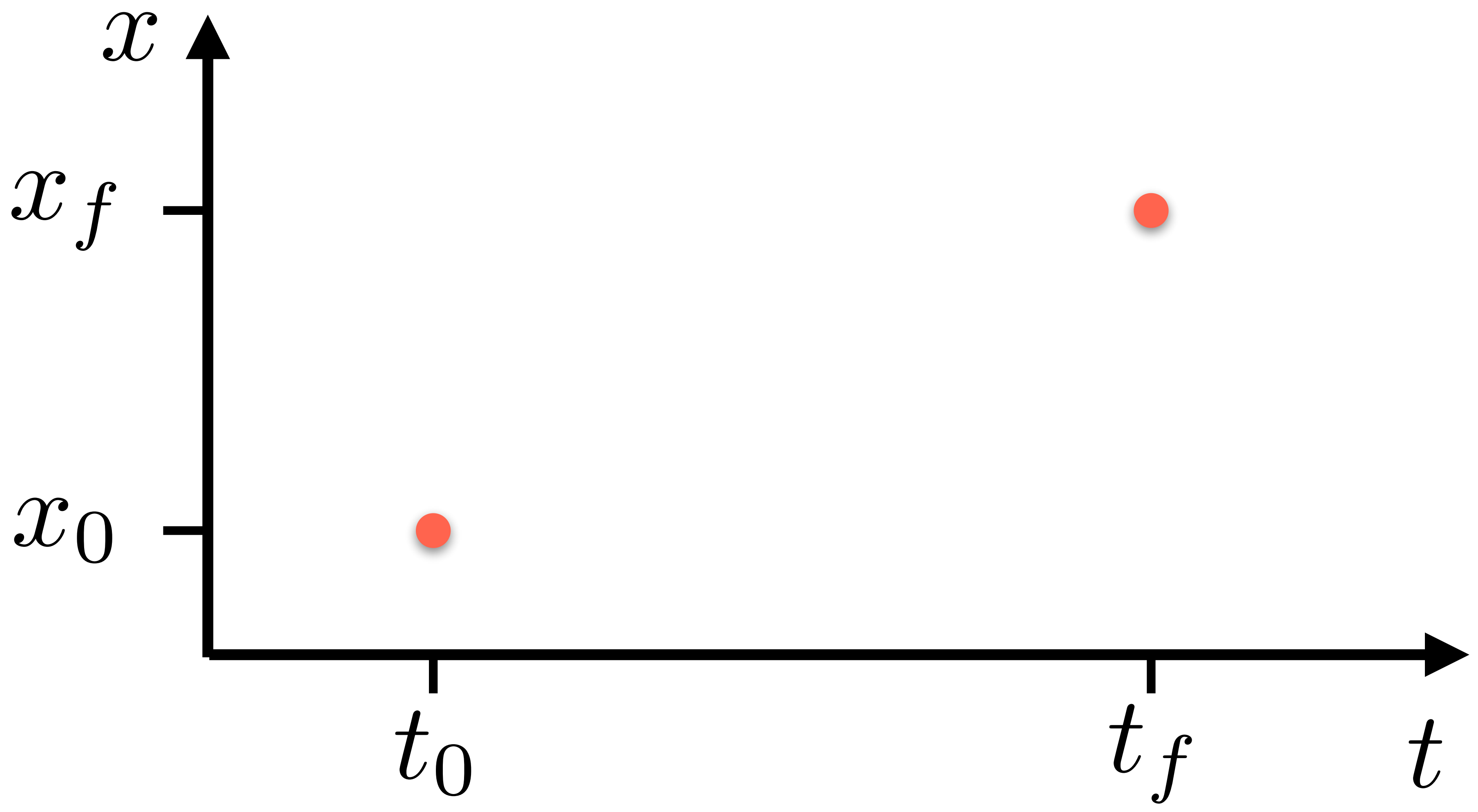
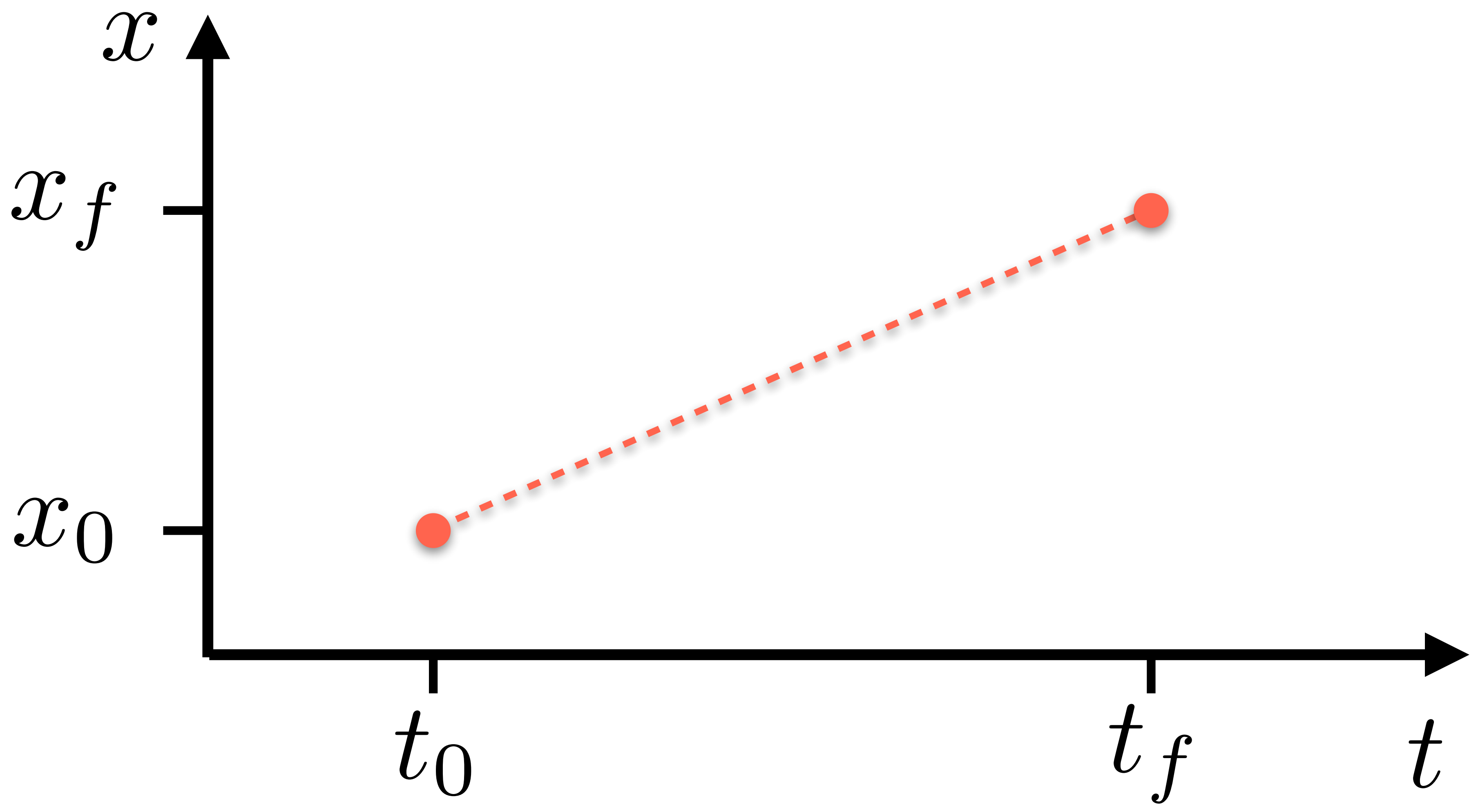
Human Robot Interaction

Trajectories


x
t

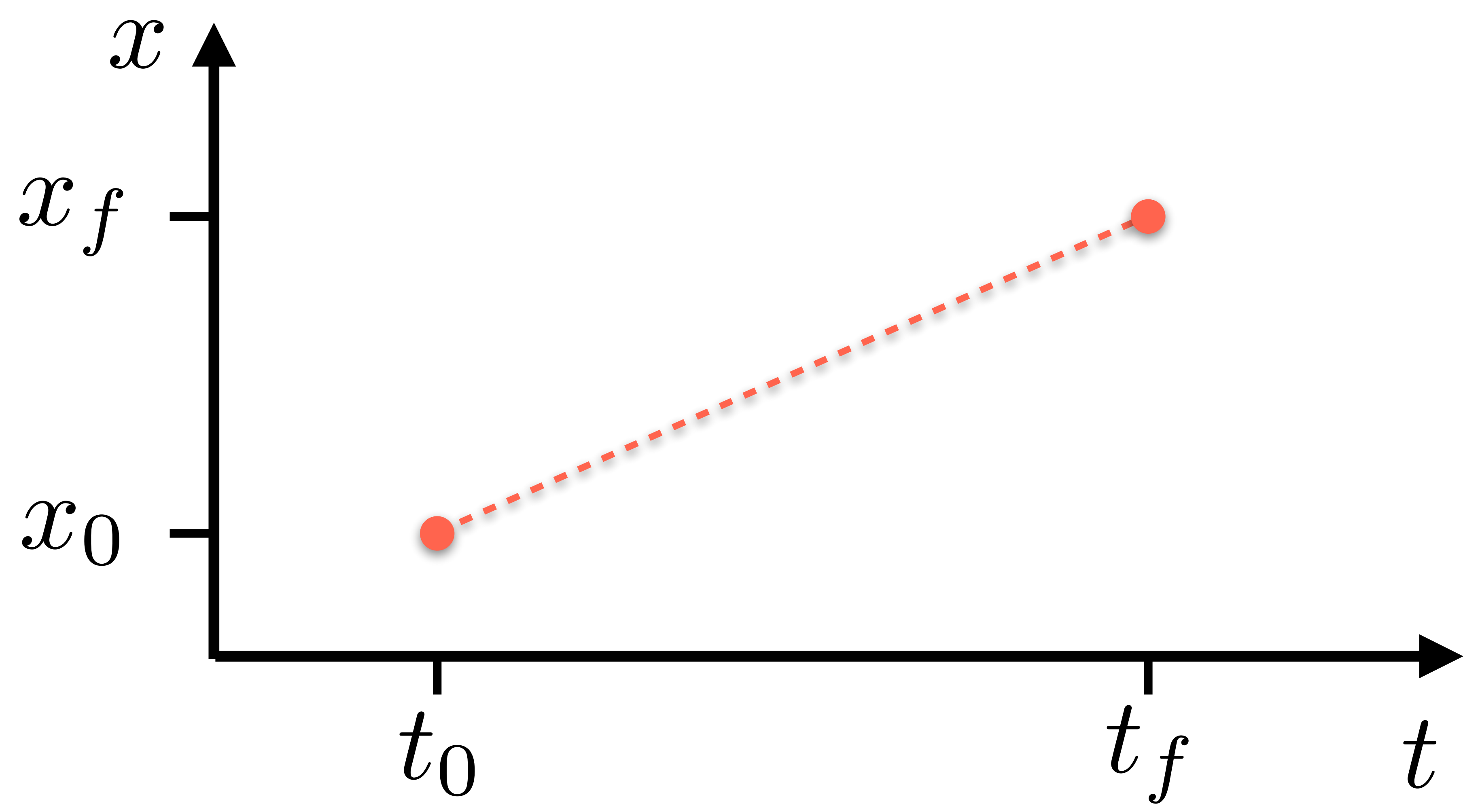

x
x_f
x_0
t_0
t_f
t


x
x_f
x_0
t_0
t_f
t

Line function:

$$x(t) = mt + n$$ (typical way of writing it)

$$x(t) = a_0 + a_1(t - t_0)$$ (polynomial way of writing it)



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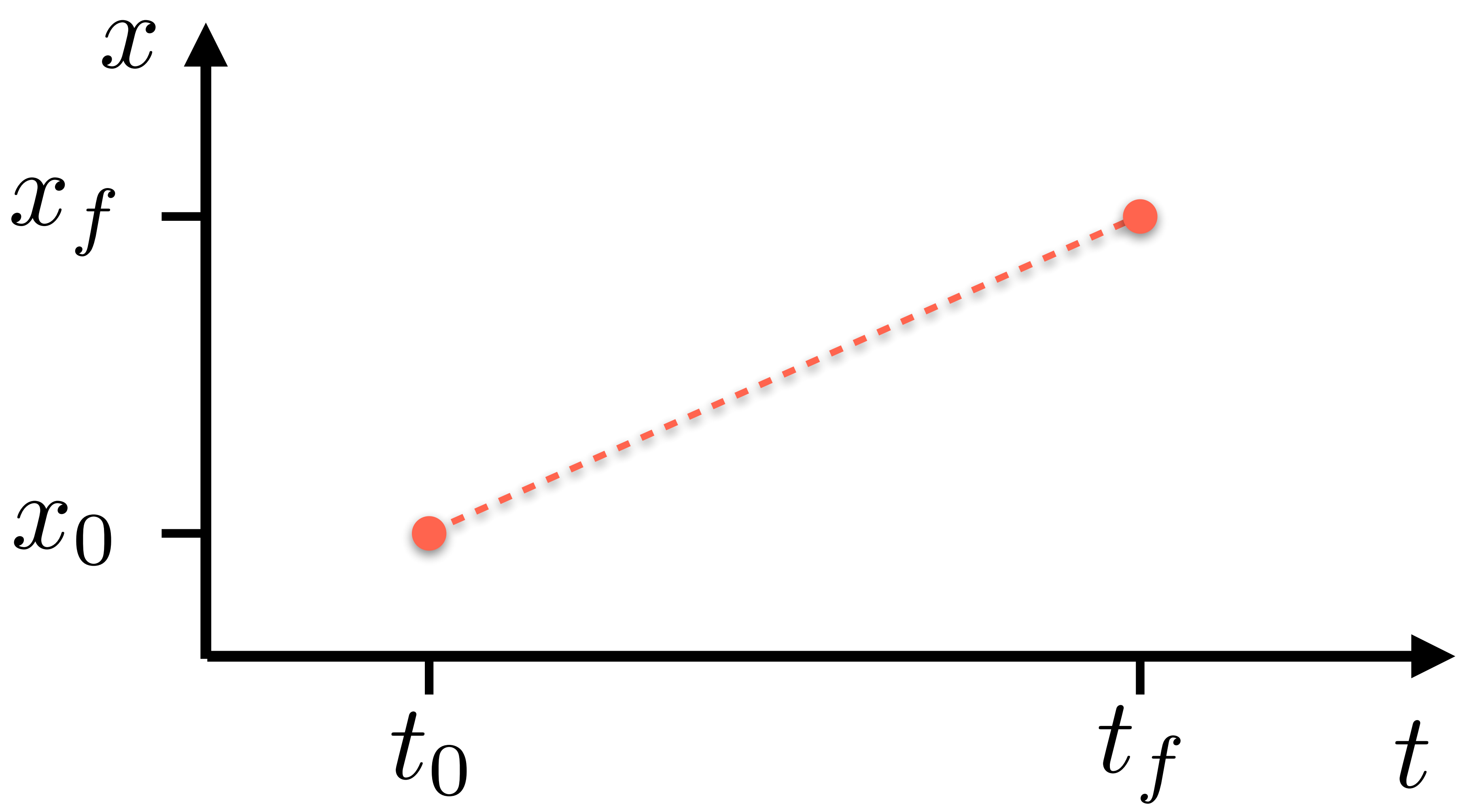


Two boundary conditions, two unknowns:

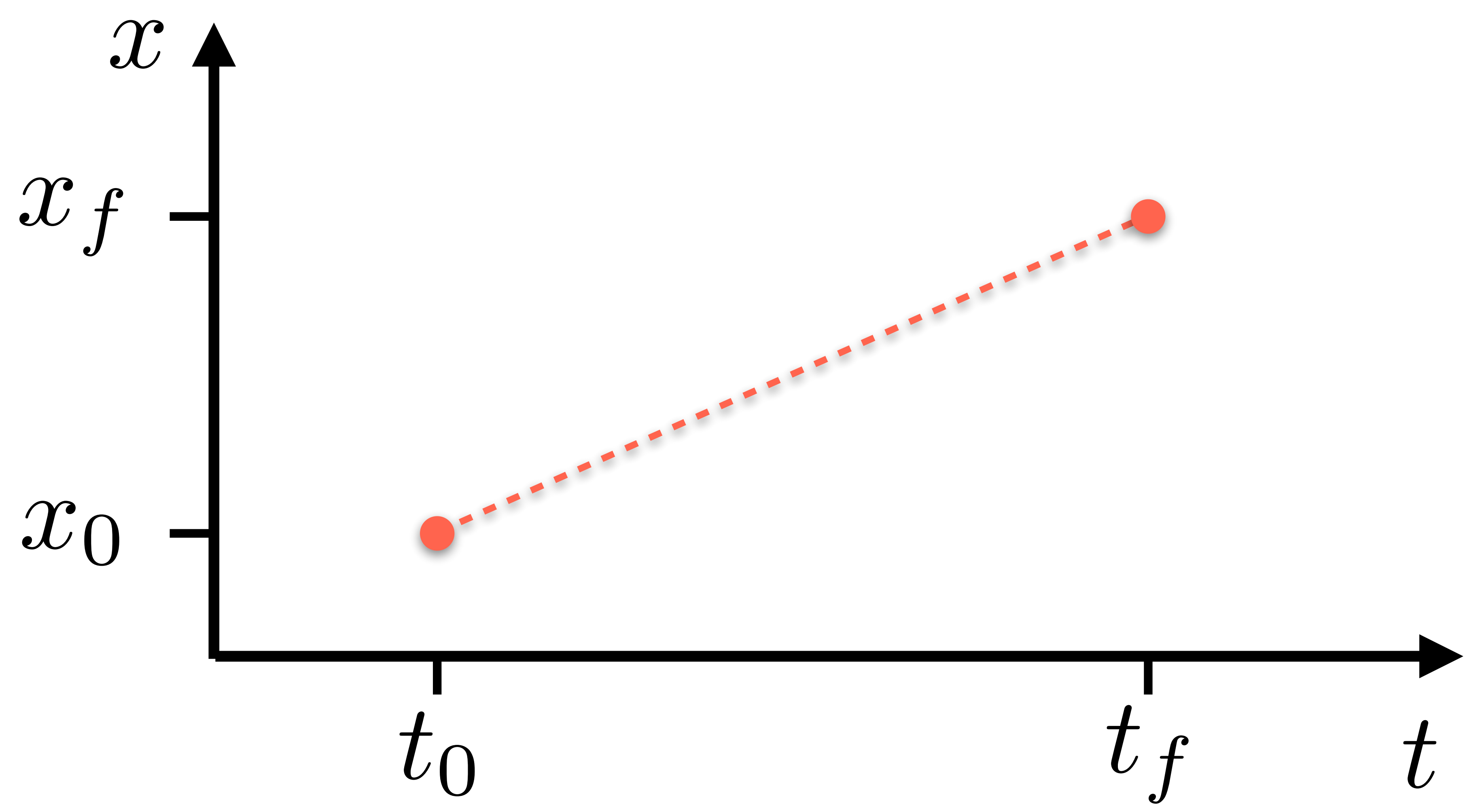
$$x(t = t_0) = x_0$$

$$x(t = t_f) = x_f$$

Line function:

$$x(t) = mt + n$$ (typical way of writing it)

$$x(t) = a_0 + a_1(t - t_0)$$ (polynomial way of writing it)



Two boundary conditions, two unknowns:

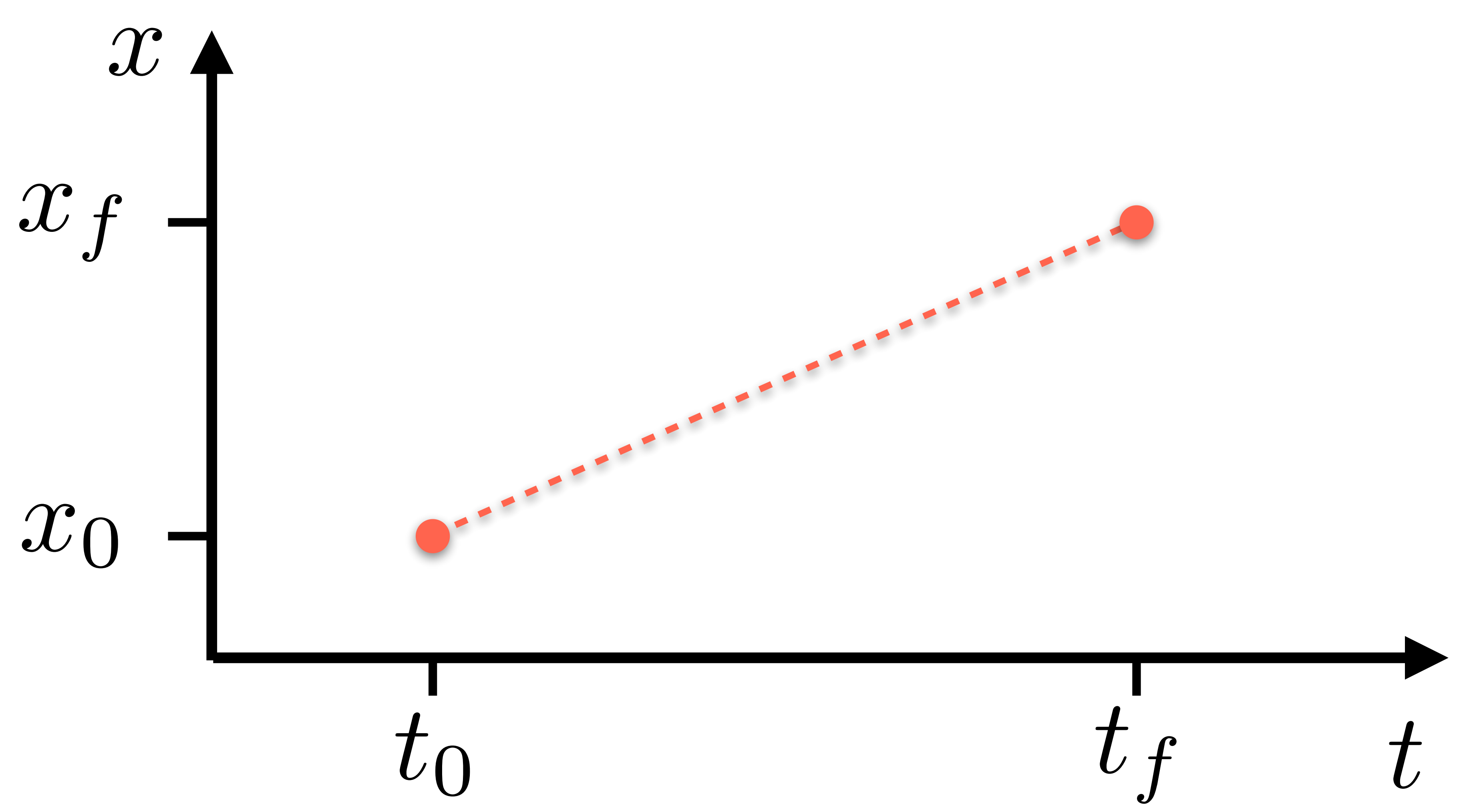
$$x(t = t_0) = x_0 \longrightarrow x(t_0) = x_0 = a_0 + a_1(t_0 - t_0)$$

$$x(t = t_f) = x_f$$

Line function:

$$x(t) = mt + n \qquad \text{(typical way of writing it)}$$

$$x(t) = a_0 + a_1(t - t_0) \qquad \text{(polynomial way of writing it)}$$



Two boundary conditions, two unknowns:

$$x(t = t_0) = x_0 \longrightarrow x(t_0) = x_0 = a_0 + a_1(t_0 - t_0) \longrightarrow a_0 = x_0$$

$$x(t = t_f) = x_f$$

Line function:

