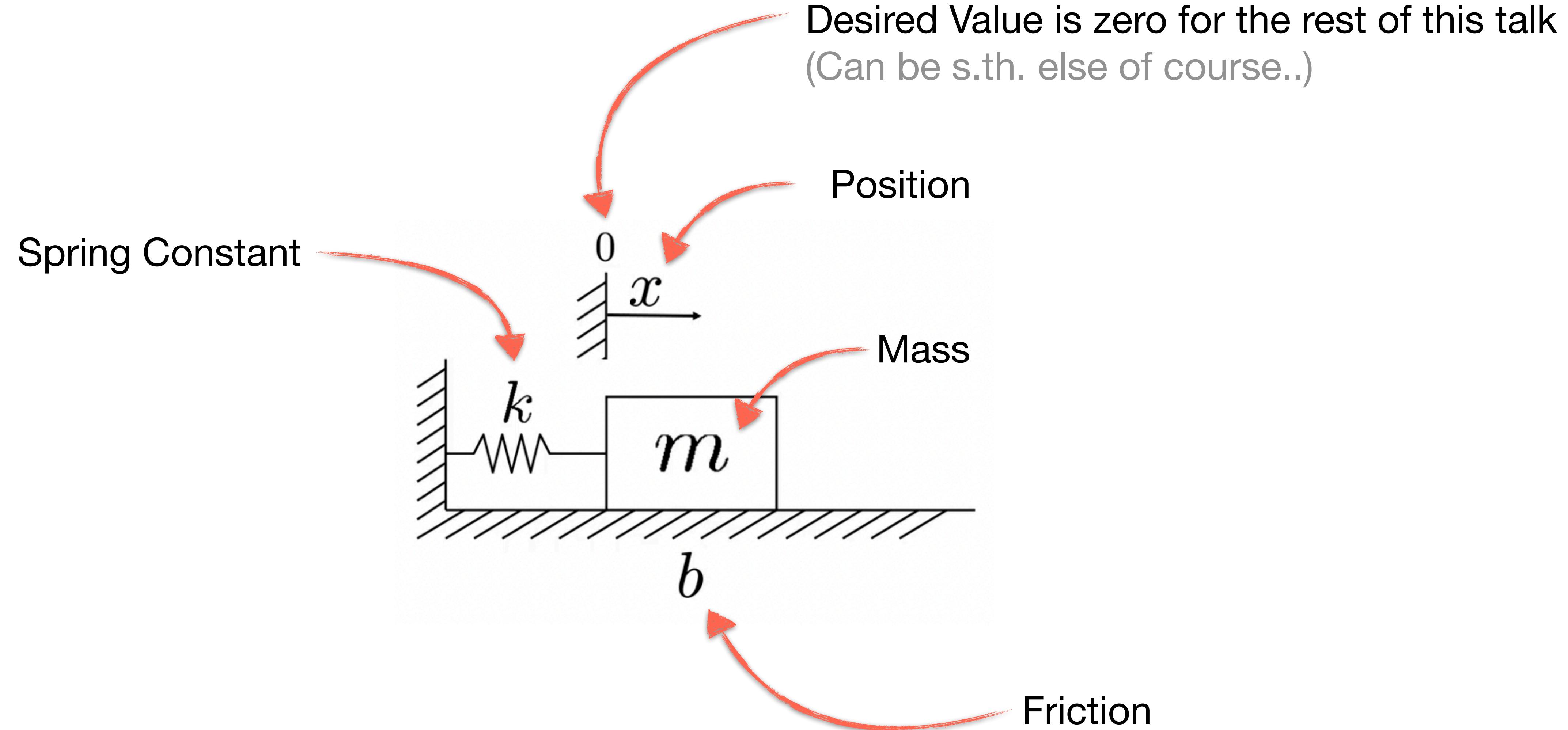


# **Human-Robot Interaction**

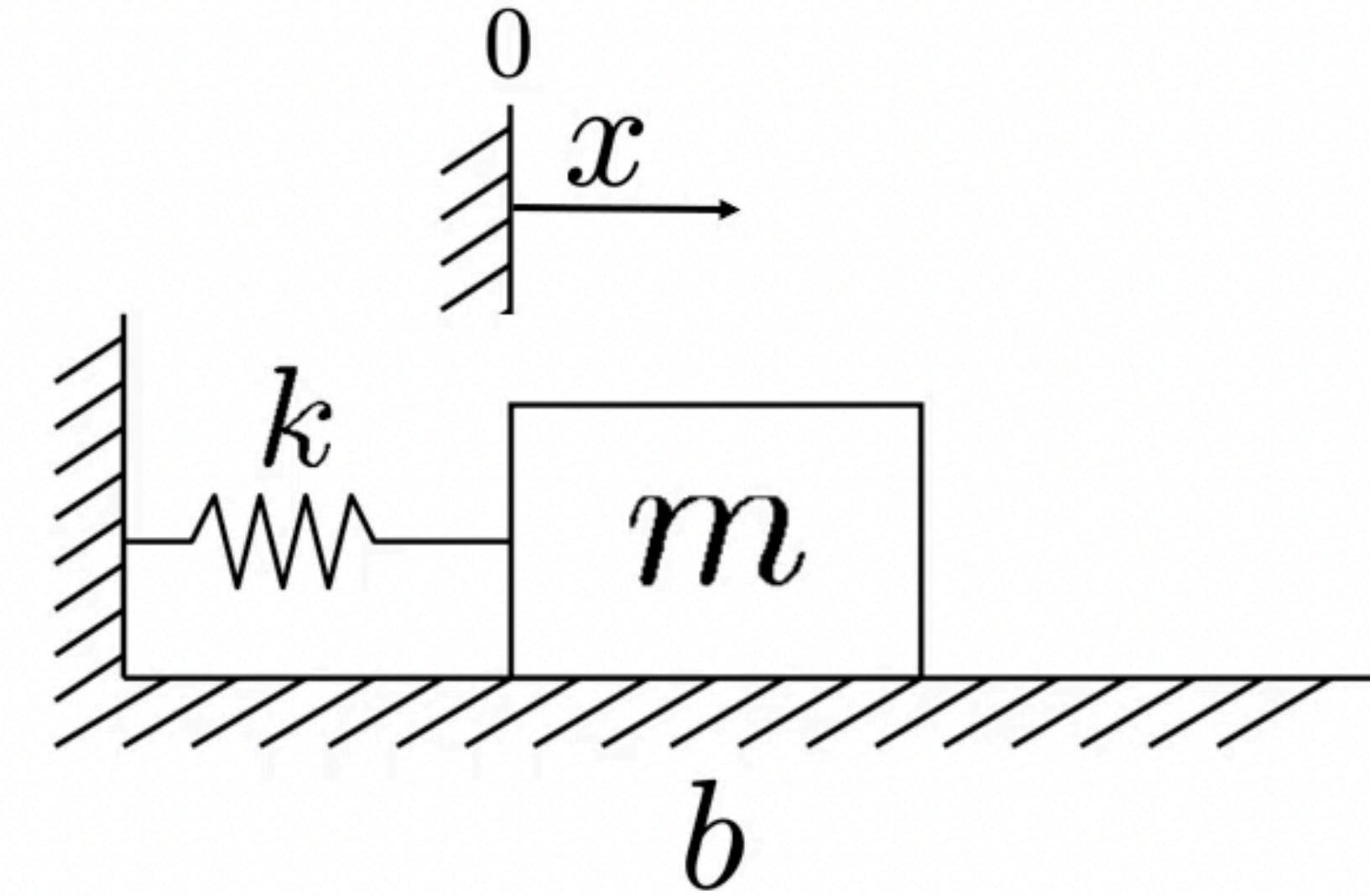
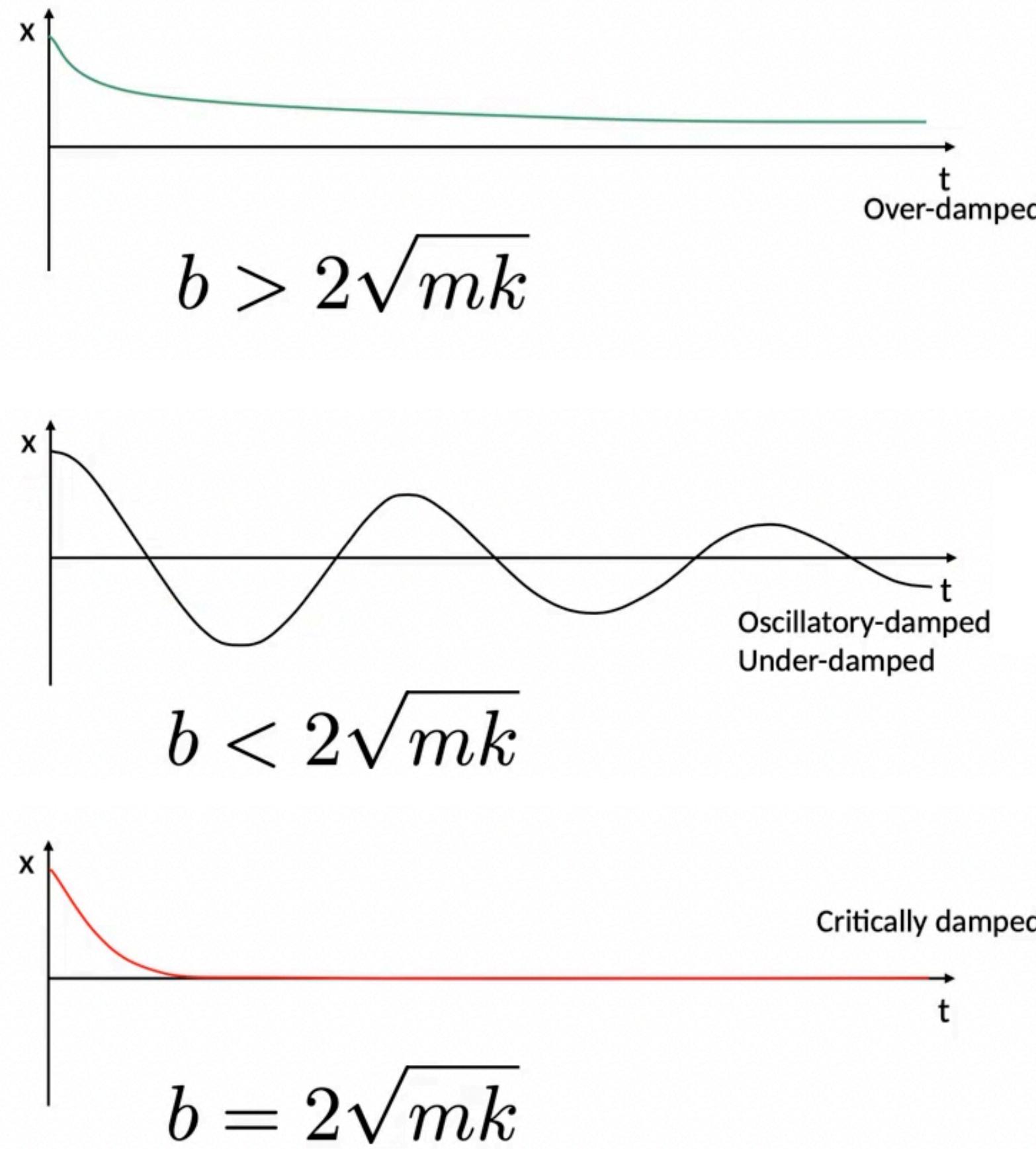
## **Impedance Control**

# **Spring-Mass-Damper System**

# Spring-Mass-Damper System

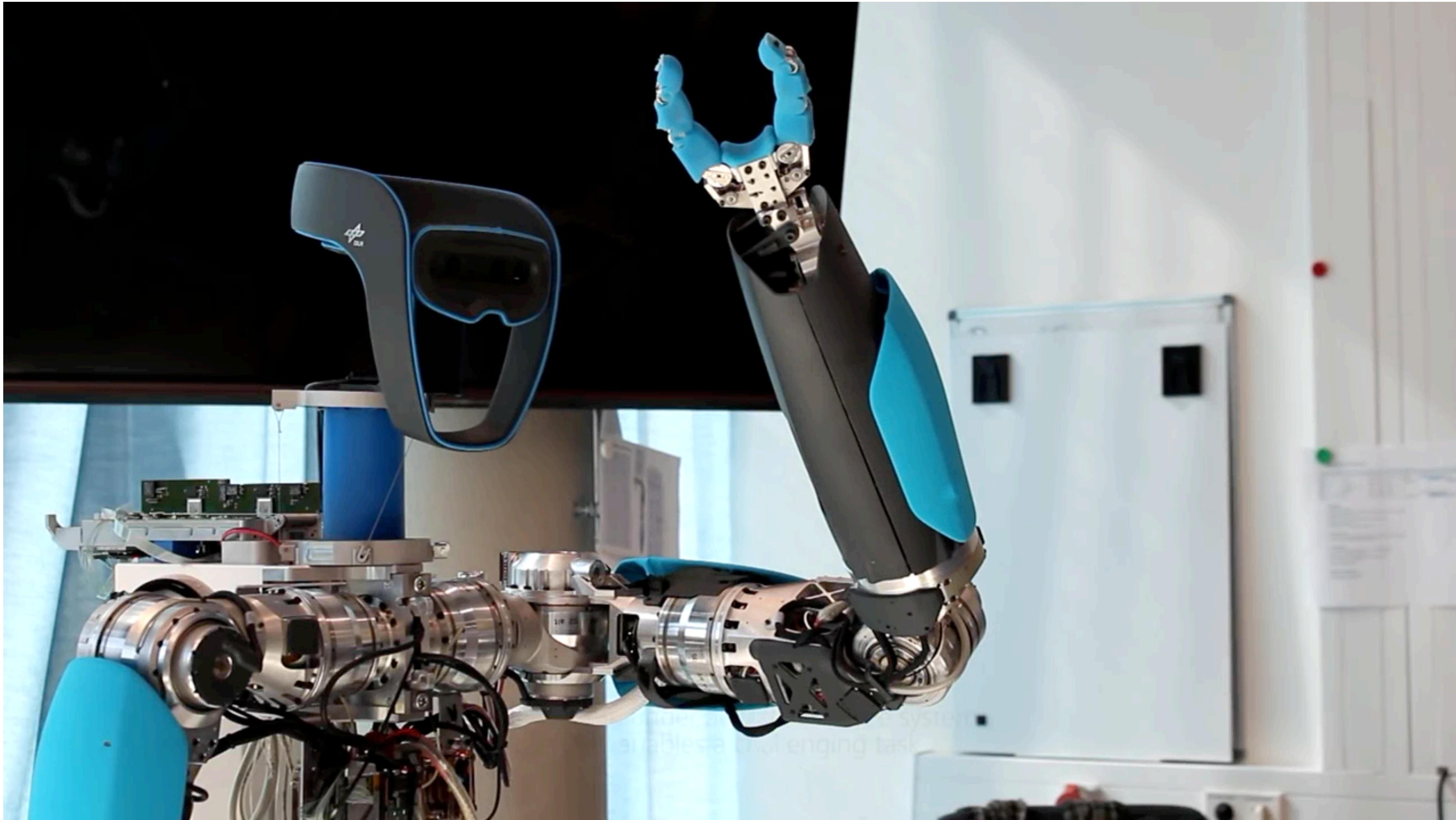


# Spring-Mass-Damper System

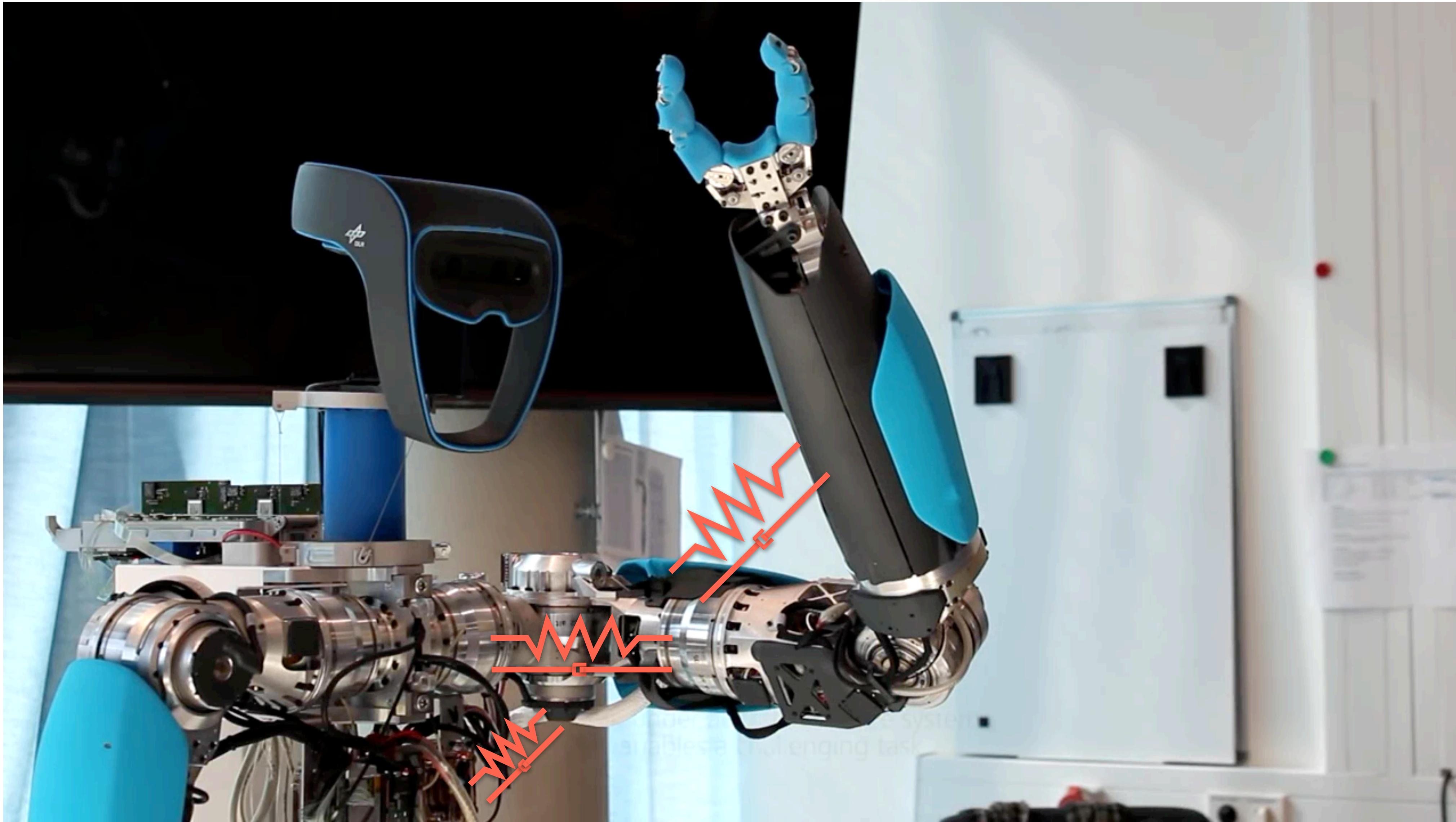


$$m\ddot{x} + b\dot{x} + kx = 0$$

# Critical Damping thanks to Spring-Mass-Damper System

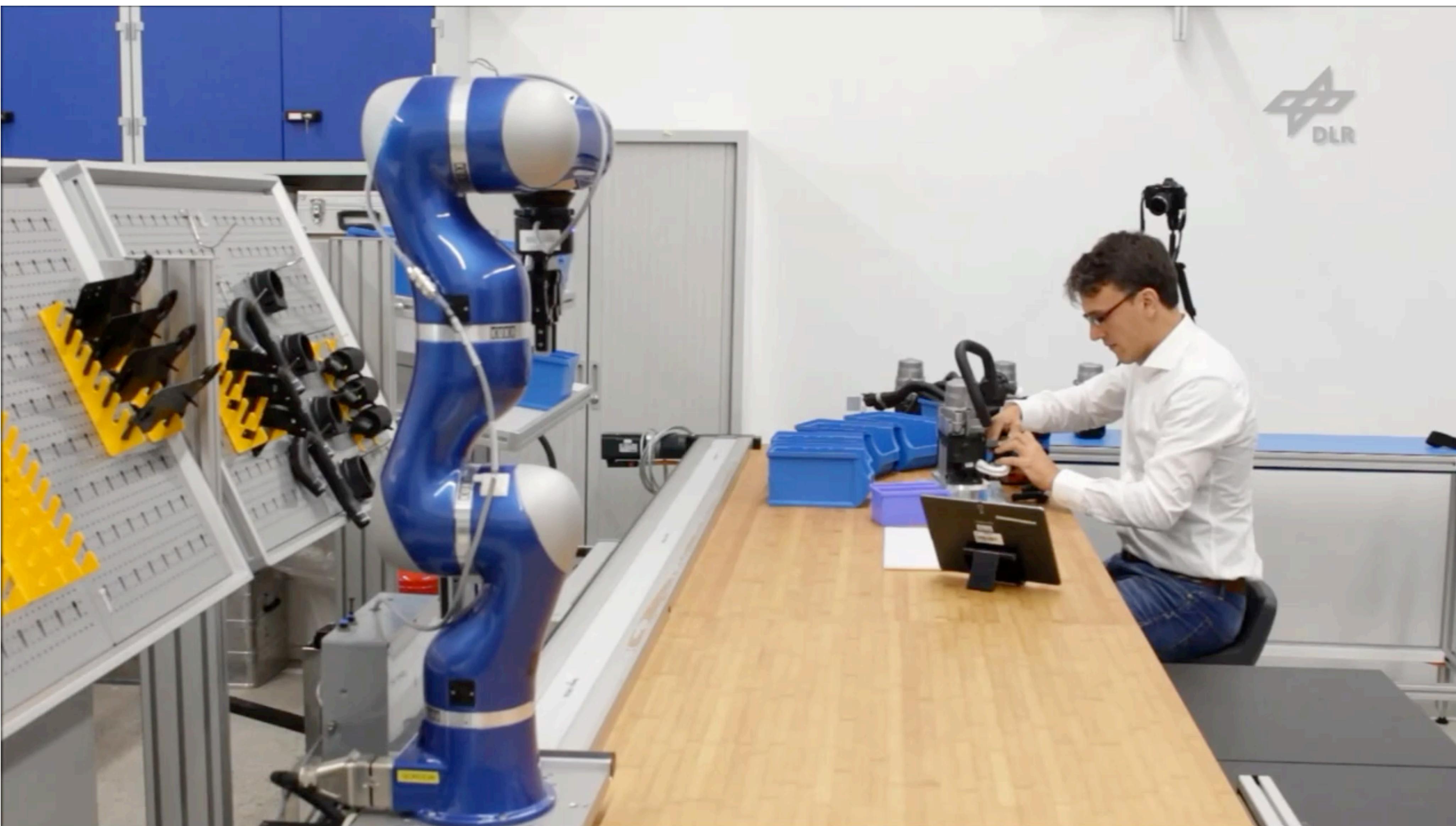


# Critical Damping thanks to Spring-Mass-Damper System

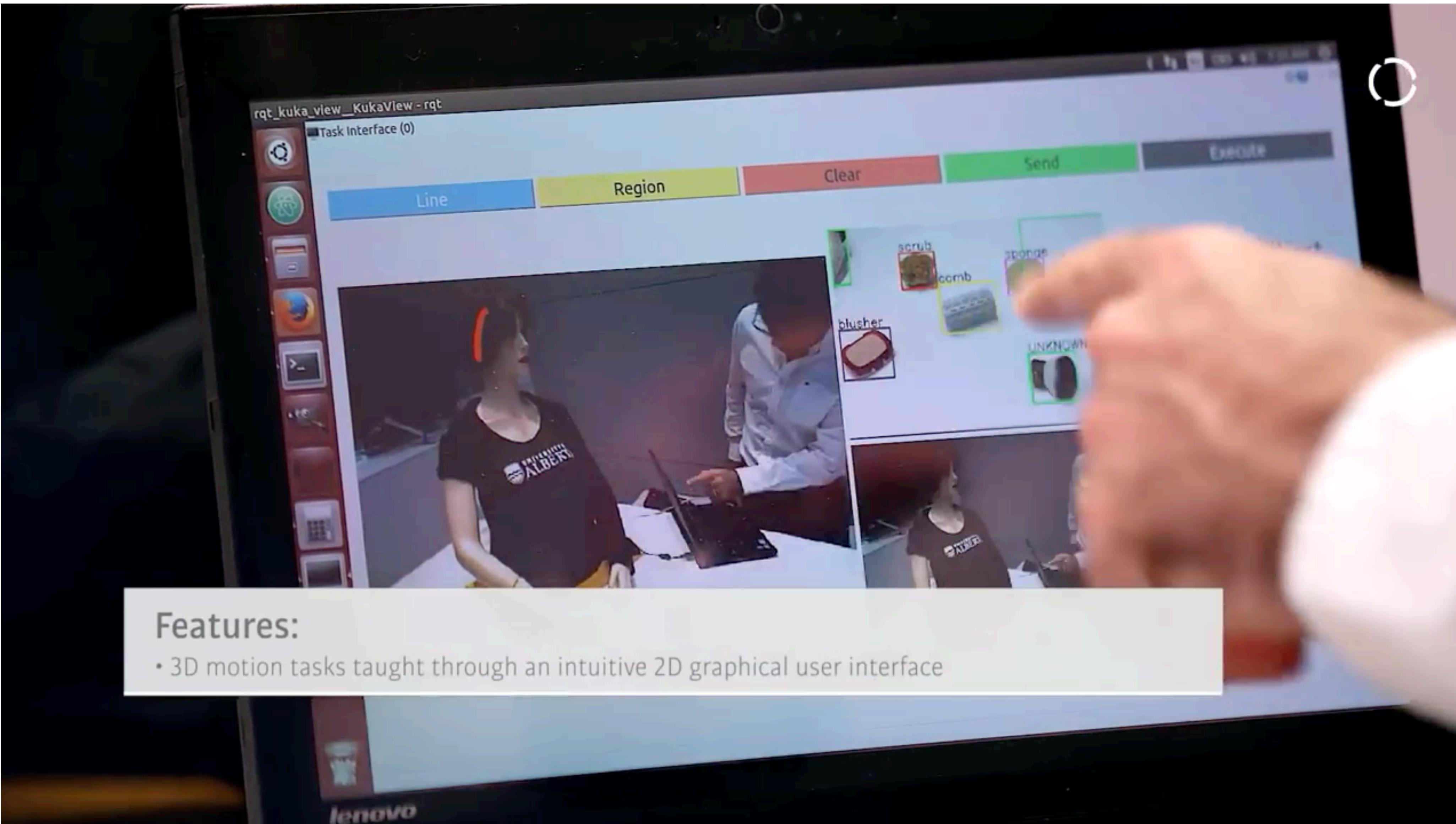


So far, everything happened in **free space**  
**without physical interaction** between  
the robot and the environment.

