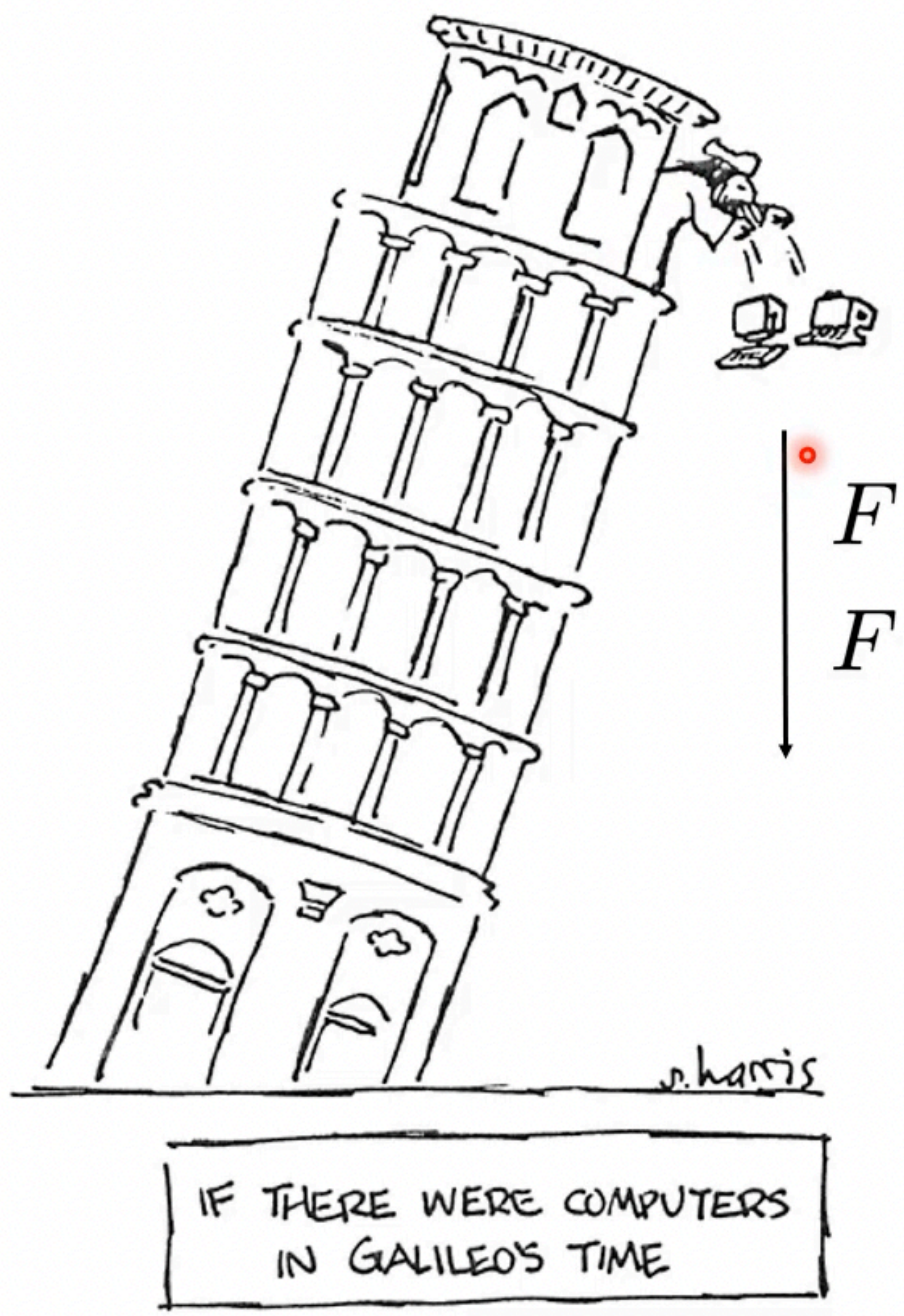
Natural System



How does it move?