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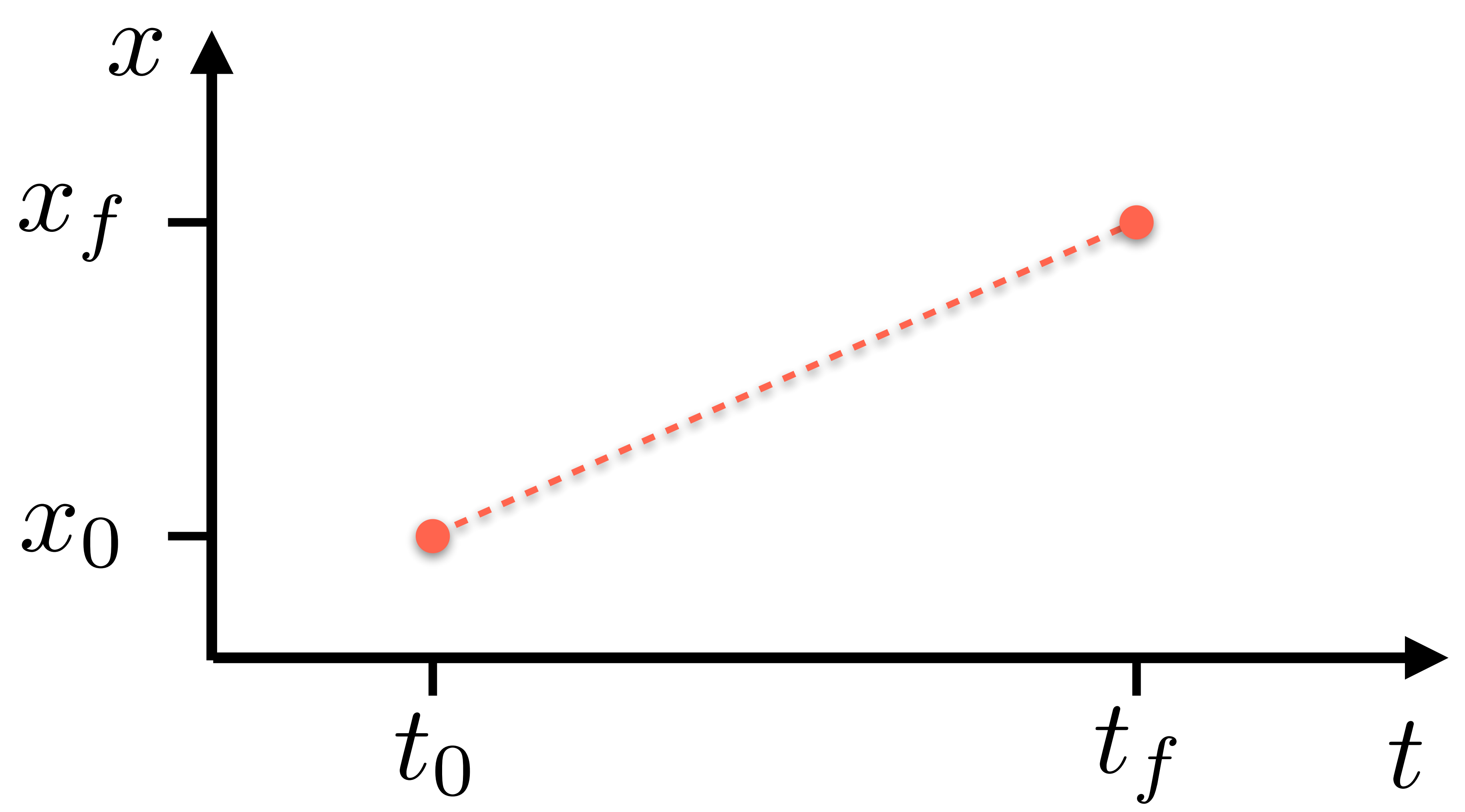

Two boundary conditions, two unknowns:

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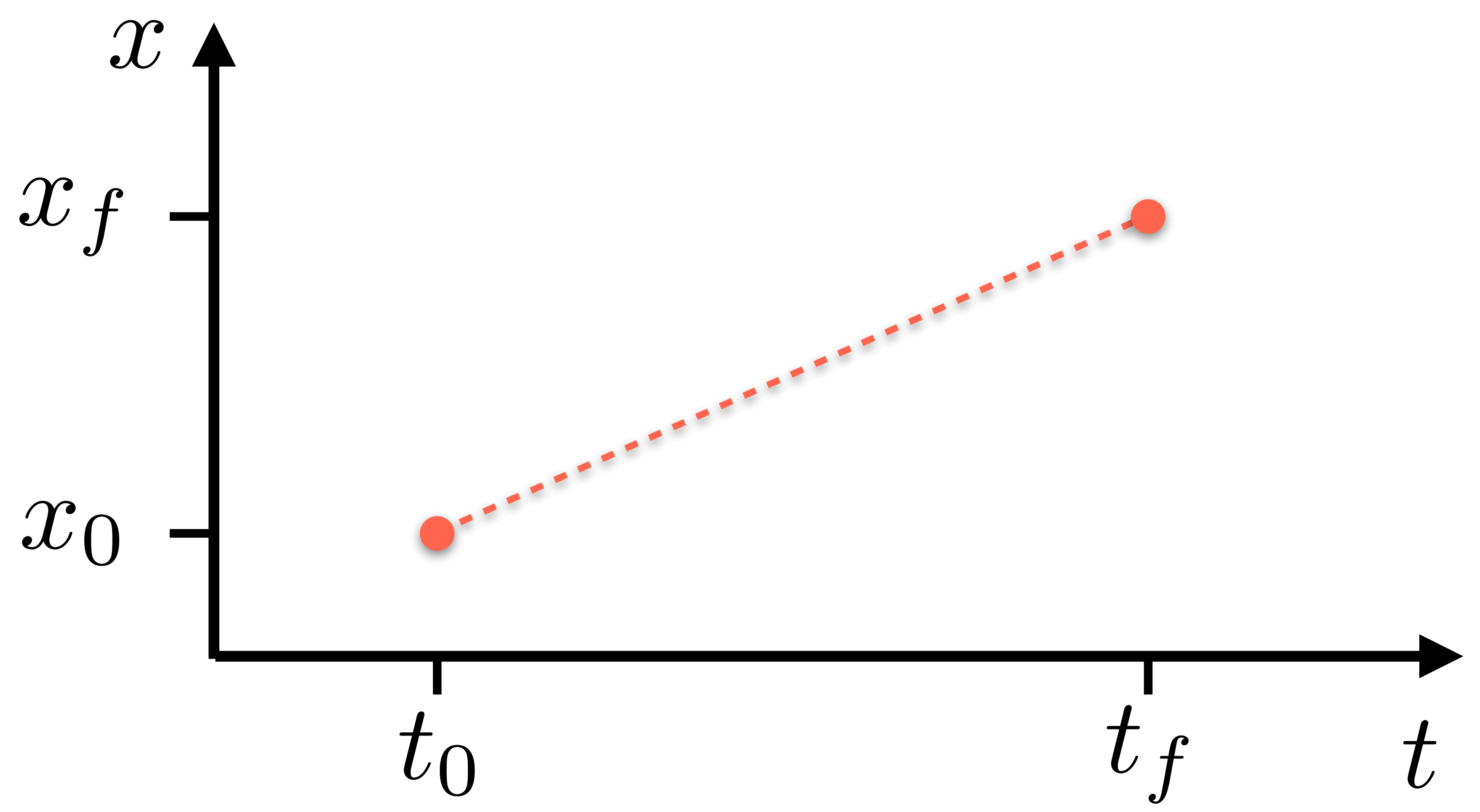
Line function:

$$x(t) = mt + n$$ (typical way of writing it)

$$x(t) = a_0 + a_1(t - t_0)$$ (polynomial way of writing it)



Two boundary conditions, two unknowns:

$$x(t = t_0) = x_0 \longrightarrow x(t_0) = x_0 = a_0 + a_1(t_0 - t_0) \longrightarrow a_0 = x_0$$

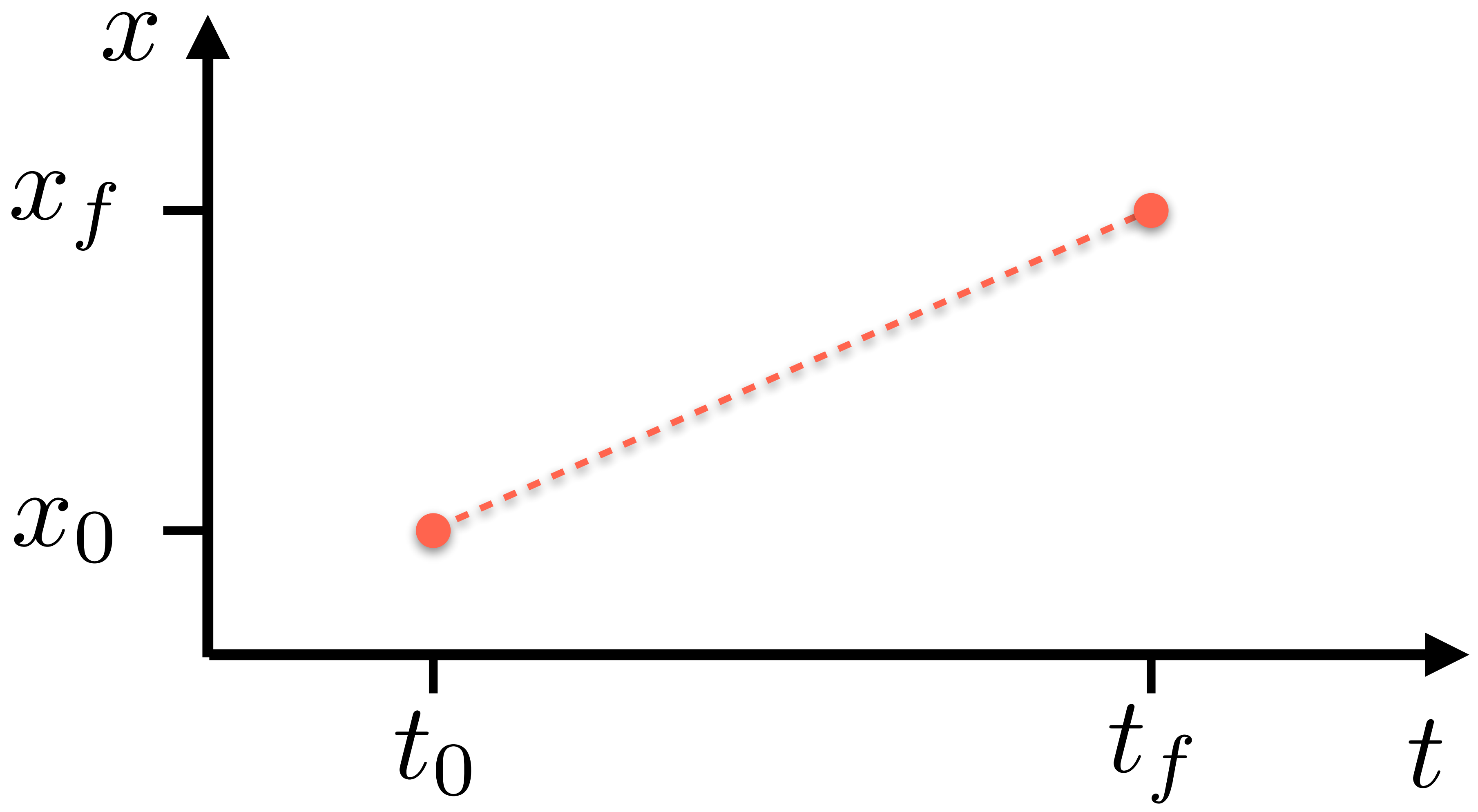
$$x(t = t_f) = x_f \longrightarrow x(t_f) = x_f = x_0 + a_1(t_f - t_0) \longrightarrow a_1 = \frac{x_f - x_0}{t_f - t_0}$$

Line function:

$$x(t) = a_0 + a_1(t - t_0)$$

$$a_0 = x_0$$

$$a_1 = \frac{x_f - x_0}{t_f - t_0}$$



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Line function:

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$$x(t = t_0) = x_0$$

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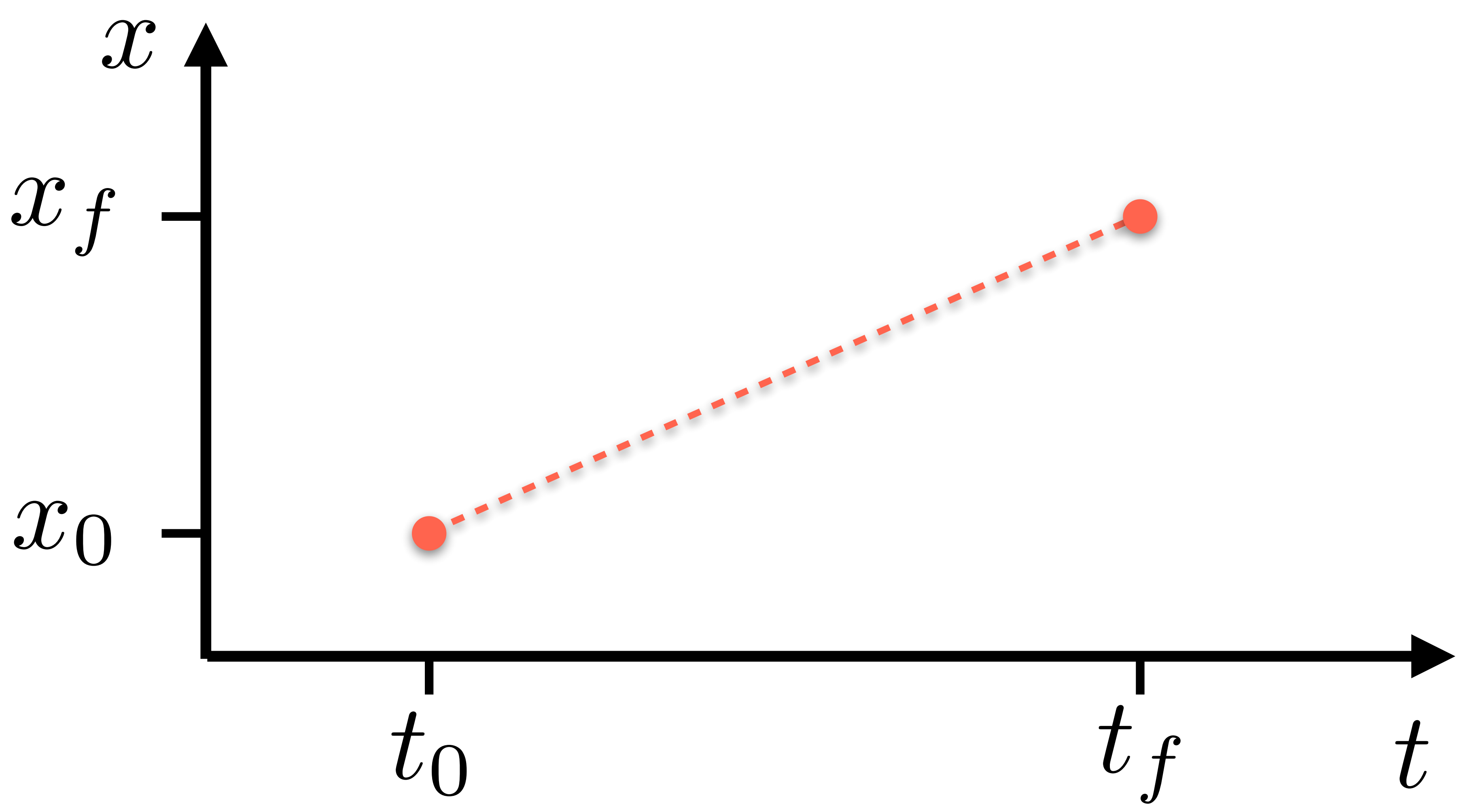
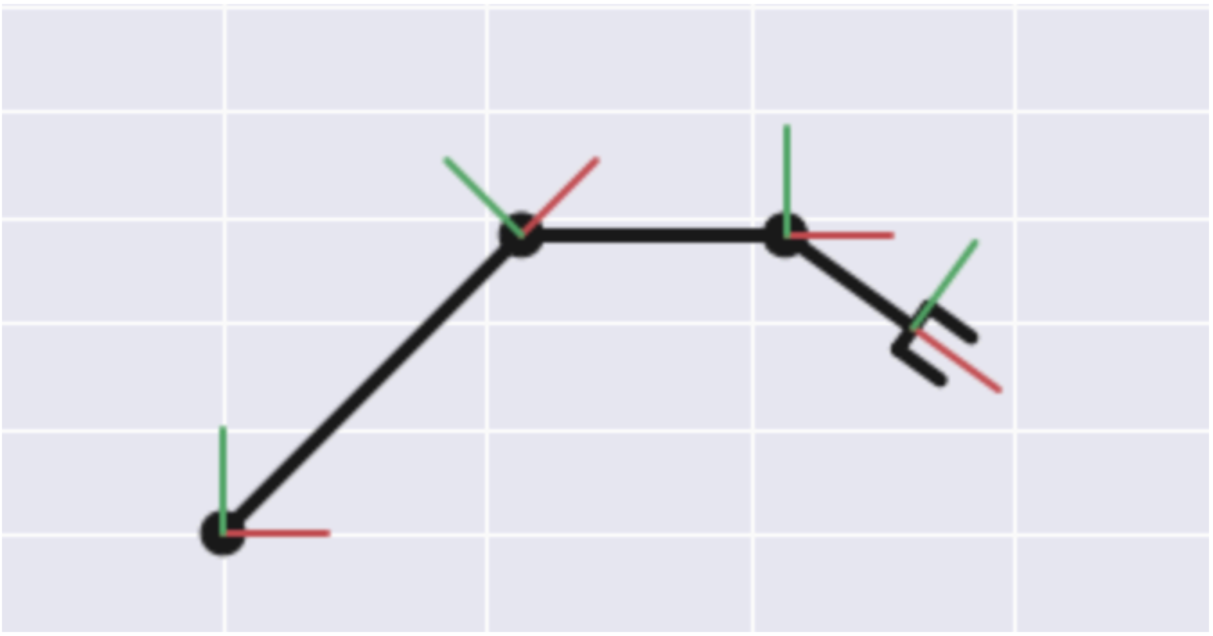




$$\mathbf{x} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

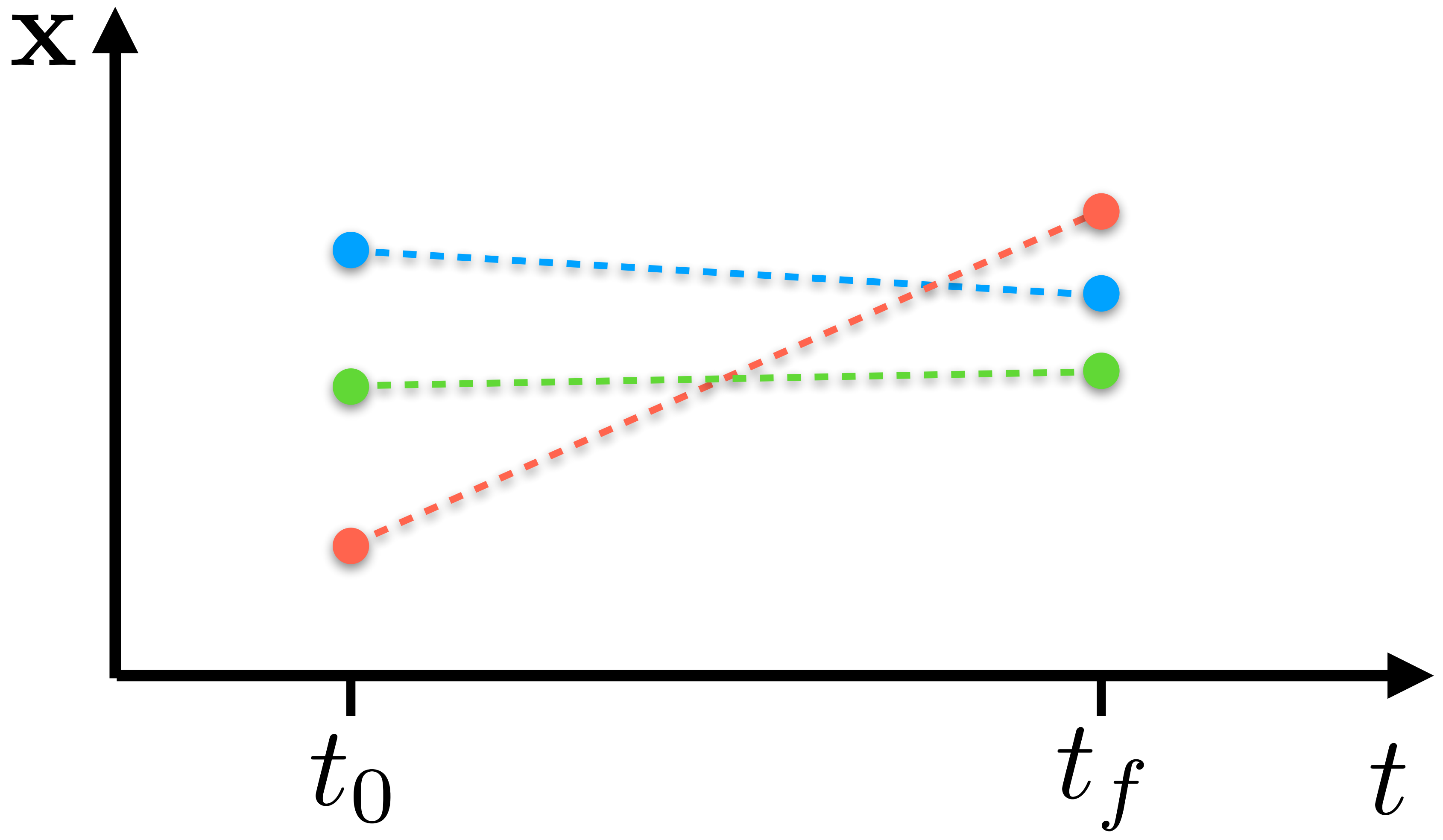
End effector pose
is multi-dimensional!