# Interactive Robotics

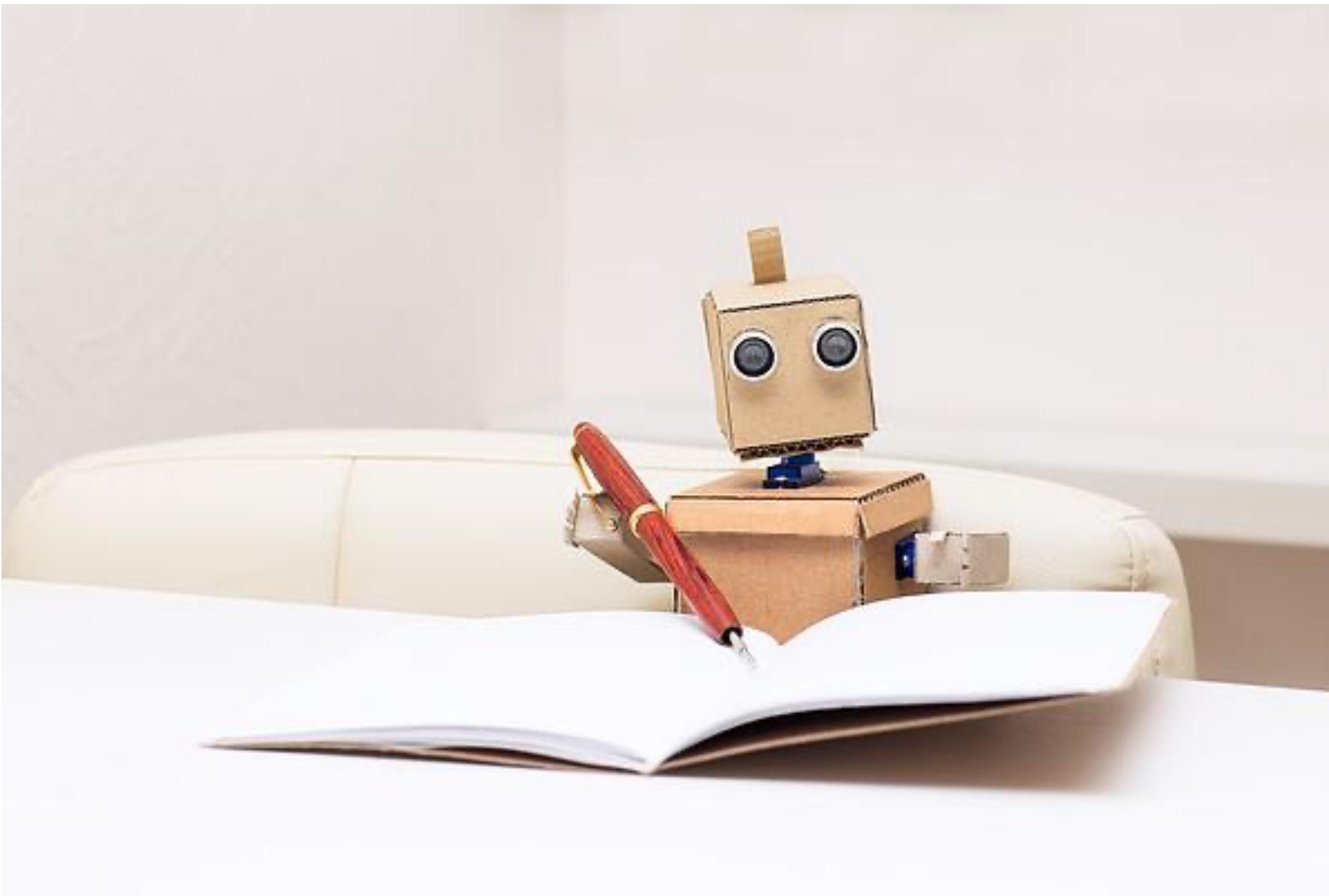


# Interactive Robotics

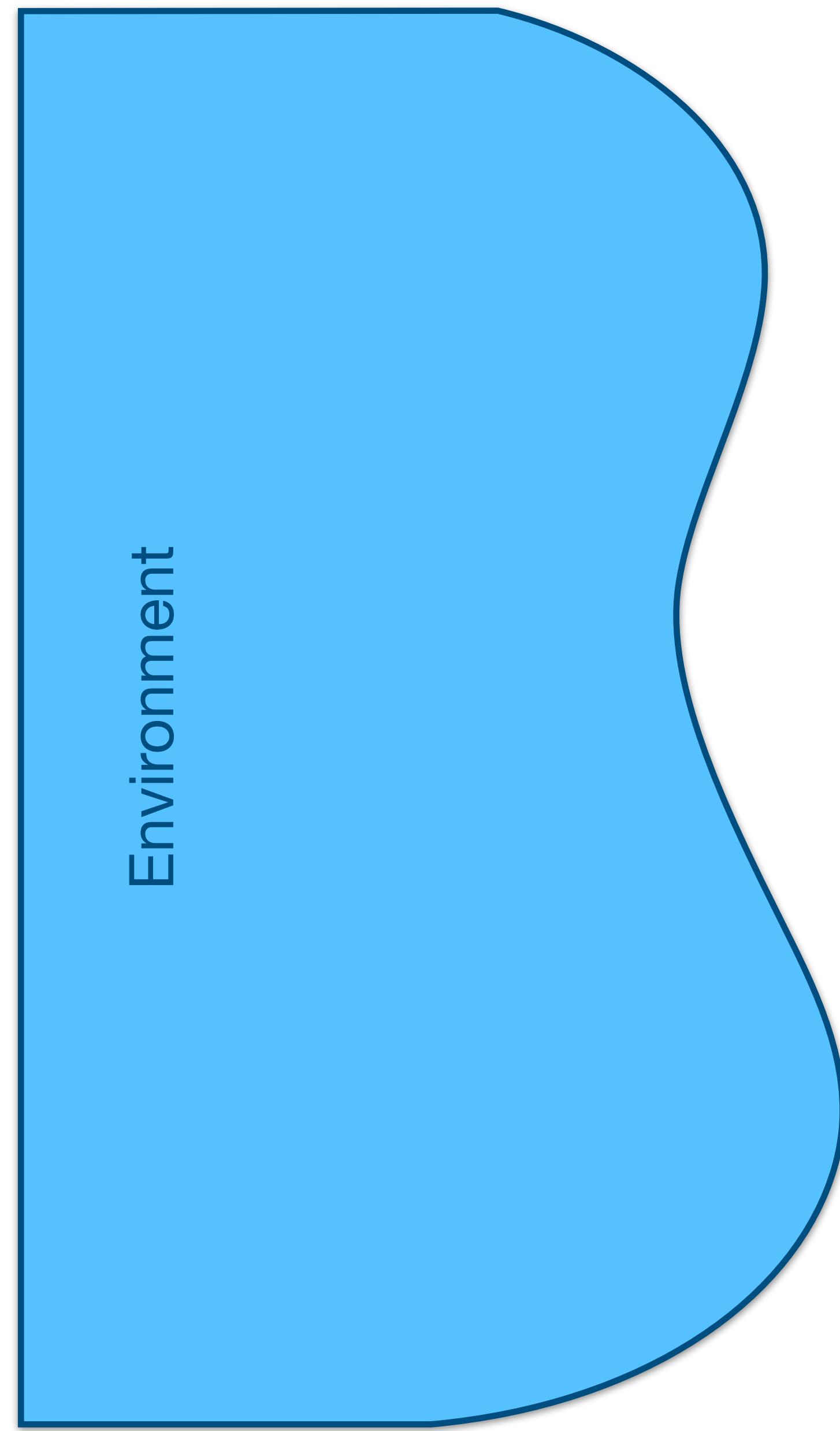


We want the robot to physically interact  
with its environment!

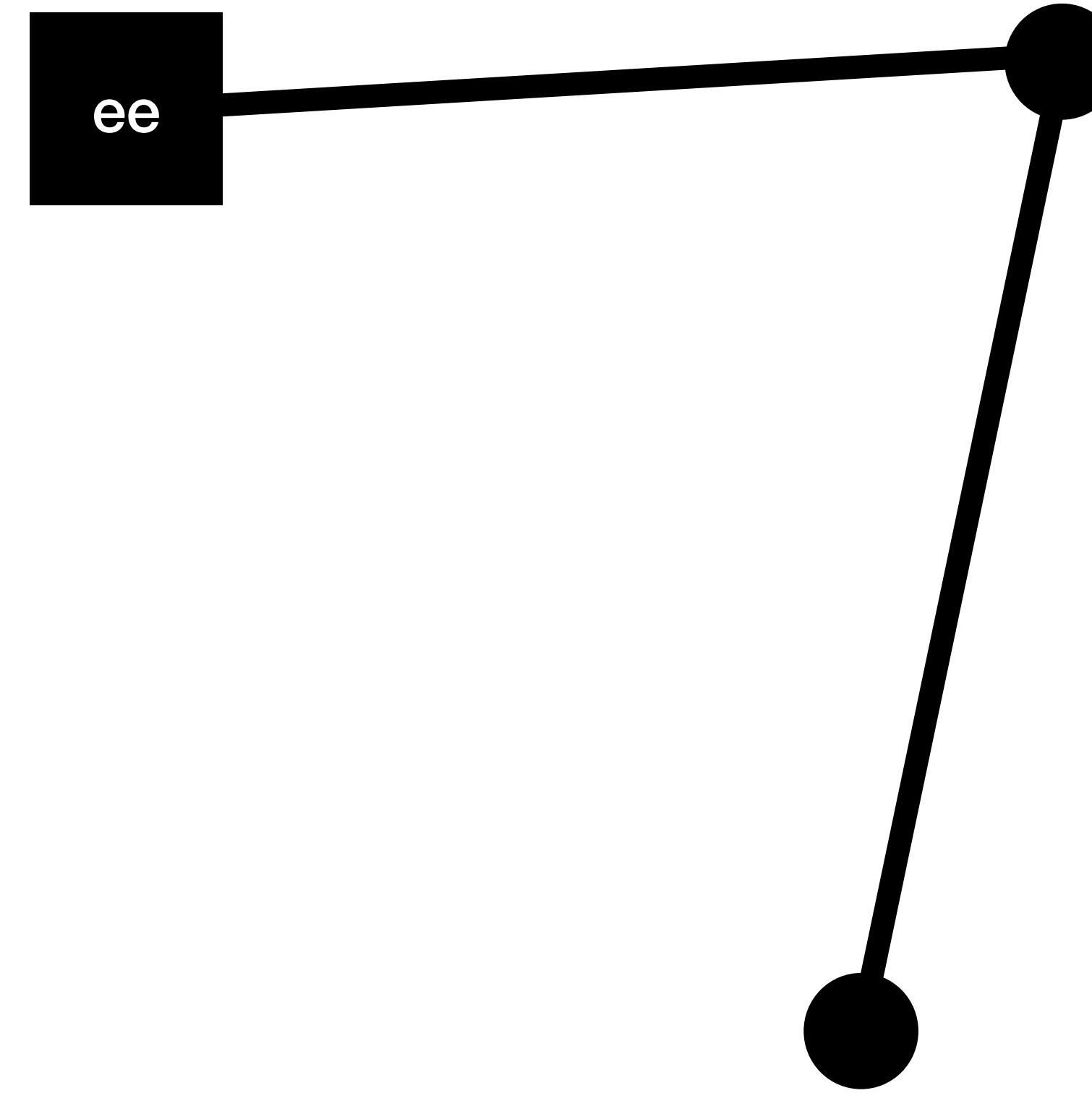
# Example of Physical Interaction between Robot and Environment



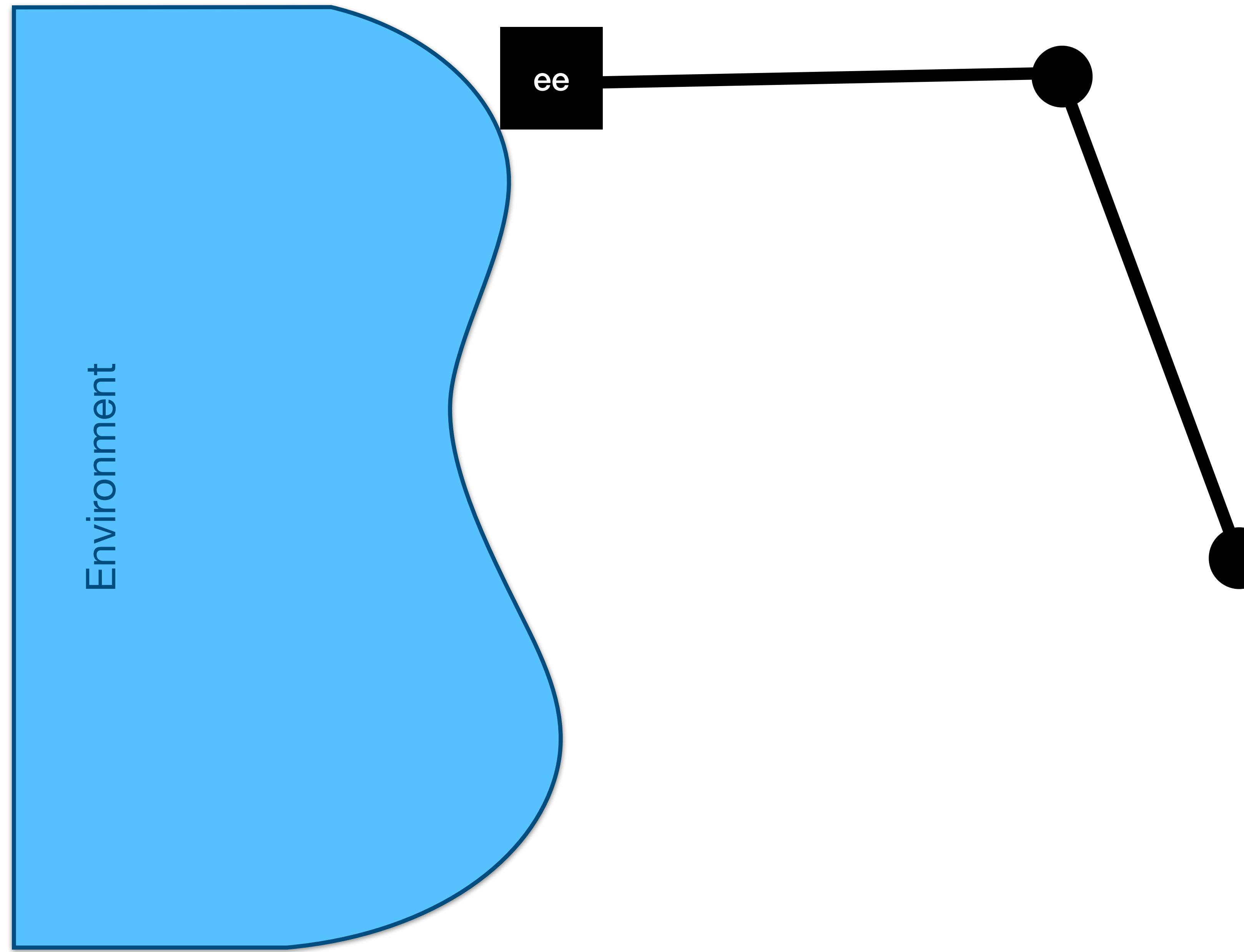
Task: Writing on a curved, compliant surface.