$$m\ddot{x} = \underline{-kx} - \underline{b\dot{x}}$$

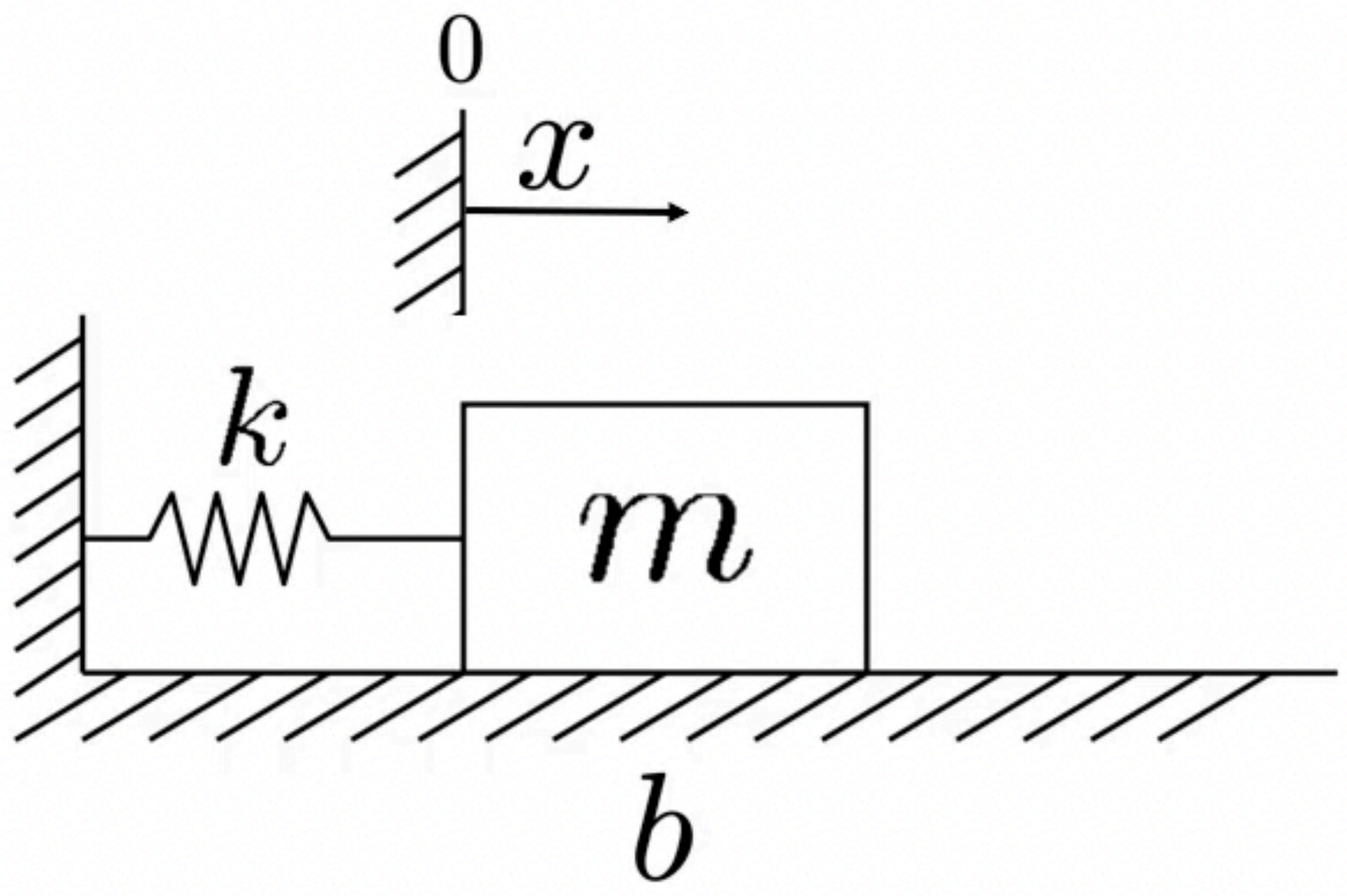
Natural System



$$F = mg$$

$$F = m\ddot{x}$$

Natural System



How does it move?