Line function:

$$x(t) = a_0 + a_1(t - t_0)$$

$$a_0 = x_0$$

$$a_1 = \frac{x_f - x_0}{t_f - t_0}$$



$$x(t = t_0) = x_0$$

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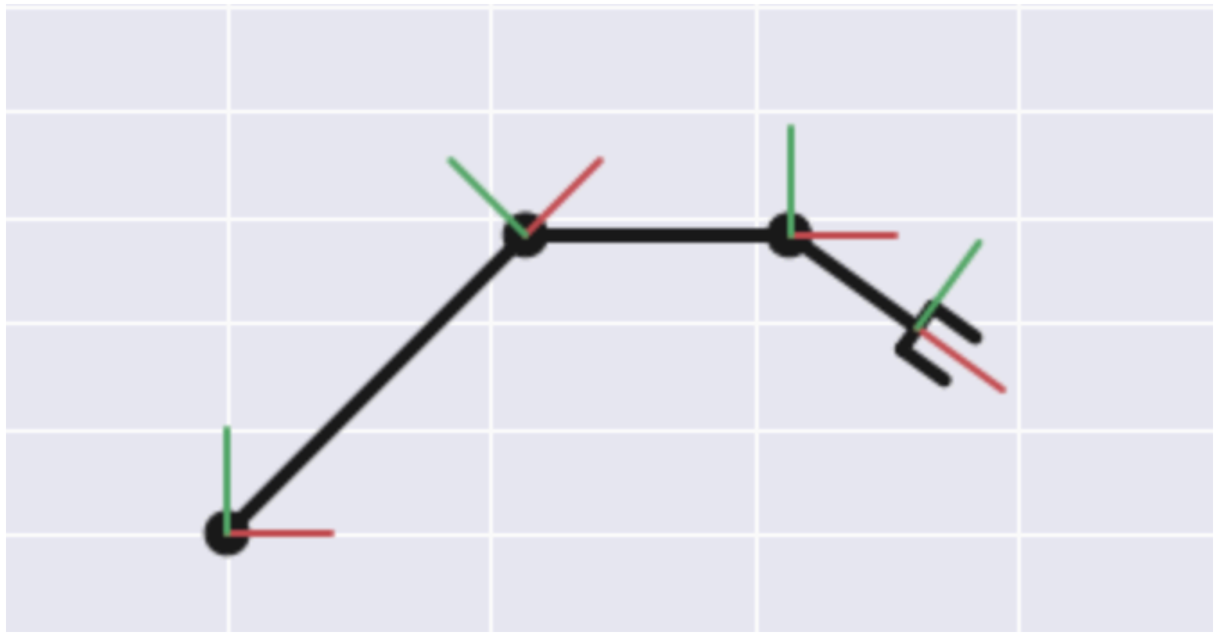

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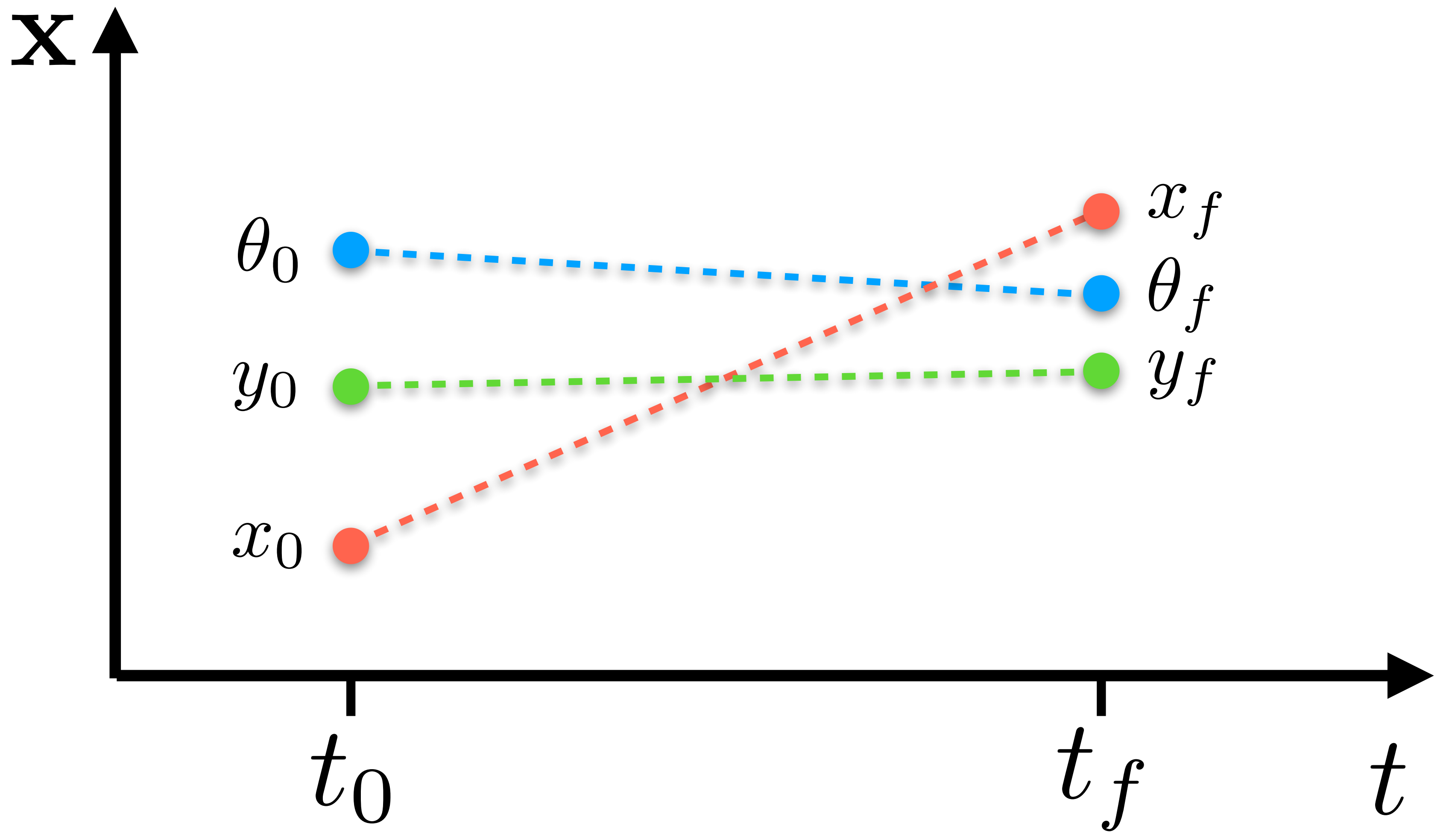

$$x(t = t_0) = x_0$$

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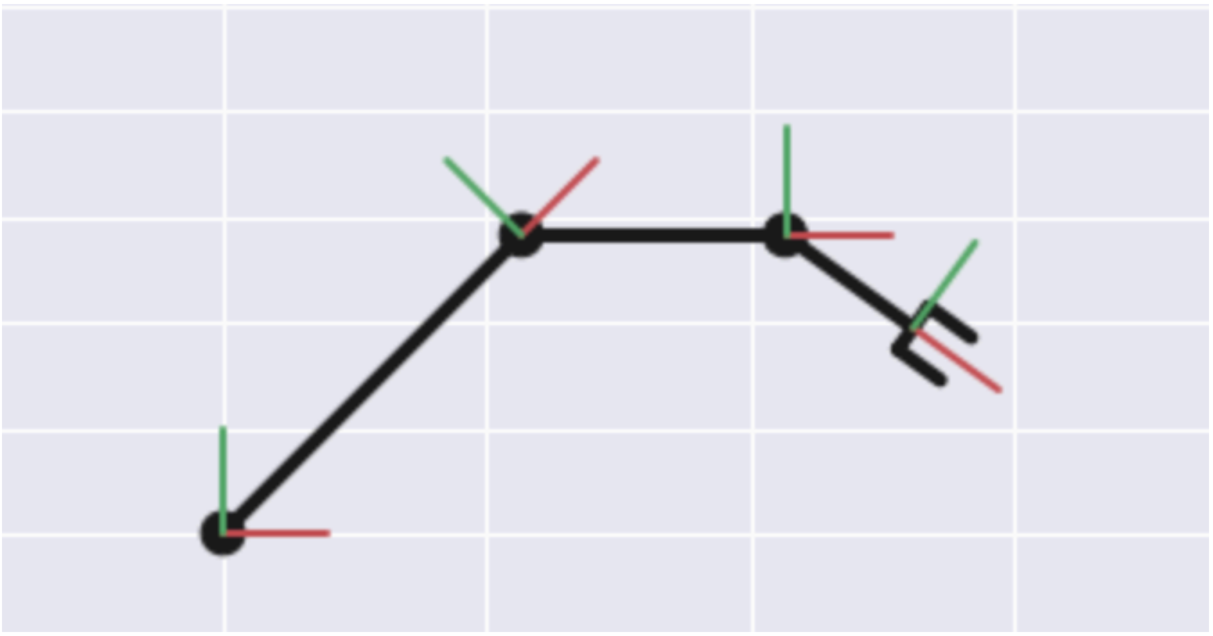

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

End effector pose
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$$x(t) = a_0 + a_1(t - t_0)$$

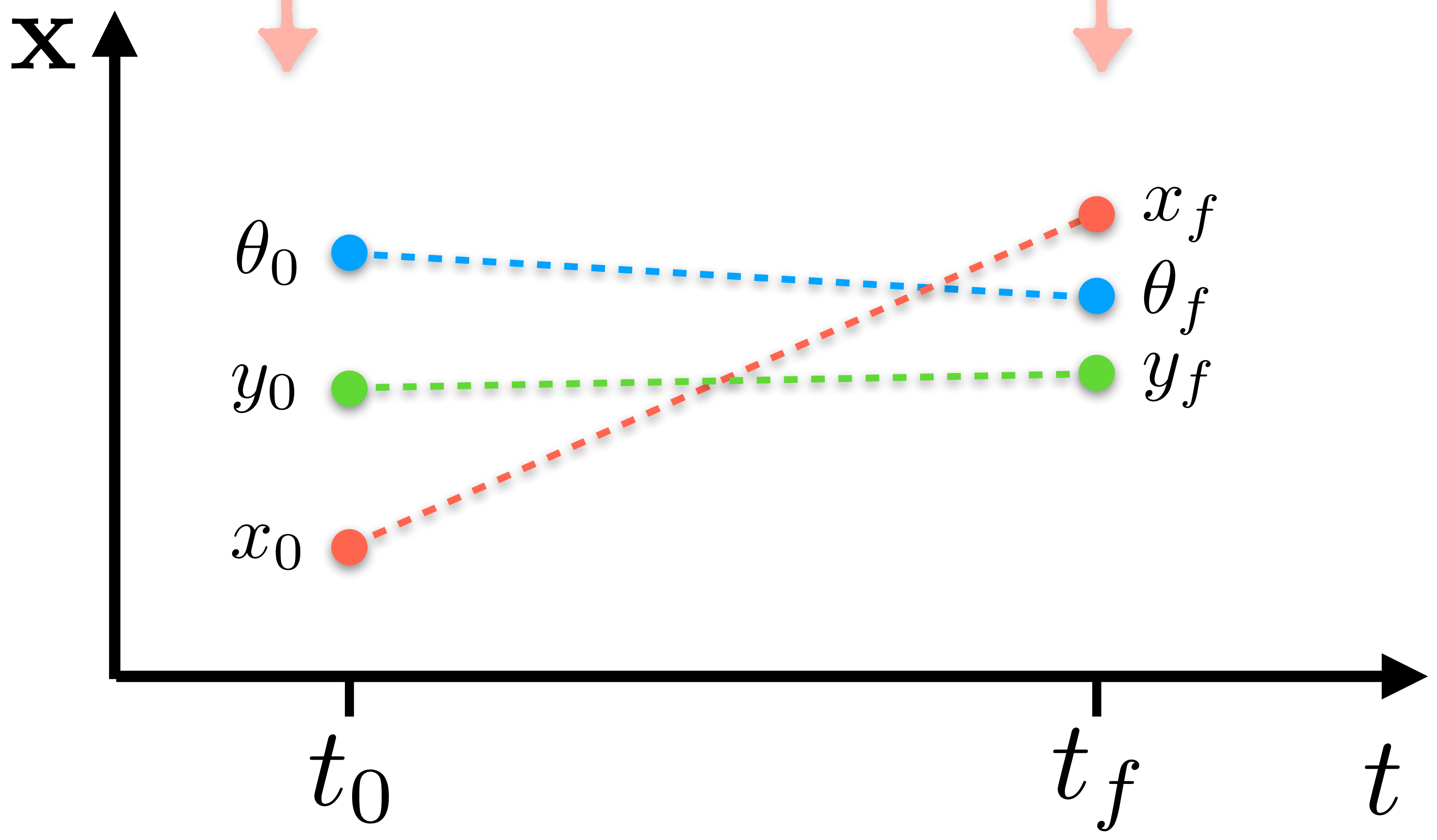
$$a_0 = x_0$$

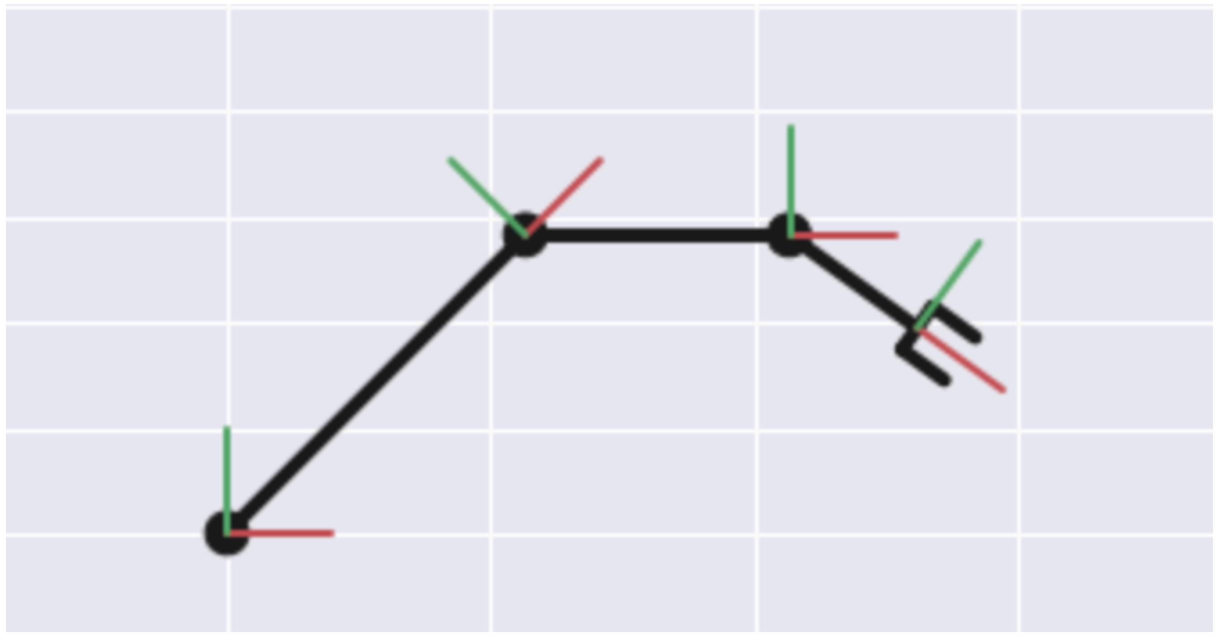
$$a_1 = \frac{x_f - x_0}{t_f - t_0}$$

$$x(t = t_0) = x_0$$

$$x(t = t_f) = x_f$$

$$\mathbf{x}_0 = \begin{bmatrix} x_0 \\ y_0 \\ \theta_0 \end{bmatrix} \qquad \mathbf{x}_f = \begin{bmatrix} x_f \\ y_f \\ \theta_f \end{bmatrix}$$





$$\mathbf{x} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

End effector pose
is multi-dimensional!

Multivariate Trajectory

Line function:

$$x(t) = a_0 + a_1(t - t_0)$$

$$a_0 = x_0$$

$$a_1 = \frac{x_f - x_0}{t_f - t_0}$$

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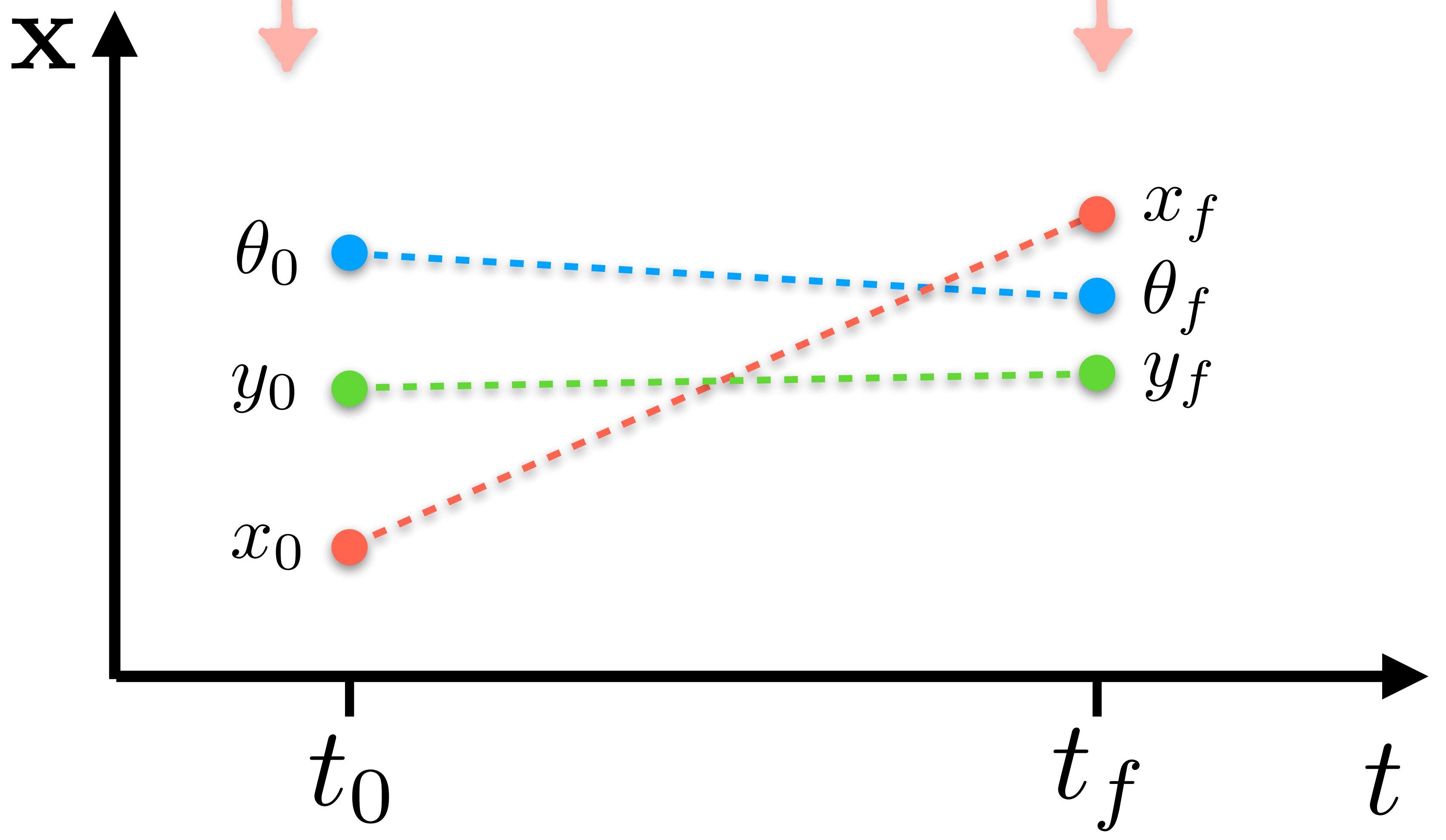
$$\mathbf{x}_0 = \begin{bmatrix} x_0 \\ y_0 \\ \theta_0 \end{bmatrix} \qquad \mathbf{x}_f = \begin{bmatrix} x_f \\ y_f \\ \theta_f \end{bmatrix}$$