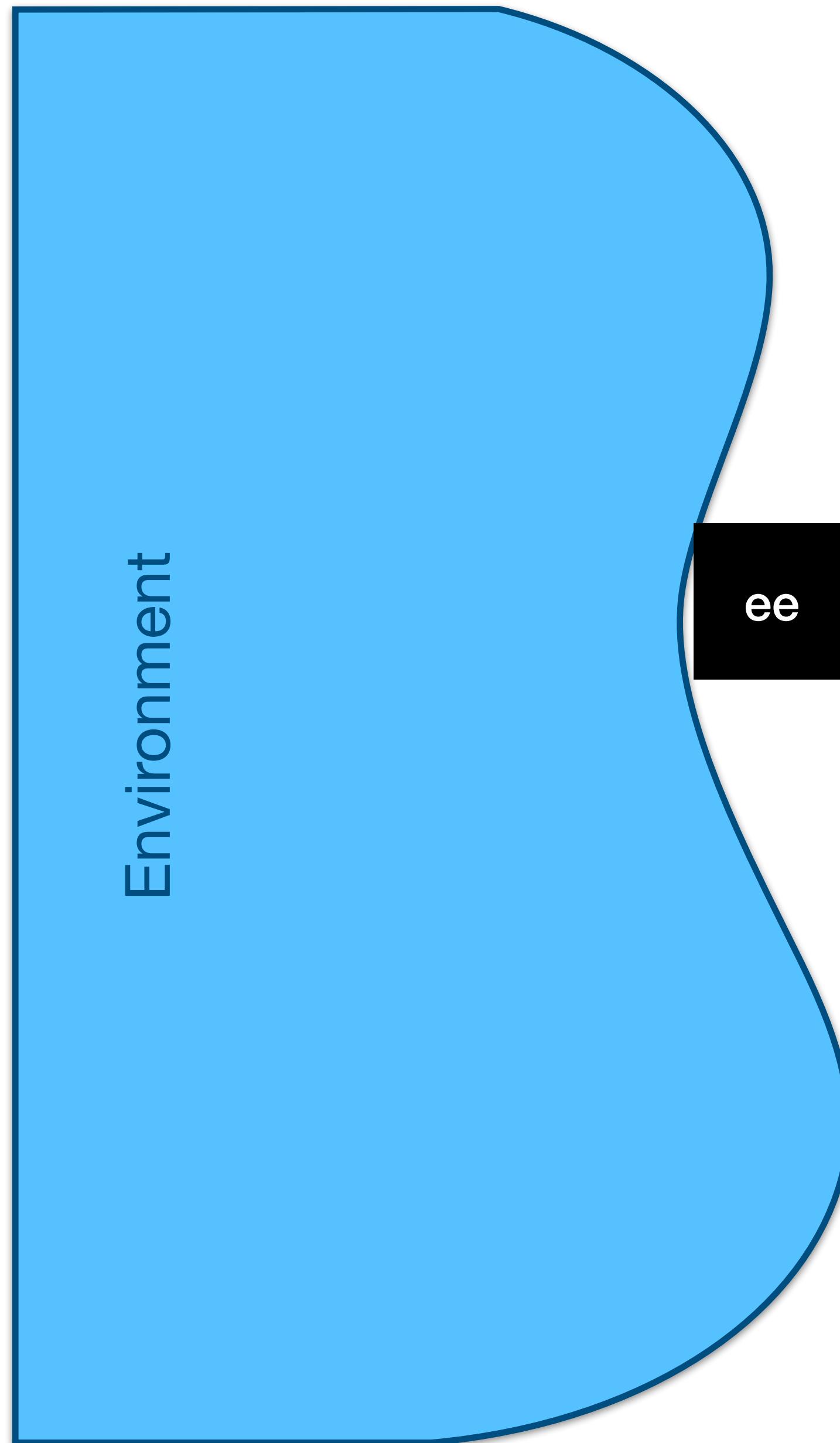
Environment



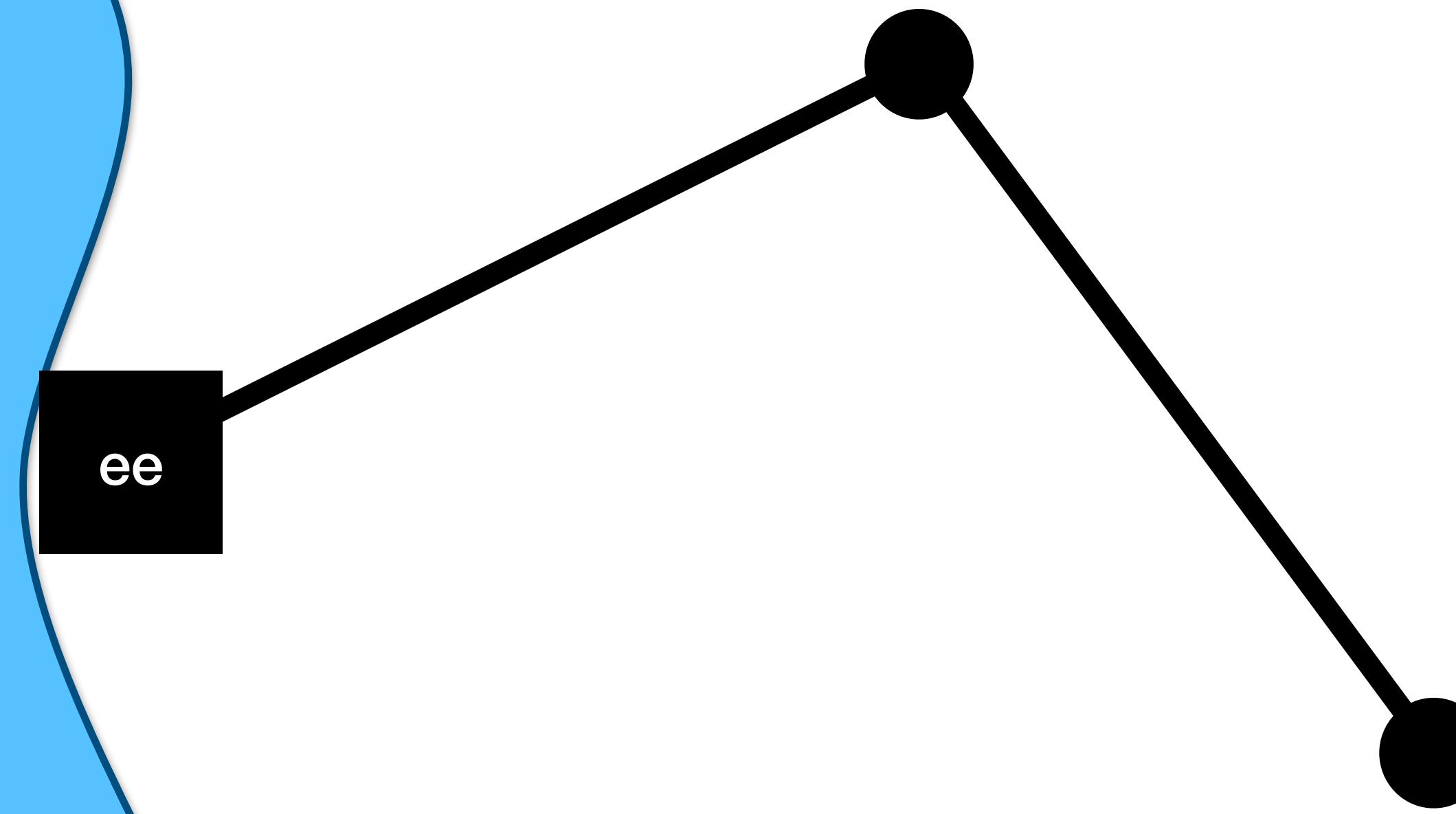
Task: Slide over surface



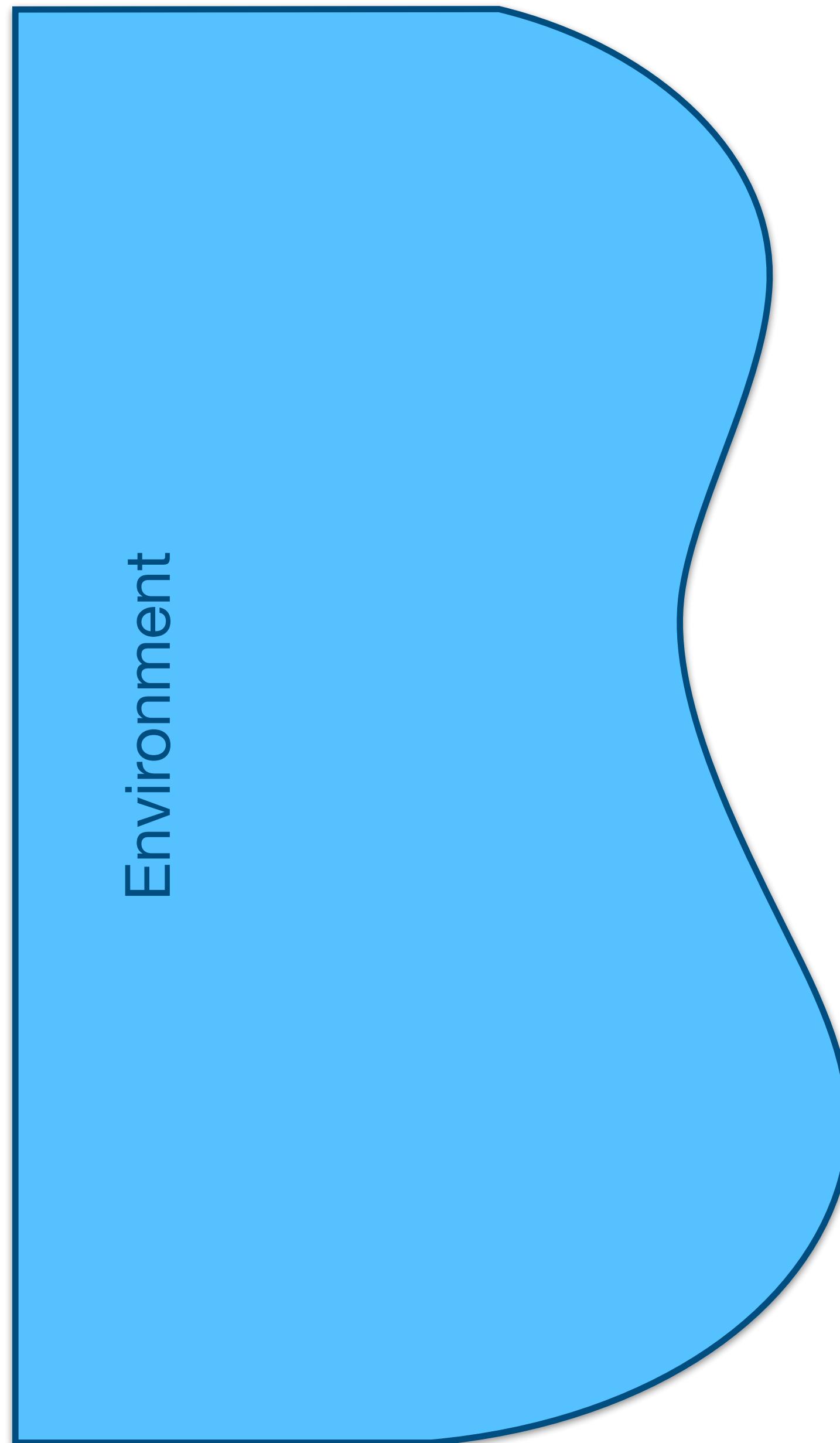
Task: Slide over surface



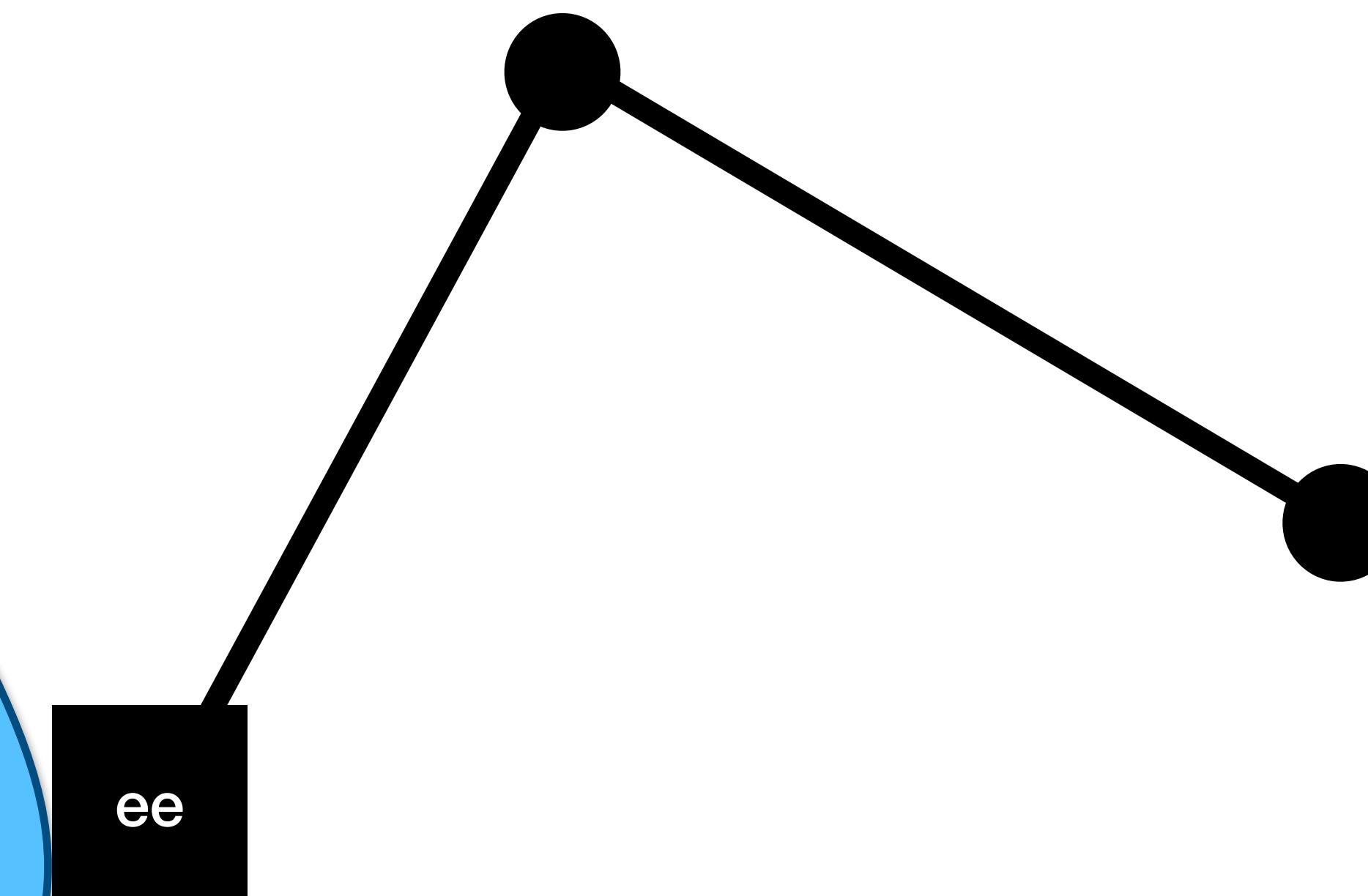
Environment



Task: Slide over surface



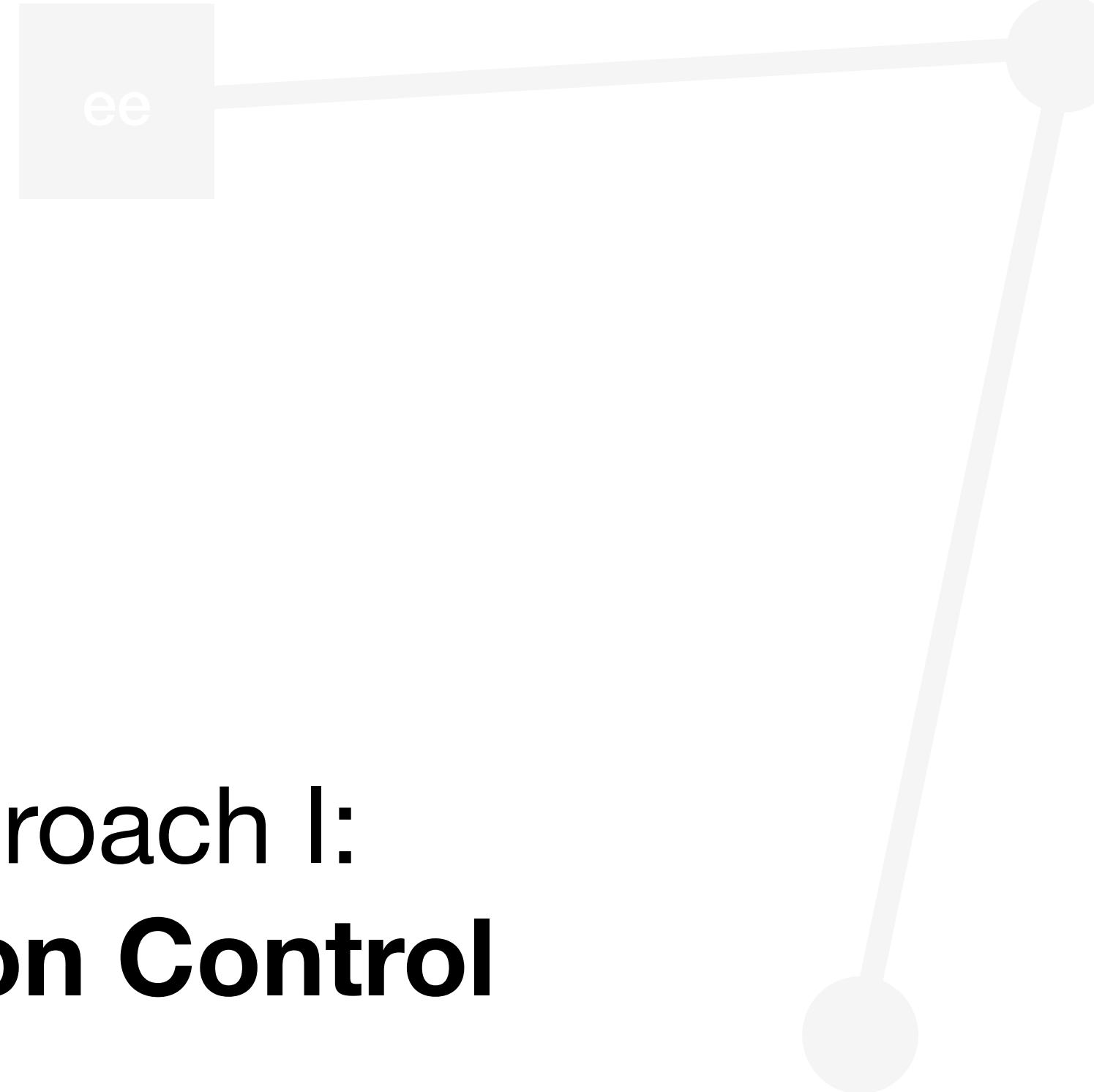
Environment



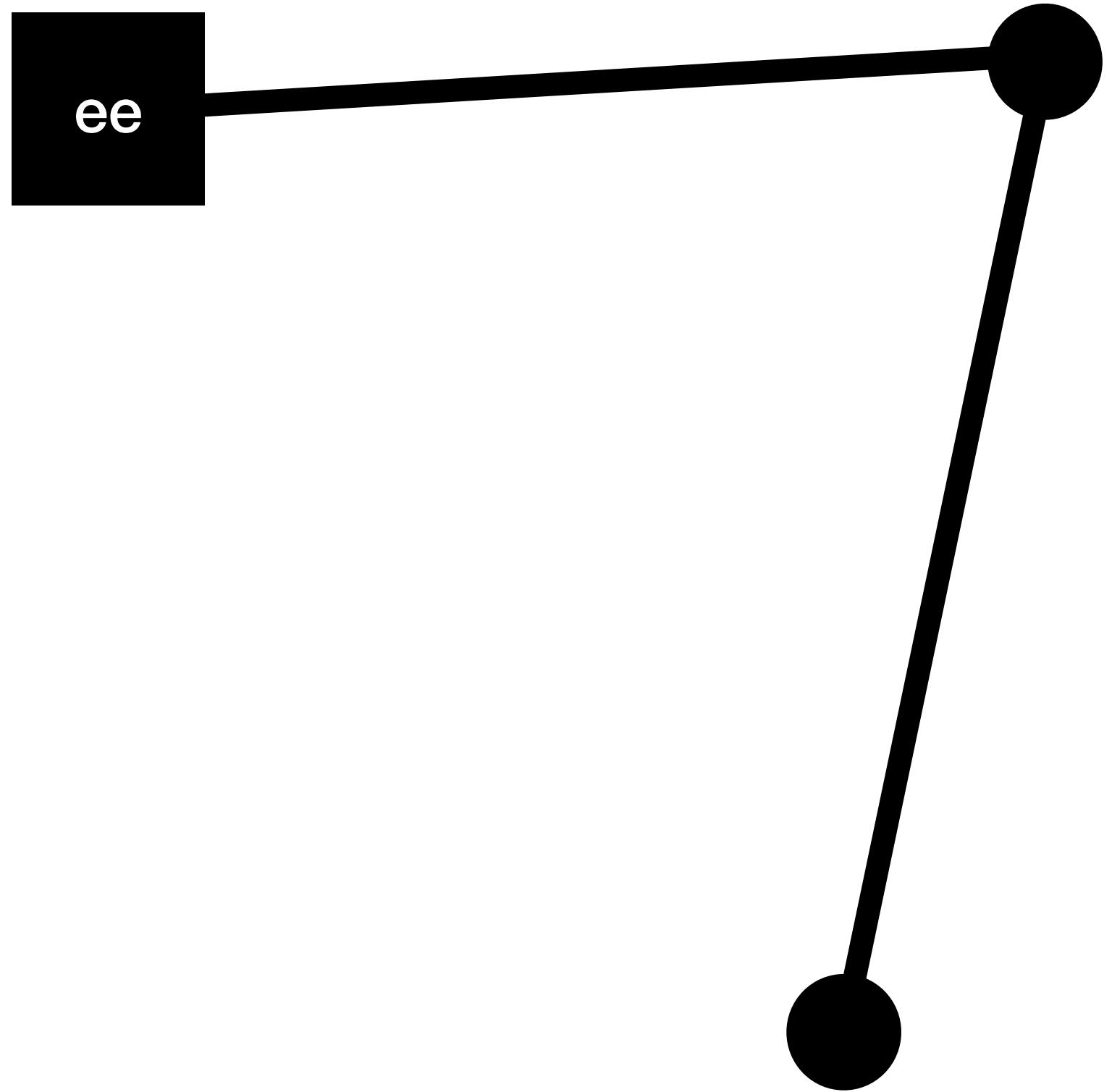
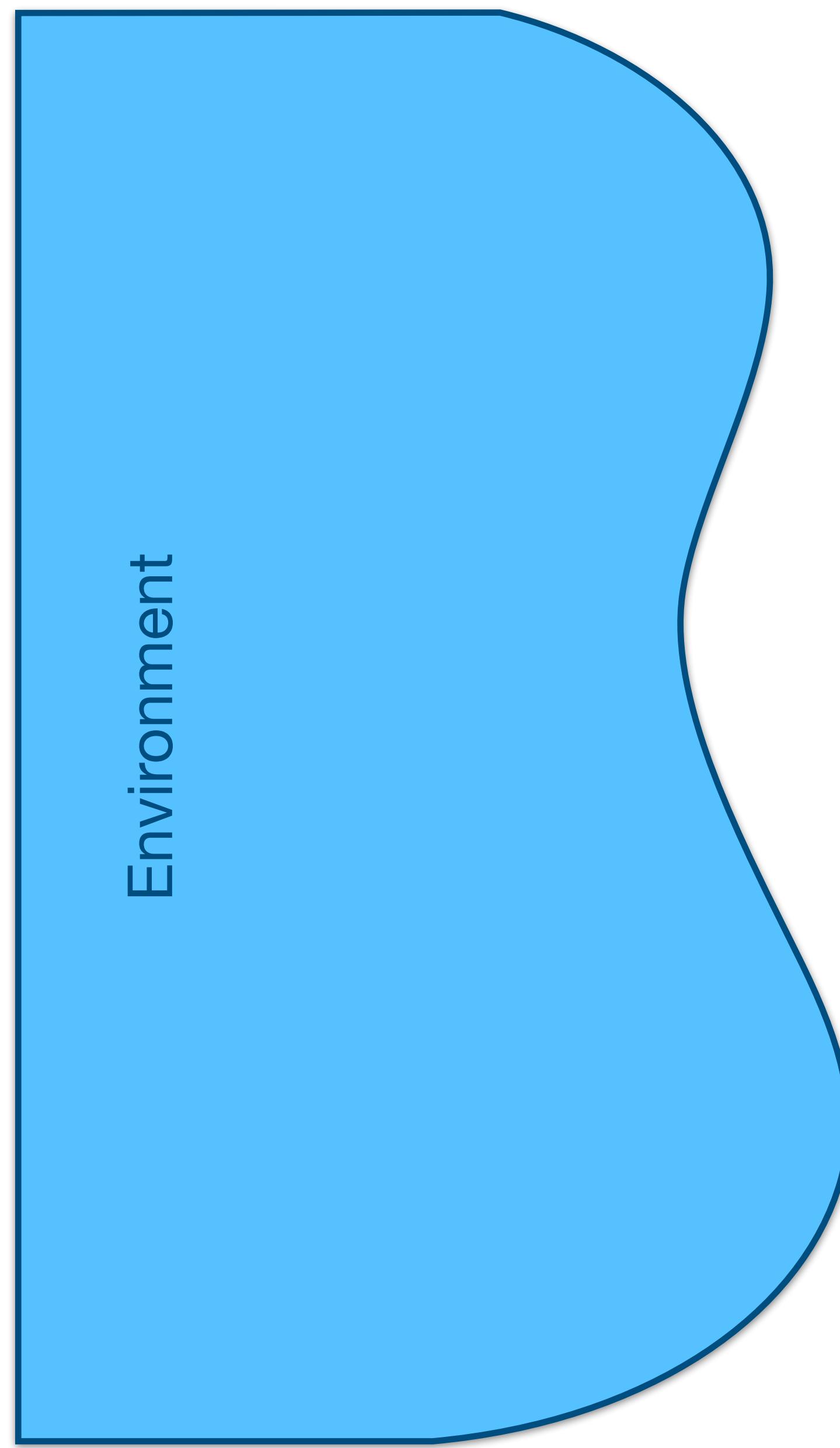
Task: Slide over surface

Environment

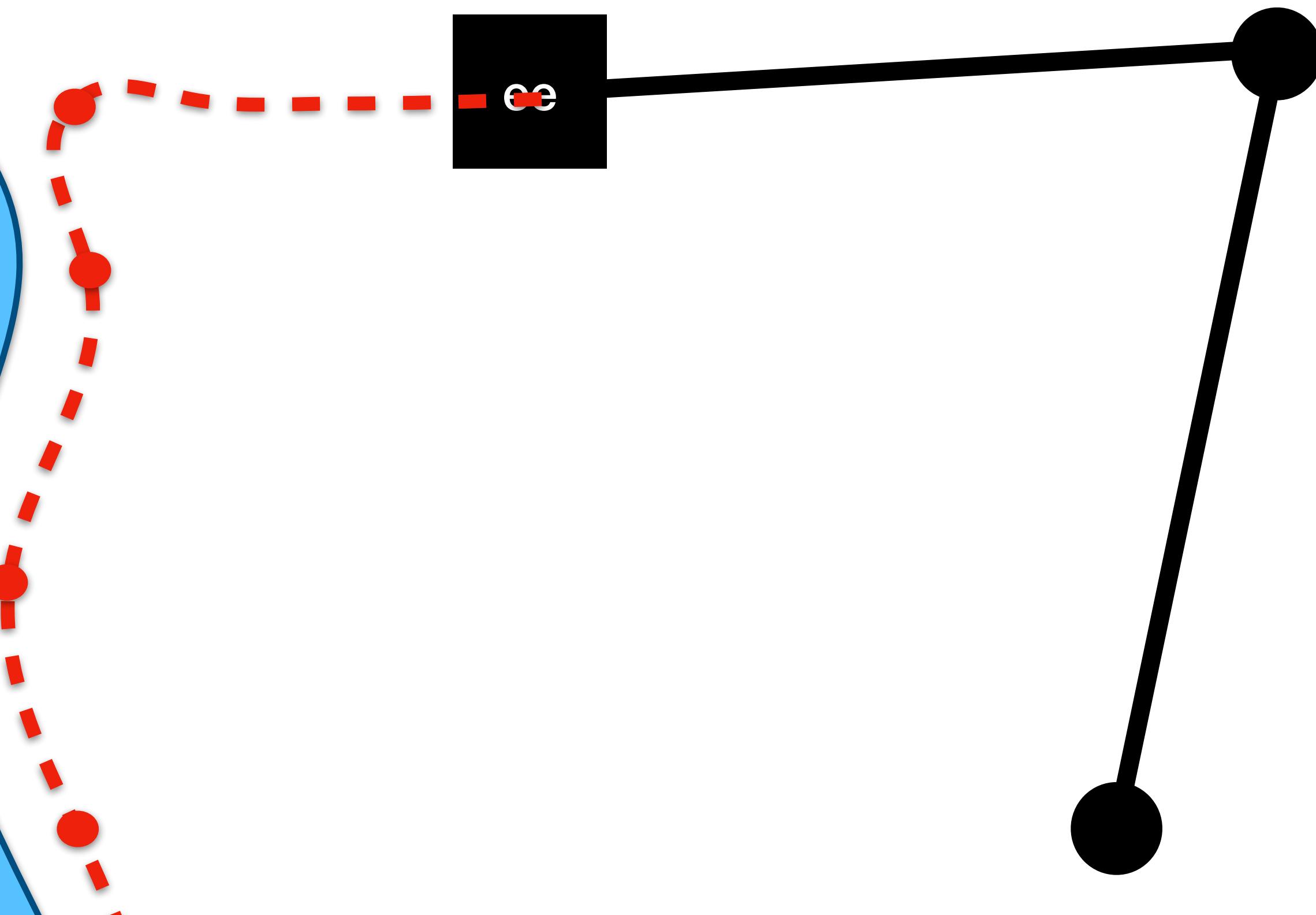
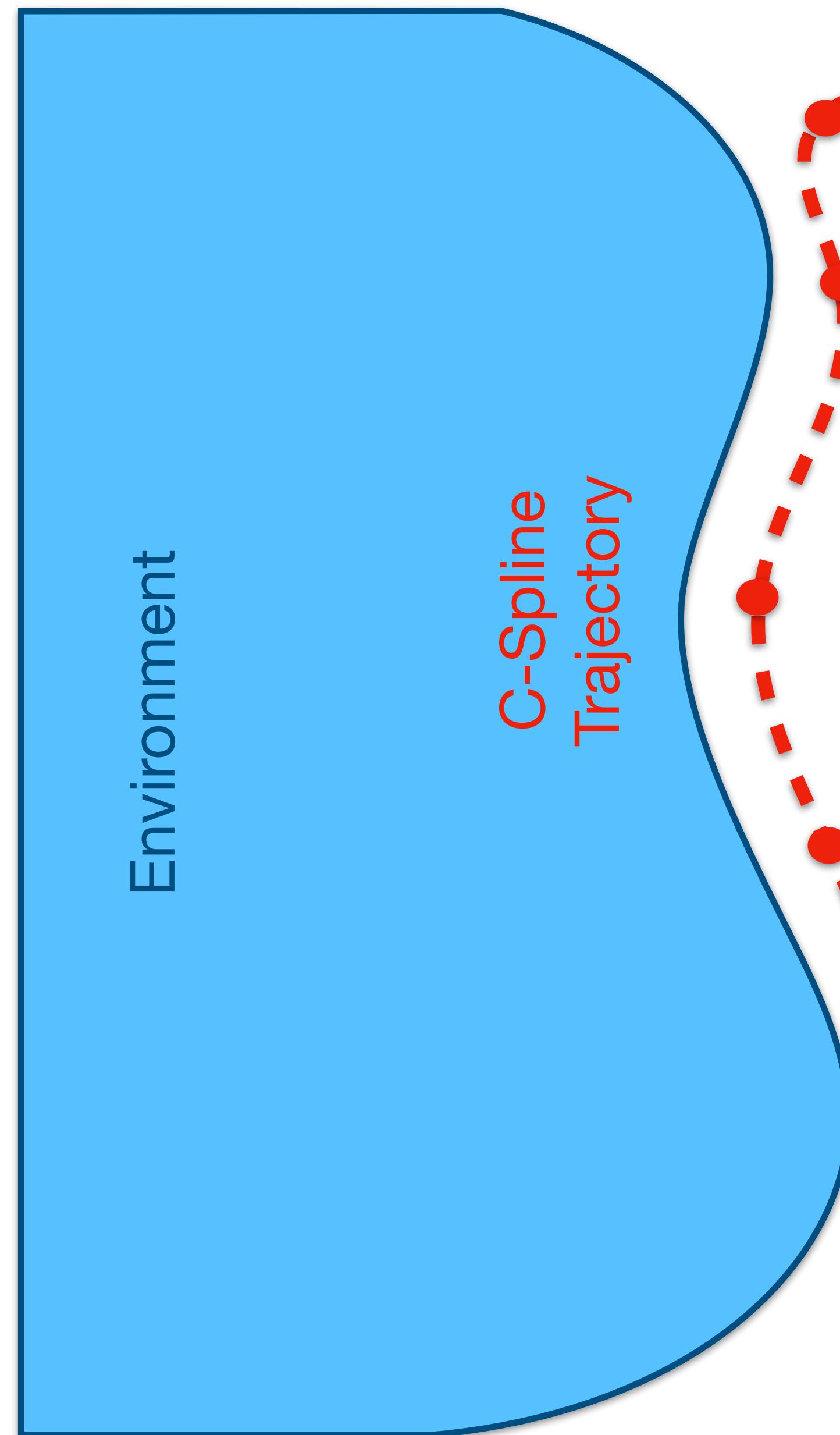
## Approach I: **Position Control**