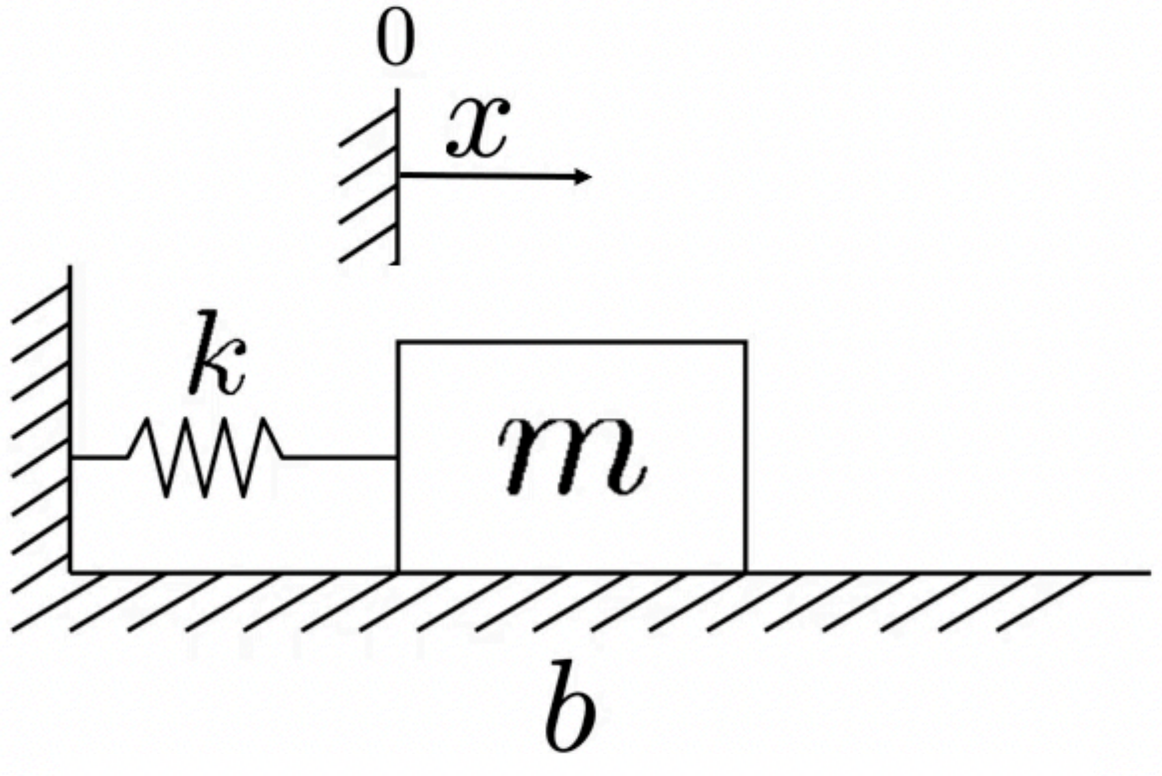
$$m\ddot{x} = -kx - b\dot{x}$$

reformulate

$$m\ddot{x} + b\dot{x} + kx = 0$$

Equation of Motion

Solving the Equation of Motion of a Second Order Linear Differential System



$$m\ddot{x} + b\dot{x} + kx = 0$$

Goal: Find trajectory $x(t)$ depending on parameters m, b, k

such that the following holds at all times: $m\ddot{x} + b\dot{x} + kx = 0$

We assume: $x = e^{st}$

Therefore: $\dot{x} = se^{st}$

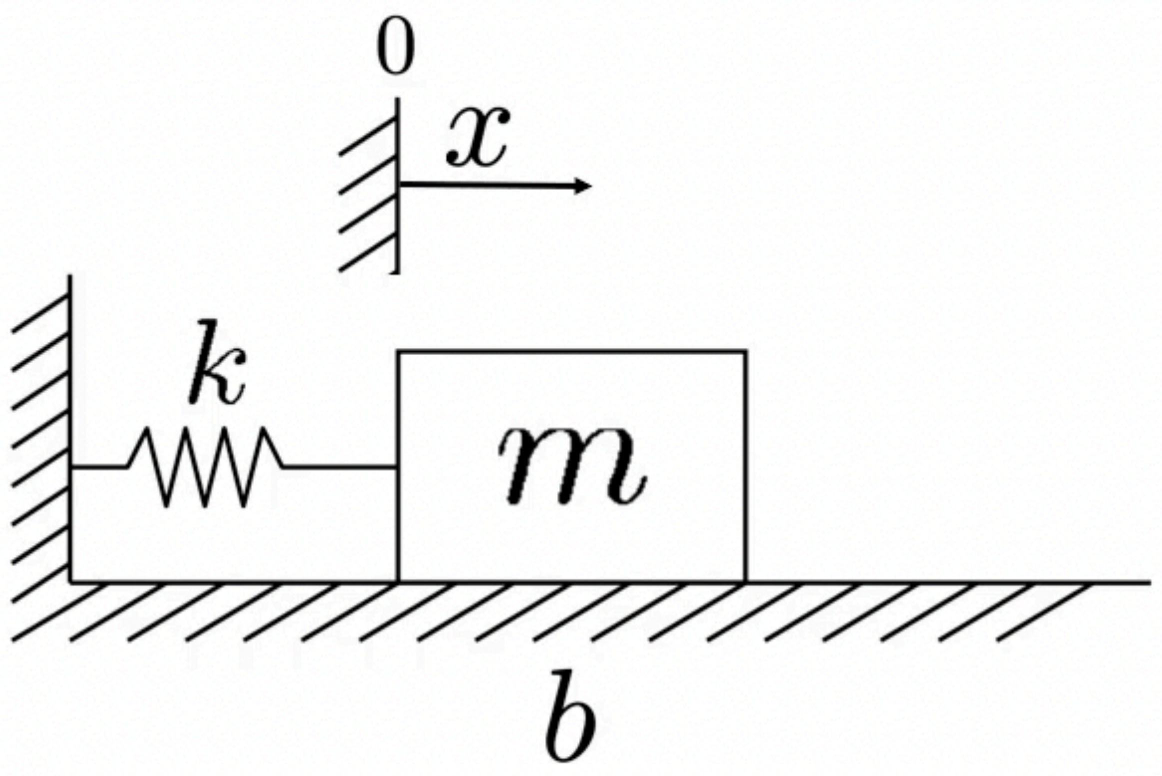
$\ddot{x} = s^2 e^{st}$

$$ms^2 e^{st} + bse^{st} + ke^{st} = 0$$ (Characteristic Equation)

$$e^{st}(ms^2 + bs + k) = 0$$

$$ms^2 + bs + k = 0$$ Now solve for s

Solving the Equation of Motion of a Second Order Linear Differential System



$$m\ddot{x} + b\dot{x} + kx = 0$$

Equation of Motion

$$m\ddot{x} + b\dot{x} + kx = 0$$

Characteristic Equation

$$ms^2 + bs + k = 0$$

Roots:

$$s_1 = -\frac{b}{2m} + \frac{\sqrt{b^2 - 4mk}}{2m}$$

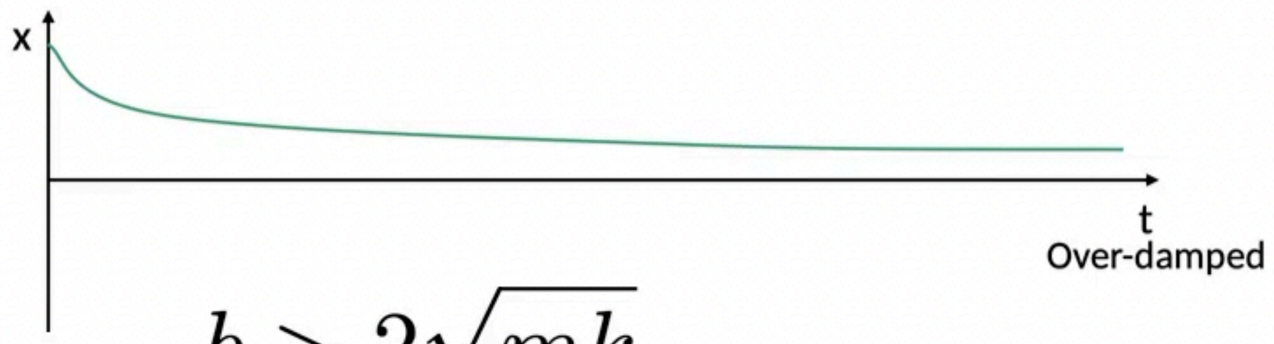
$$s_2 = -\frac{b}{2m} - \frac{\sqrt{b^2 - 4mk}}{2m}$$

Solution:

$$x(t) = c_1 e^{s_1 t} + c_2 e^{s_2 t}$$

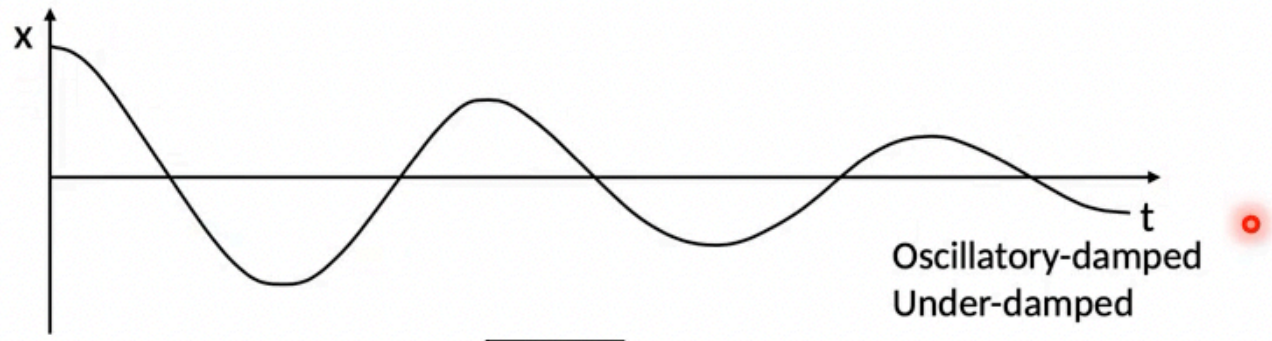
Possible Behaviors:

1. *$b^2 > 4mk$*
   real and unequal roots
   overdamped



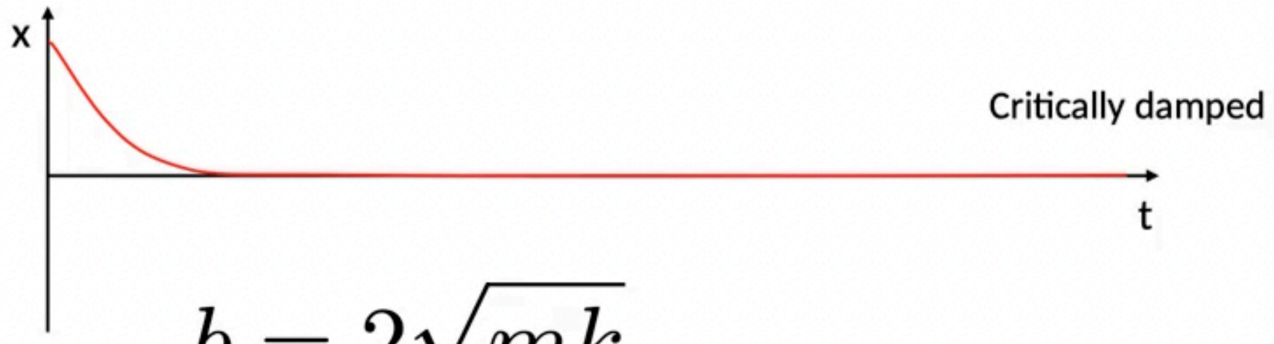
$$b > 2\sqrt{mk}$$

2. *$b^2 < 4mk$*
   complex roots
   underdamped



$$b < 2\sqrt{mk}$$

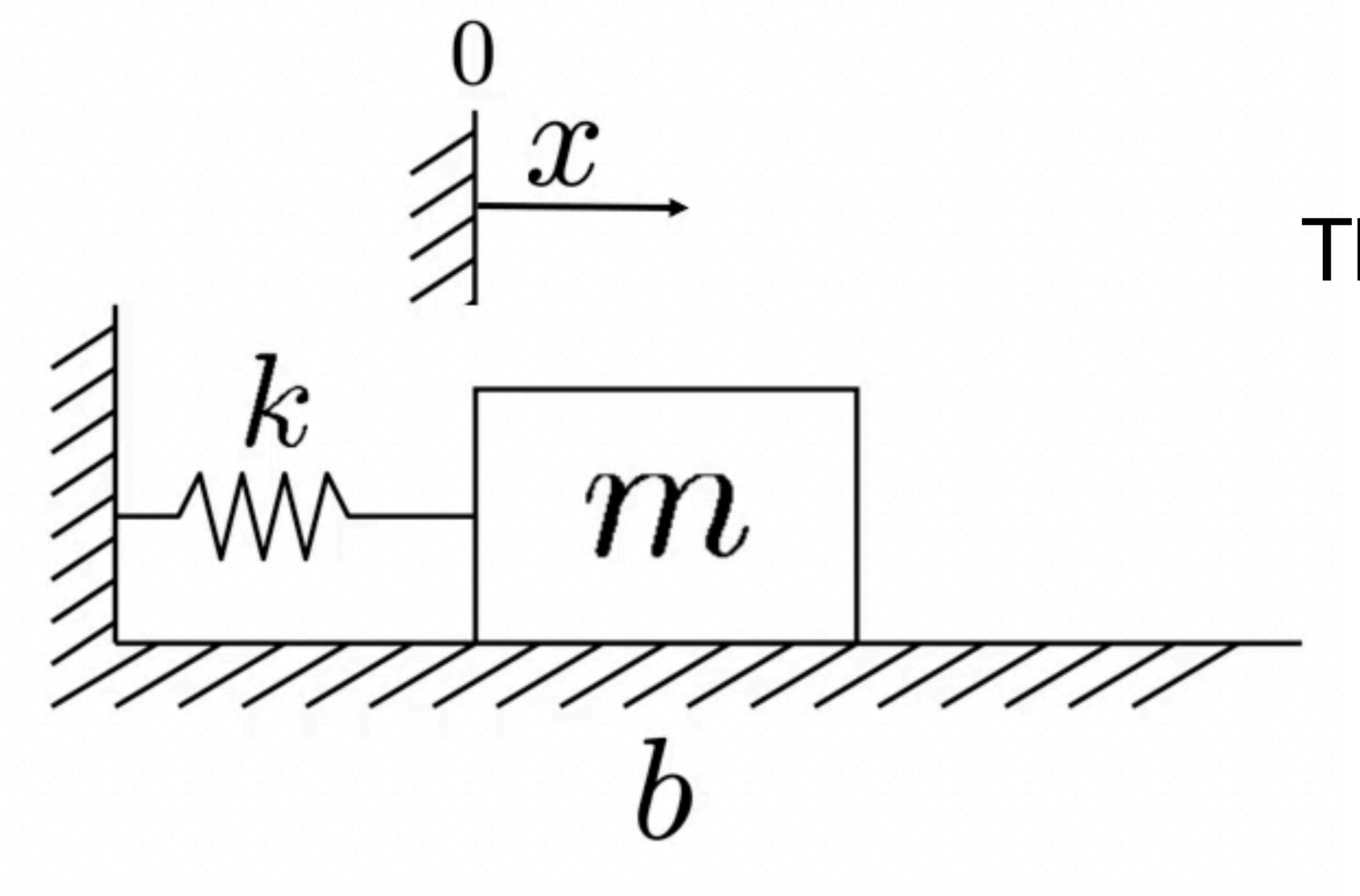
3. *$b^2 = 4mk$*
   real and equal roots
   critically damped