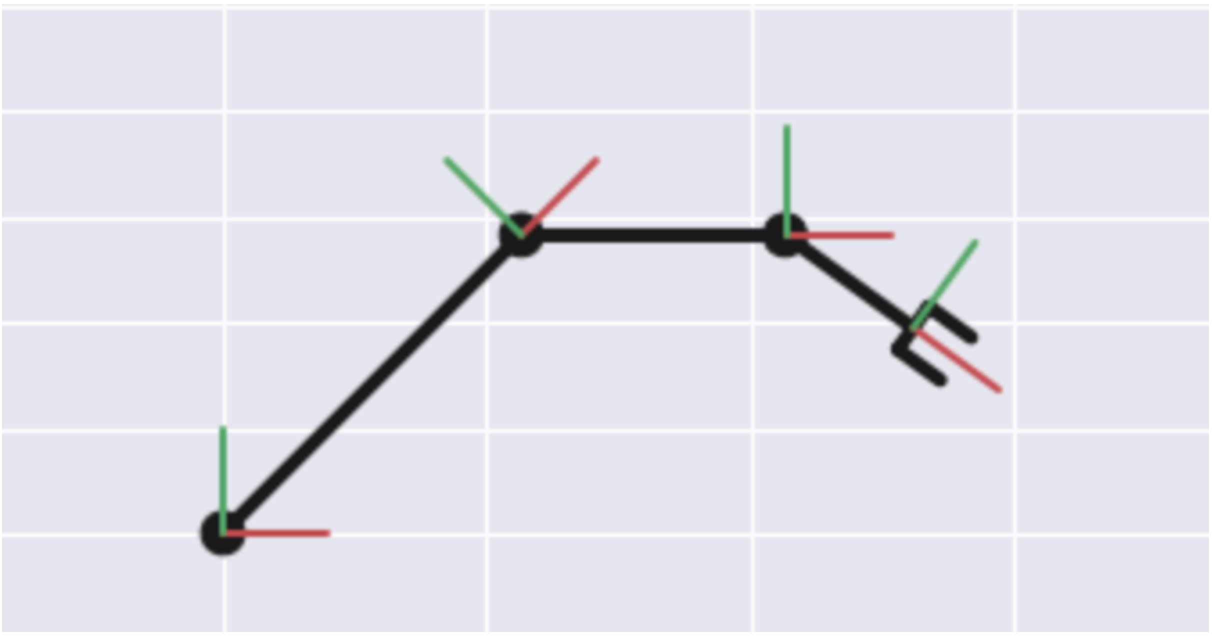


Separate line function for each dimension!

$$x(t) = a_0^x + a_1^x(t - t_0)$$

$$y(t) = a_0^y + a_1^y(t - t_0)$$

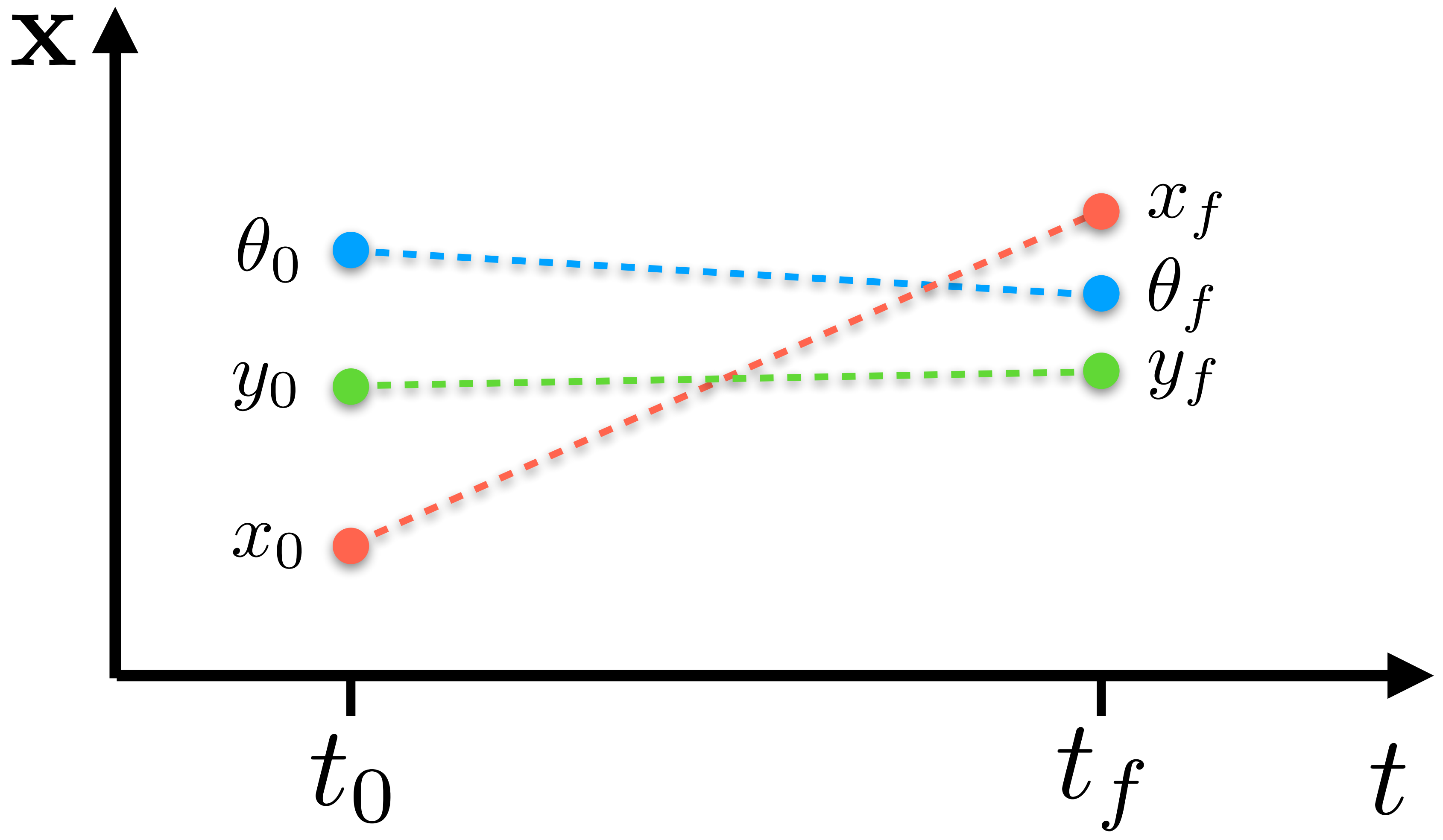
$$\theta(t) = a_0^\theta + a_1^\theta(t - t_0)$$



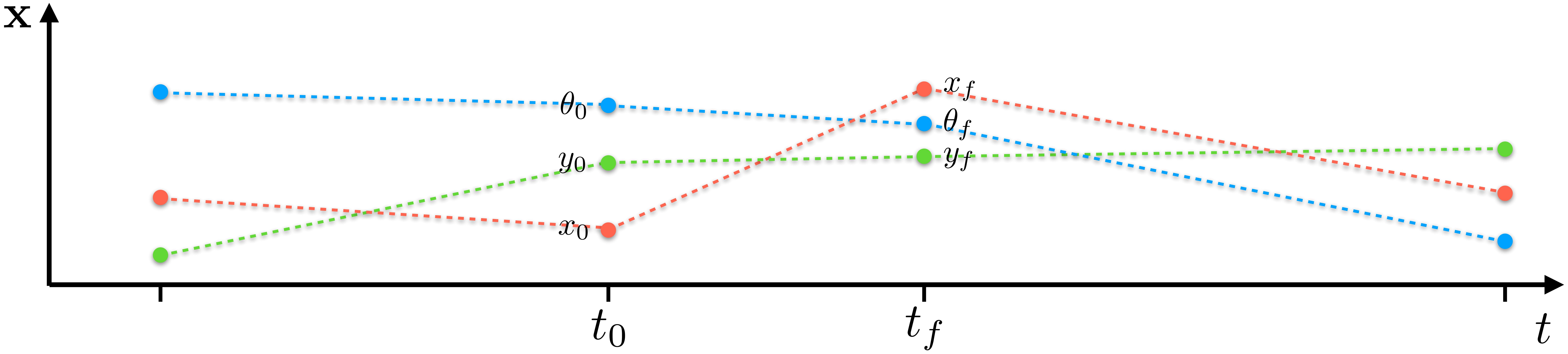
$$\mathbf{x} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

End effector pose is multi-dimensional!

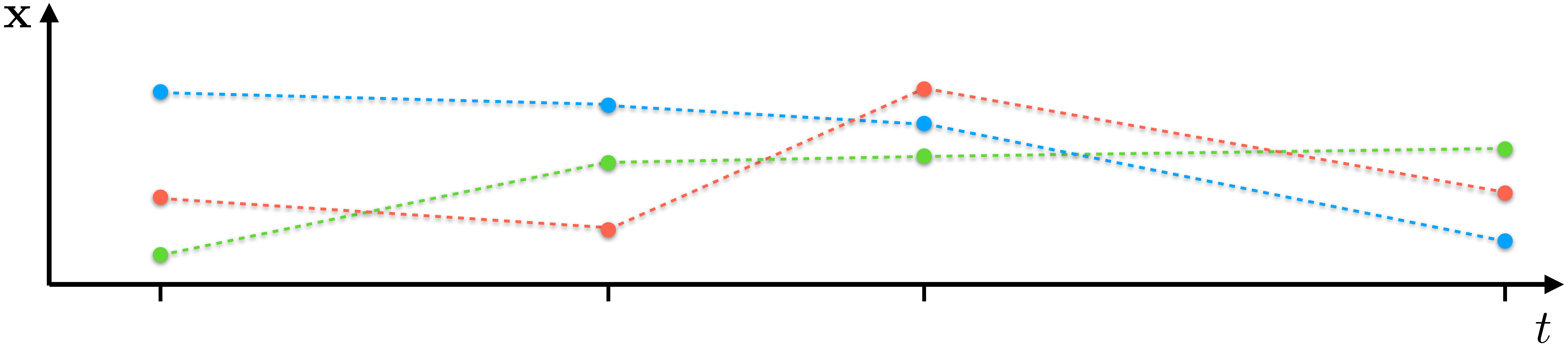
Multivariate Trajectory



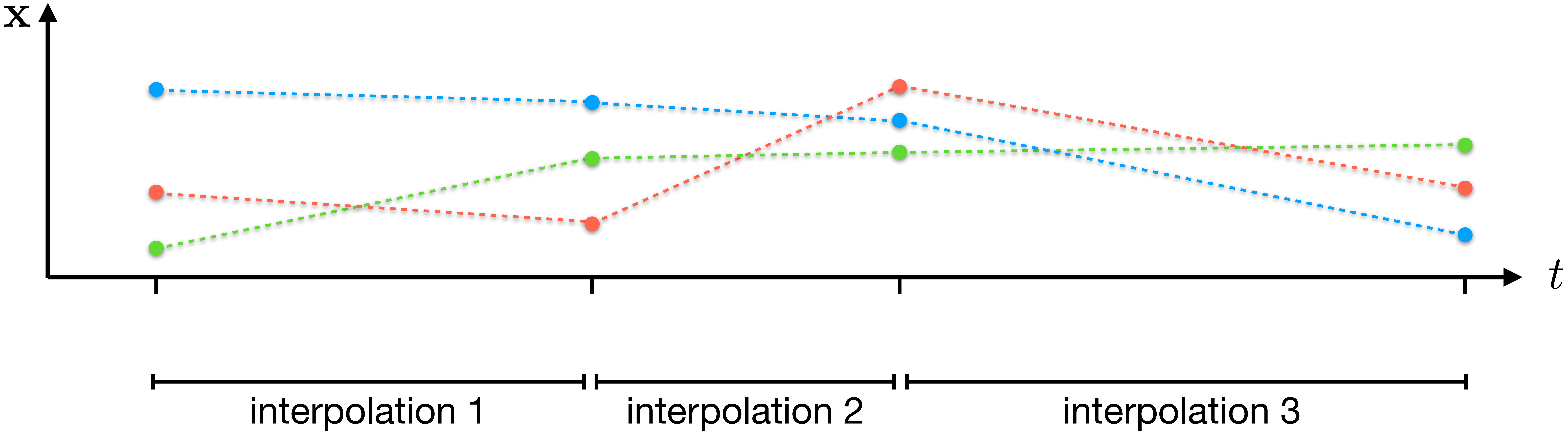
Multivariate Trajectory



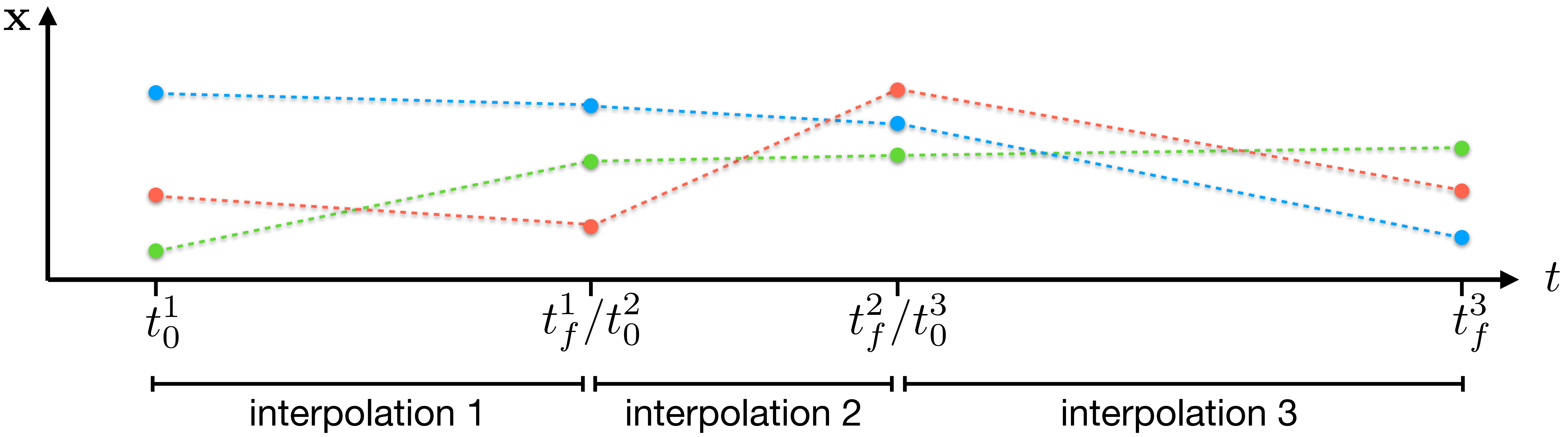
Multivariate Trajectory



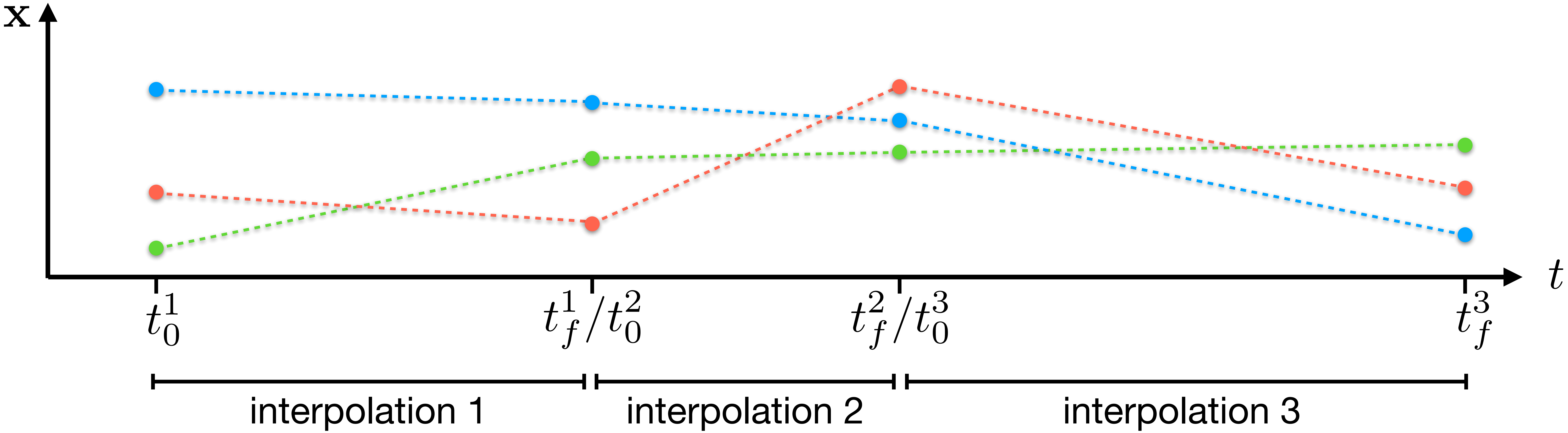
Multivariate Trajectory



Multivariate Trajectory



Multivariate Trajectory

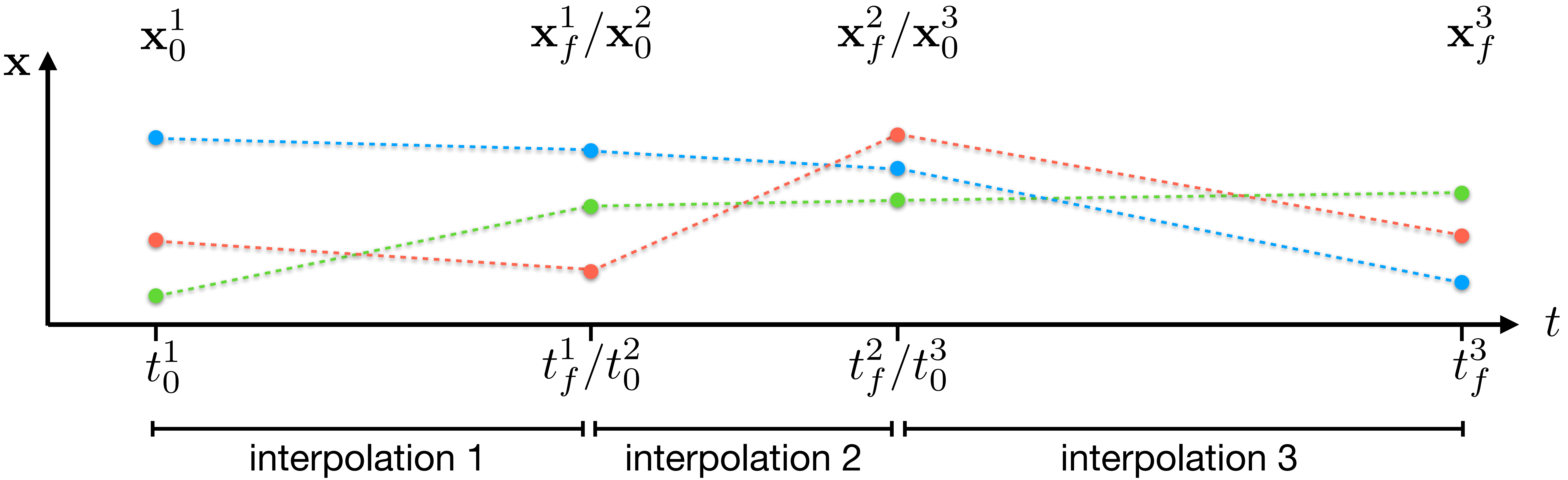


$$x^1(t) = a_0^{1,x} + a_1^{1,x}(t - t_0^1) \qquad x^2(t) = a_0^{2,x} + a_1^{2,x}(t - t_0^2) \qquad x^3(t) = a_0^{3,x} + a_1^{3,x}(t - t_0^3)$$

$$y^1(t) = a_0^{1,y} + a_1^{1,y}(t - t_0^1) \qquad y^2(t) = a_0^{2,y} + a_1^{2,y}(t - t_0^2) \qquad y^3(t) = a_0^{3,y} + a_1^{3,y}(t - t_0^3)$$

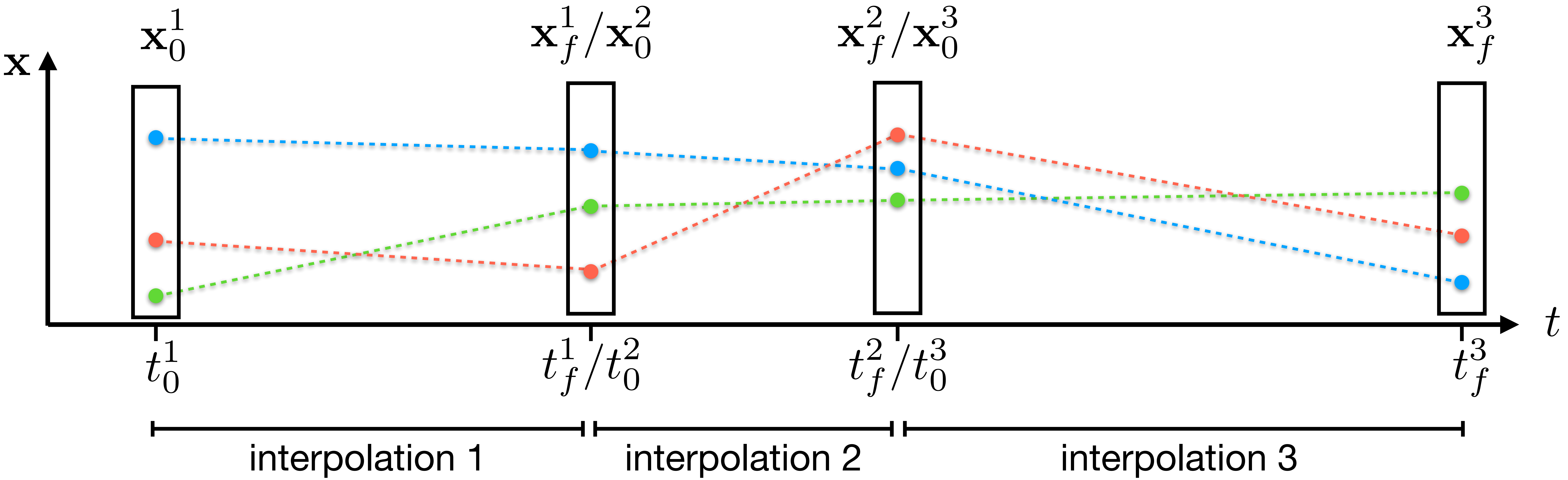
$$\theta^1(t) = a_0^{1,\theta} + a_1^{1,\theta}(t - t_0^1) \qquad \theta^2(t) = a_0^{2,\theta} + a_1^{2,\theta}(t - t_0^2) \qquad \theta^3(t) = a_0^{3,\theta} + a_1^{3,\theta}(t - t_0^3)$$