Task: Slide over surface

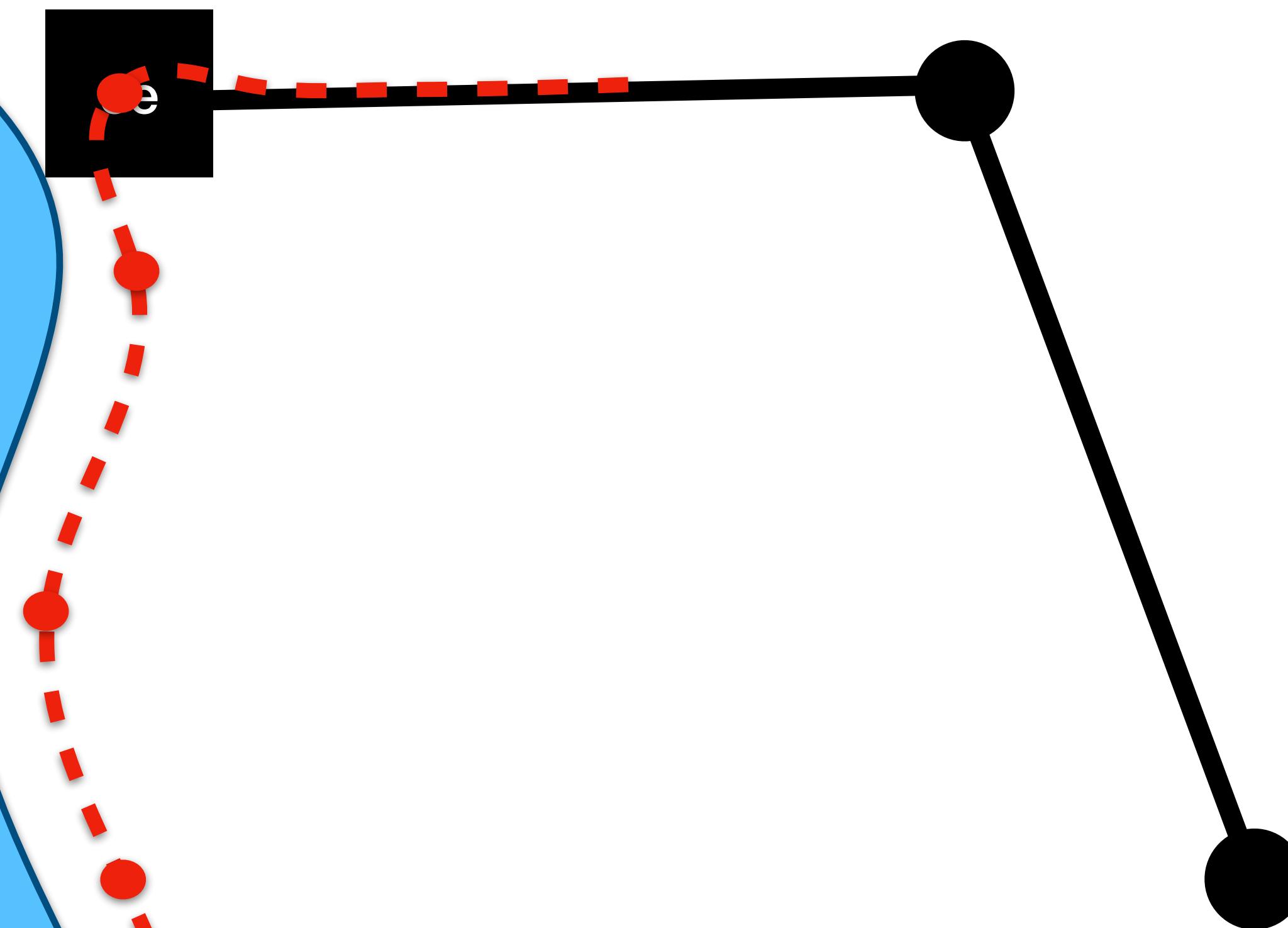
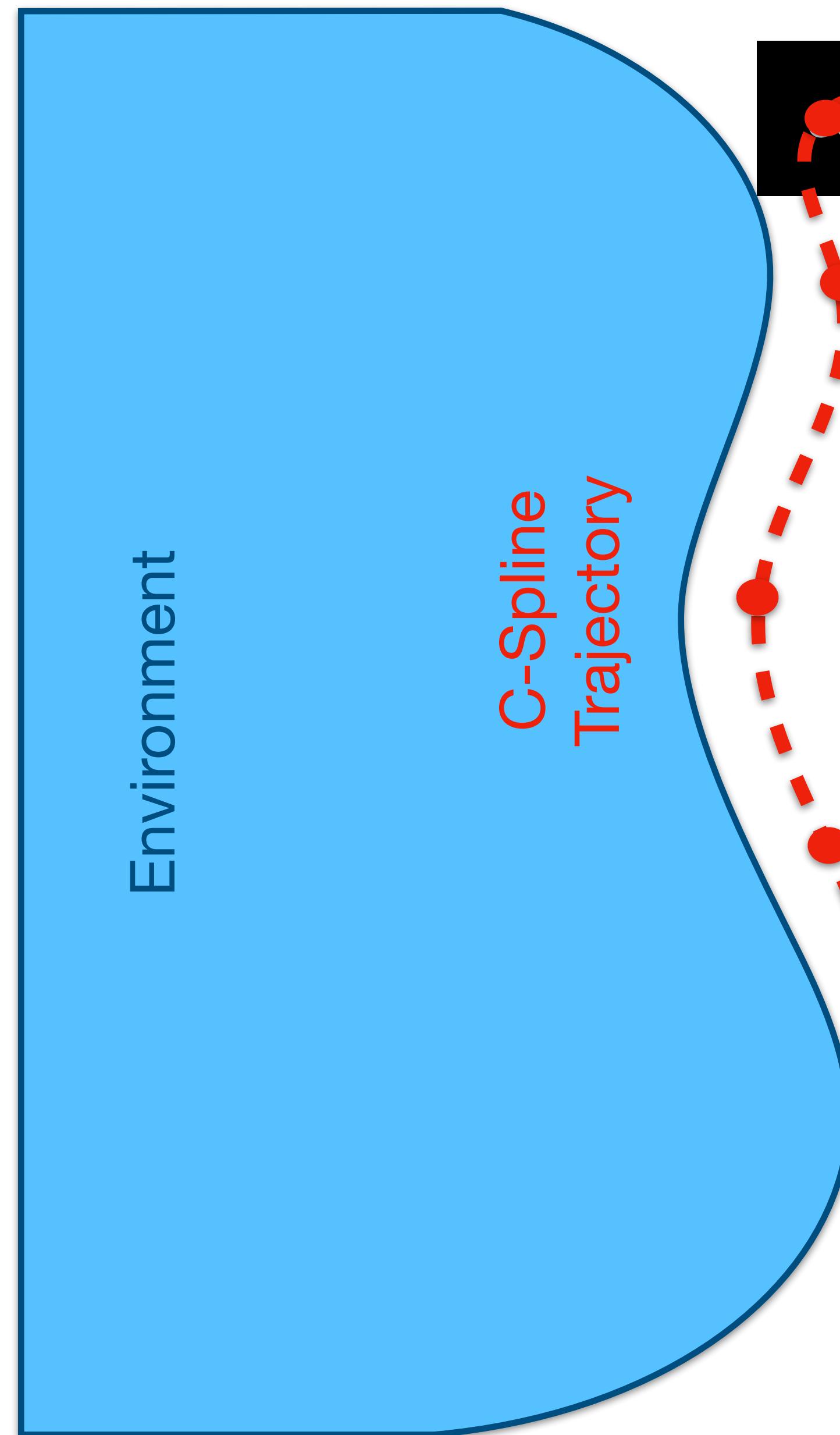


Robot tries to move along a desired trajectory with high precision



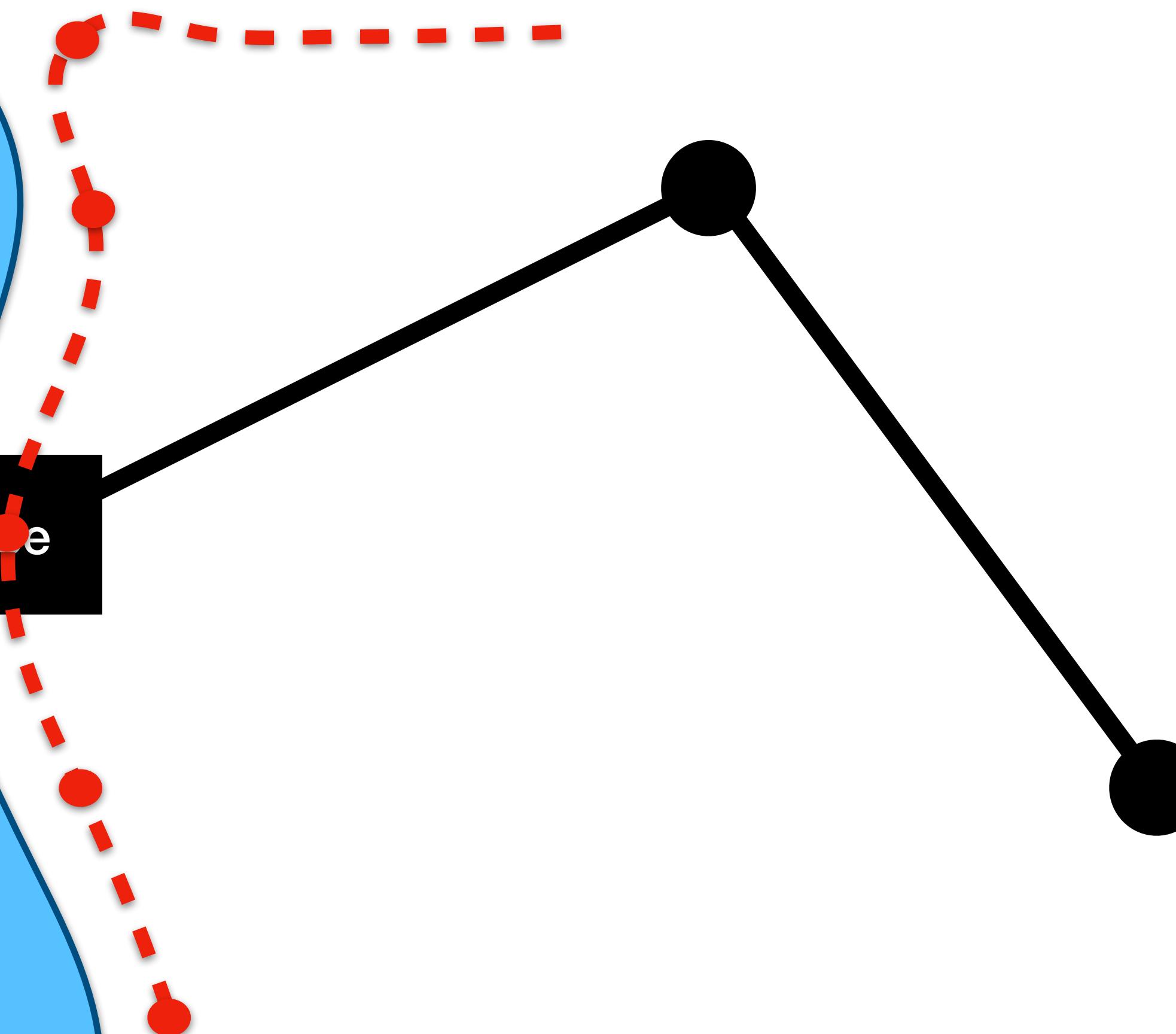
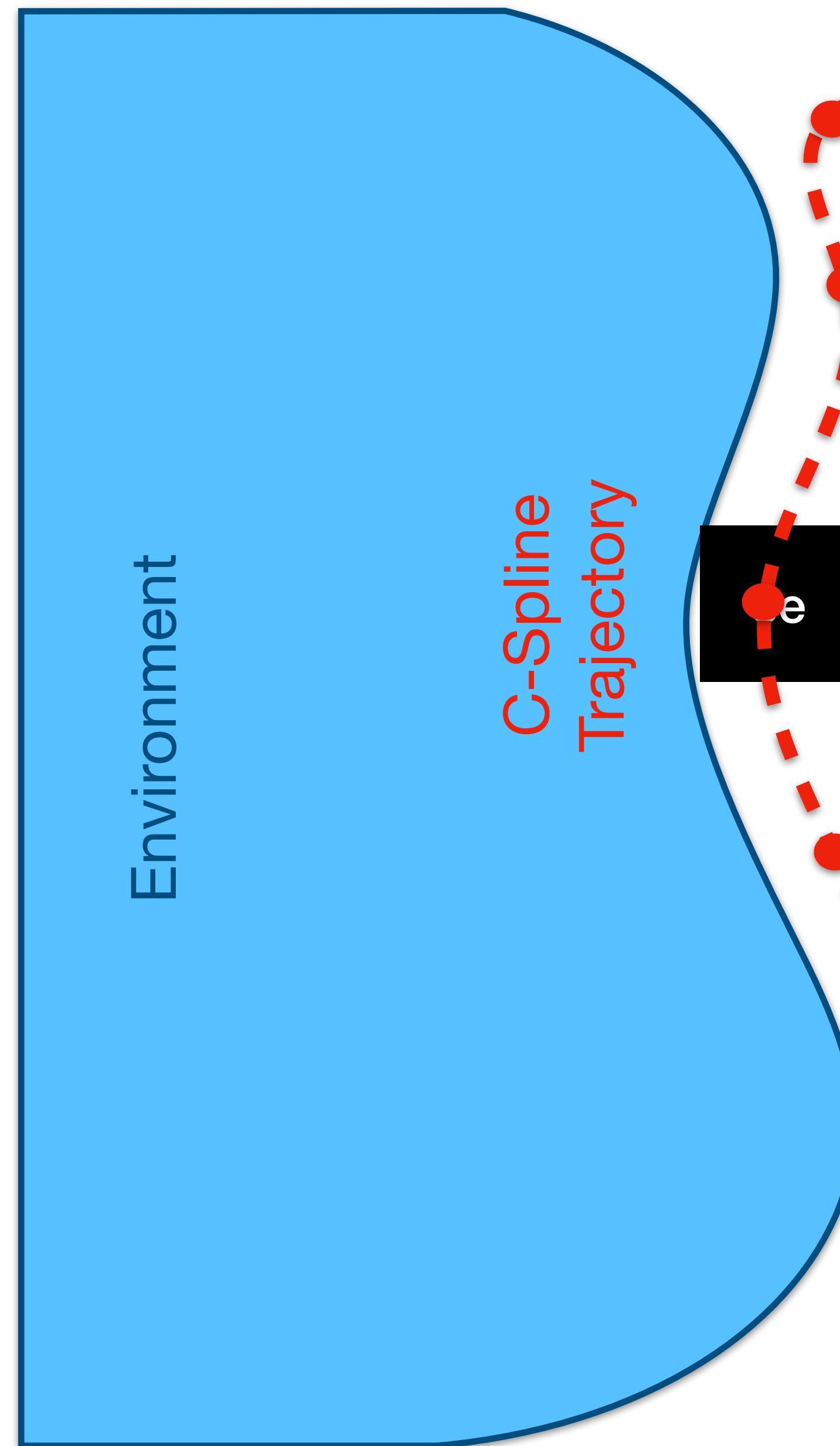
Robot tries to move along a desired trajectory with high precision

High precision requires high gains!



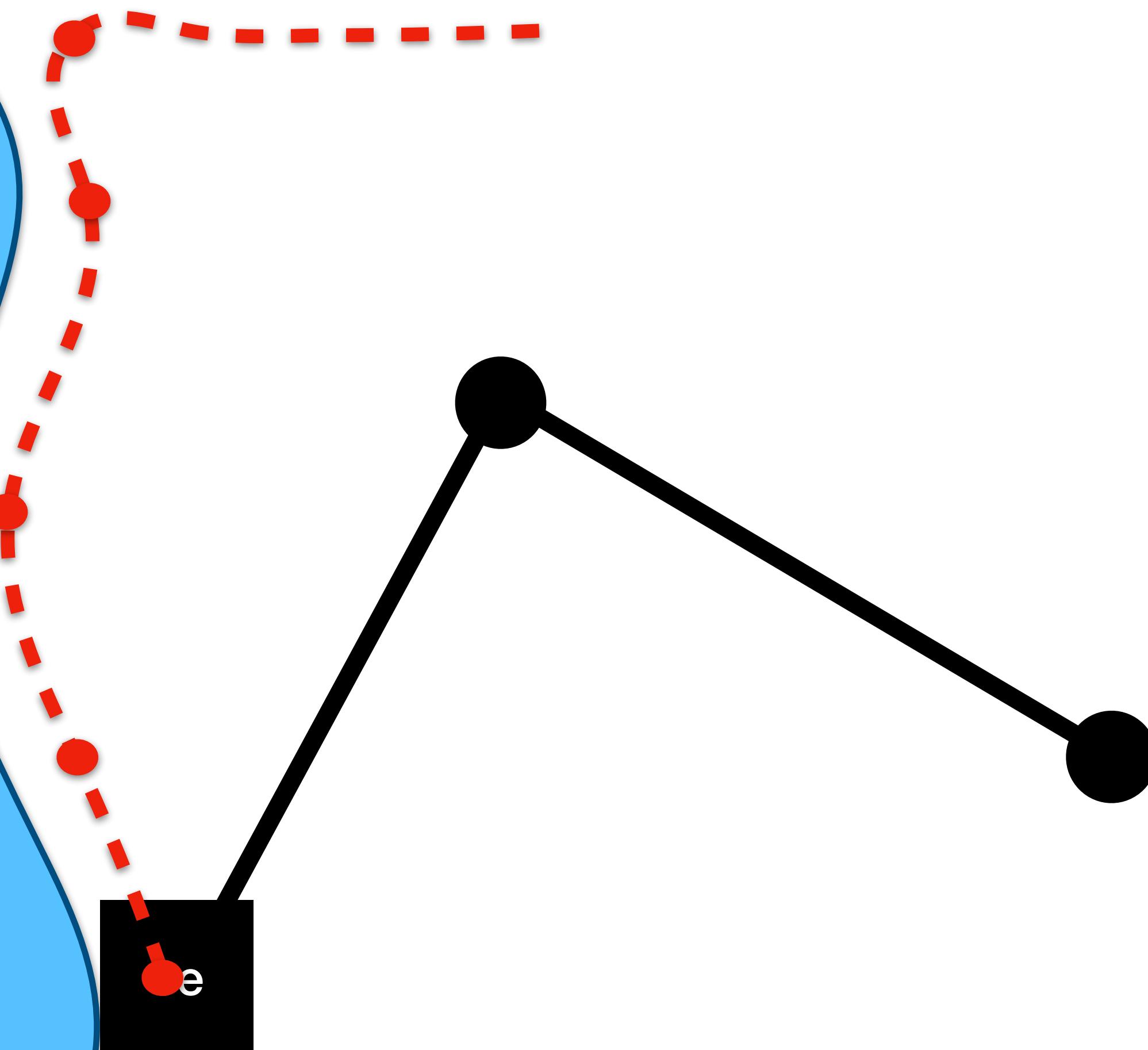
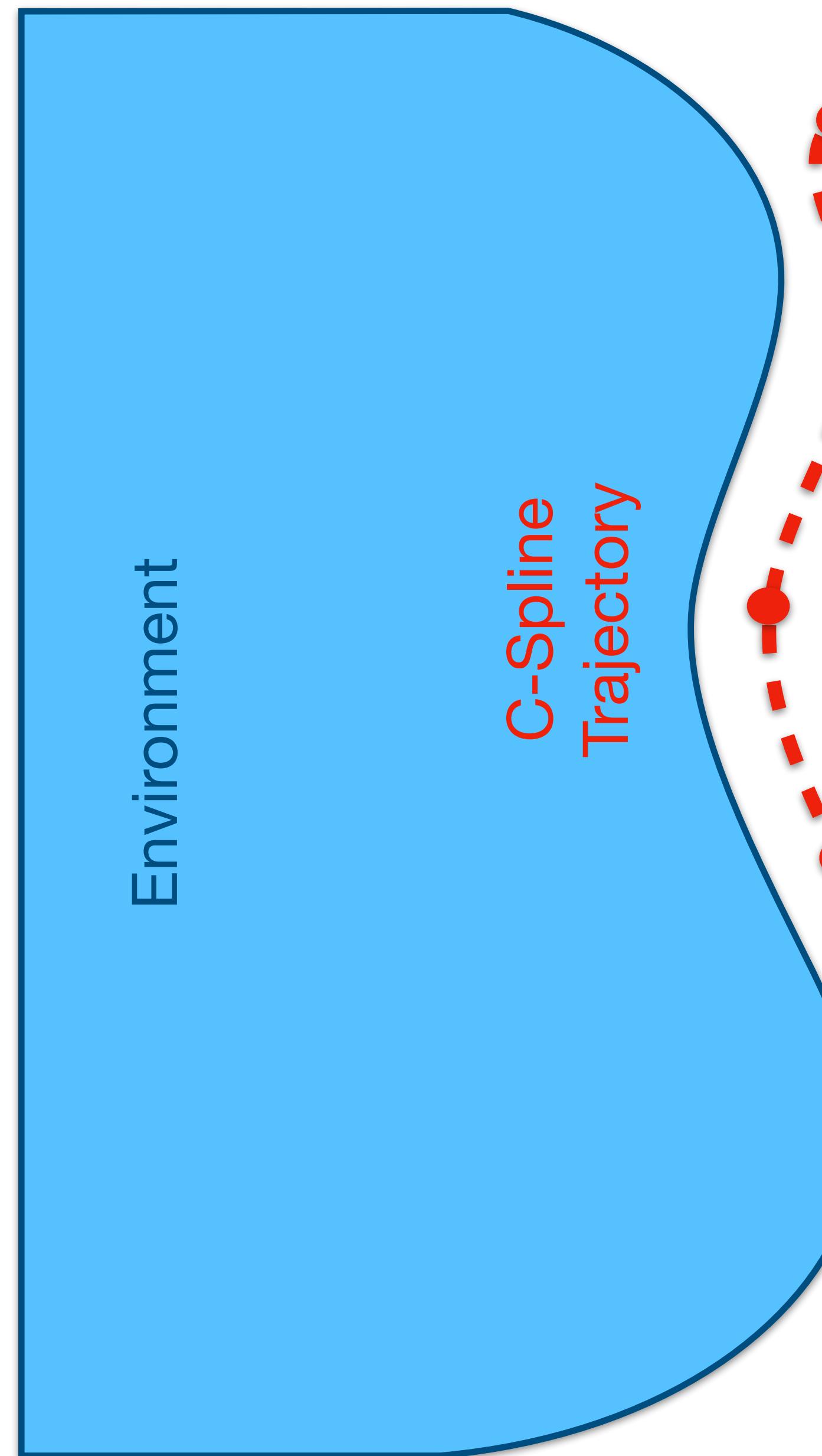
Robot tries to move along a desired trajectory with high precision

High precision requires high gains!



Robot tries to move along a desired trajectory with high precision

High precision requires high gains!

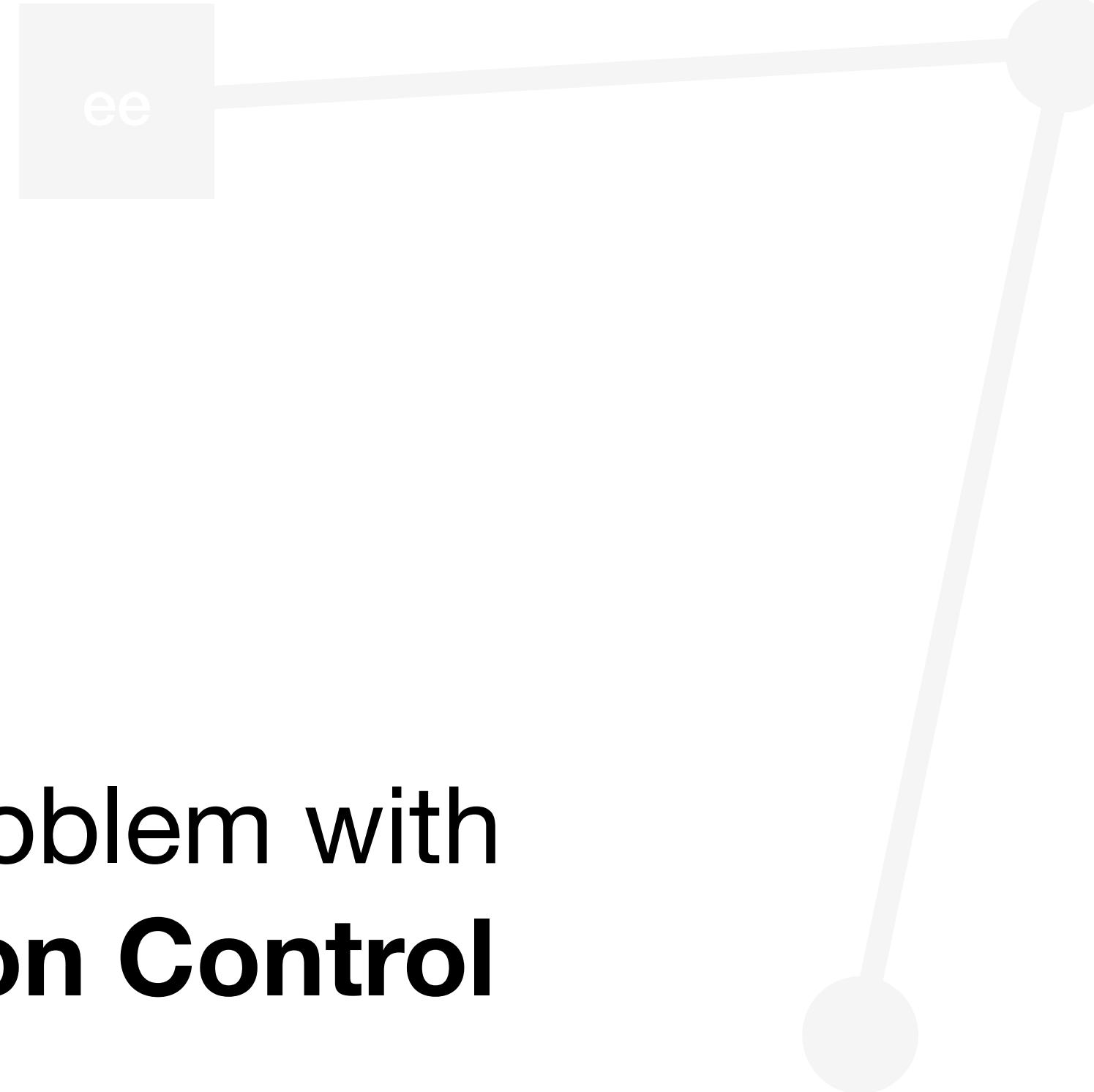


Robot tries to move along a desired trajectory with high precision

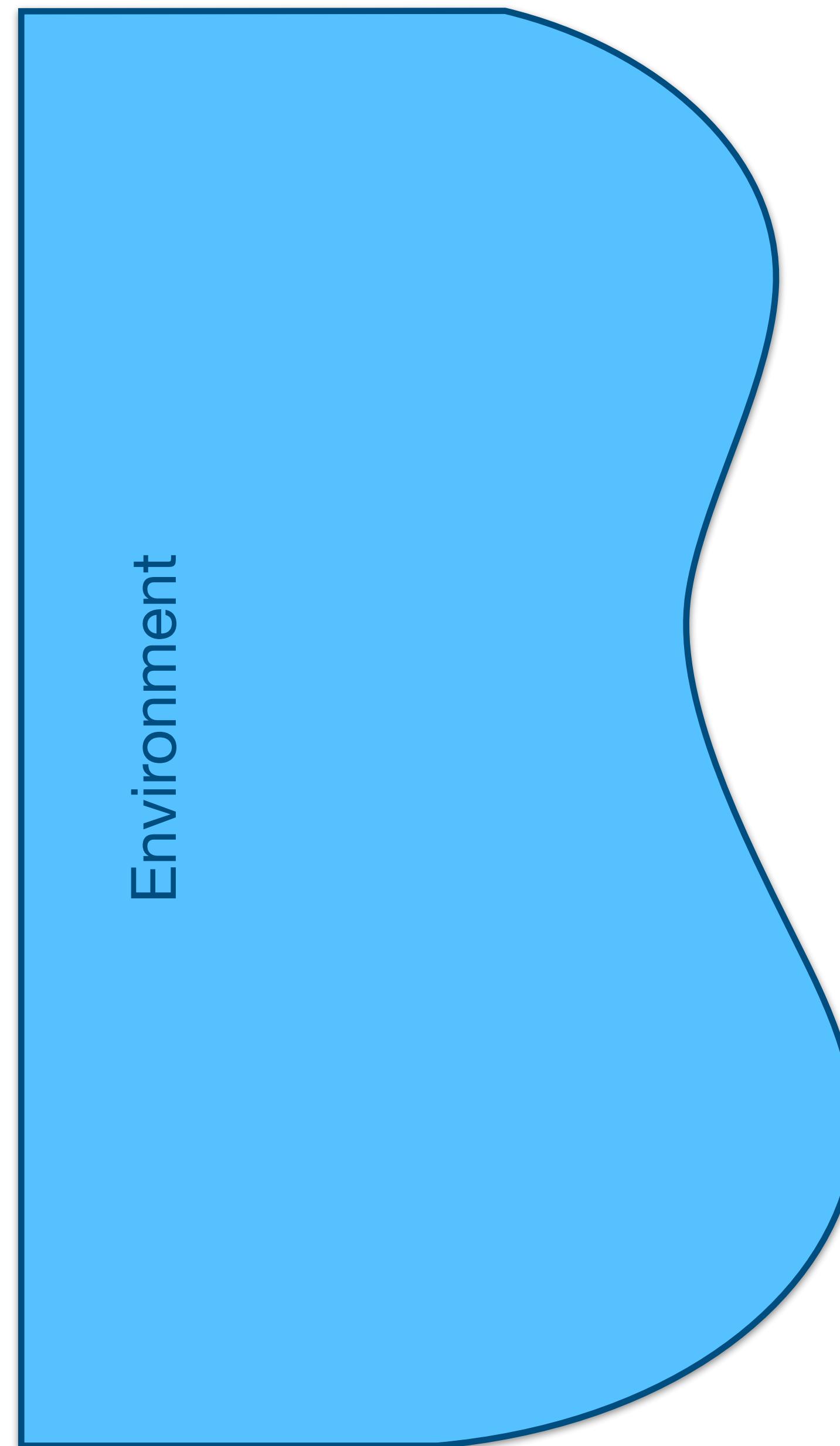
High precision requires high gains!