$$b = 2\sqrt{mk}$$

Control of 2nd-Order Linear System

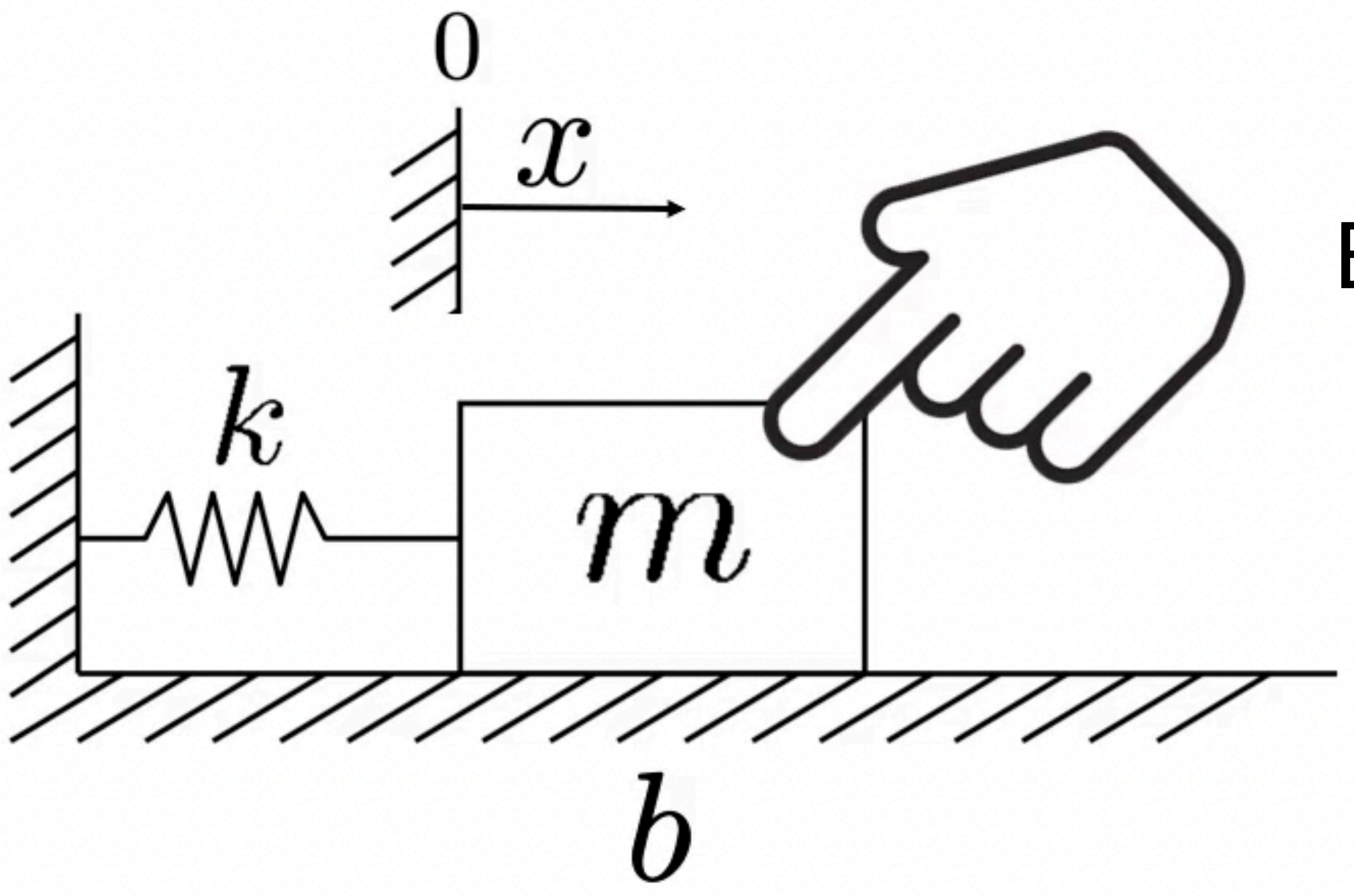
Natural System



$$m\ddot{x} = -kx - b\dot{x}$$

There is nothing to control…

Control of 2nd-Order Linear System

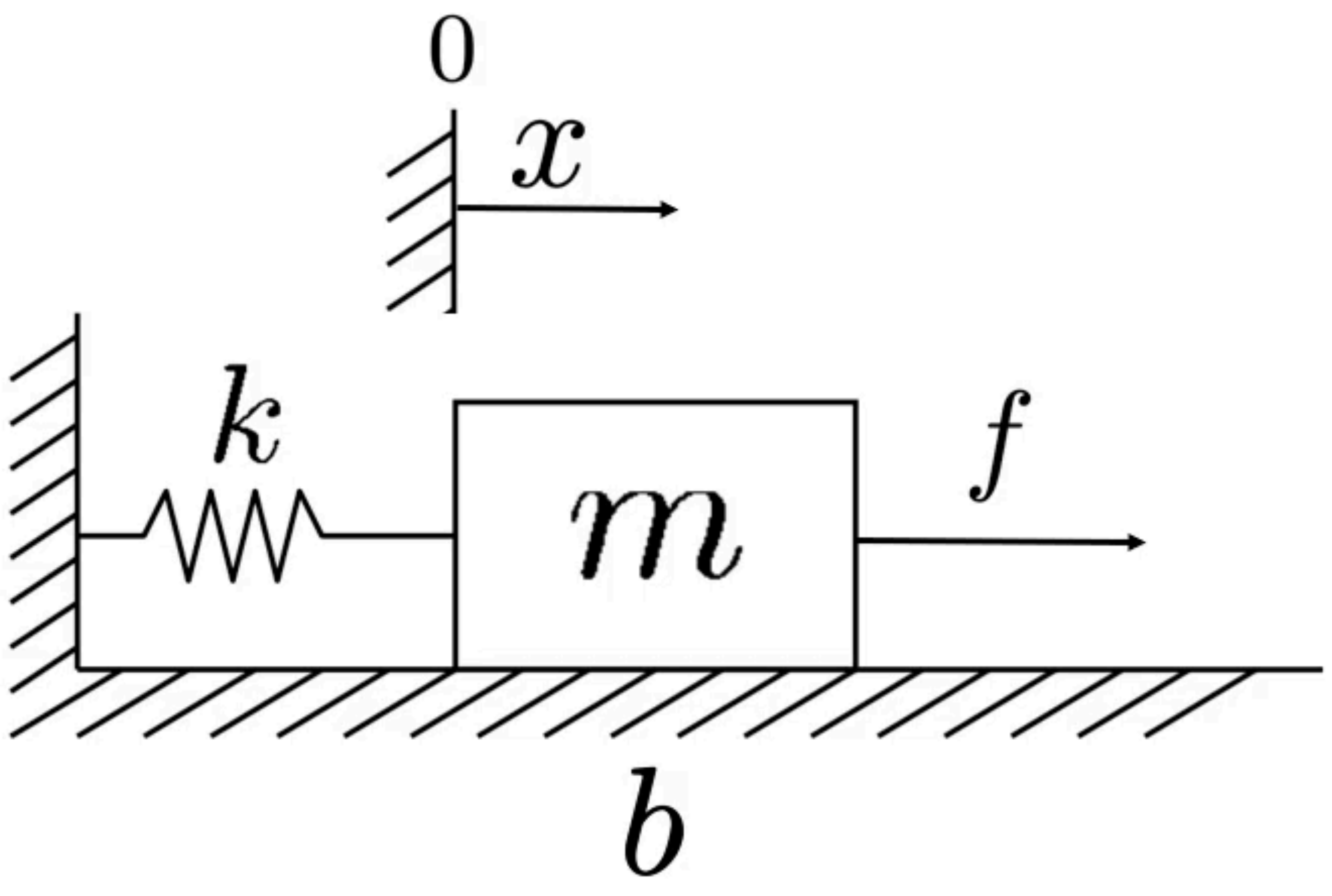


But we want to control it …

$$m\ddot{x} = -kx - b\dot{x}$$