Multivariate Trajectory



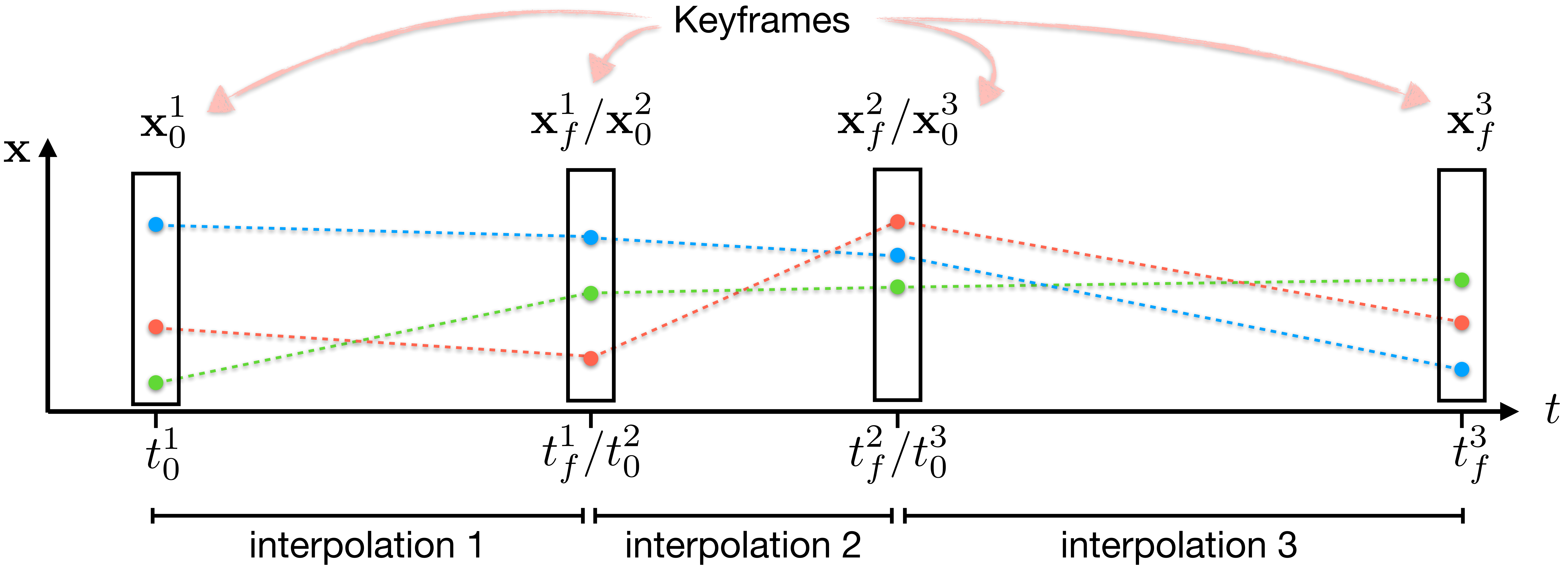
$$x^1(t) = a_0^{1,x} + a_1^{1,x}(t - t_0^1) \qquad x^2(t) = a_0^{2,x} + a_1^{2,x}(t - t_0^2) \qquad x^3(t) = a_0^{3,x} + a_1^{3,x}(t - t_0^3)$$
$$y^1(t) = a_0^{1,y} + a_1^{1,y}(t - t_0^1) \qquad y^2(t) = a_0^{2,y} + a_1^{2,y}(t - t_0^2) \qquad y^3(t) = a_0^{3,y} + a_1^{3,y}(t - t_0^3)$$
$$\theta^1(t) = a_0^{1,\theta} + a_1^{1,\theta}(t - t_0^1) \qquad \theta^2(t) = a_0^{2,\theta} + a_1^{2,\theta}(t - t_0^2) \qquad \theta^3(t) = a_0^{3,\theta} + a_1^{3,\theta}(t - t_0^3)$$

Multivariate Trajectory



$$
\begin{aligned}
x^1(t) &= a_0^{1,x} + a_1^{1,x}(t - t_0^1) \\
y^1(t) &= a_0^{1,y} + a_1^{1,y}(t - t_0^1) \\
\theta^1(t) &= a_0^{1,\theta} + a_1^{1,\theta}(t - t_0^1)
\end{aligned}
\qquad
\begin{aligned}
x^2(t) &= a_0^{2,x} + a_1^{2,x}(t - t_0^2) \\
y^2(t) &= a_0^{2,y} + a_1^{2,y}(t - t_0^2) \\
\theta^2(t) &= a_0^{2,\theta} + a_1^{2,\theta}(t - t_0^2)
\end{aligned}
\qquad
\begin{aligned}
x^3(t) &= a_0^{3,x} + a_1^{3,x}(t - t_0^3) \\
y^3(t) &= a_0^{3,y} + a_1^{3,y}(t - t_0^3) \\
\theta^3(t) &= a_0^{3,\theta} + a_1^{3,\theta}(t - t_0^3)
\end{aligned}
$$

Multivariate Trajectory

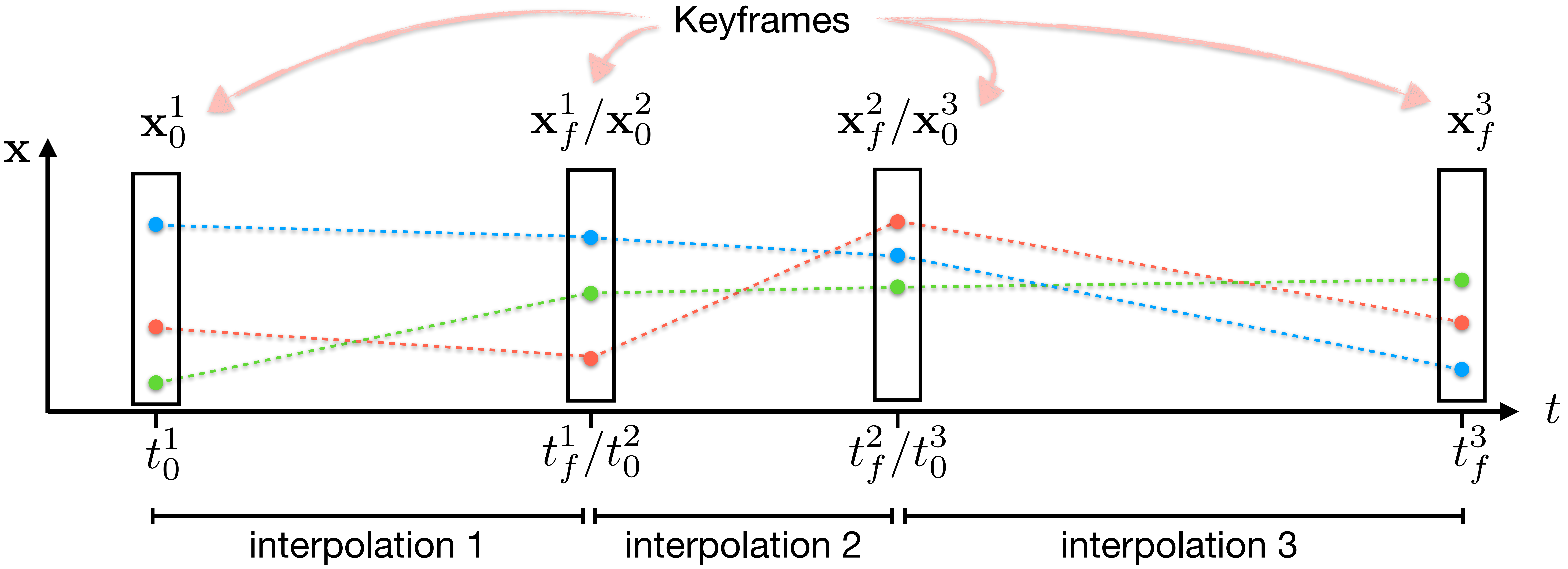


$$x^1(t) = a_0^{1,x} + a_1^{1,x}(t - t_0^1) \qquad x^2(t) = a_0^{2,x} + a_1^{2,x}(t - t_0^2) \qquad x^3(t) = a_0^{3,x} + a_1^{3,x}(t - t_0^3)$$

$$y^1(t) = a_0^{1,y} + a_1^{1,y}(t - t_0^1) \qquad y^2(t) = a_0^{2,y} + a_1^{2,y}(t - t_0^2) \qquad y^3(t) = a_0^{3,y} + a_1^{3,y}(t - t_0^3)$$

$$\theta^1(t) = a_0^{1,\theta} + a_1^{1,\theta}(t - t_0^1) \qquad \theta^2(t) = a_0^{2,\theta} + a_1^{2,\theta}(t - t_0^2) \qquad \theta^3(t) = a_0^{3,\theta} + a_1^{3,\theta}(t - t_0^3)$$

Multivariate Trajectory
Linear Keyframe Interpolation

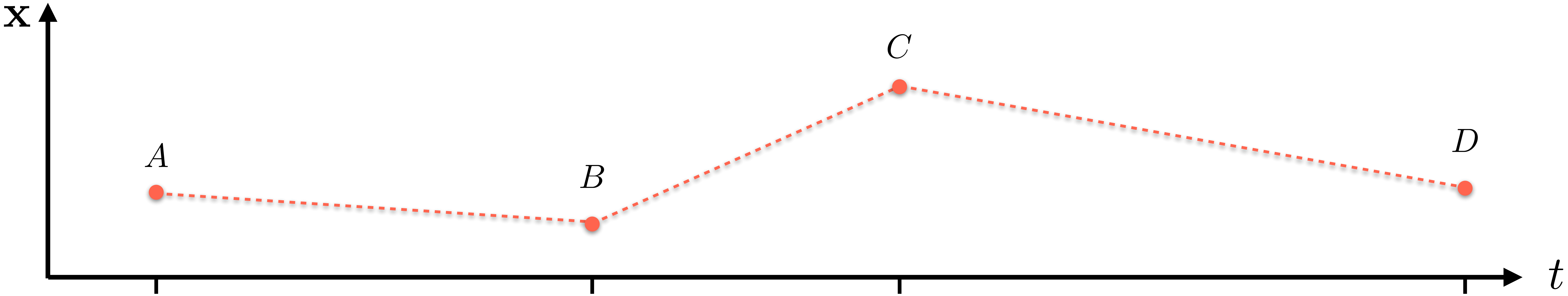


$$x^1(t) = a_0^{1,x} + a_1^{1,x}(t - t_0^1) \qquad x^2(t) = a_0^{2,x} + a_1^{2,x}(t - t_0^2) \qquad x^3(t) = a_0^{3,x} + a_1^{3,x}(t - t_0^3)$$

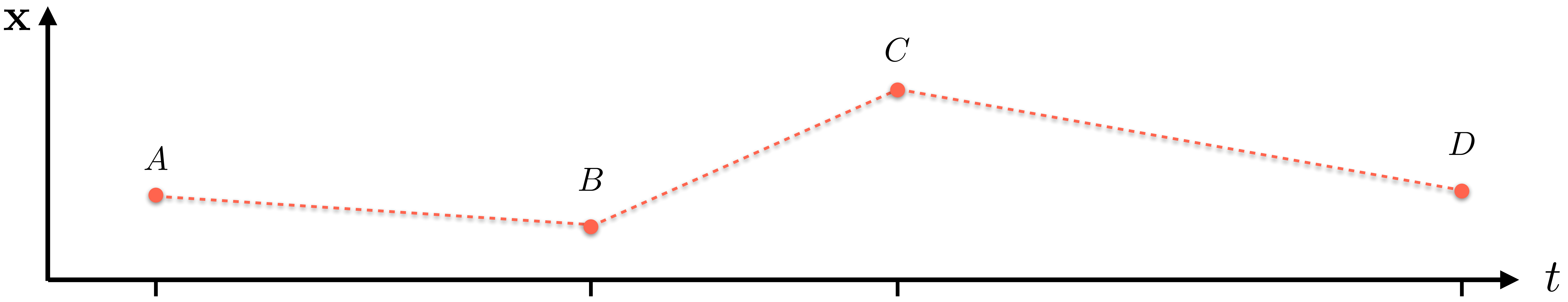
$$y^1(t) = a_0^{1,y} + a_1^{1,y}(t - t_0^1) \qquad y^2(t) = a_0^{2,y} + a_1^{2,y}(t - t_0^2) \qquad y^3(t) = a_0^{3,y} + a_1^{3,y}(t - t_0^3)$$

$$\theta^1(t) = a_0^{1,\theta} + a_1^{1,\theta}(t - t_0^1) \qquad \theta^2(t) = a_0^{2,\theta} + a_1^{2,\theta}(t - t_0^2) \qquad \theta^3(t) = a_0^{3,\theta} + a_1^{3,\theta}(t - t_0^3)$$

Problems of Linear Interpolation

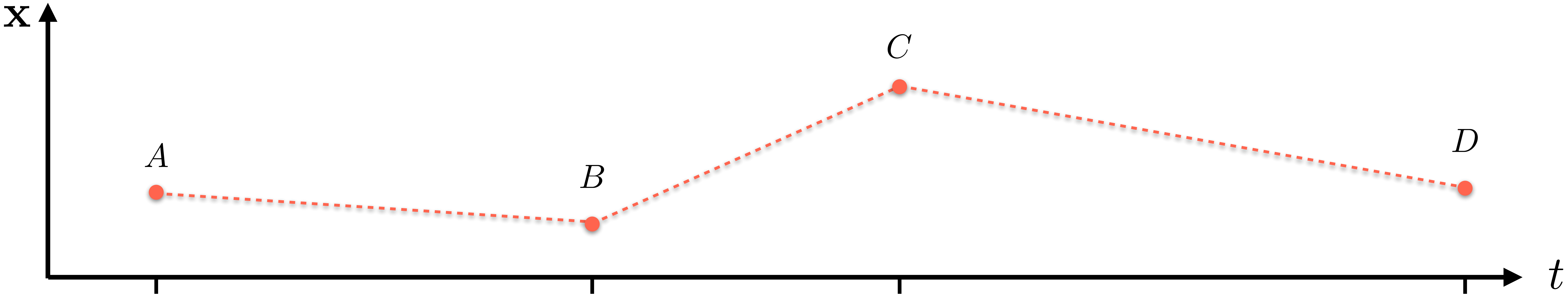


Problems of Linear Interpolation



Discontinuities at Keyframes

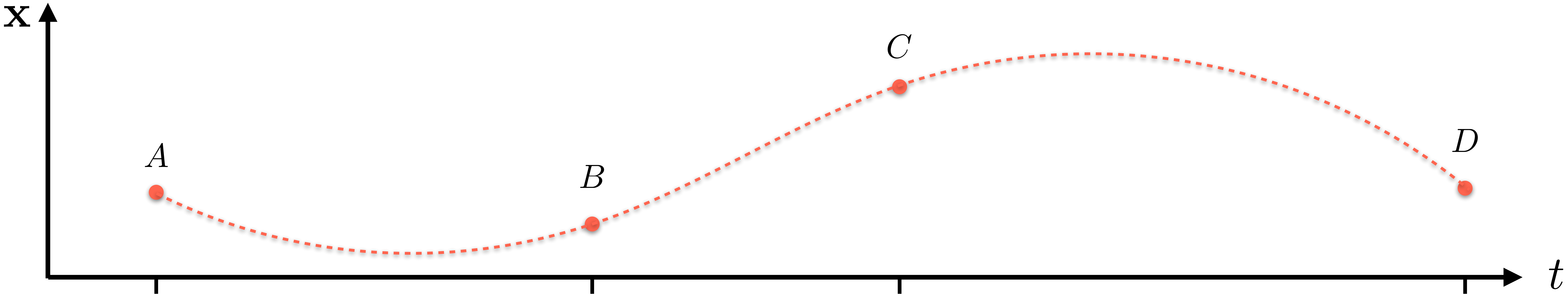
Problems of Linear Interpolation



Discontinuities at Keyframes

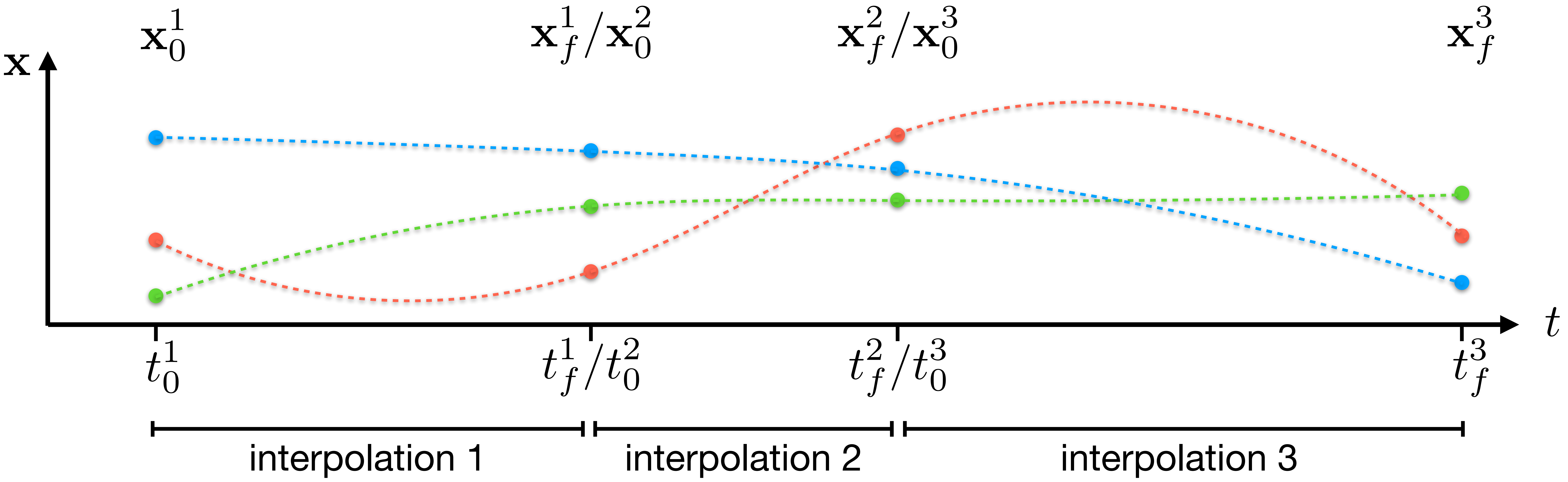
We need control over velocities at Keyframes!

We want continuous (smooth) transitions!



We want continuous (smooth) transitions!

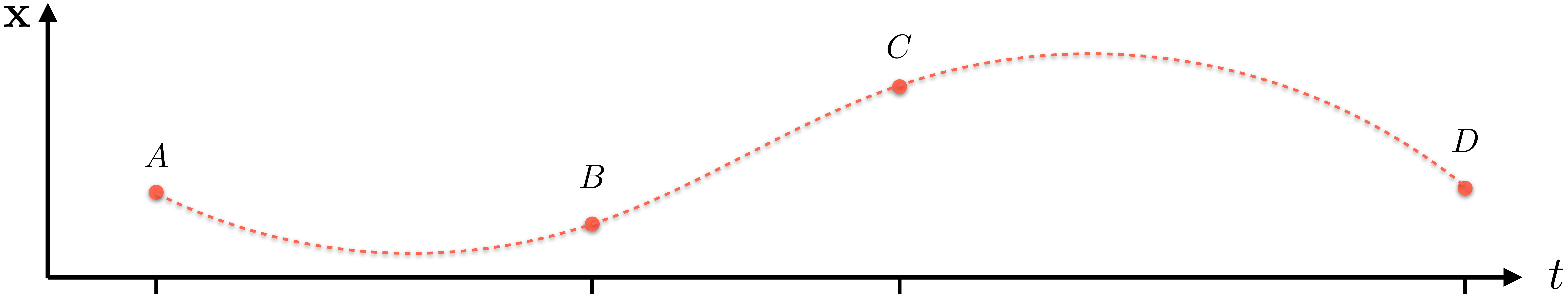
(also for multivariate trajectories)



Solution:

Cubic Splines

(also called „csplines“)



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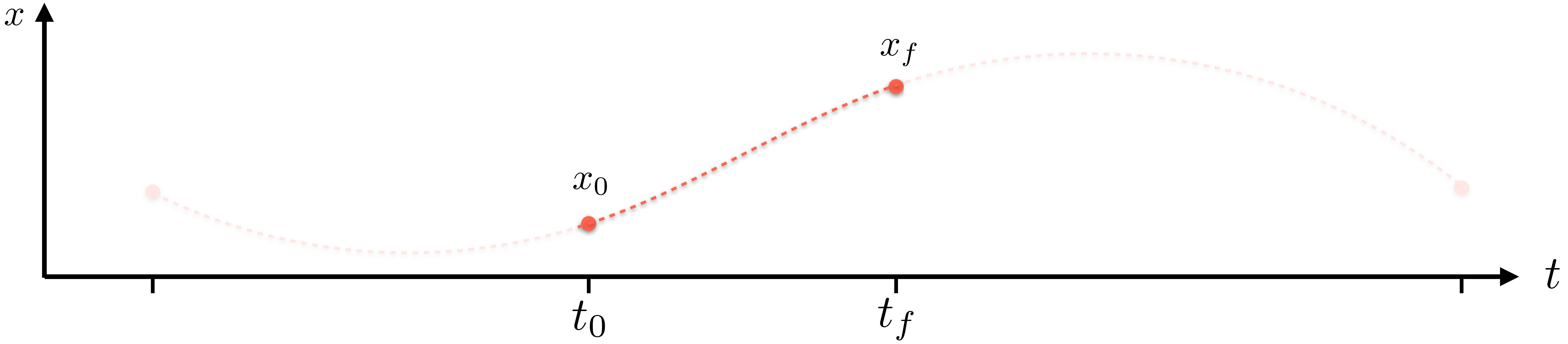
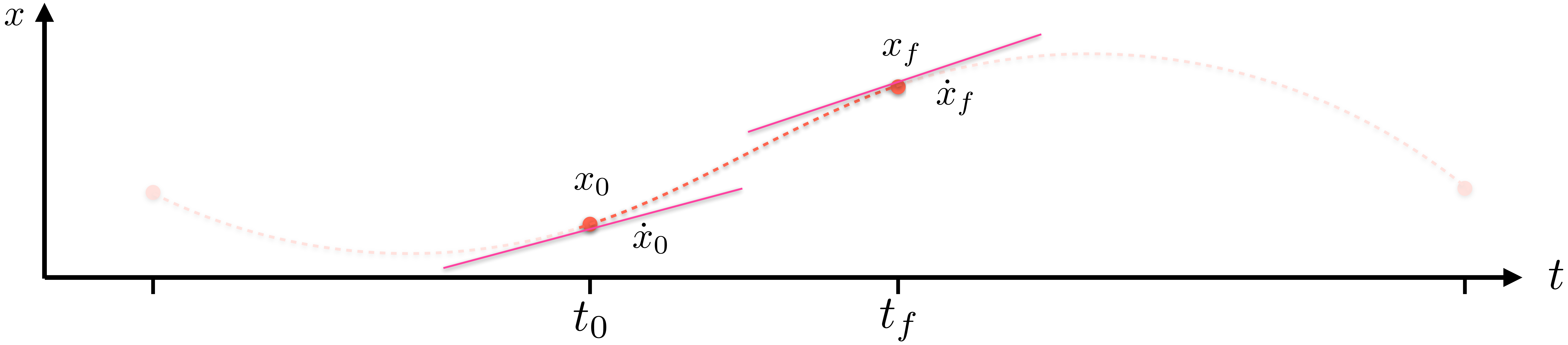