Environment

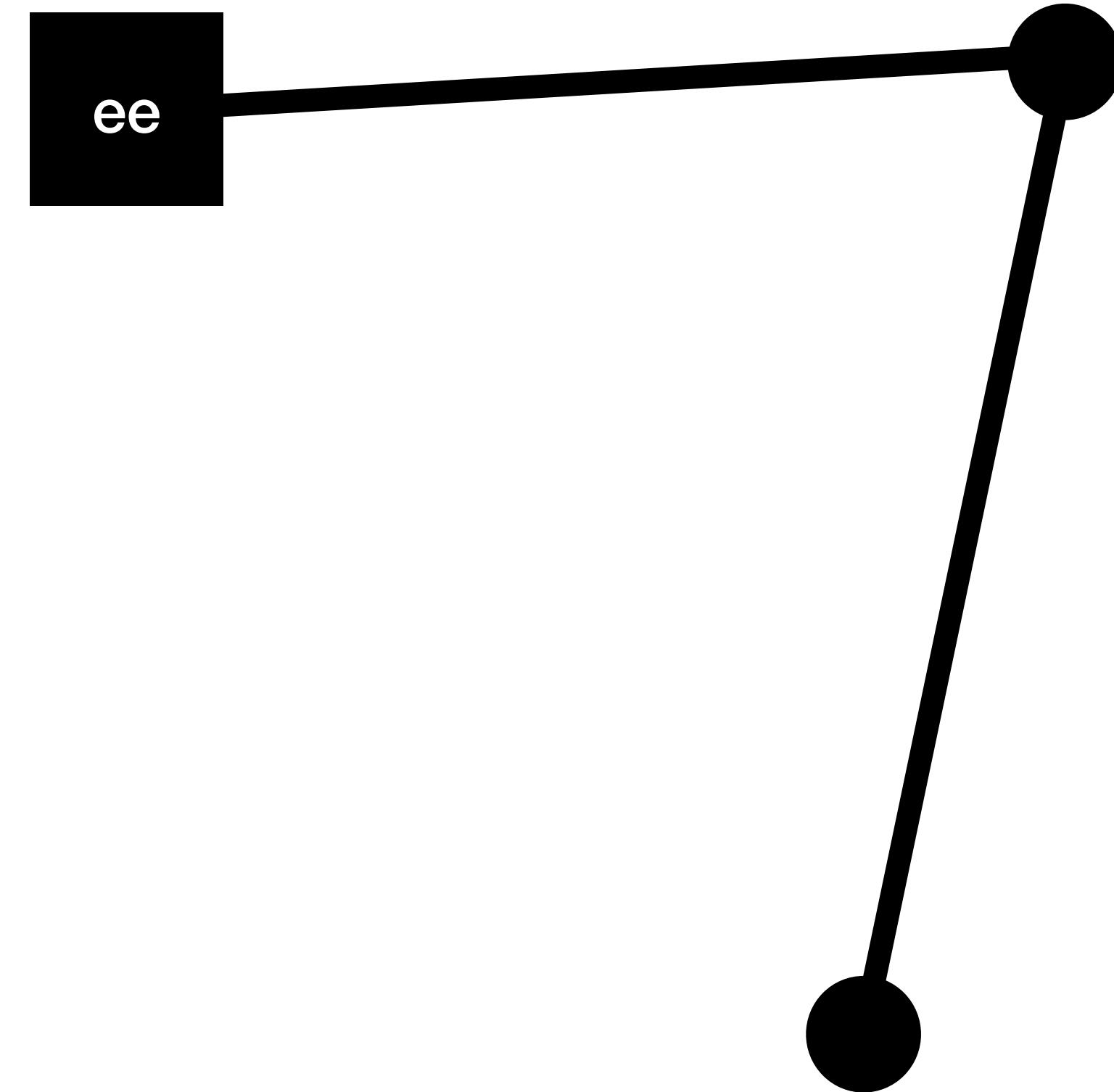
## The Problem with Position Control



Task: Slide over surface

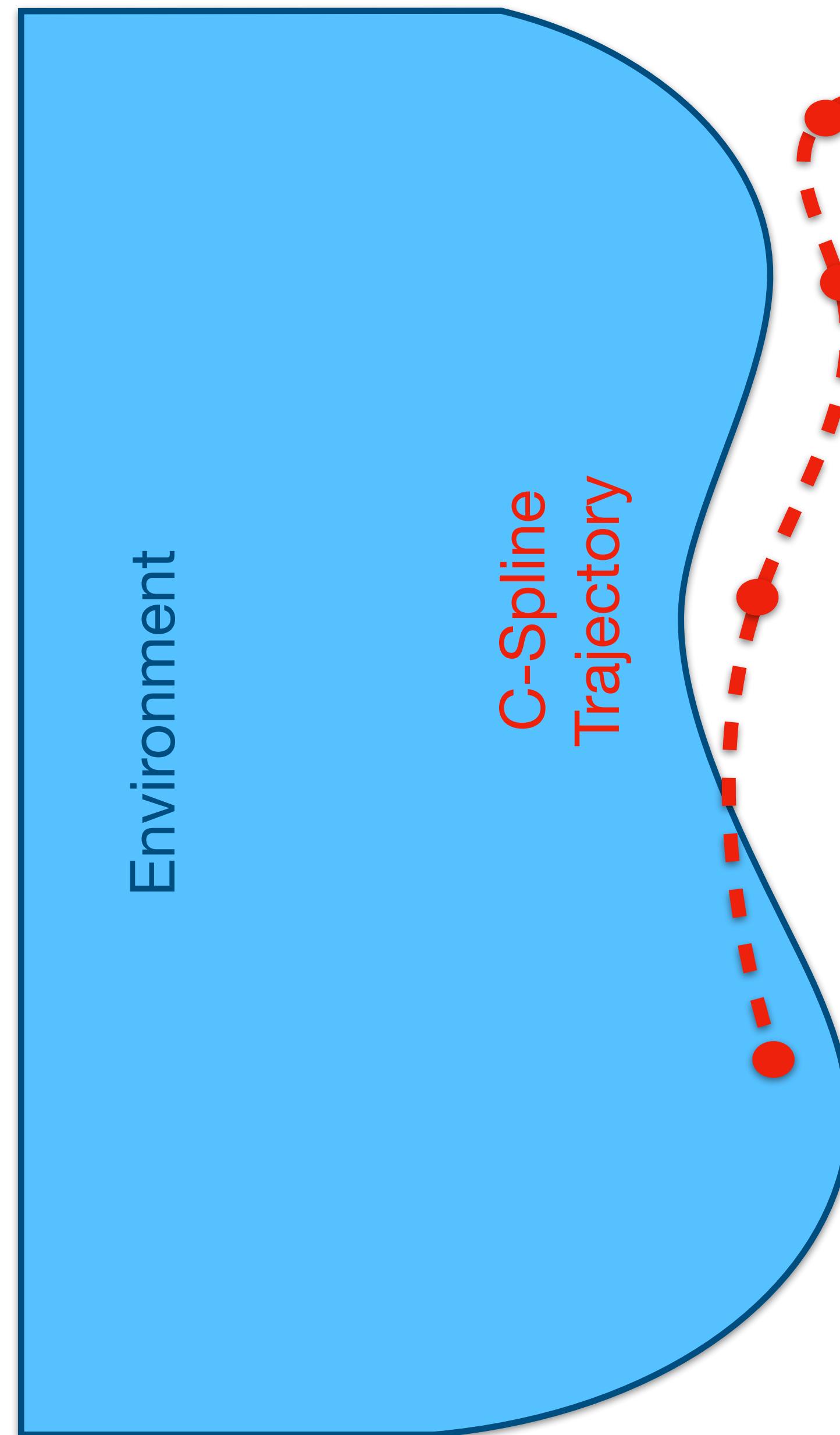


Environment

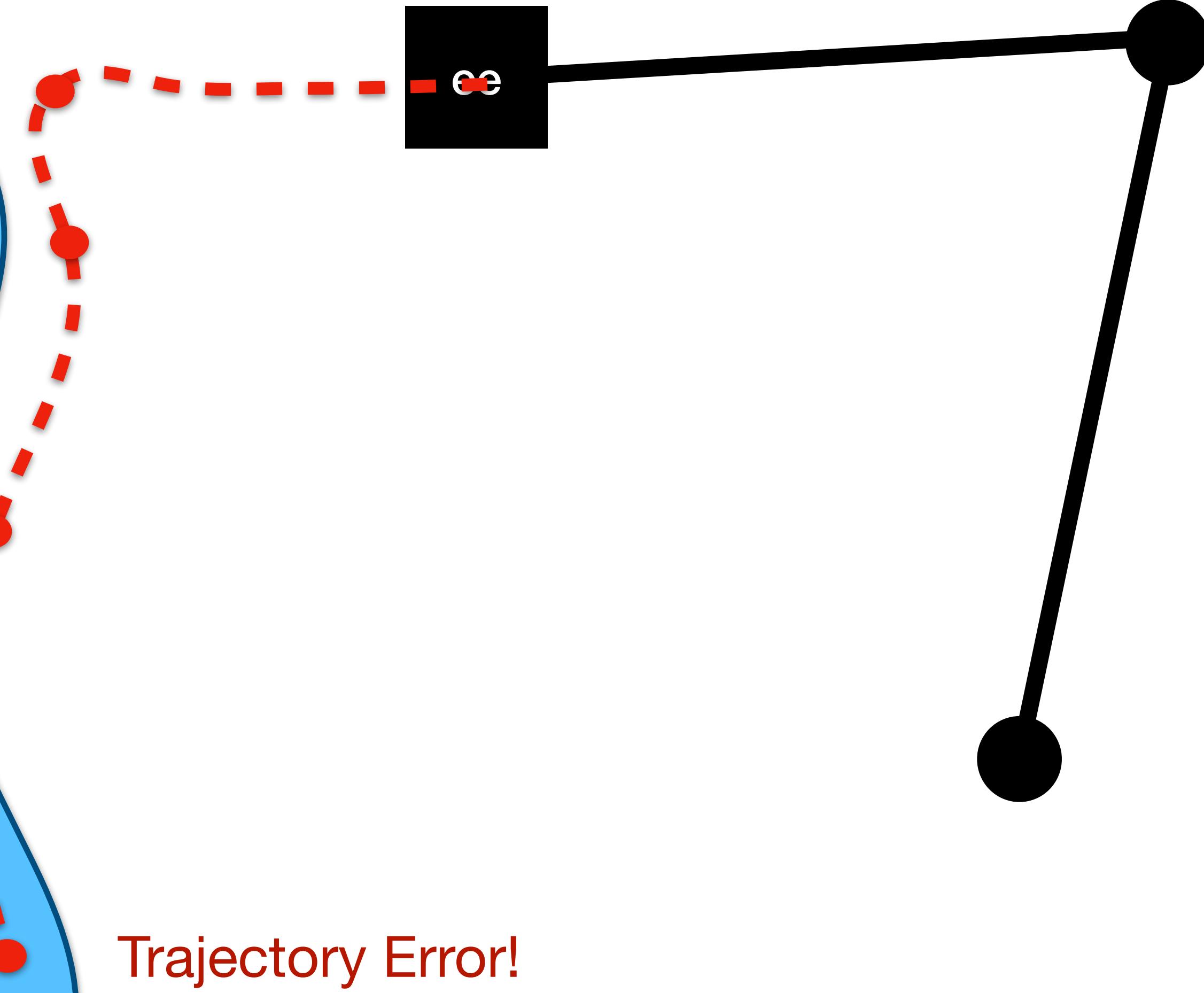


Robot tries to move along a desired trajectory with high precision

High precision requires high gains!

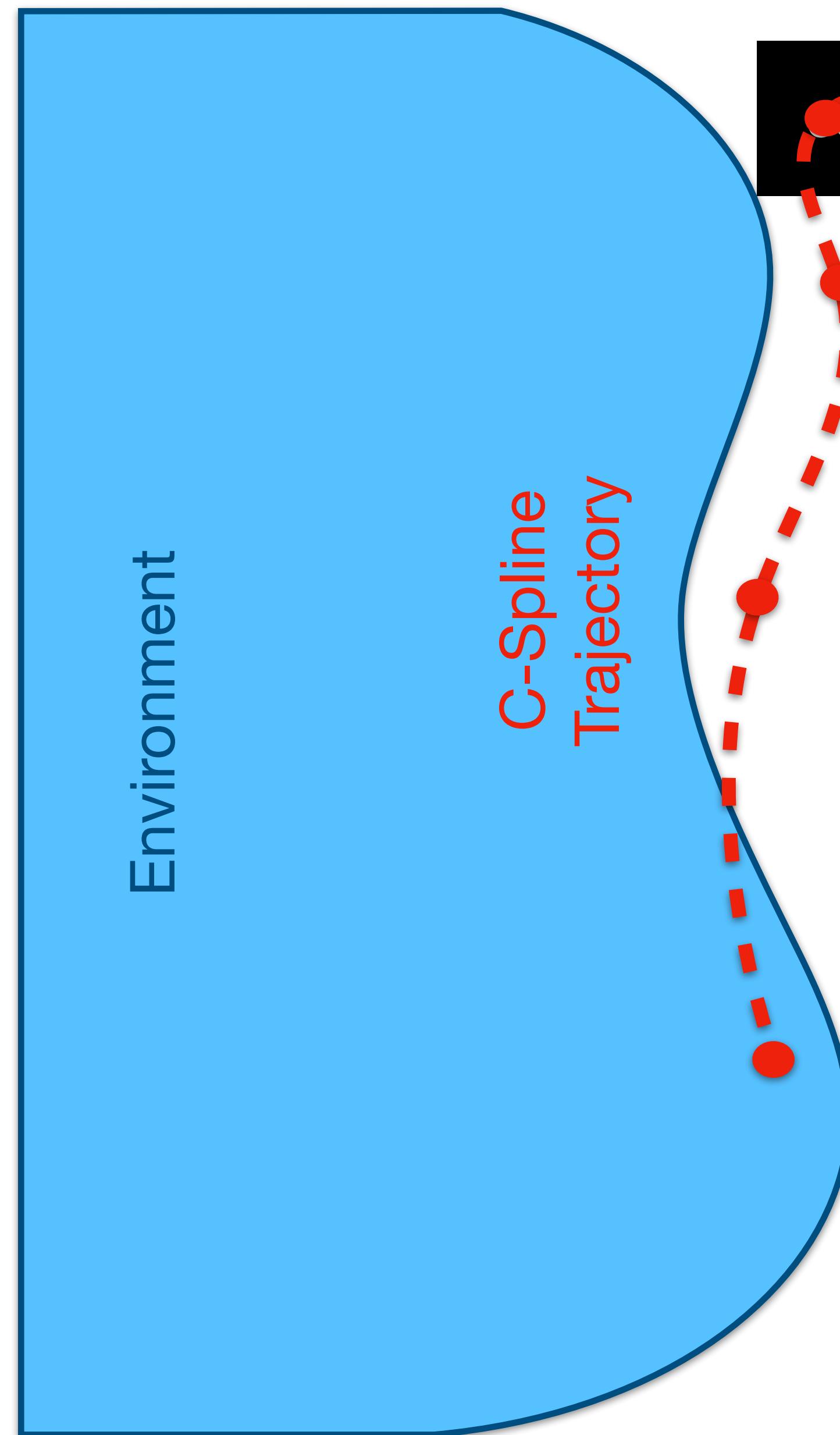


C-Spline  
Trajectory



High precision requires high gains!

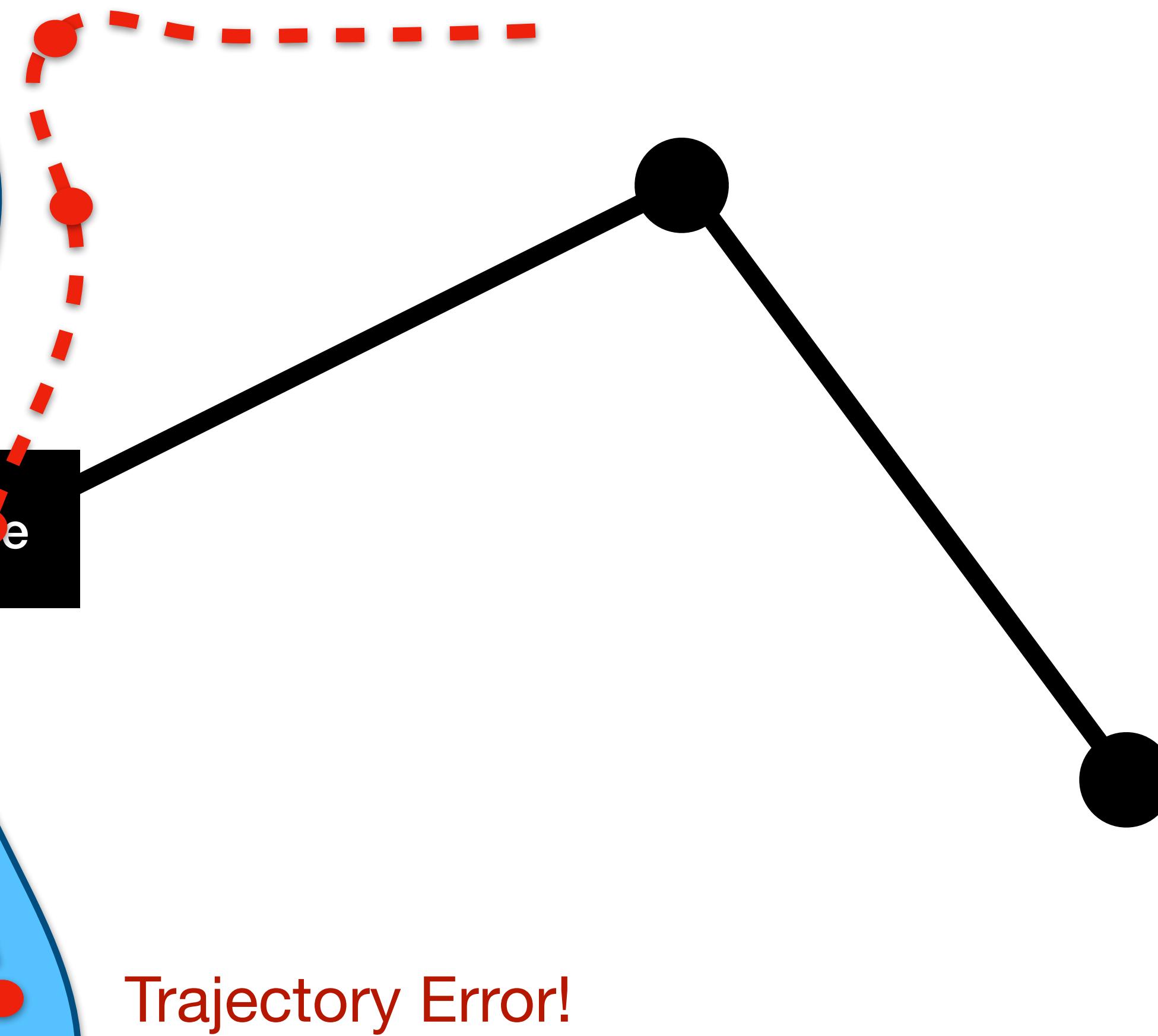
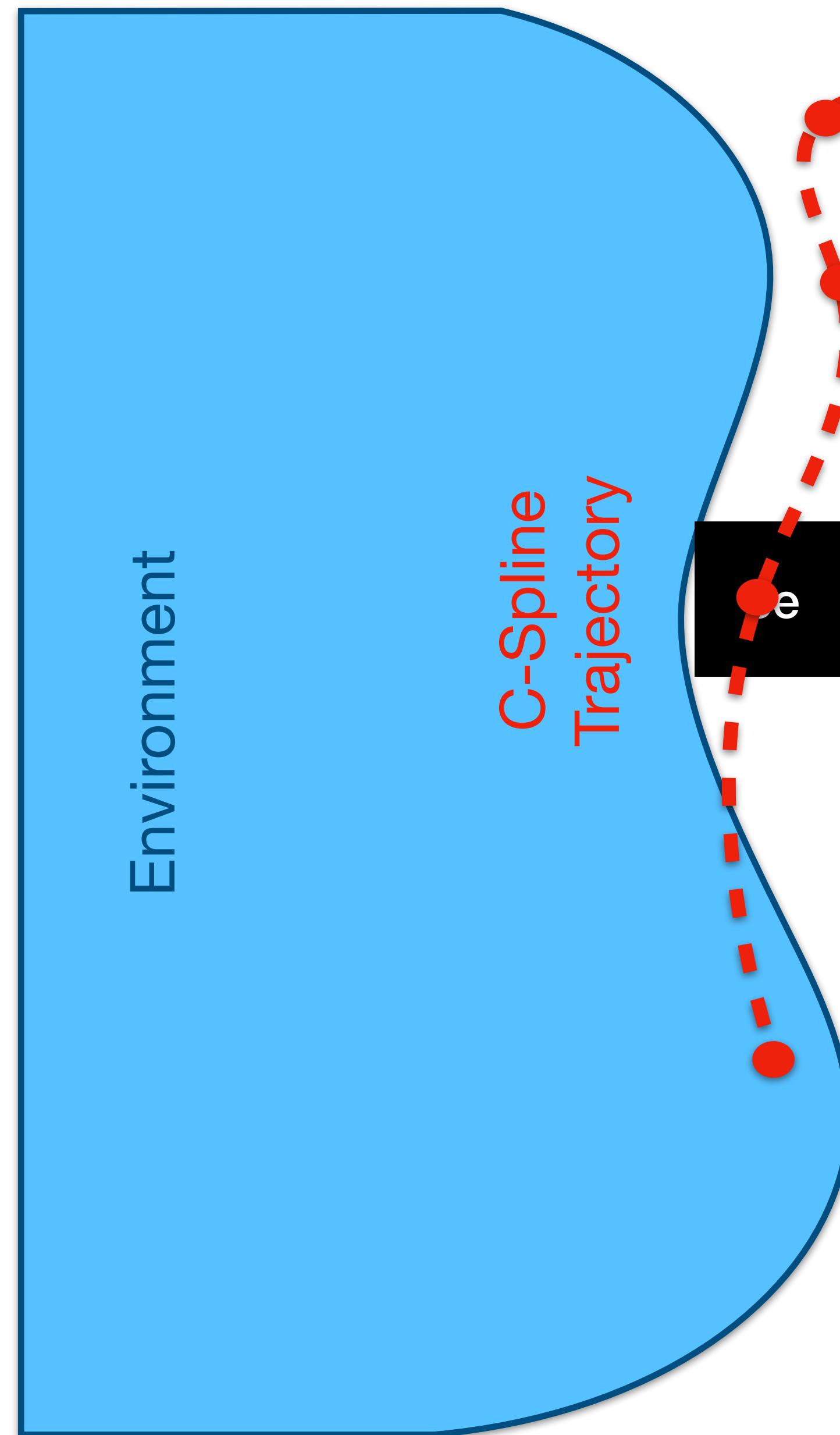
Robot tries to move along a desired trajectory with high precision



Trajectory Error!