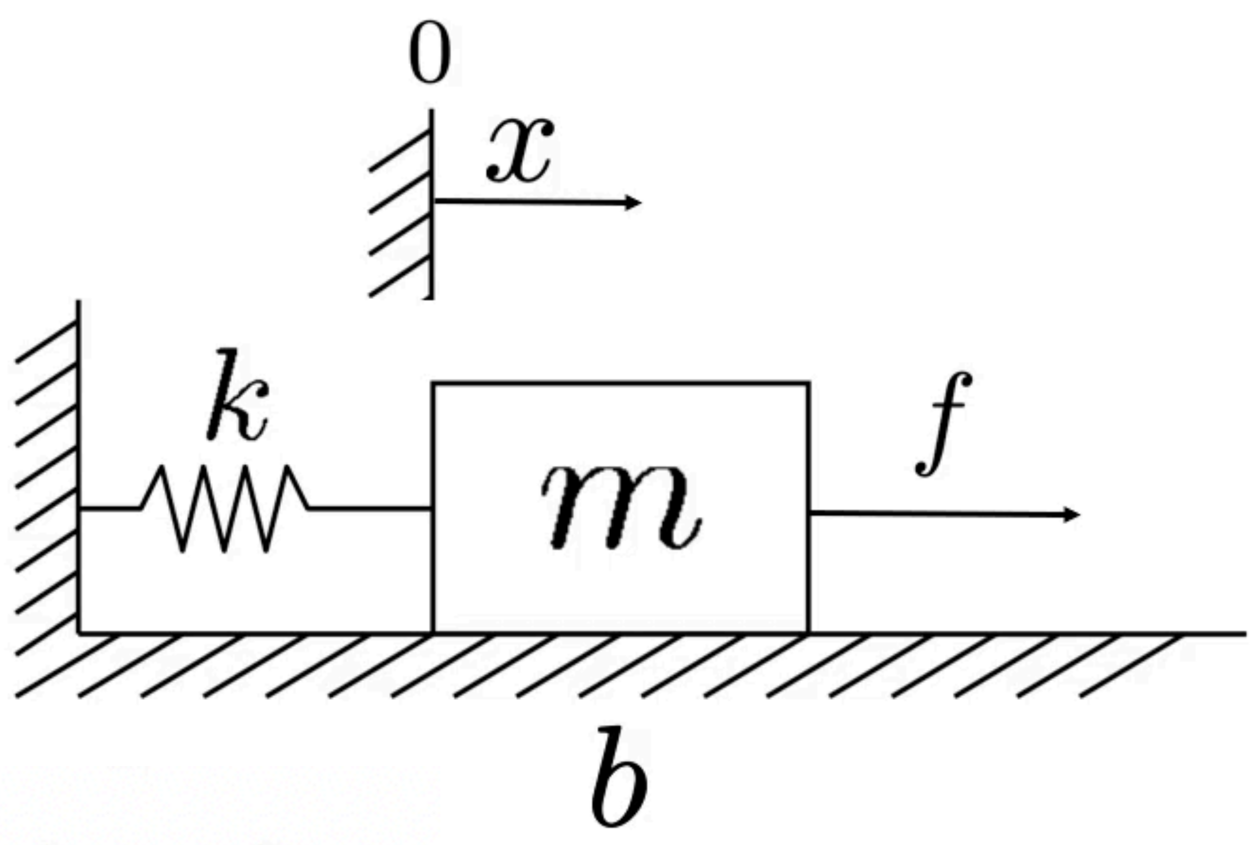
Control of 2nd-Order Linear System

Closed-Loop System



$$m\ddot{x} = -kx - b\dot{x} + f$$

Critical Damping in Closed-Loop System



$$m\ddot{x} = -kx - b\dot{x}$$

add control force

$$m\ddot{x} = -kx - b\dot{x} + [f]$$

We can define this to be whatever we want!