Third oder polynomial: $x(t) = a_0 + a_1 t + a_2 t^2 + a3 t^3$

Solution:

Cubic Splines

(also called „csplines“)

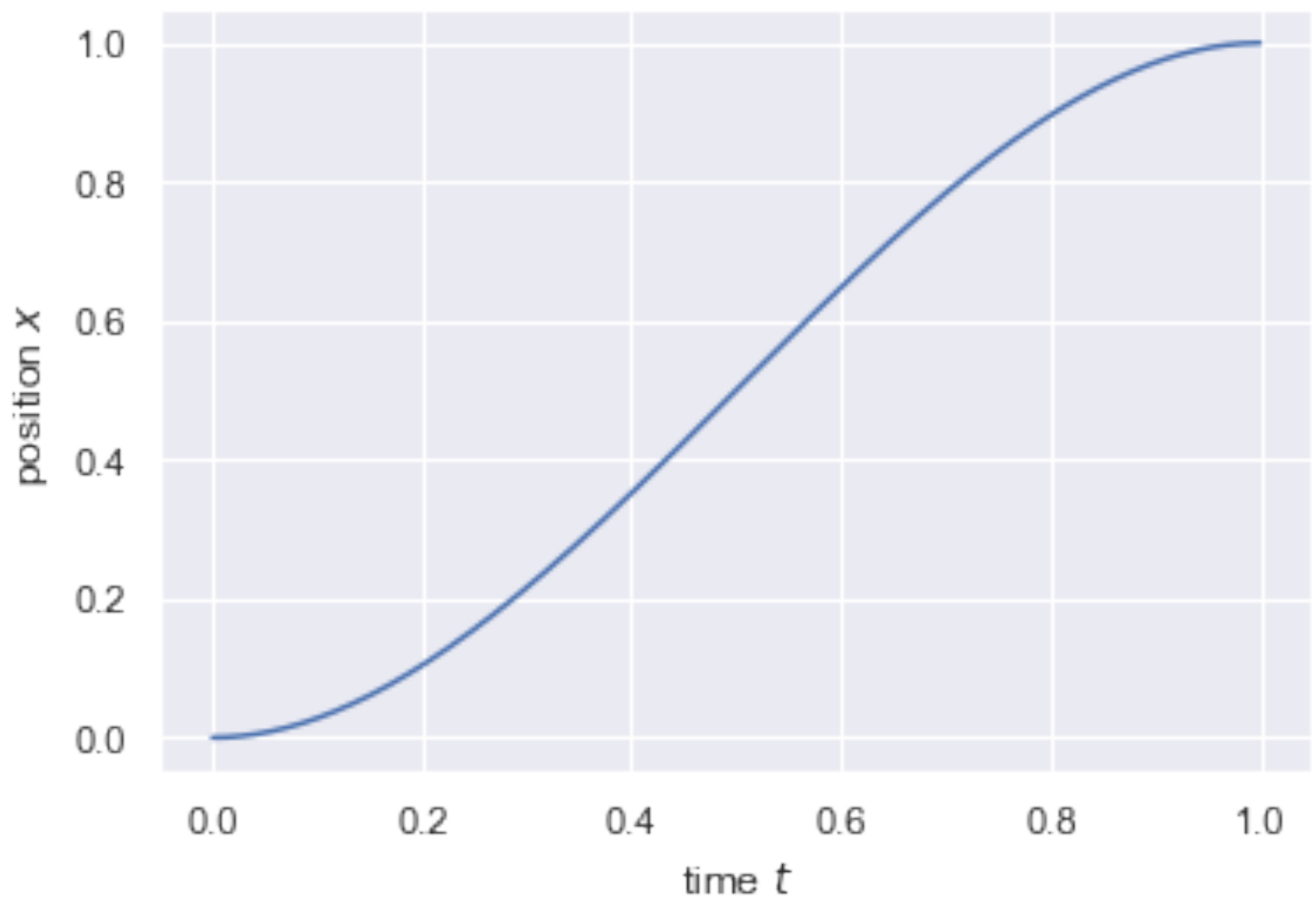


Third oder polynomial: $x(t) = a_0 + a_1 t + a_2 t^2 + a3t^3$

Solution:

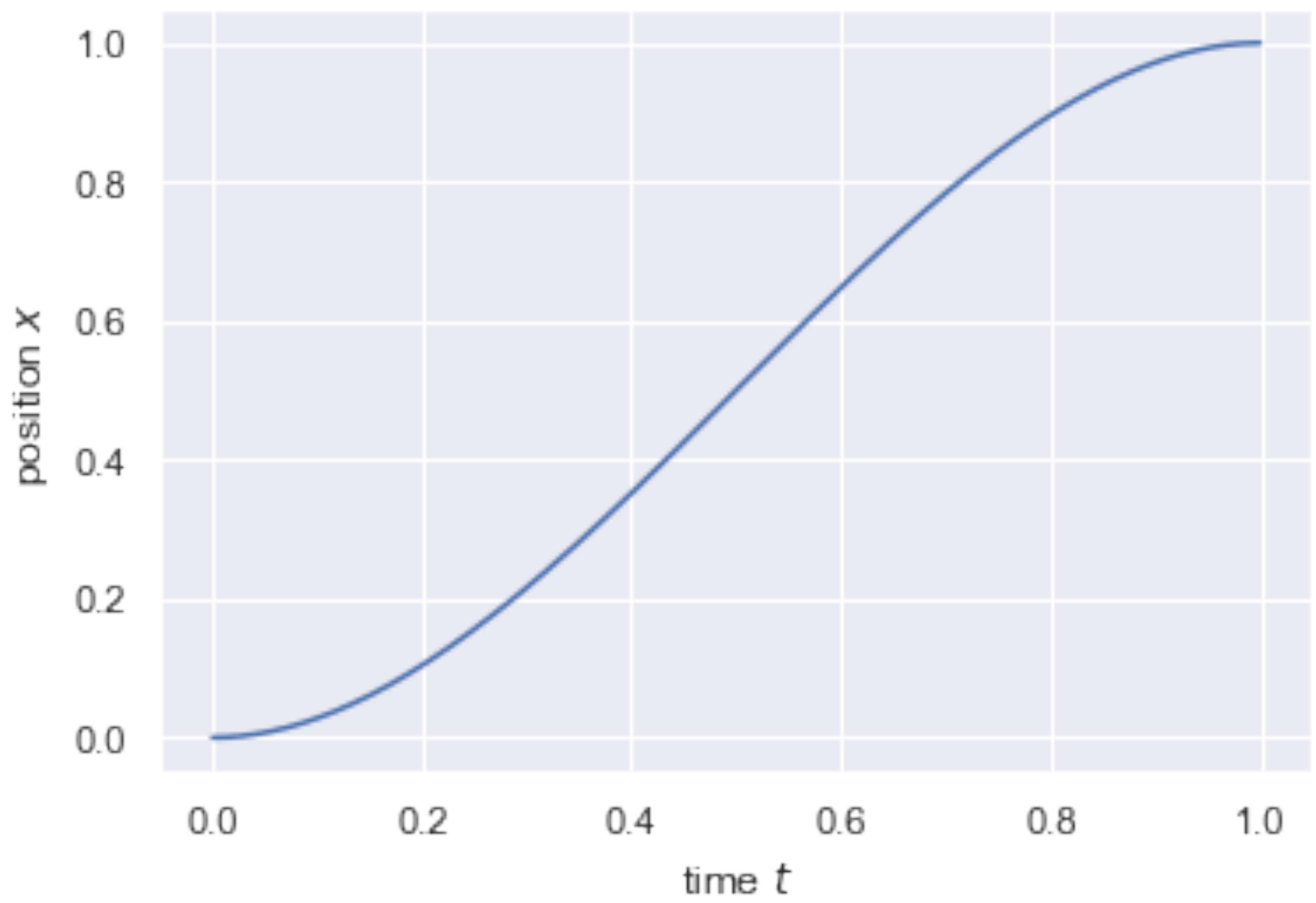
Cubic Splines

(also called „csplines“)

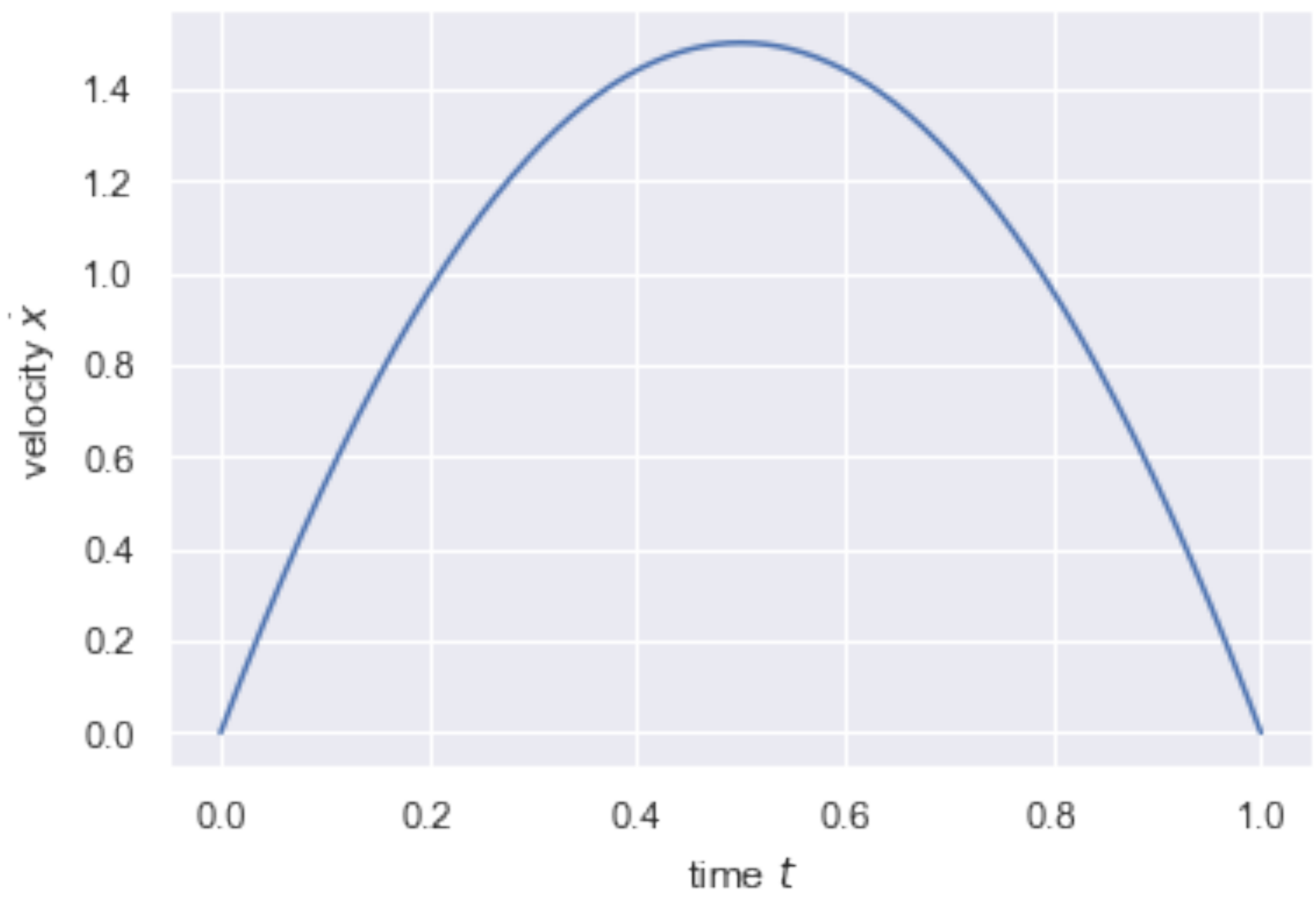


$$x(t) = a_0 + a_1 t + a_2 t^2 + a3 t^3$$

Solution:
Cubic Splines
(also called „csplines“)



$$x(t) = a_0 + a_1 t + a_2 t^2 + a3t^3$$

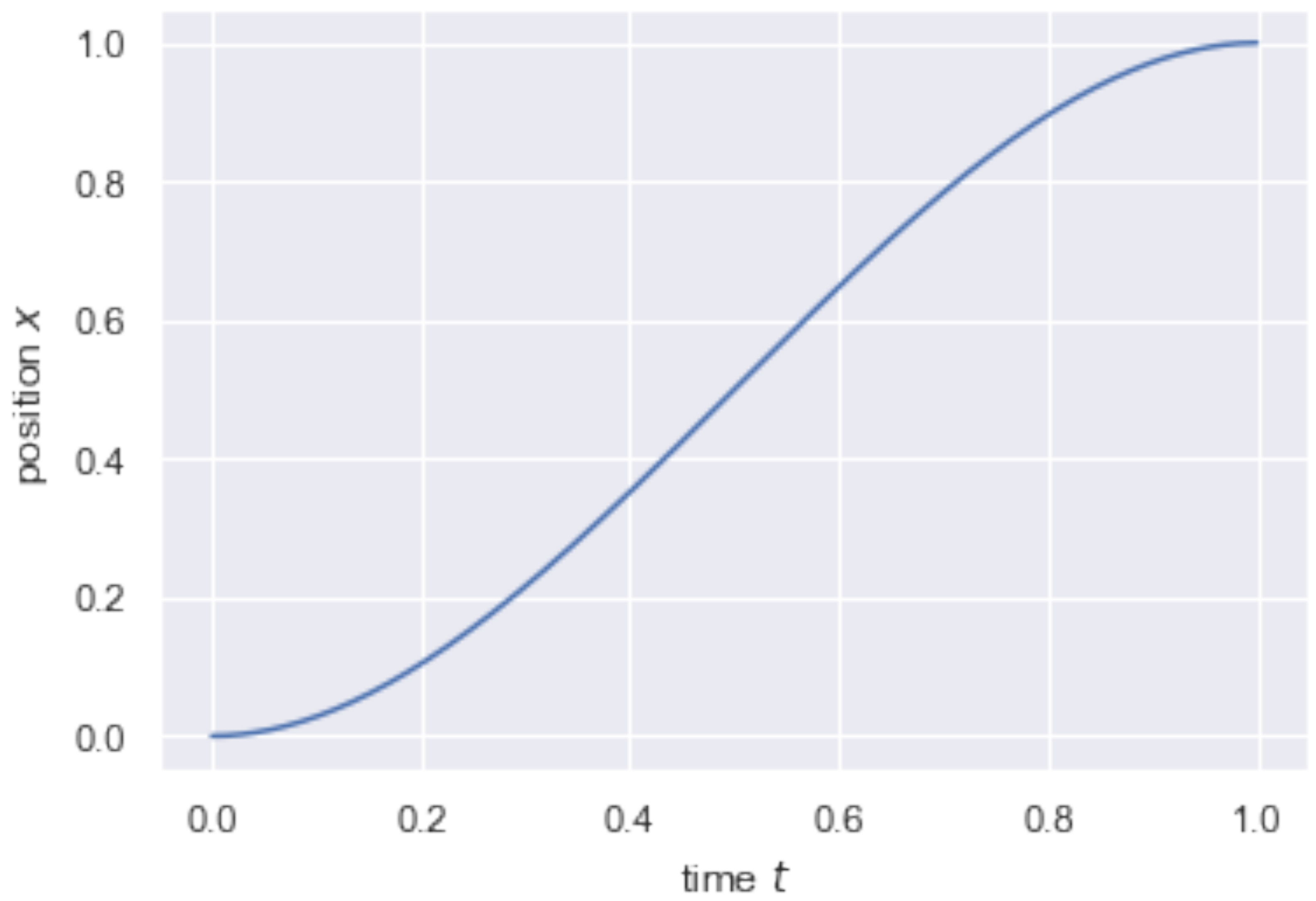


$$\dot{x}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

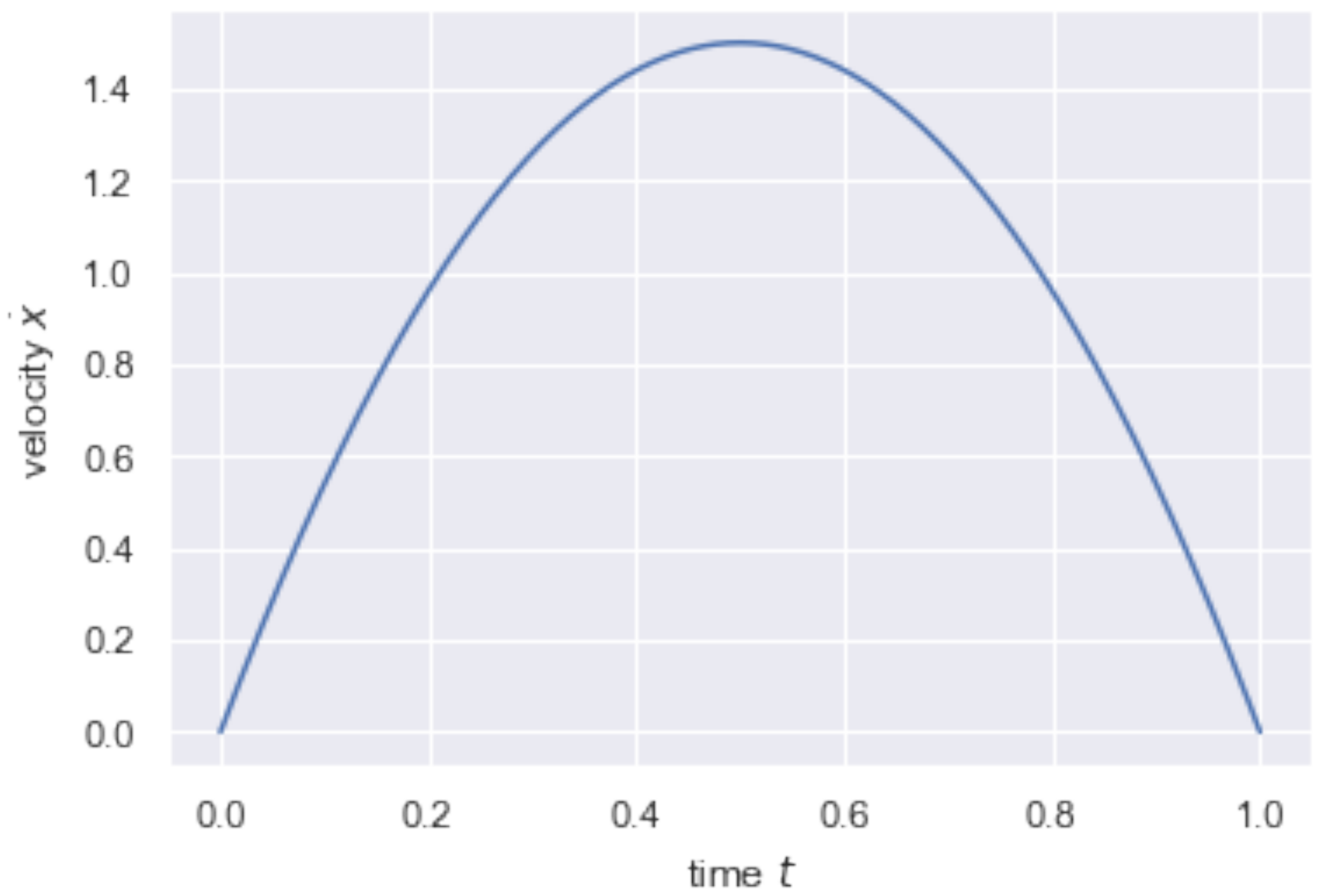
Solution:

Cubic Splines

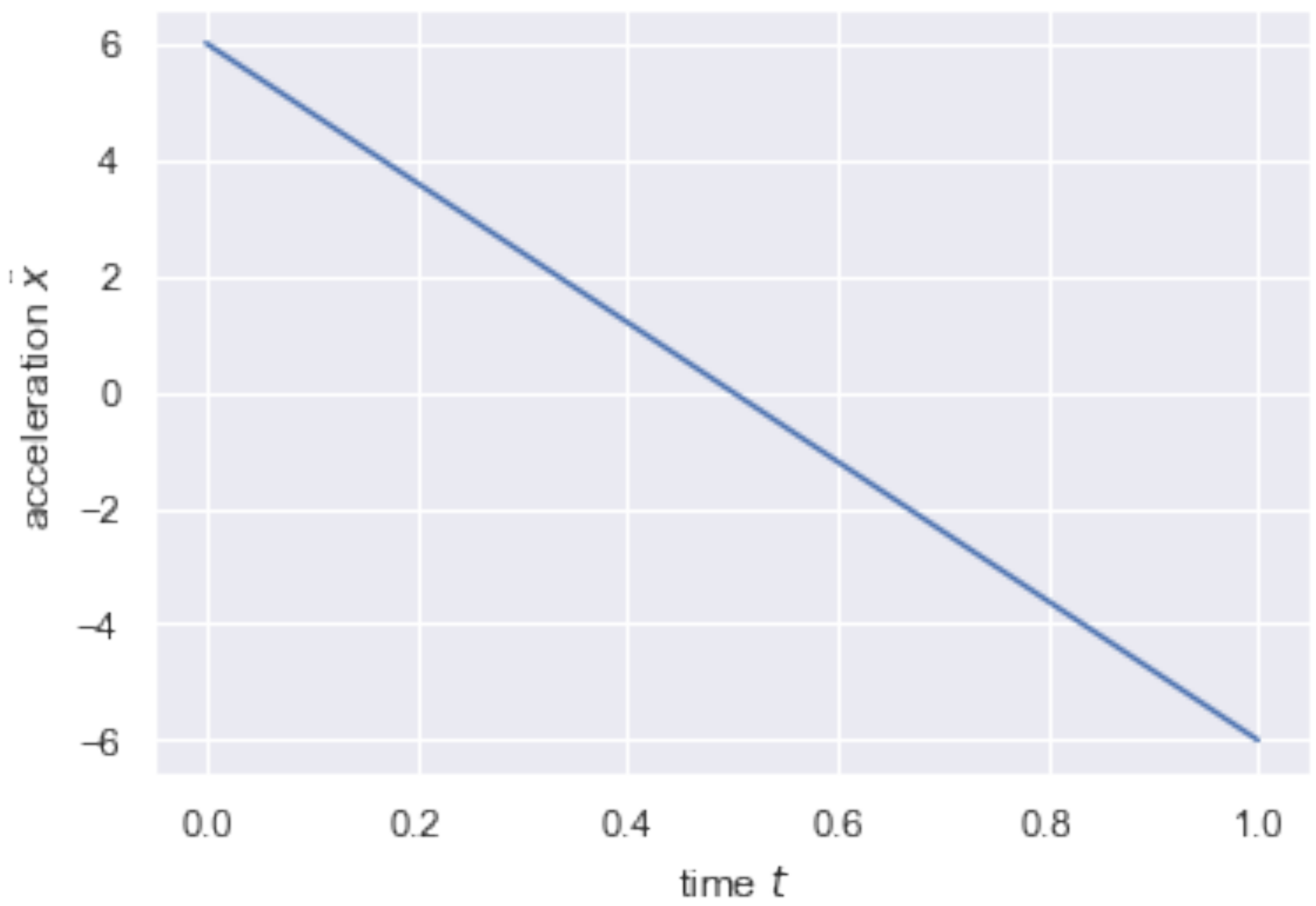
(also called „csplines“)



$$x(t) = a_0 + a_1 t + a_2 t^2 + a3t^3$$



$$\dot{x}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

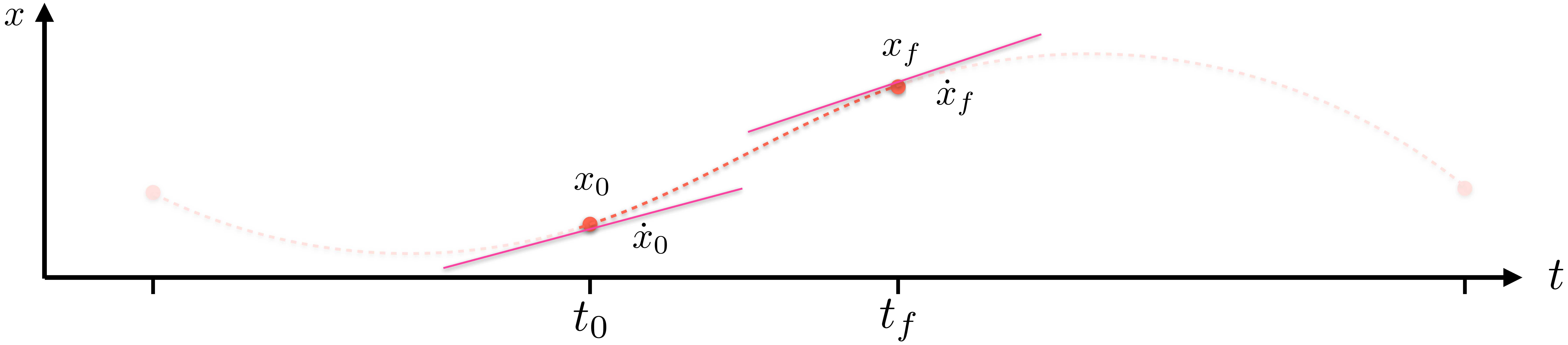


$$\ddot{x}(t) = 2a_2 + 6a_3 t$$

Solution:

Cubic Splines

(also called „csplines“)

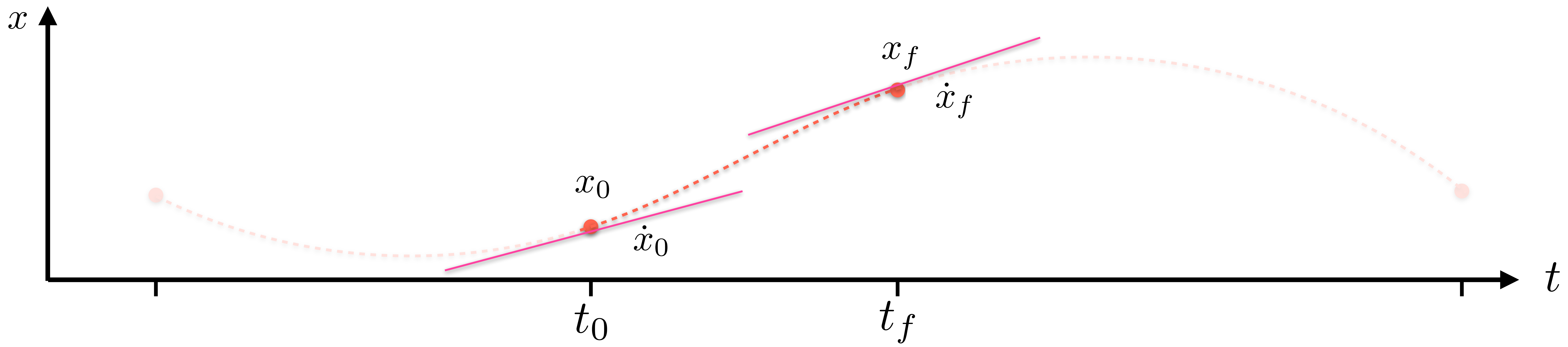


Third oder polynomial: $x(t) = a_0 + a_1 t + a_2 t^2 + a3t^3$

Solution:

Cubic Splines

(also called „csplines“)



Third oder polynomial: $x(t) = a_0 + a_1 t + a_2 t^2 + a3t^3$

Four unknown, four boundary conditions:

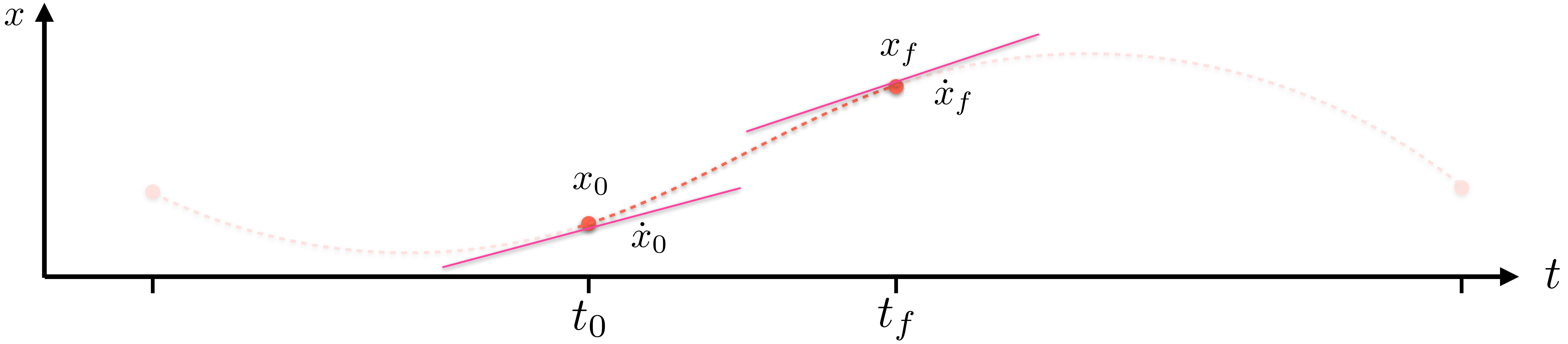
$$x(t = t_0) = x_0 \qquad \dot{x}(t = t_0) = \dot{x}_0$$

$$x(t = t_f) = x_f \qquad \dot{x}(t = t_f) = \dot{x}_f$$

Solution:

Cubic Splines

(also called „csplines“)



Third oder polynomial: $x(t) = a_0 + a_1 t + a_2 t^2 + a3t^3$

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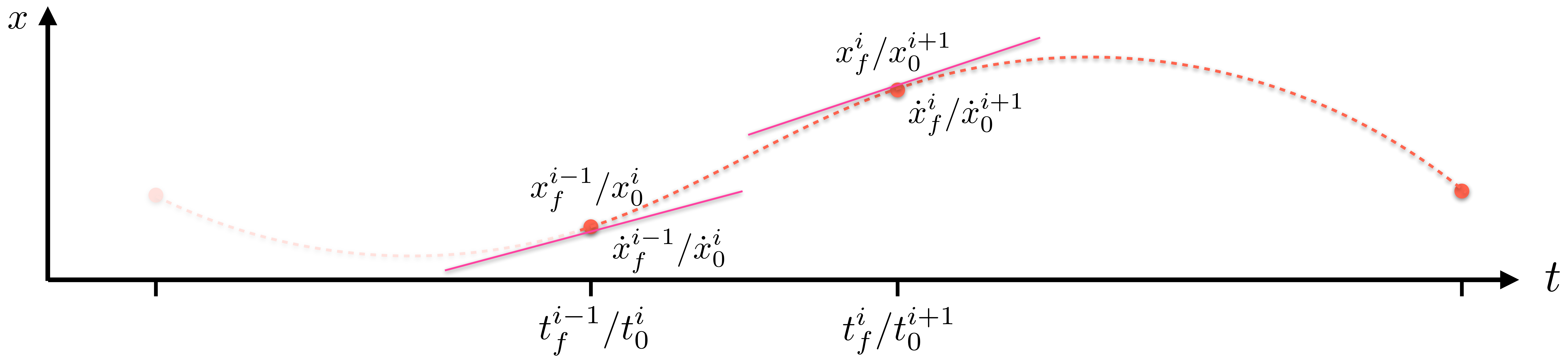
How to choose?

Solution:

Cubic Splines

(also called „csplines“)



Third oder polynomial: $x(t) = a_0 + a_1 t + a_2 t^2 + a3t^3$

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$$x(t = t_f) = x_f \qquad \dot{x}(t = t_f) = \dot{x}_f$$

$$\dot{x}_0^i = \begin{cases} 0 & \text{if i = 1} \\ \dot{x}_0^{i-1} & \text{else} \end{cases} \qquad \dot{x}_f^i = \begin{cases} 0 & \text{if i = n} \\ \dot{x}_0^{i+1} & \text{else} \end{cases}$$

How to choose velocities?

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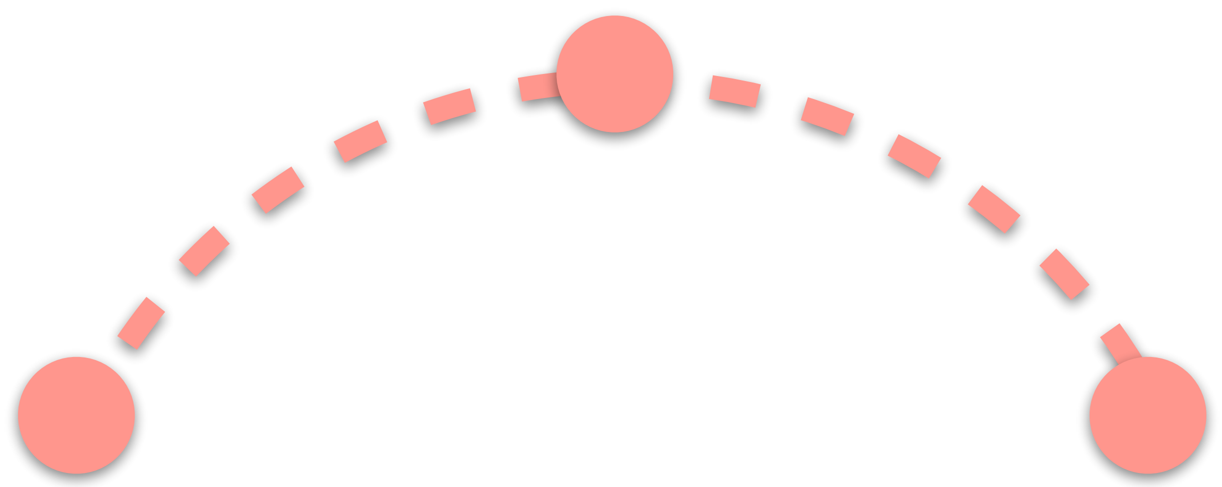
Case A



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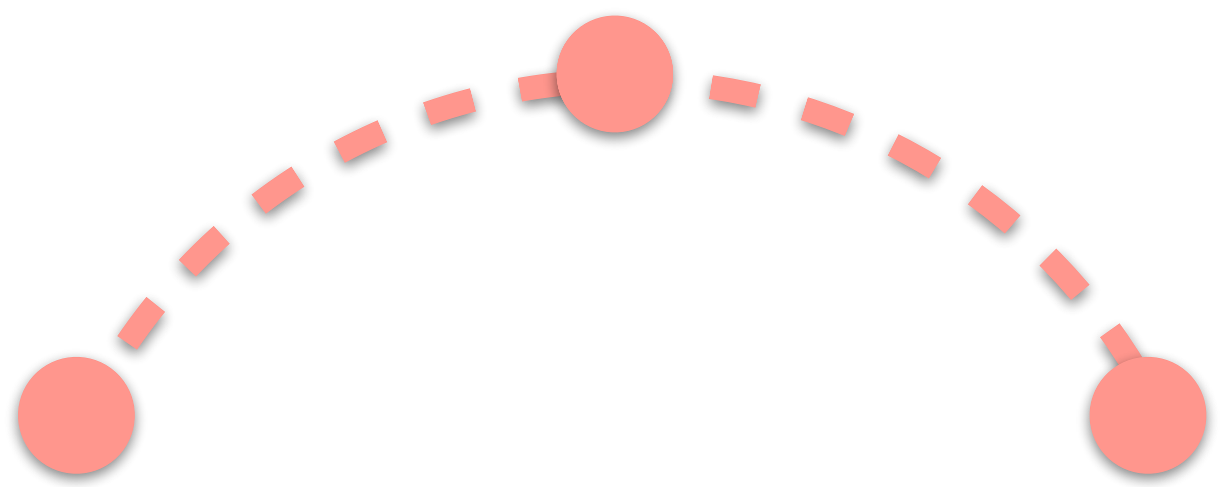
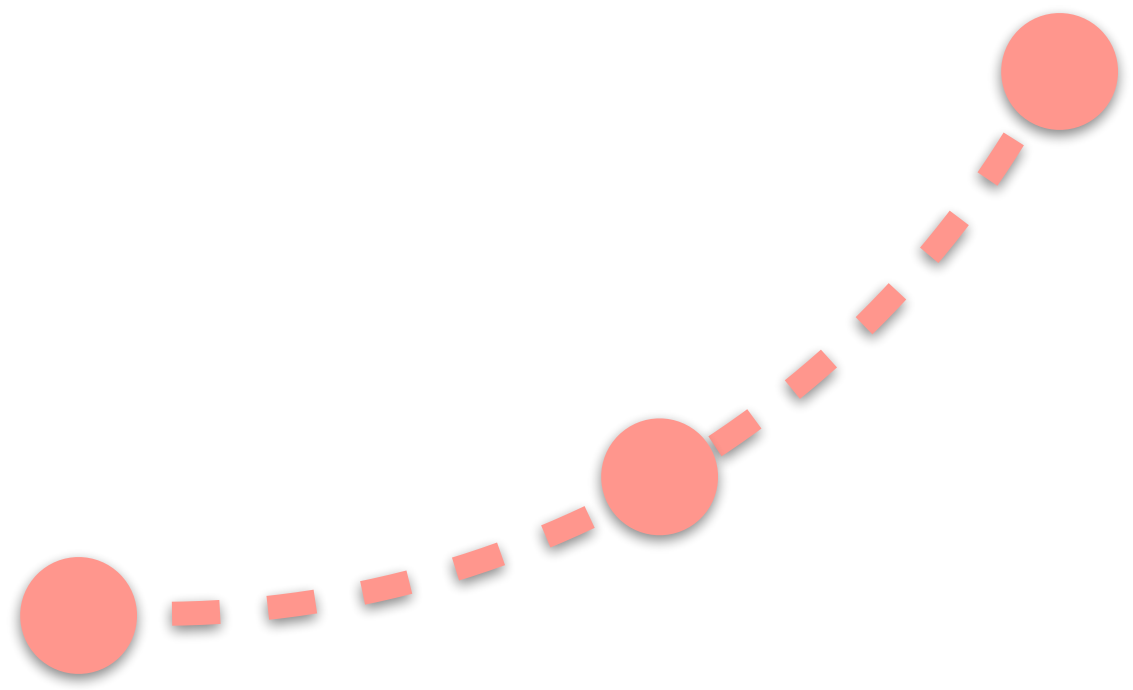


Case B



How to choose velocities?

$$\dot{x}_0^i = \begin{cases} 0 & \text{if i = 1} \\ \dot{x}_f^{i-1} & \text{else} \end{cases} \qquad \dot{x}_f^i = \begin{cases} 0 & \text{if i = n} \\ \dot{x}_0^{i+1} & \text{else} \end{cases}$$

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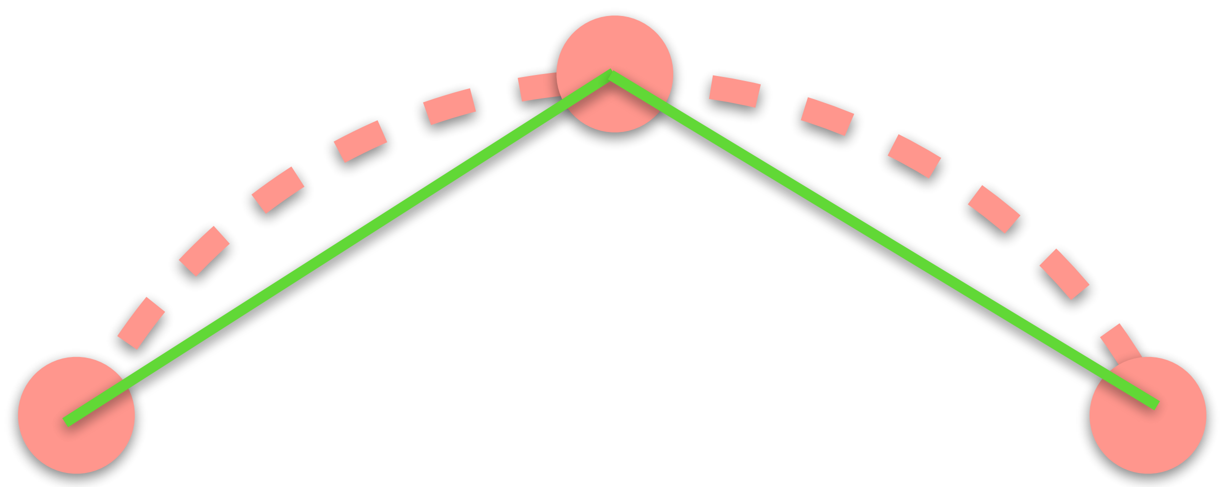
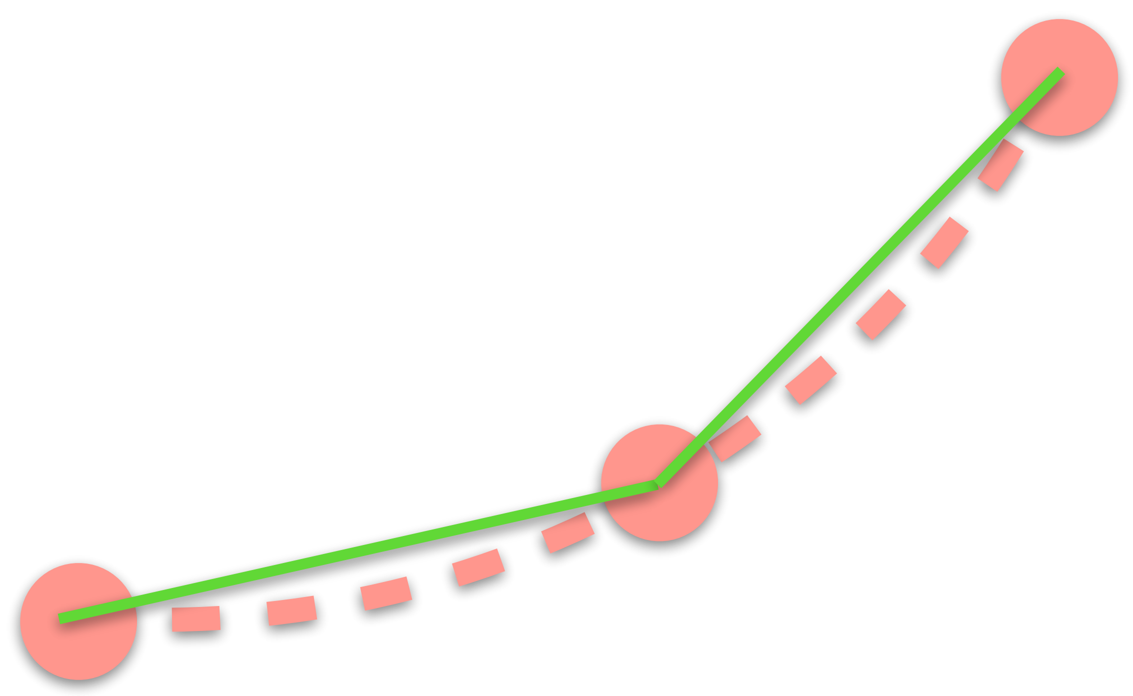


Linearized System

Case B



Linearized System

How to choose velocities?