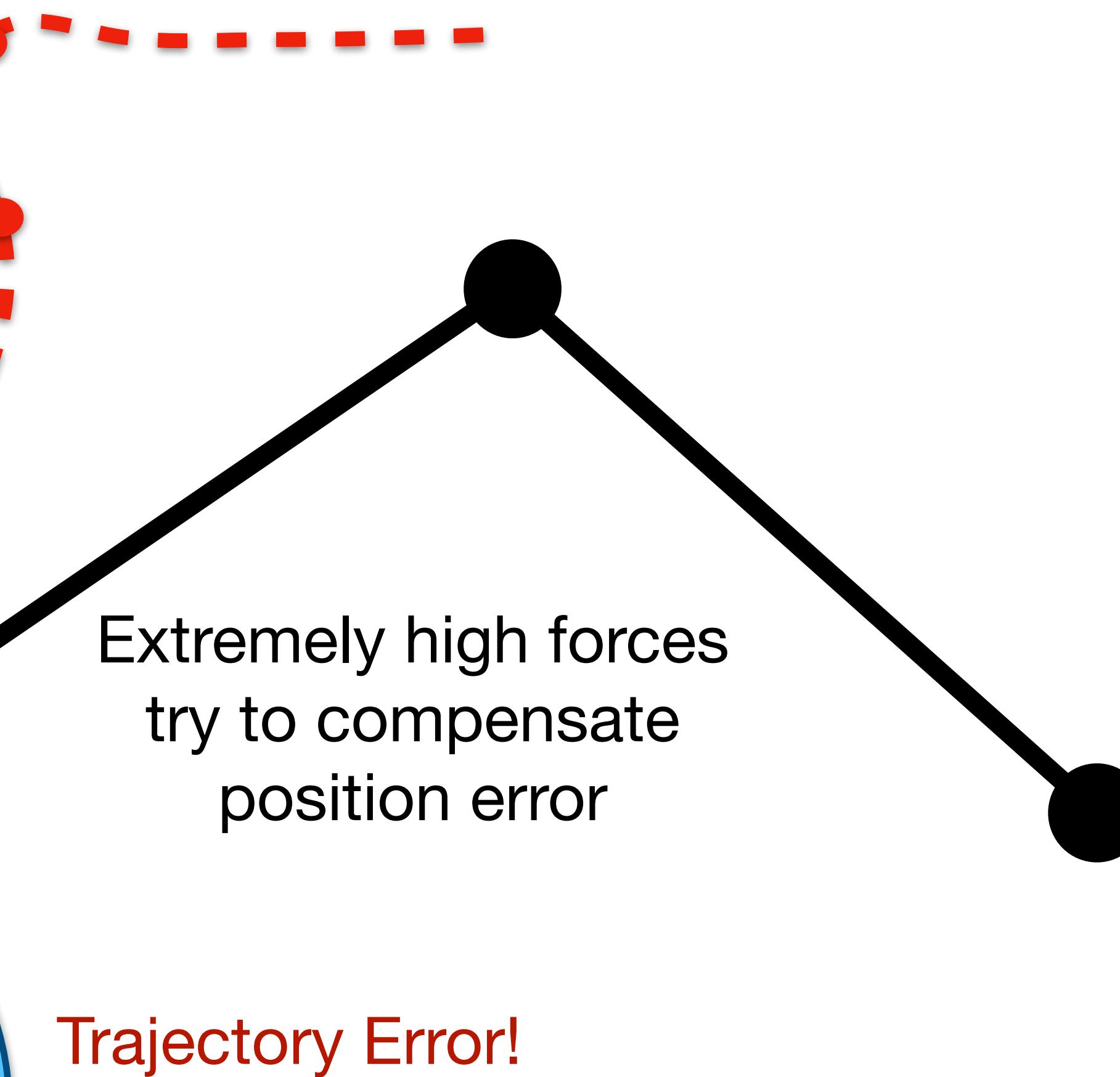
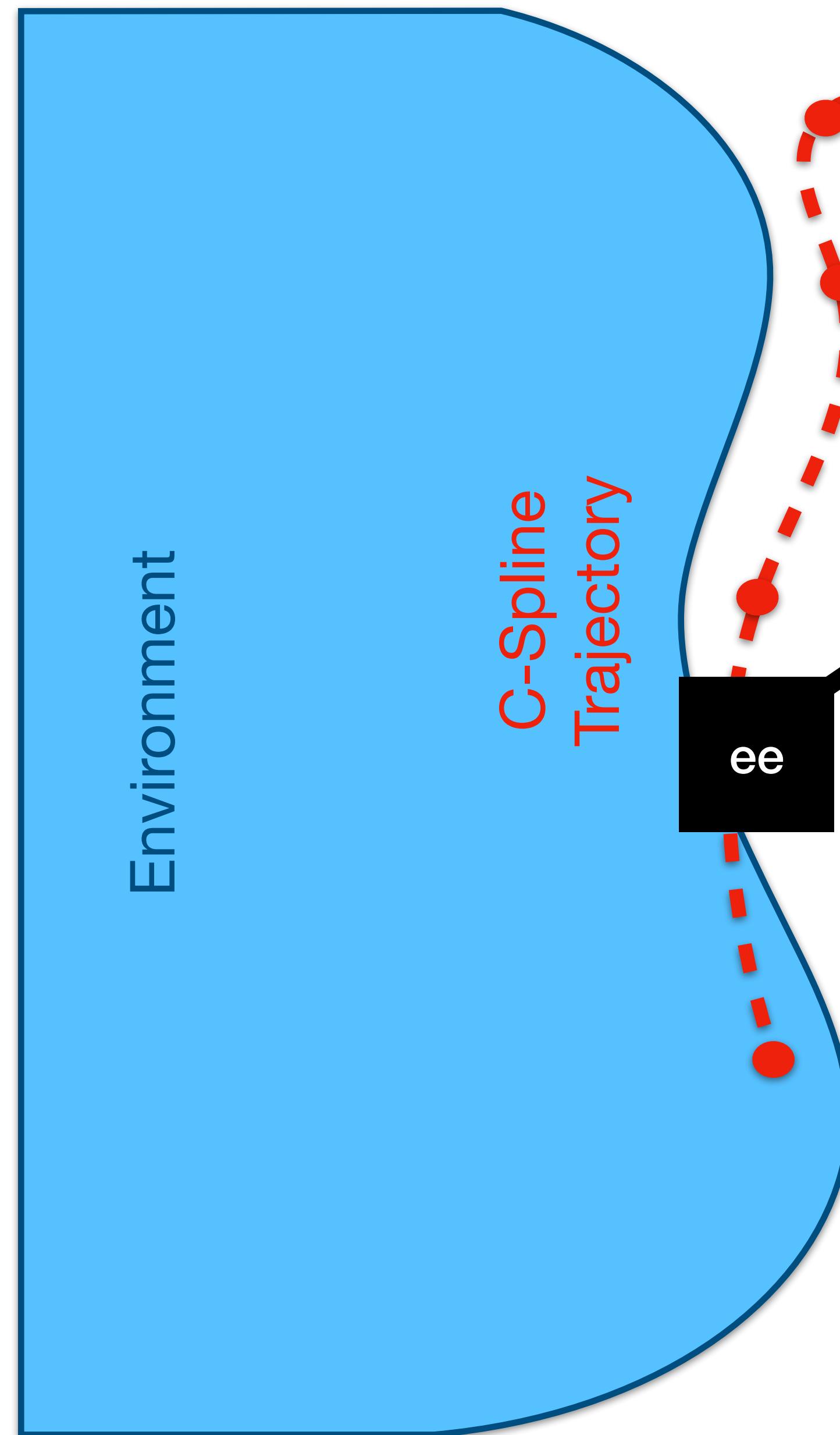
High precision requires high gains!

Robot tries to move along a desired trajectory with high precision



High precision requires high gains!

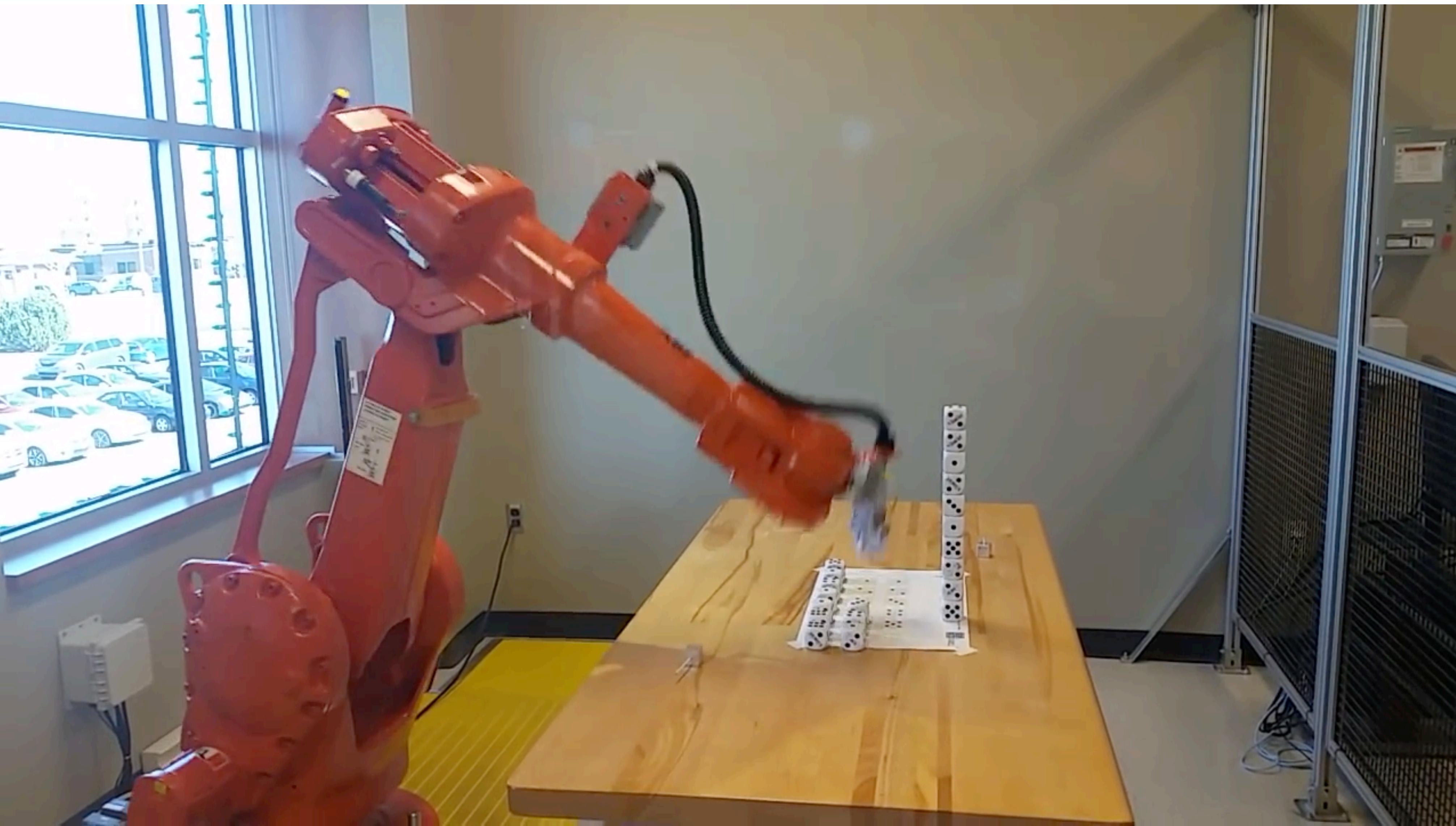
Robot tries to move along a desired trajectory with high precision



Robot tries to move along a desired trajectory with high precision

High precision requires high gains!

# The Problem with Position Control



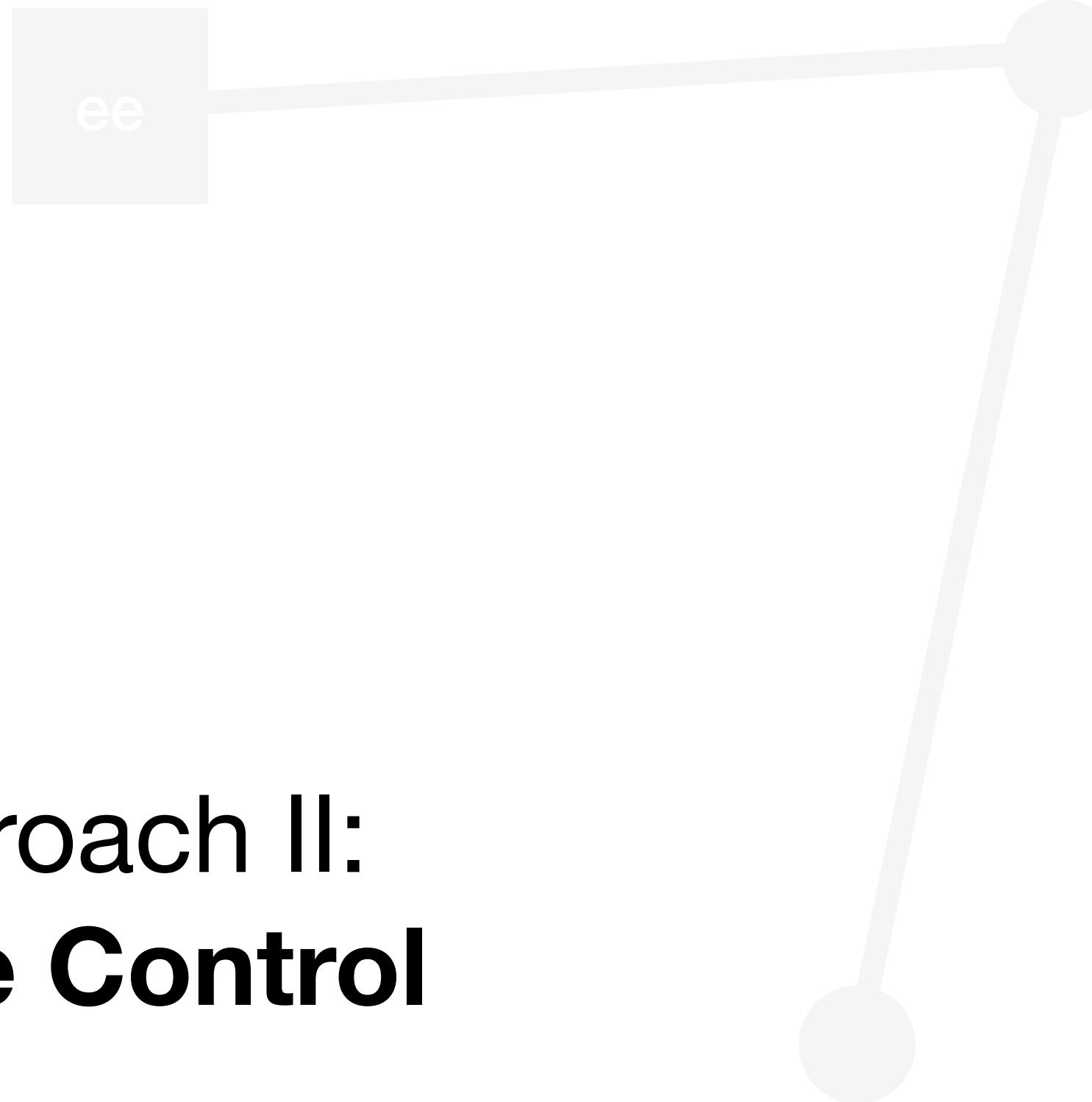
# The Problem with Position Control

The logo for The Guardian newspaper, featuring the word "The" above "Guardian" in a white serif font.

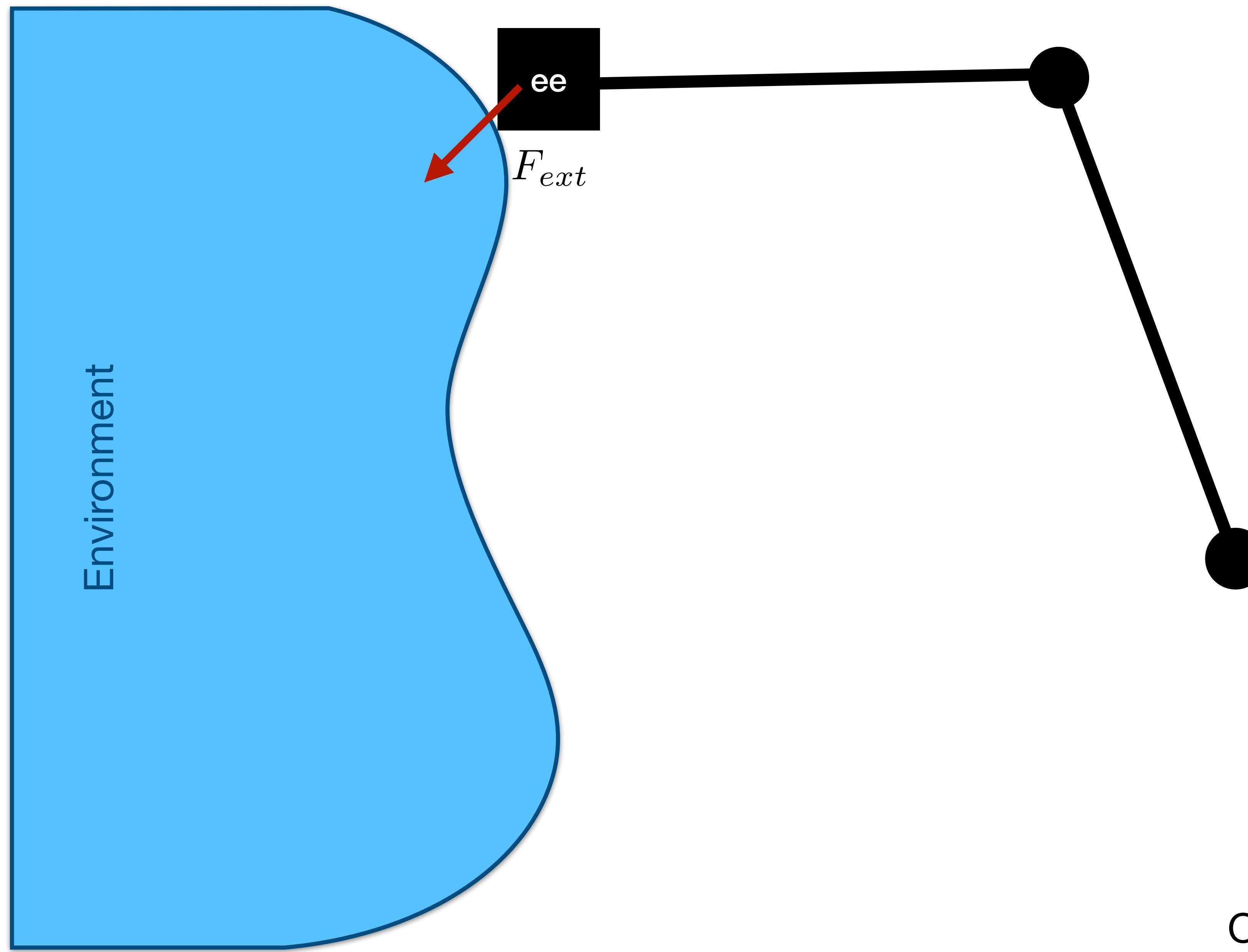
The  
Guardian

Environment

## Approach II: **Force Control**

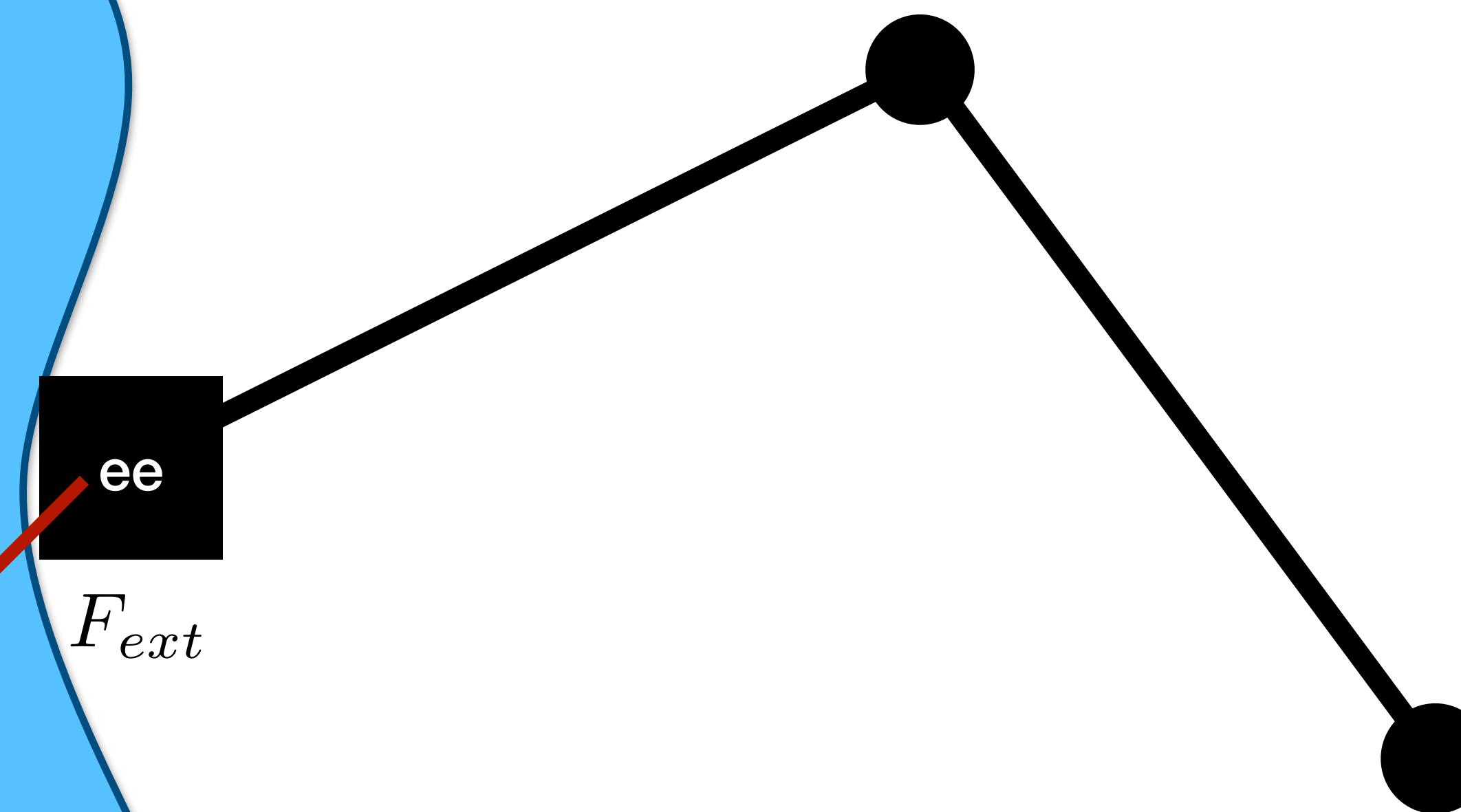
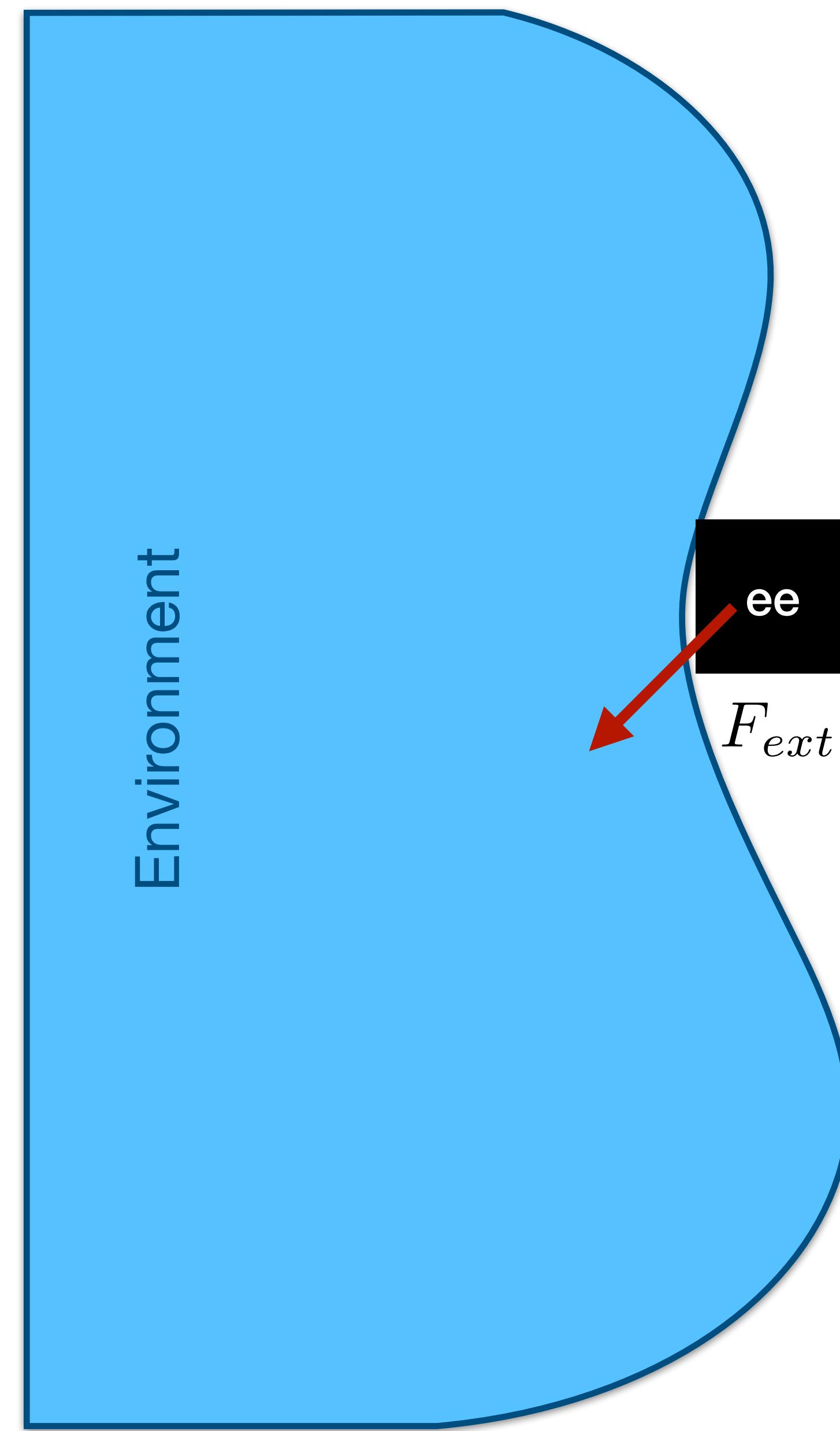