$f := -k_p x - k_v \dot{x}$

$$m\ddot{x} = -kx - b\dot{x} + [-k_p x - k_v \dot{x}]$$

reformulate

$$m\ddot{x} = -\underbrace{(b + k_v)}_{b_{cls}}\dot{x} - \underbrace{(k + k_p)}_{k_{cls}}x$$

reformulate

$$m\ddot{x} + b_{cls}\dot{x} + k_{cls}x = 0$$

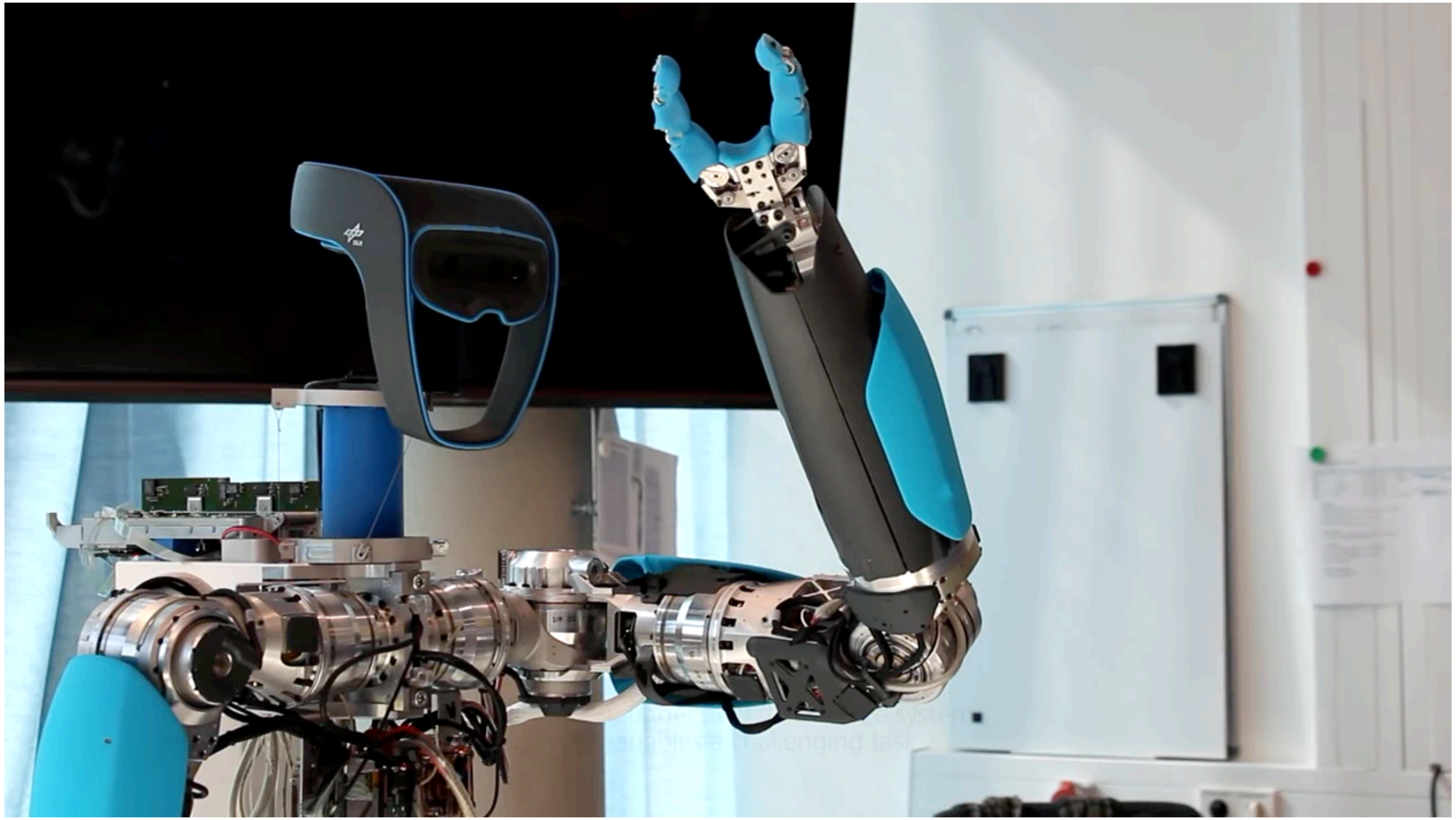
Remember:

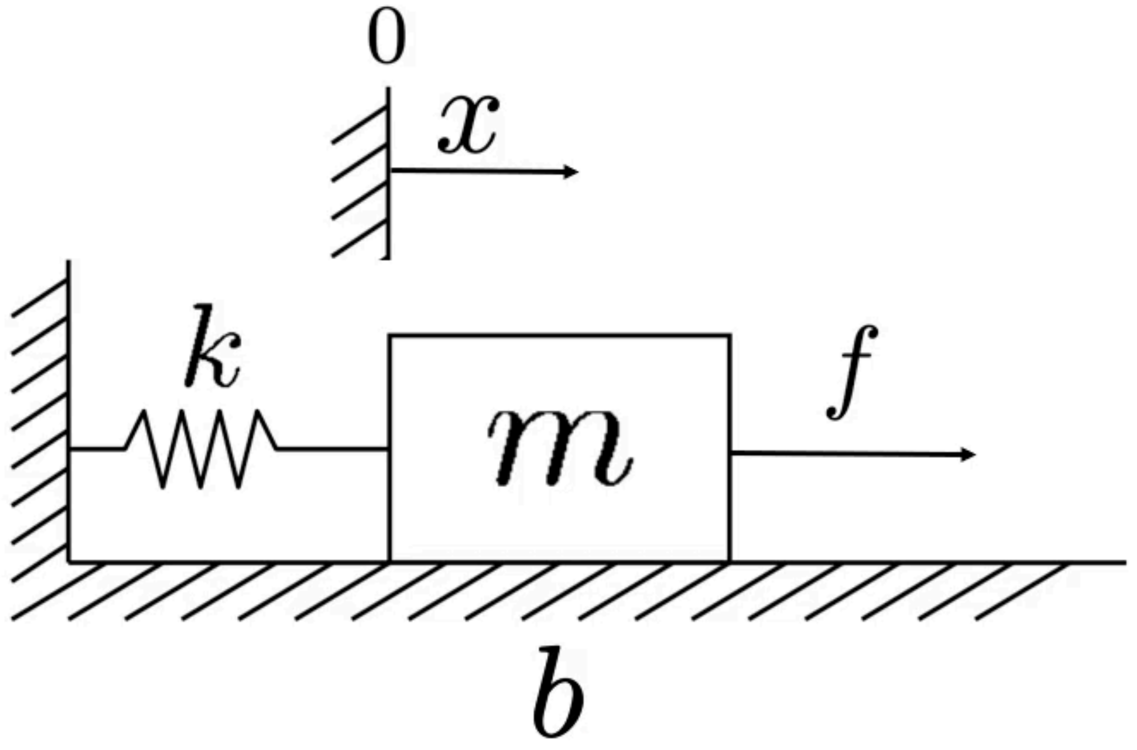
For: $m\ddot{x} + b\dot{x} + kx = 0$

Critical damping: $b = 2\sqrt{mk}$

Critical Damping in Closed-Loop System

$$b_{cls} = 2\sqrt{mk_{cls}}$$





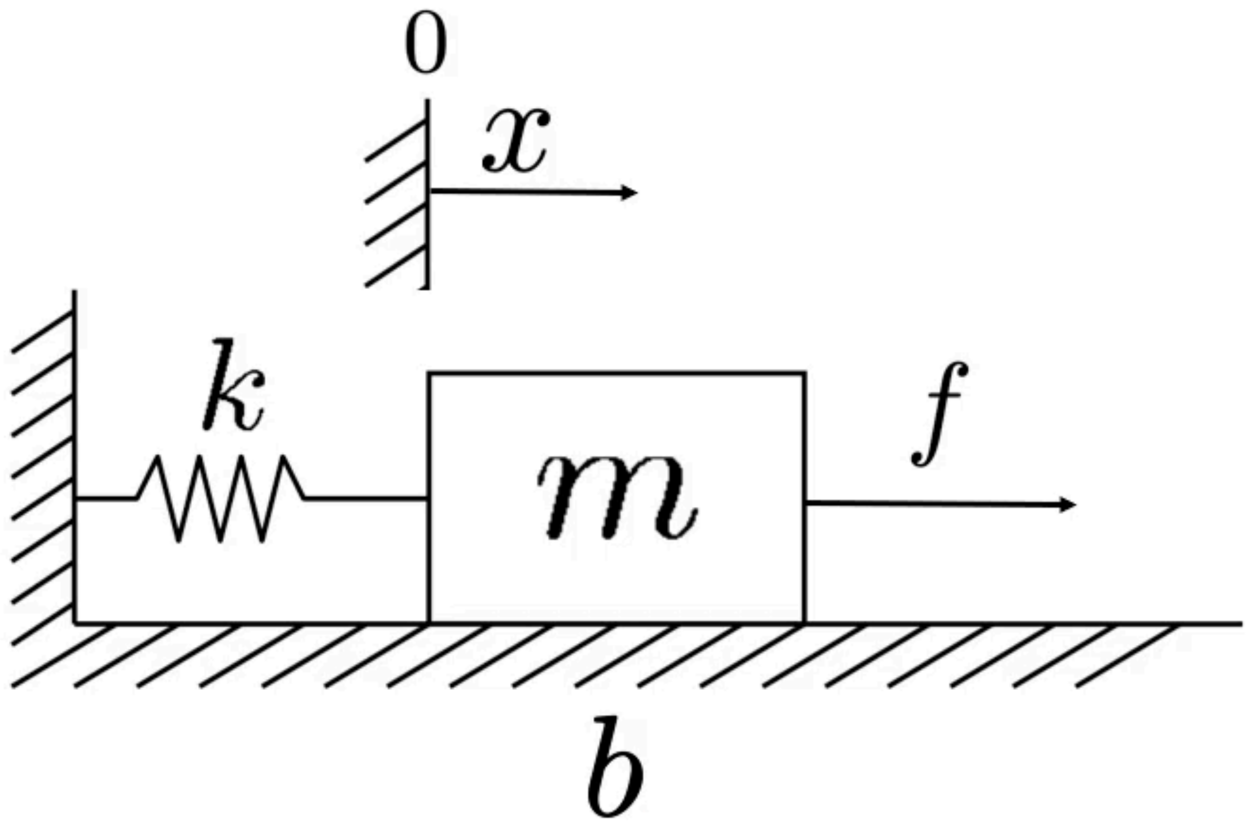
Closed-Loop System

$$m\ddot{x} = -kx - b\dot{x} + f$$

Critical Damping

$$b_{cls} = 2\sqrt{m k_{cls}}$$

Control-Law Partitioning



Equation of Motion in SMD-System

$$\tau = m\ddot{q} + b\dot{q} + kq$$

Actual Equation of Motion

$$\tau = M(q)\ddot{q} + C(q,\dot{q}) + B(q)\dot{q} + K(q) + G(q)$$

Reformulate

$$M(q)\ddot{q} = -C(q,\dot{q}) - B(q)\dot{q} - K(q) - G(q) + [\tau]$$

Simplify

$$m\ddot{q} = [\text{complicated dynamics}] + [\tau]$$

$\tau = \alpha f' + \beta$ (control-law partitioning)

$$m\ddot{q} = [\text{complicated dynamics}] + [\alpha f' + \beta]$$

$\alpha = m$

$\beta = -[\text{complicated dynamics}]$

$$m\ddot{q} = [\text{complicated dynamics}] + [m f' - [\text{complicated dynamics}]]$$

Reformulate

$\ddot{q} = f'$ (unit-mass system)

$f' = -k_p q - k_v \dot{q}$

$$\ddot{q} + k_v\dot{q} + k_p q = 0$$

$k_v = 2\sqrt{k_p}$ (critical damping)

Remember:

For: $m\ddot{x} + b\dot{x} + kx = 0$

Critical damping: $b = 2\sqrt{mk}$

Coding Exercise

Download Juypter Notebook from Moodle