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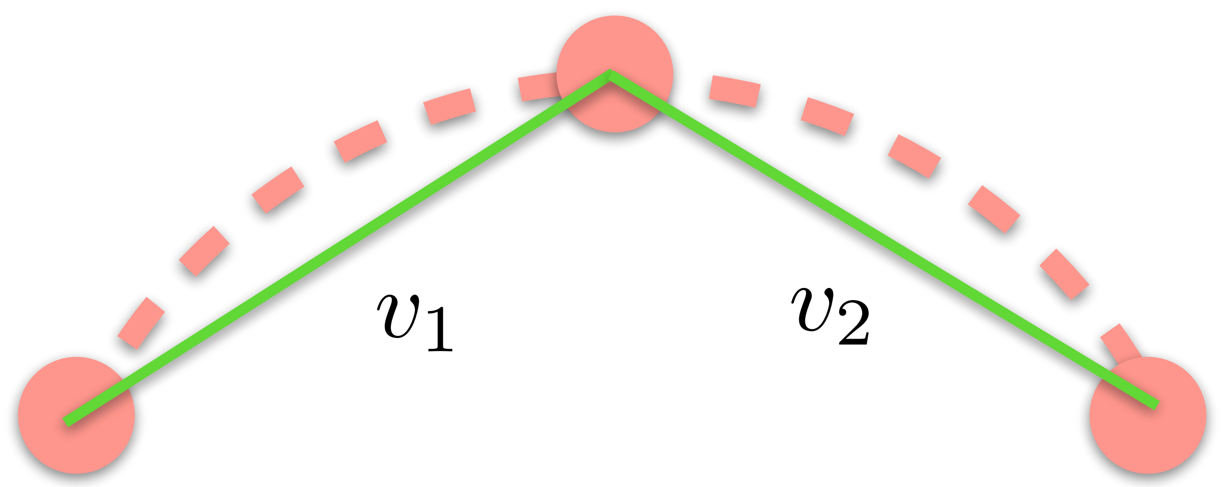
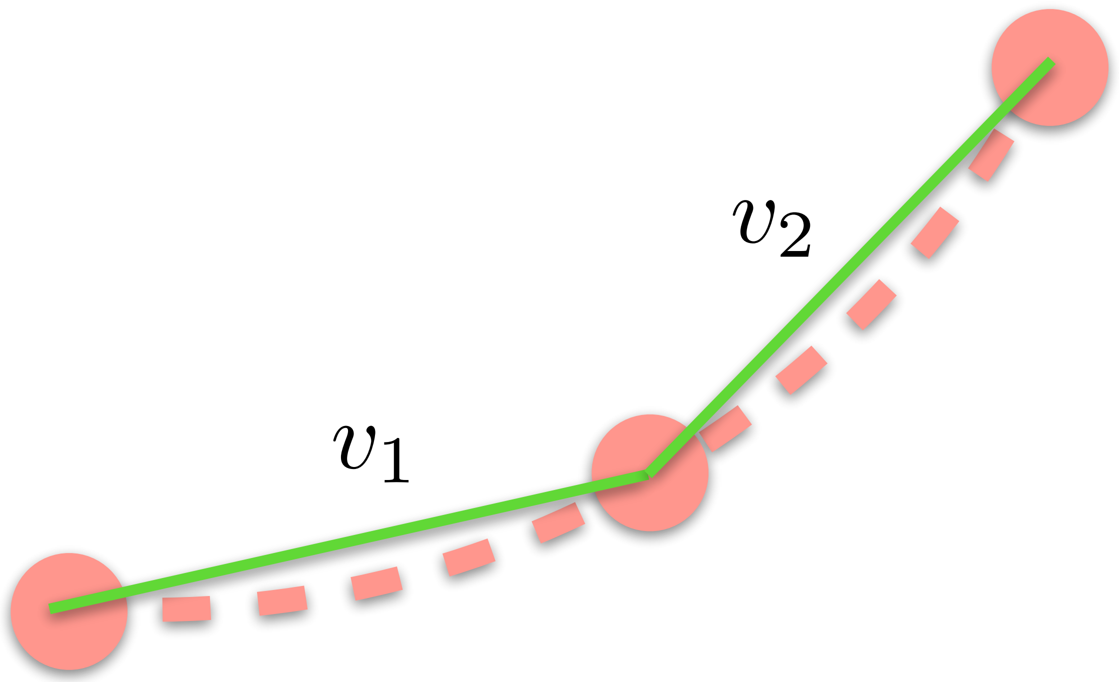
Case A



Linearized System

Case B



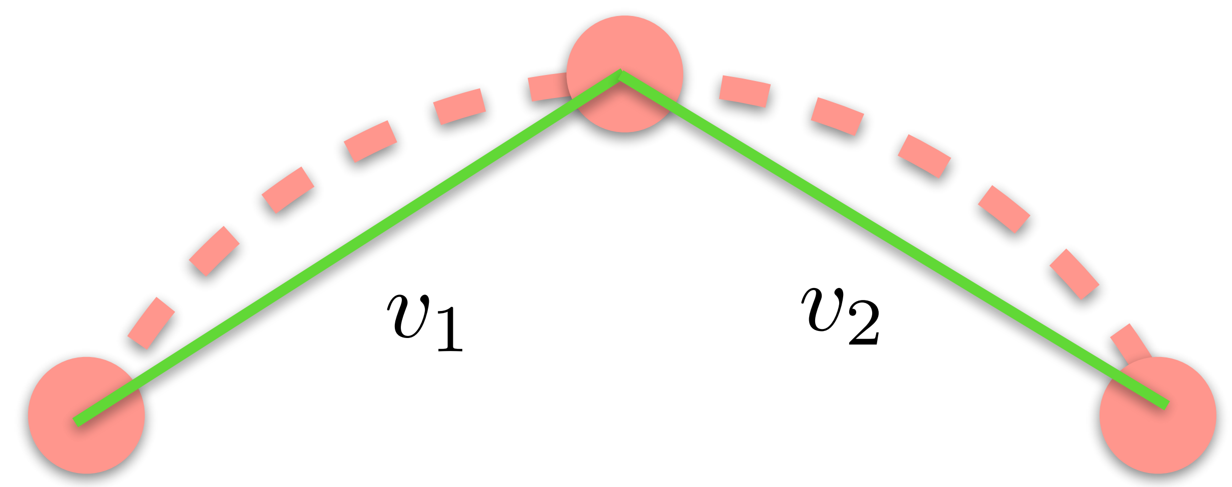
Linearized System

How to choose velocities?

$$\dot{x}_0^i = \begin{cases} 0 & \text{if i} = 1 \\ \dot{x}_f^{i-1} & \text{else} \end{cases} \qquad \dot{x}_f^i = \begin{cases} 0 & \text{if i} = \text{n} \\ \dot{x}_0^{i+1} & \text{else} \end{cases}$$

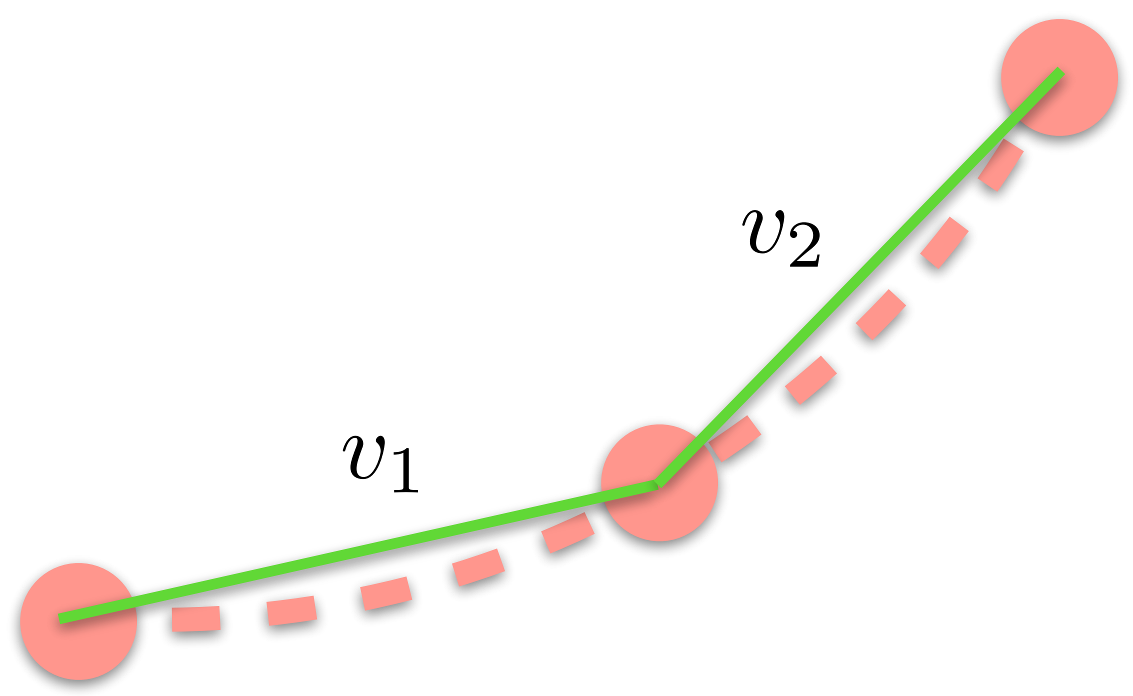
$$x_f^i / x_0^{i+1} = \begin{cases} 0 & \text{if } \operatorname{sgn}(v_i) \neq \operatorname{sgn}(v_{i+1}) \\ (v_i + v_{i+1})/2 & \text{else} \end{cases}$$

Case A

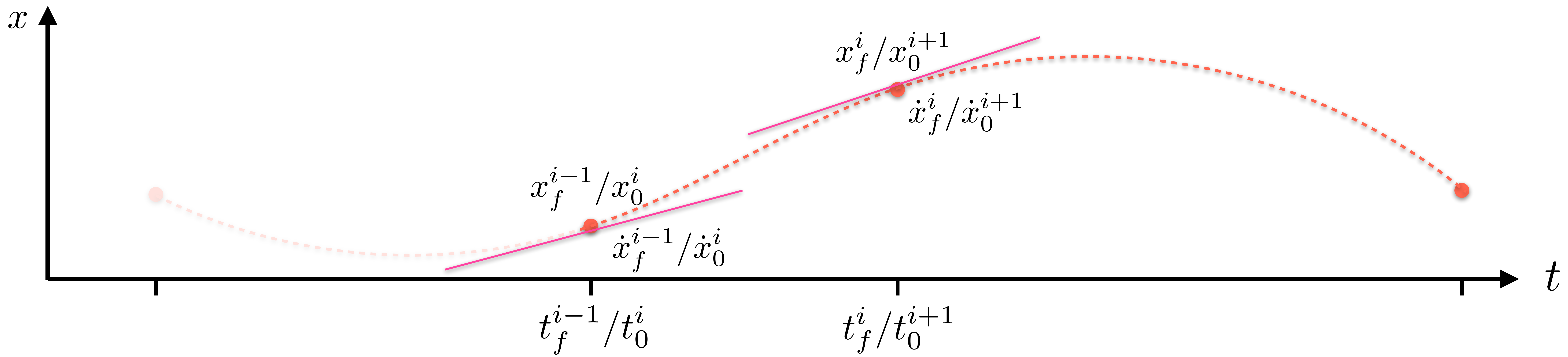


Linearized System

Case B



Linearized System



Third order polynomial:

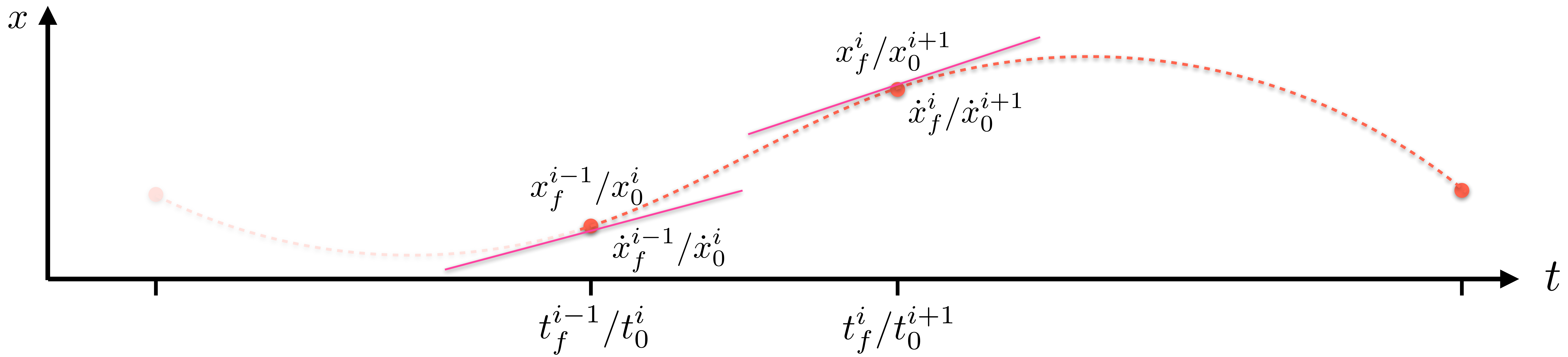
$$x(t) = a_0 + a_1 t + a_2 t^2 + a3t^3 \qquad \dot{x}(t) = a_1 + 2a_2 t + 3a_3 t^2 \qquad \ddot{x}(t) = 2a_2 + 6a_3 t$$

Positon constraints:

$$x(t = t_0) = x_0 \qquad x(t = t_f) = x_f$$

Velocity constraints:

$$x_f^i/x_0^{i+1} = \begin{cases} 0 & \text{if } \mathrm{sgn}(v_i) \neq \mathrm{sgn}(v_{i+1}) \\ (v_i + v_{i+1})/2 & \text{else} \end{cases} \qquad \dot{x}_0^i = \begin{cases} 0 & \text{if i = 1} \\ \dot{x}_0^{i-1} & \text{else} \end{cases} \qquad \dot{x}_f^i = \begin{cases} 0 & \text{if i = n} \\ \dot{x}_0^{i+1} & \text{else} \end{cases}$$



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Positon constraints:

$$x(t = t_0) = x_0 \qquad x(t = t_f) = x_f$$

How to compute coefficients???

Velocity constraints:

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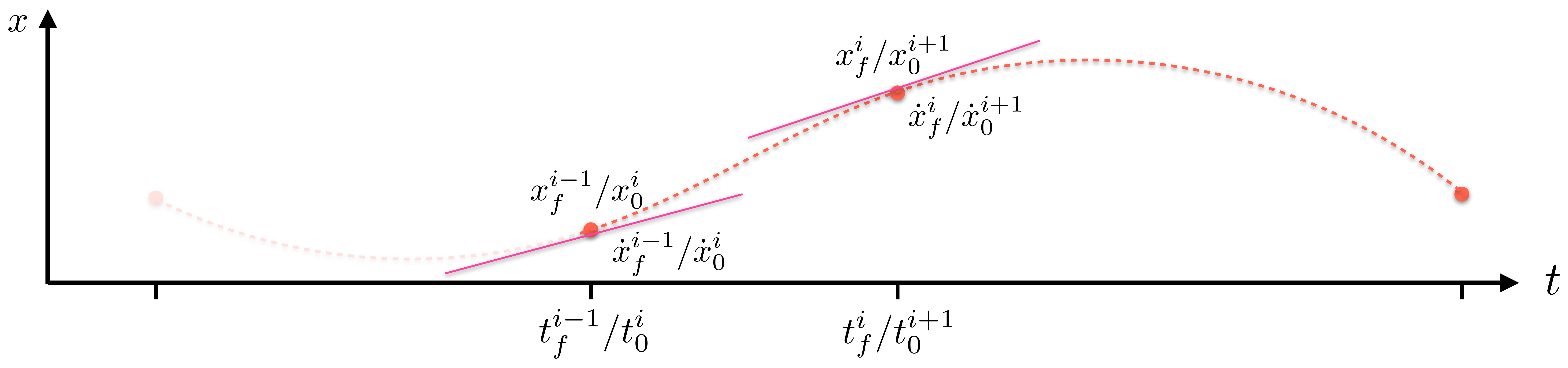


Third order polynomial:

$$x(t) = a_0 + a_1 t + a_2 t^2 + a3t^3$$

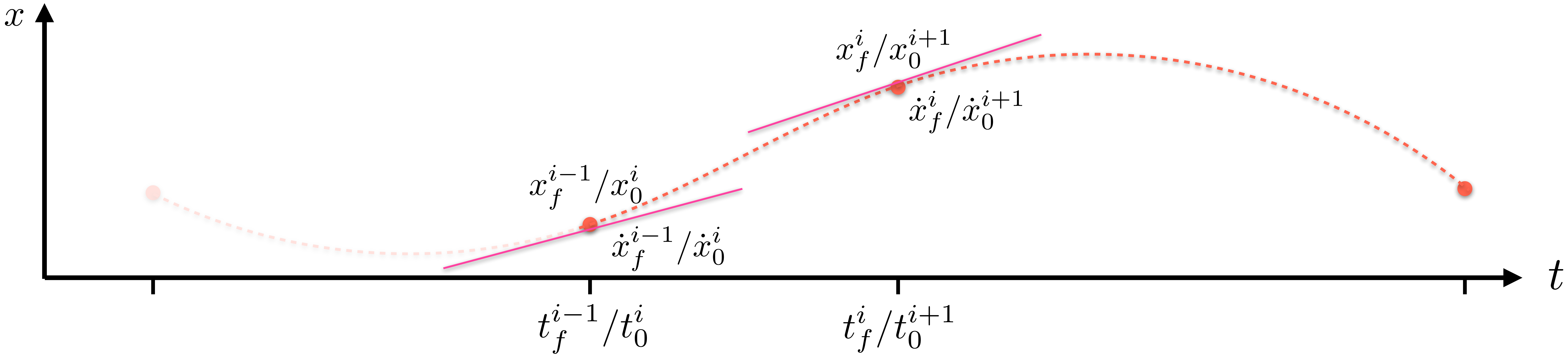
Coefficients:

$$a_0 = x_0$$

$$a_1 = \dot{x}_0$$

$$a_2 = \frac{3(x_f - x_0)}{t_f^2} - \frac{2\dot{x}_0 + \dot{x}_f}{t_f}$$

$$a_3 = -\frac{2(x_f - x_0)}{t_f^3} + \frac{\dot{x}_f + \dot{x}_0}{t_f^2}$$



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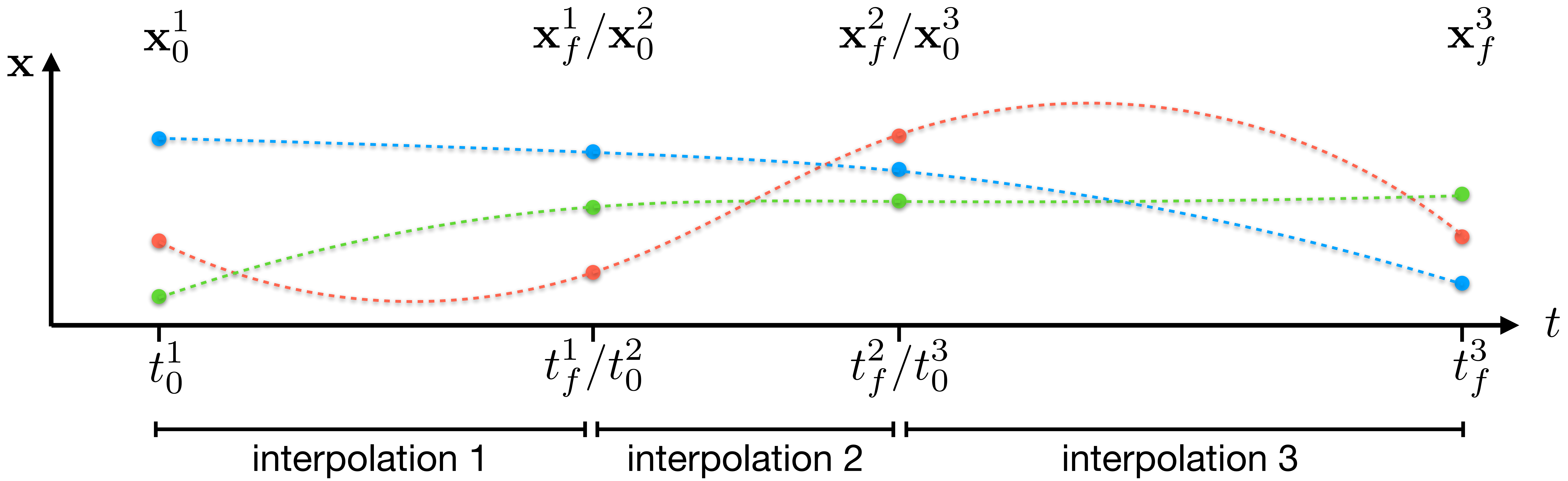
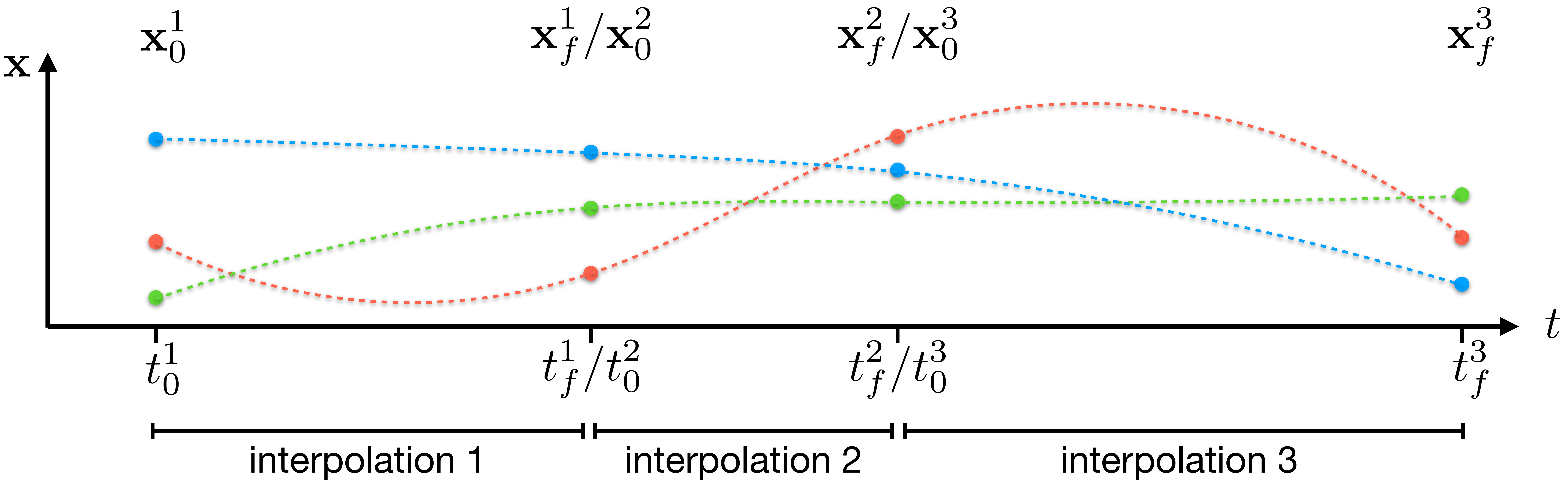
Multivariate Time Series



Multivariate Time Series



Each dimension of each segment
is modeled by its own (separate) third order polynomial!