Task: Slide over surface



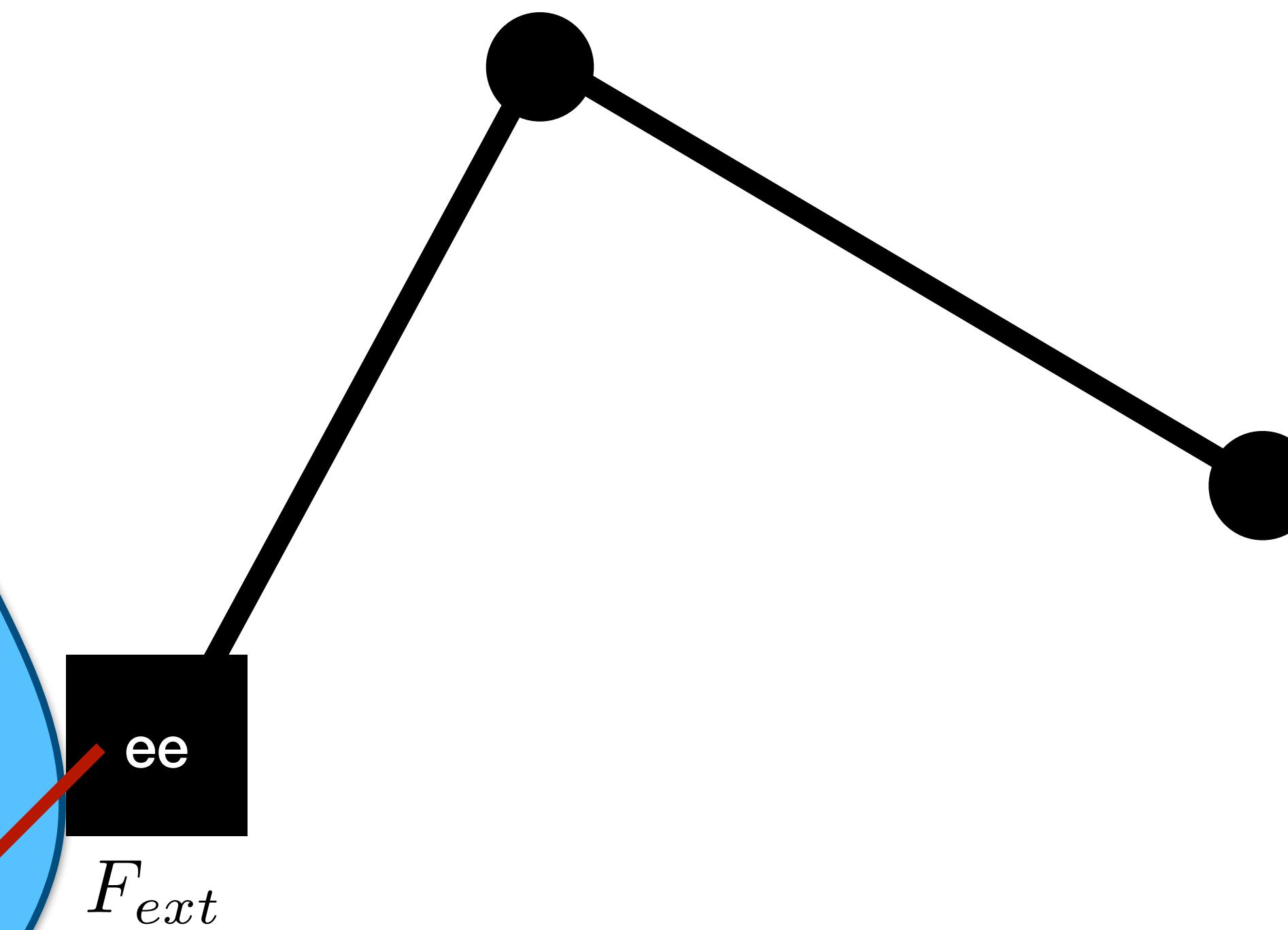
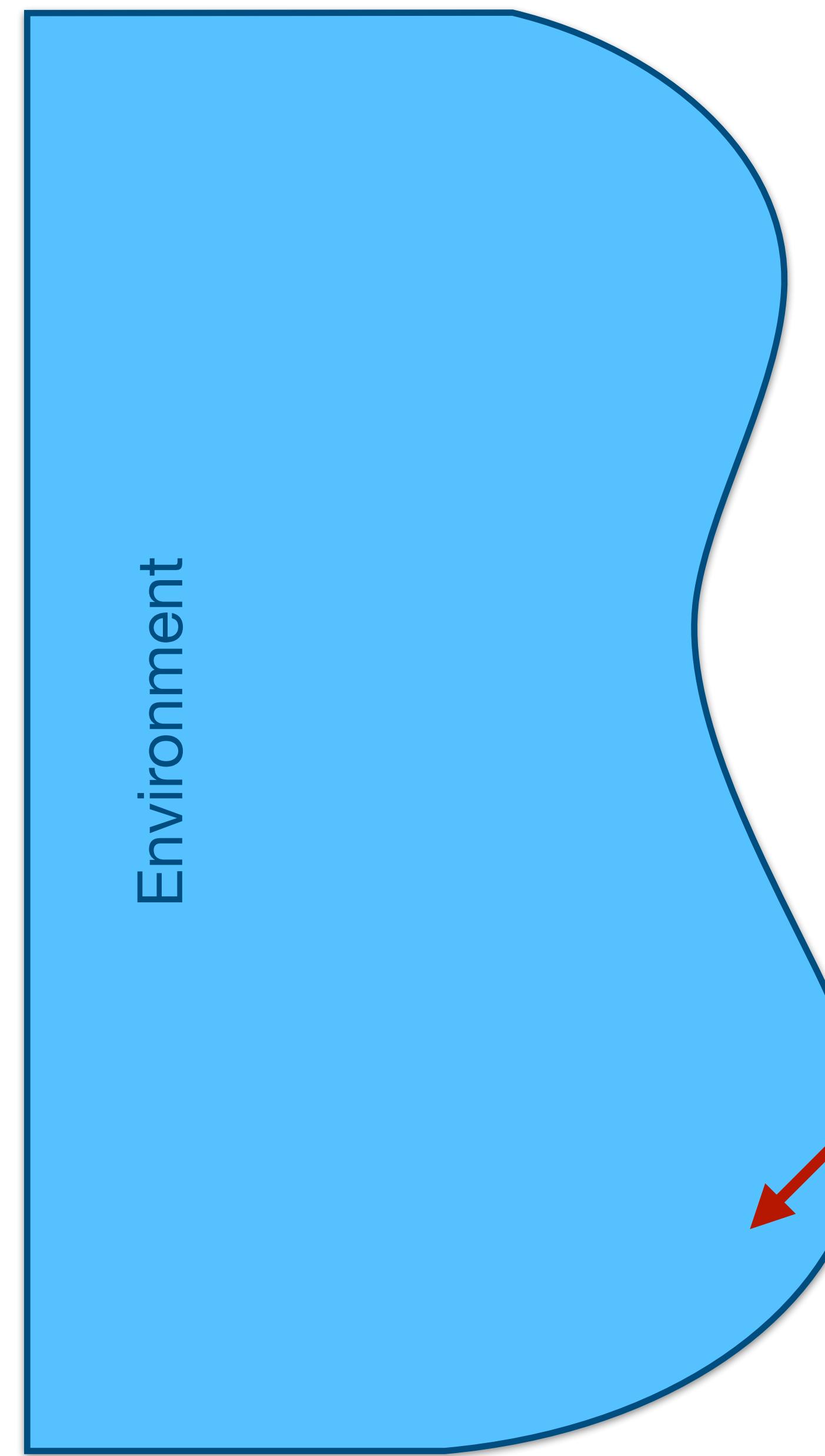
Robot tries to establish and maintain a desired contact force between EE and env.

Only works when contact has already been established!



Robot tries to establish  
and maintain a  
desired contact force  
between EE and env.

Only works when contact has  
already been established!



Robot tries to establish  
and maintain a  
desired contact force  
between EE and env.

Only works when contact has  
already been established!

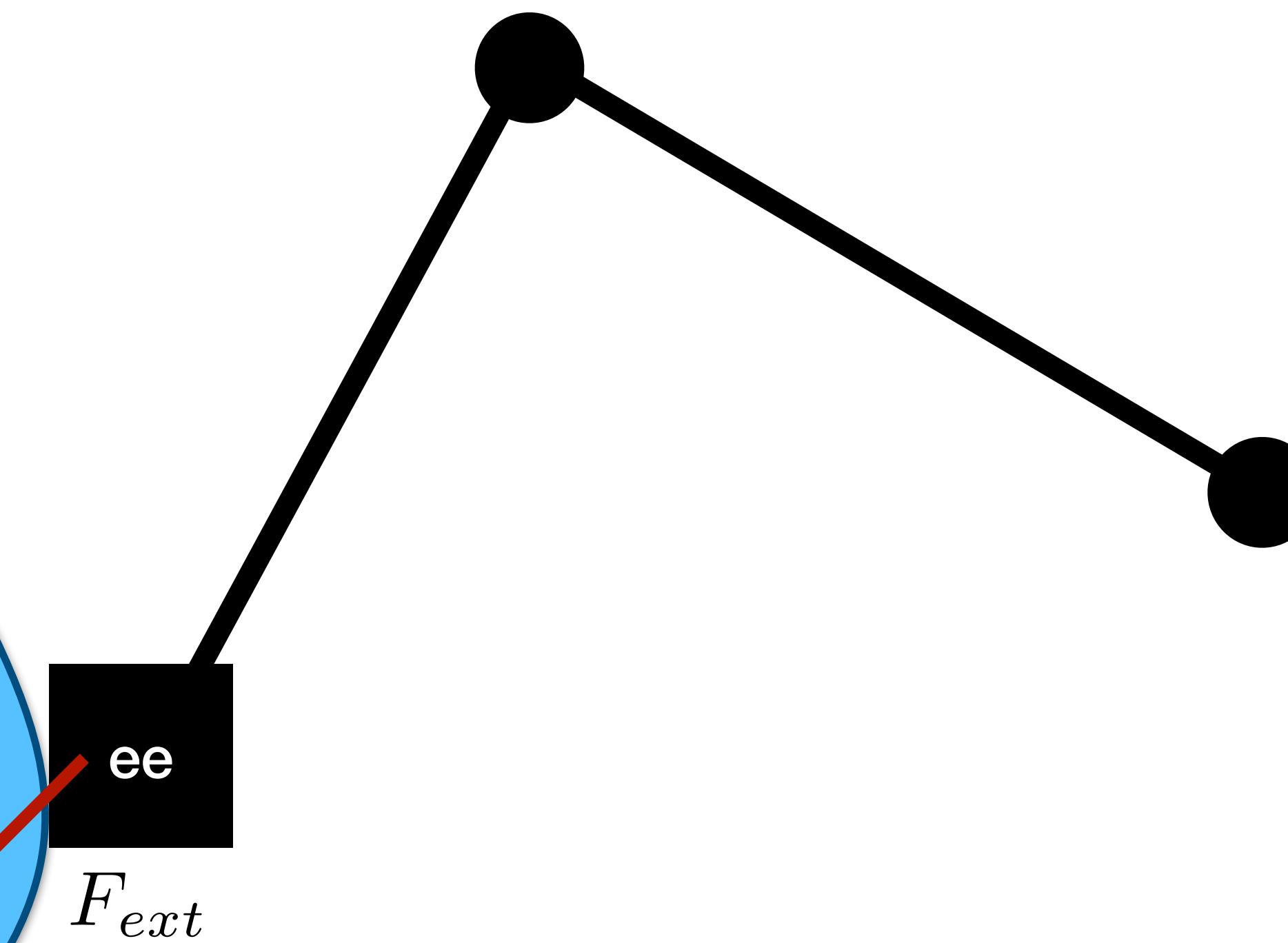
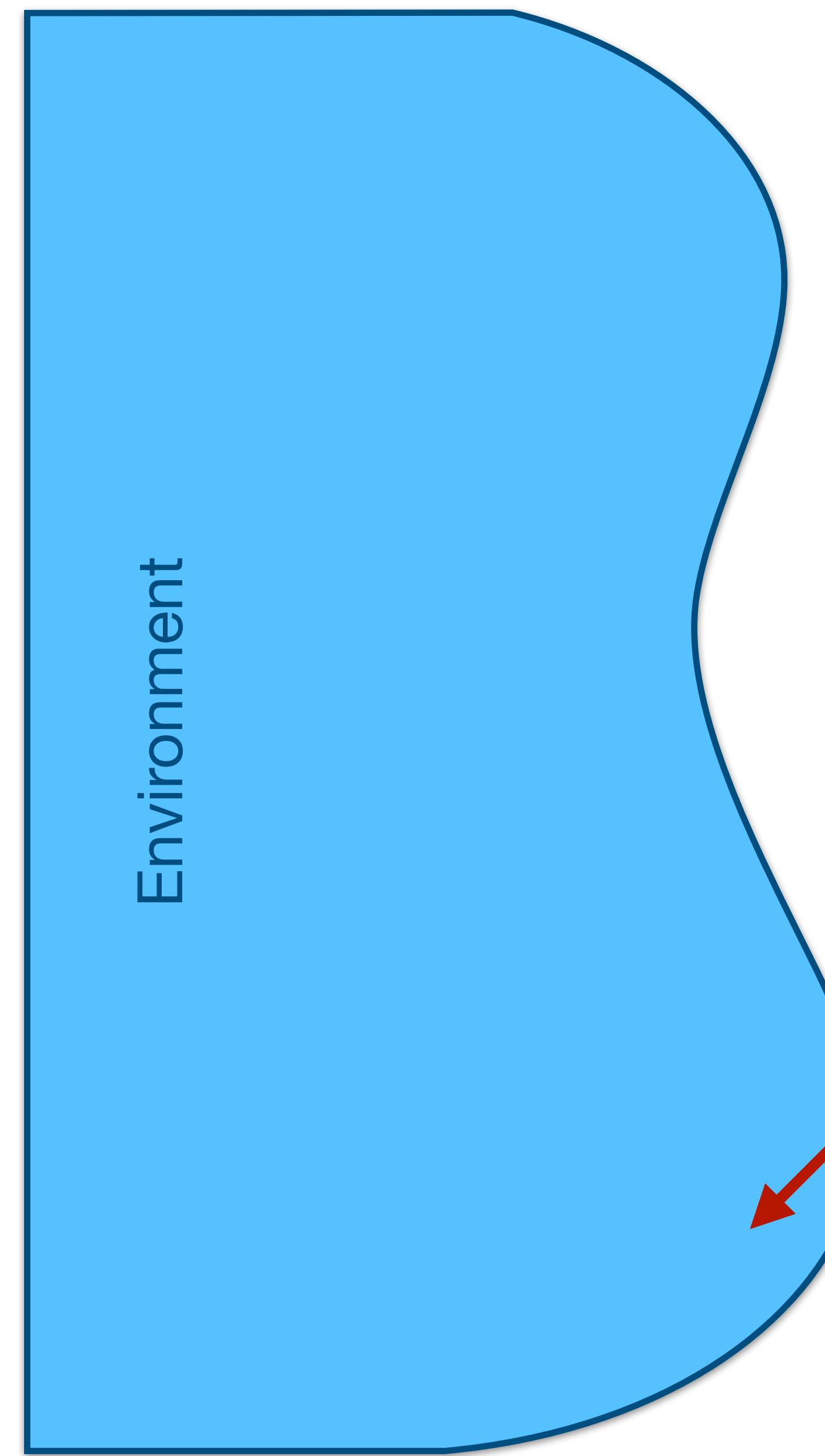
Environment

## The Problem with Force Control



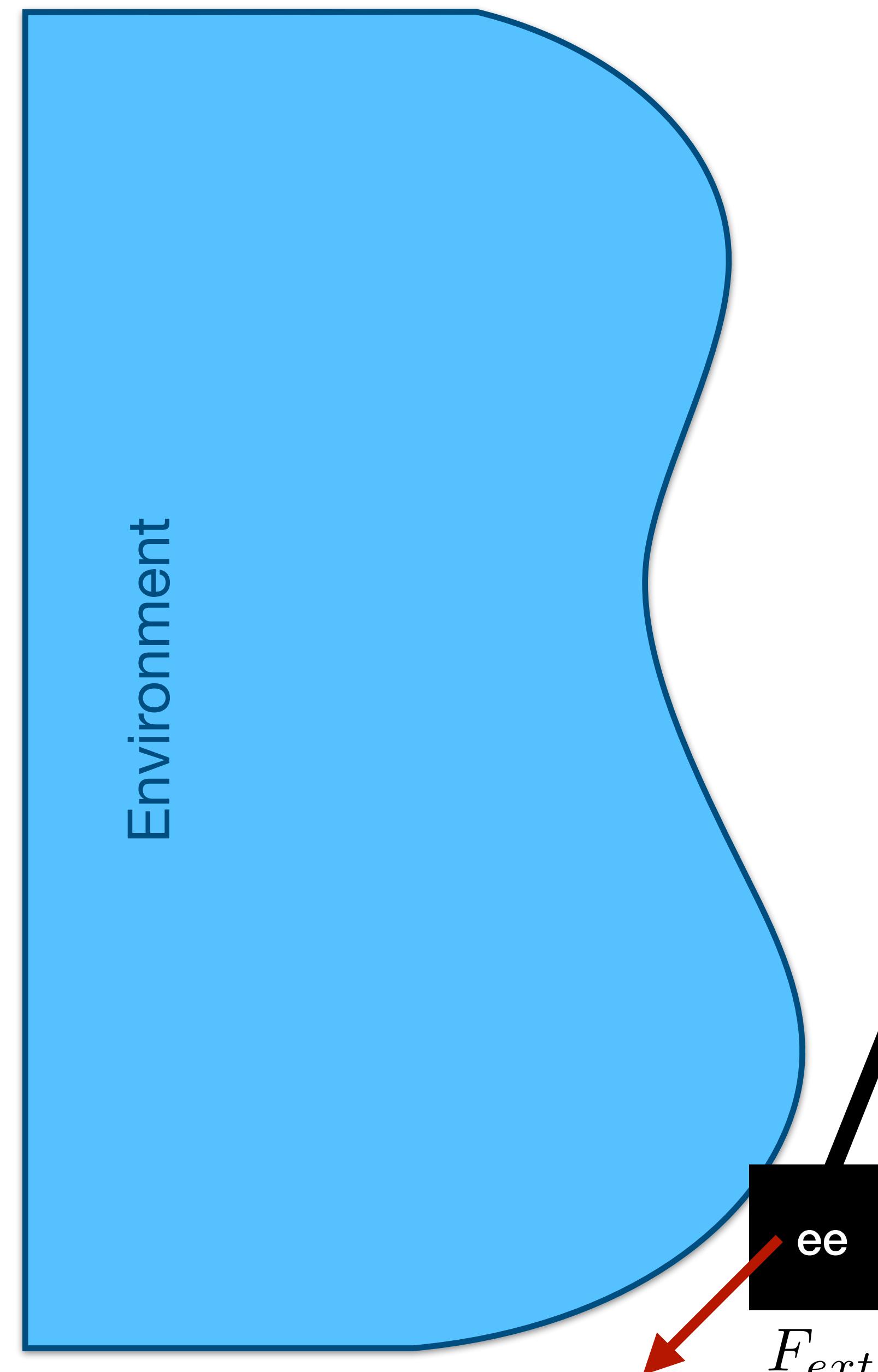
Only works when contact has  
already been established!

Task: Slide over surface  
Approach: Force Control



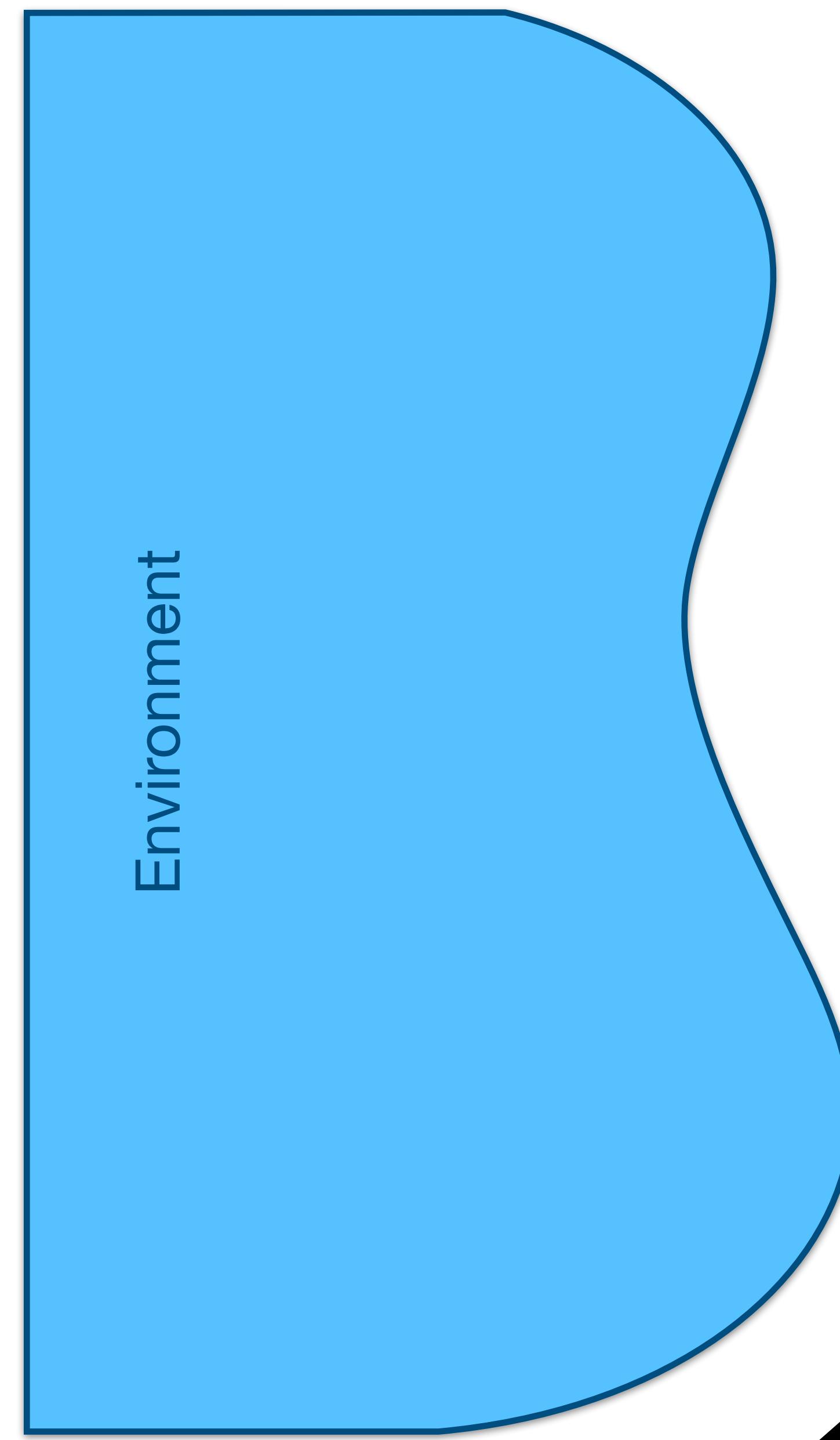
Robot tries to establish  
and maintain a  
desired contact force  
between EE and env.

Only works when contact has  
already been established!

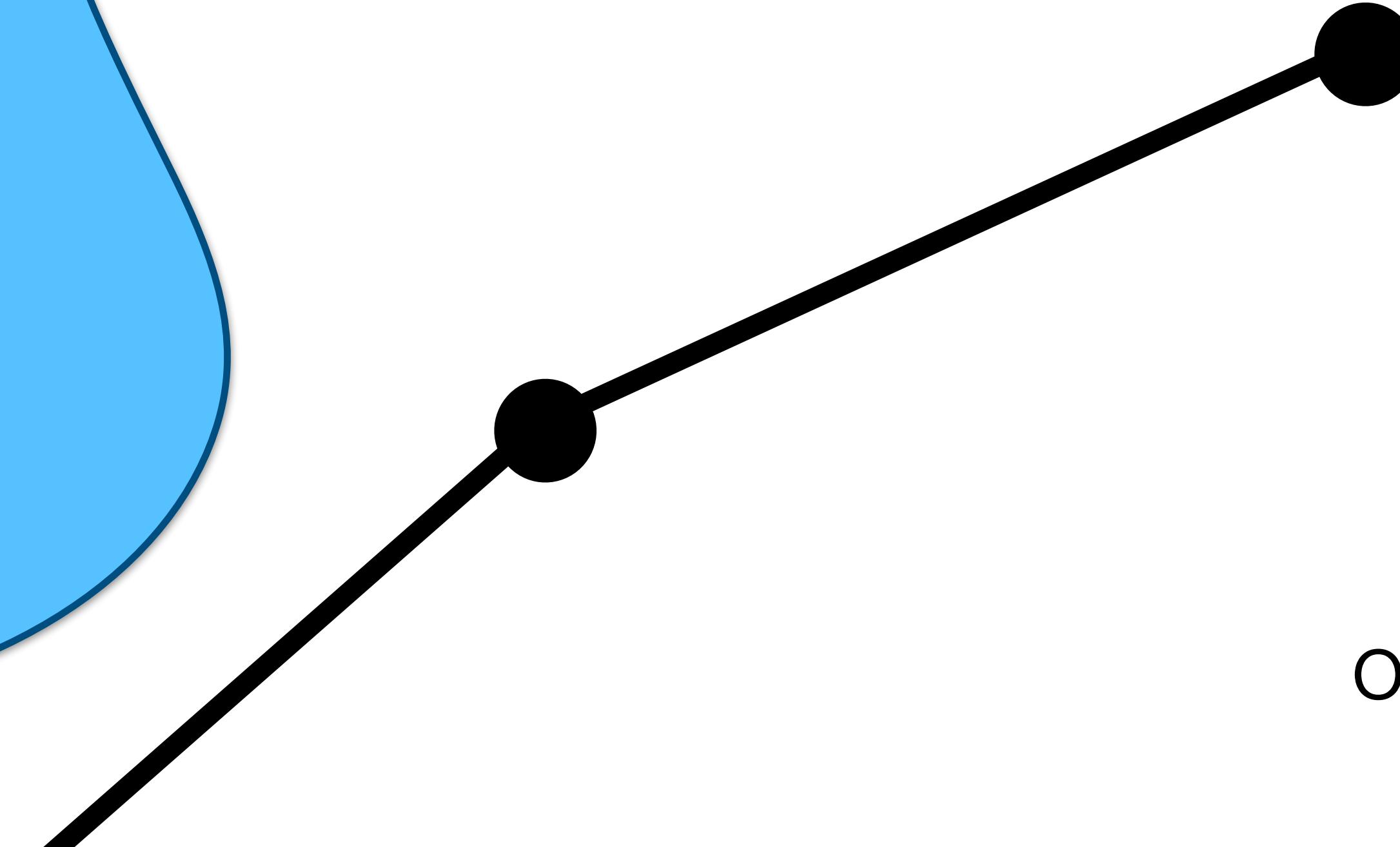


Robot tries to establish  
and maintain a  
desired contact force  
between EE and env.

Only works when contact has  
already been established!

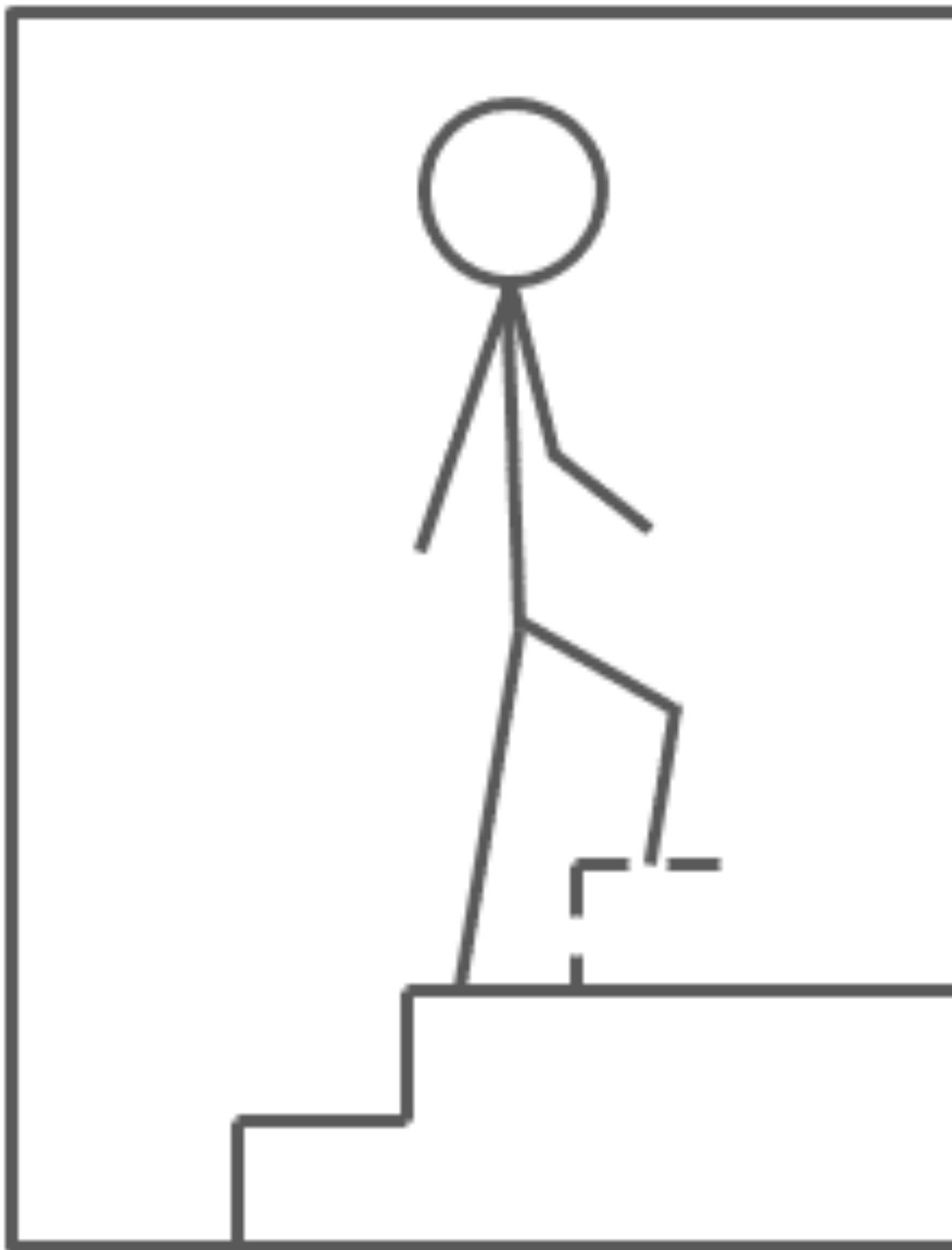


Environment



Robot tries to establish  
and maintain a  
desired contact force  
between EE and env.

Only works when contact has  
already been established!



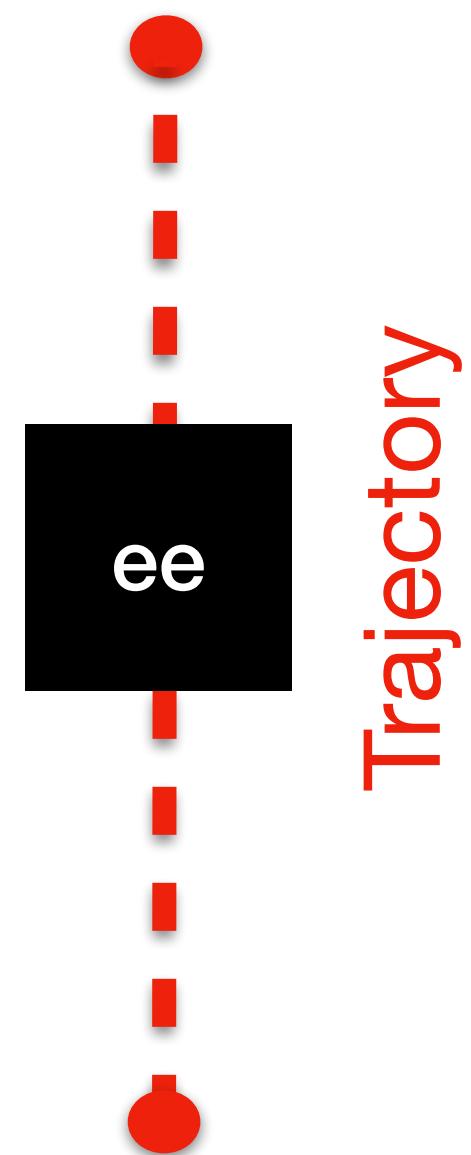
Bad Idea:  
Climbing Stairs with Force Control

Approach III:  
**Hybrid Position/Force Control**

Workspace is divided into two subspaces:

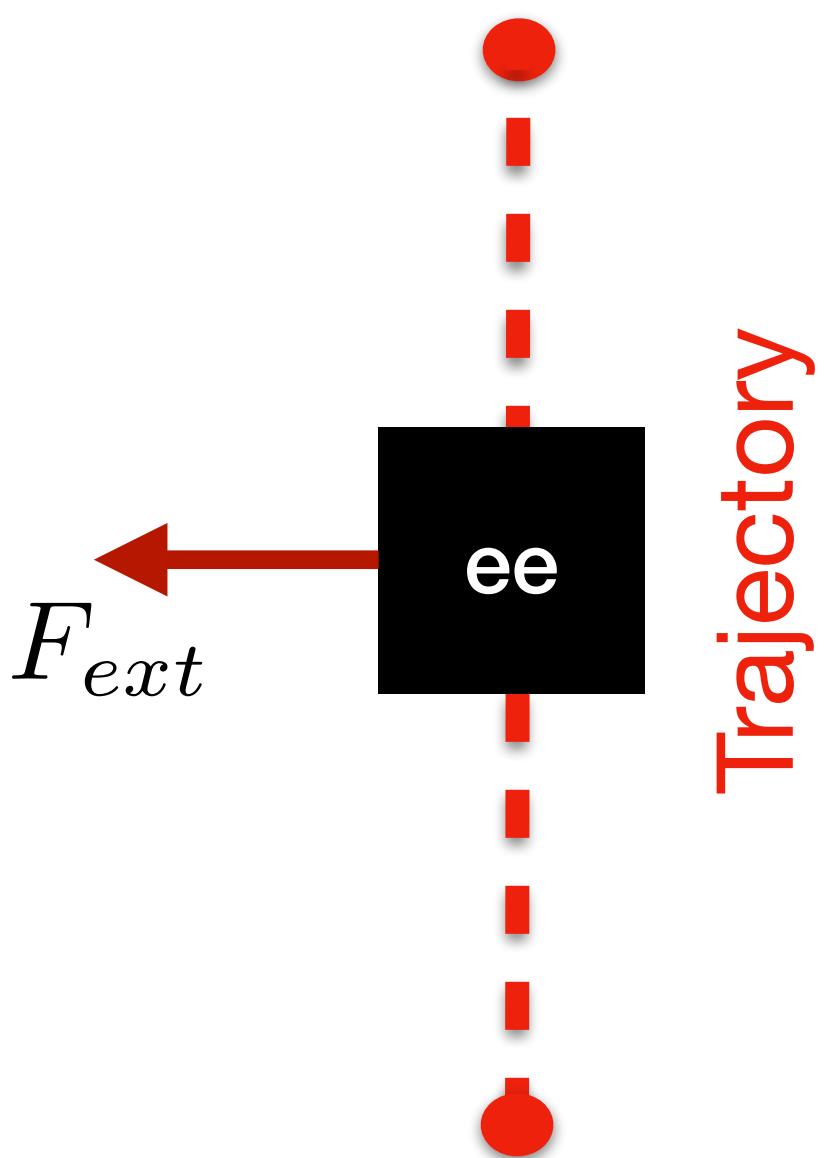
ee

Workspace is divided into two subspaces:  
\* Subspace for Position Control



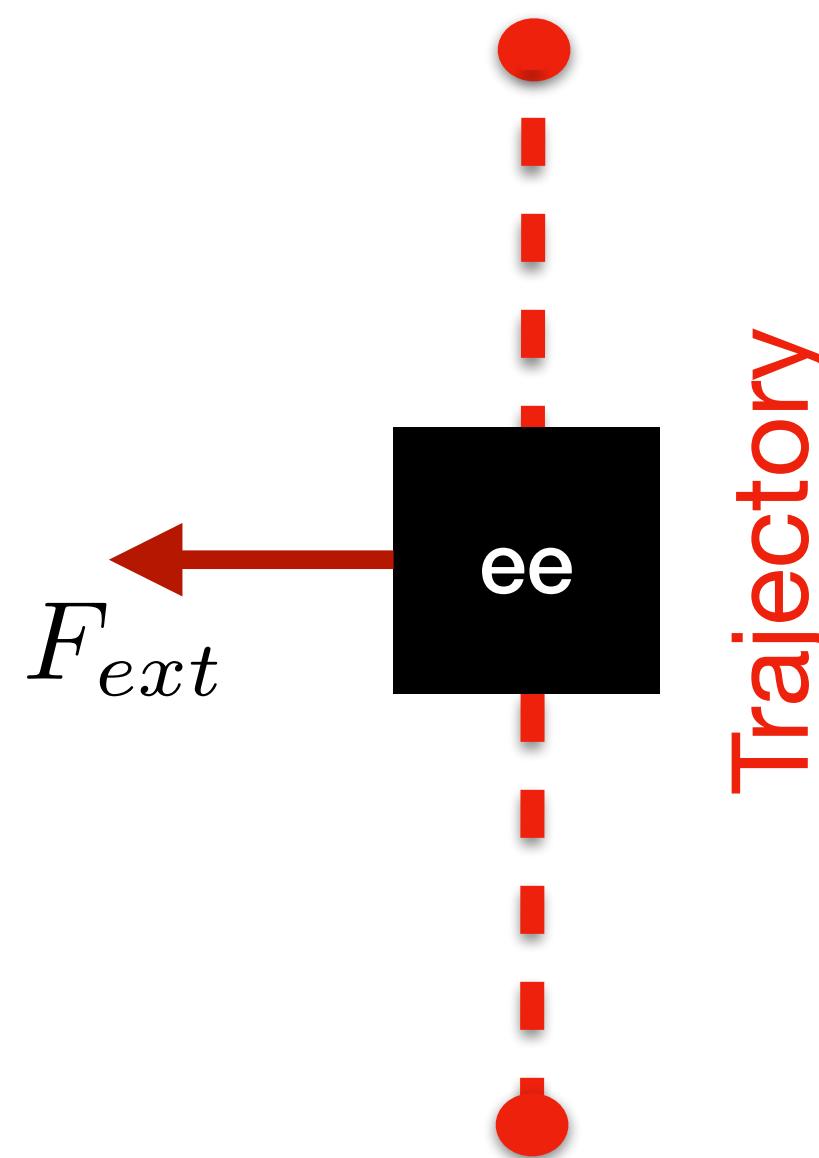
Workspace is divided into two subspaces:

- \* Subspace for Position Control
- \* Subspace for Force Control



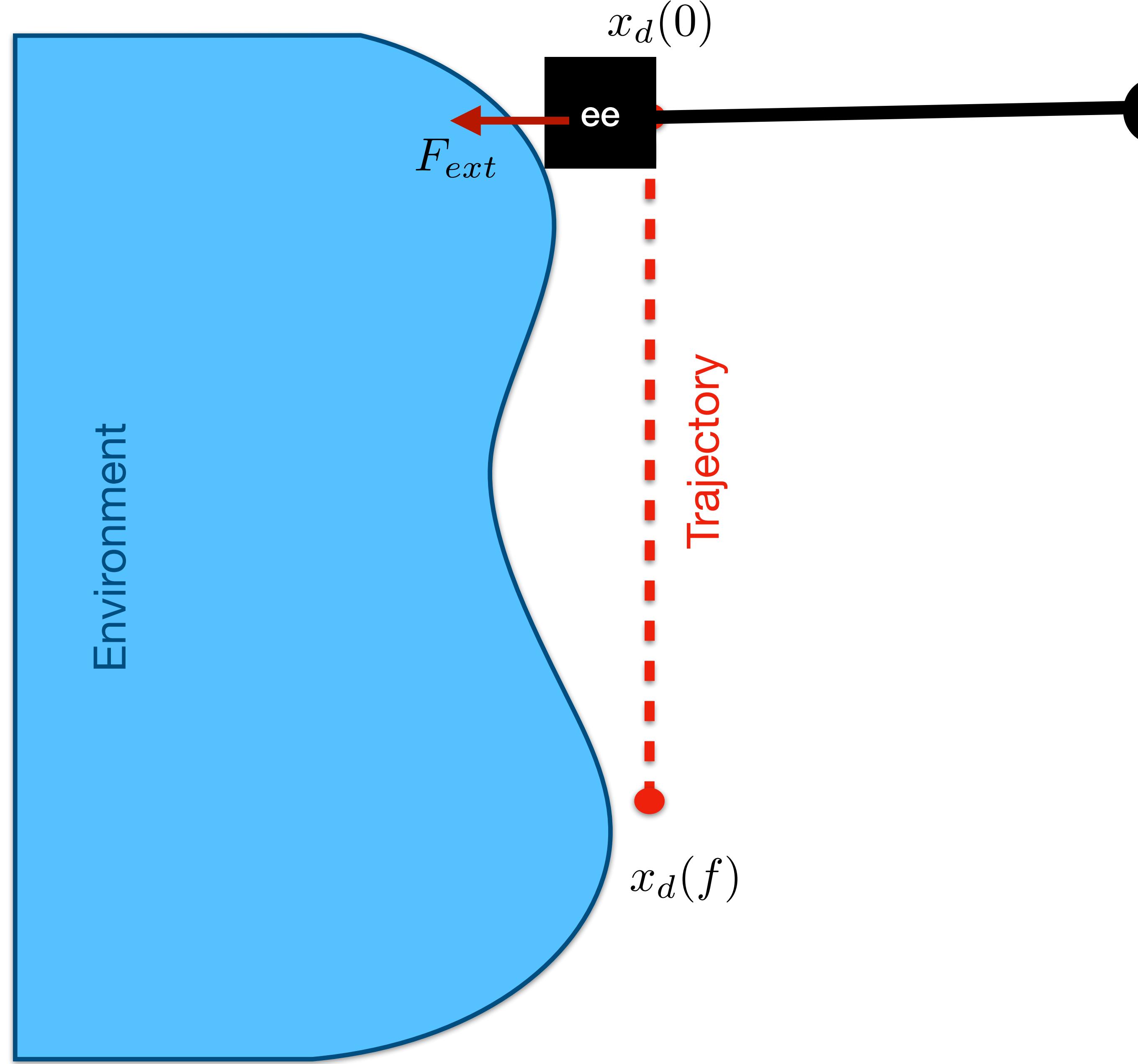
Workspace is divided into two subspaces:

- \* Subspace for Position Control
- \* Subspace for Force Control



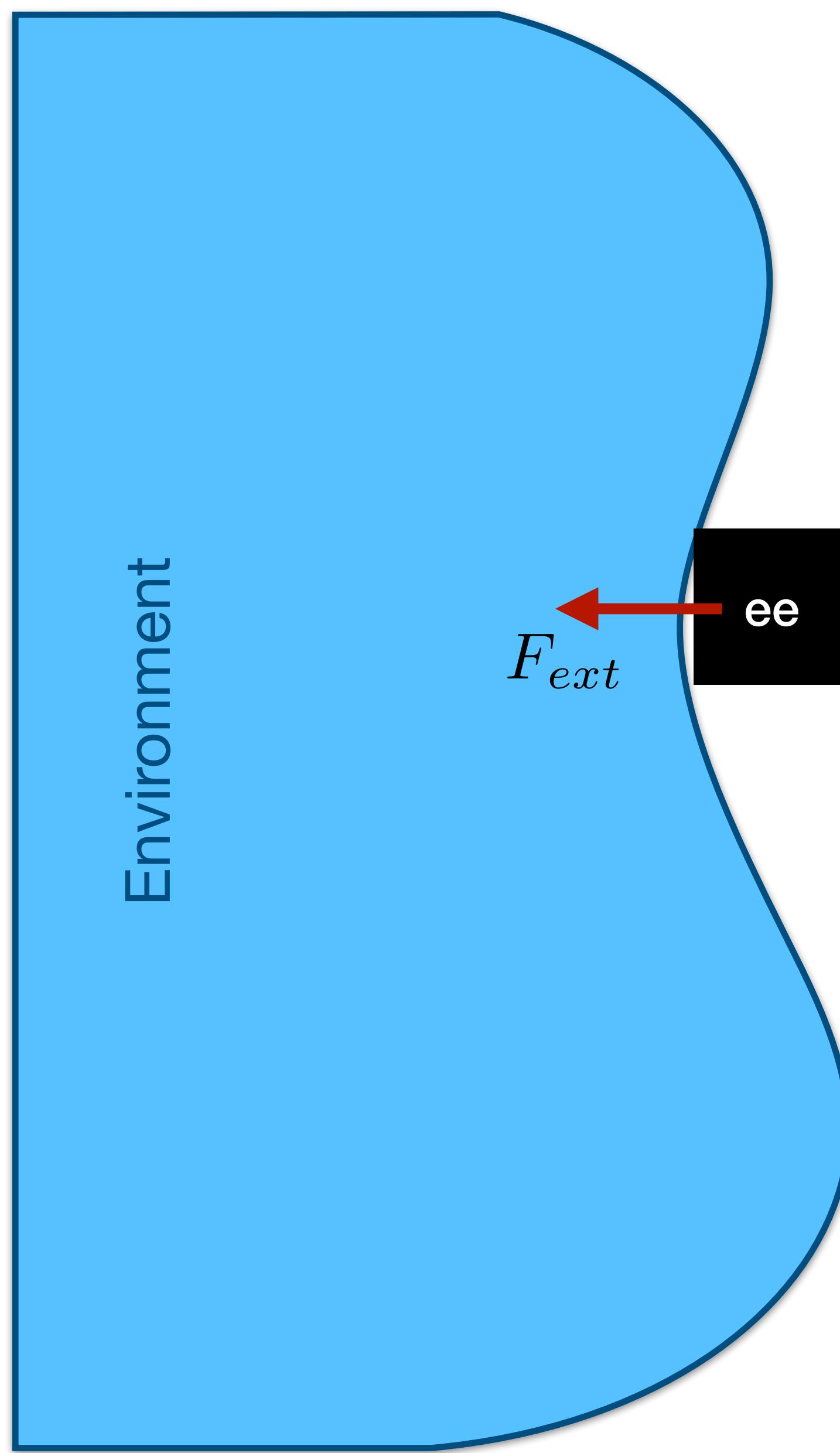
Robot moves along desired trajectory with high precision in the one subspace

and tries to maintain desired contact force between EE and env in the other subspace.



Robot moves along desired trajectory with high precision in the one subspace and tries to maintain desired contact force between EE and env in the other subspace.

Only works when contact has already been established!



$x_d(0)$

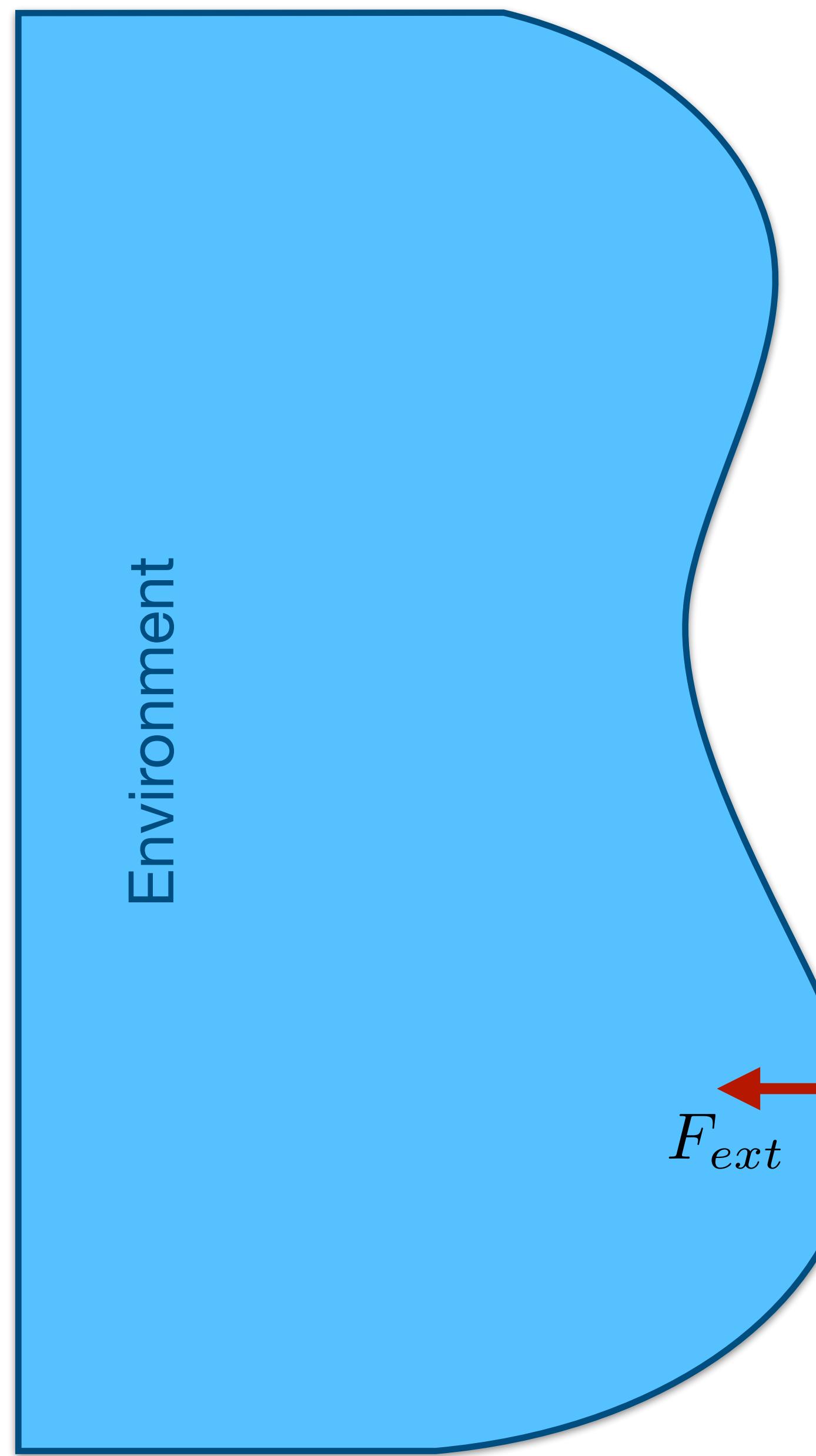
Trajectory

$x_d(f)$

Robot moves along desired trajectory with high precision in the one subspace

and tries to maintain desired contact force between EE and env in the other subspace.

Only works when contact has already been established!



$x_d(0)$

Trajectory

$x_d(f)$

ee

$F_{ext}$

Robot moves along desired trajectory with high precision in the one subspace

and tries to maintain desired contact force between EE and env in the other subspace.

Only works when contact has already been established!

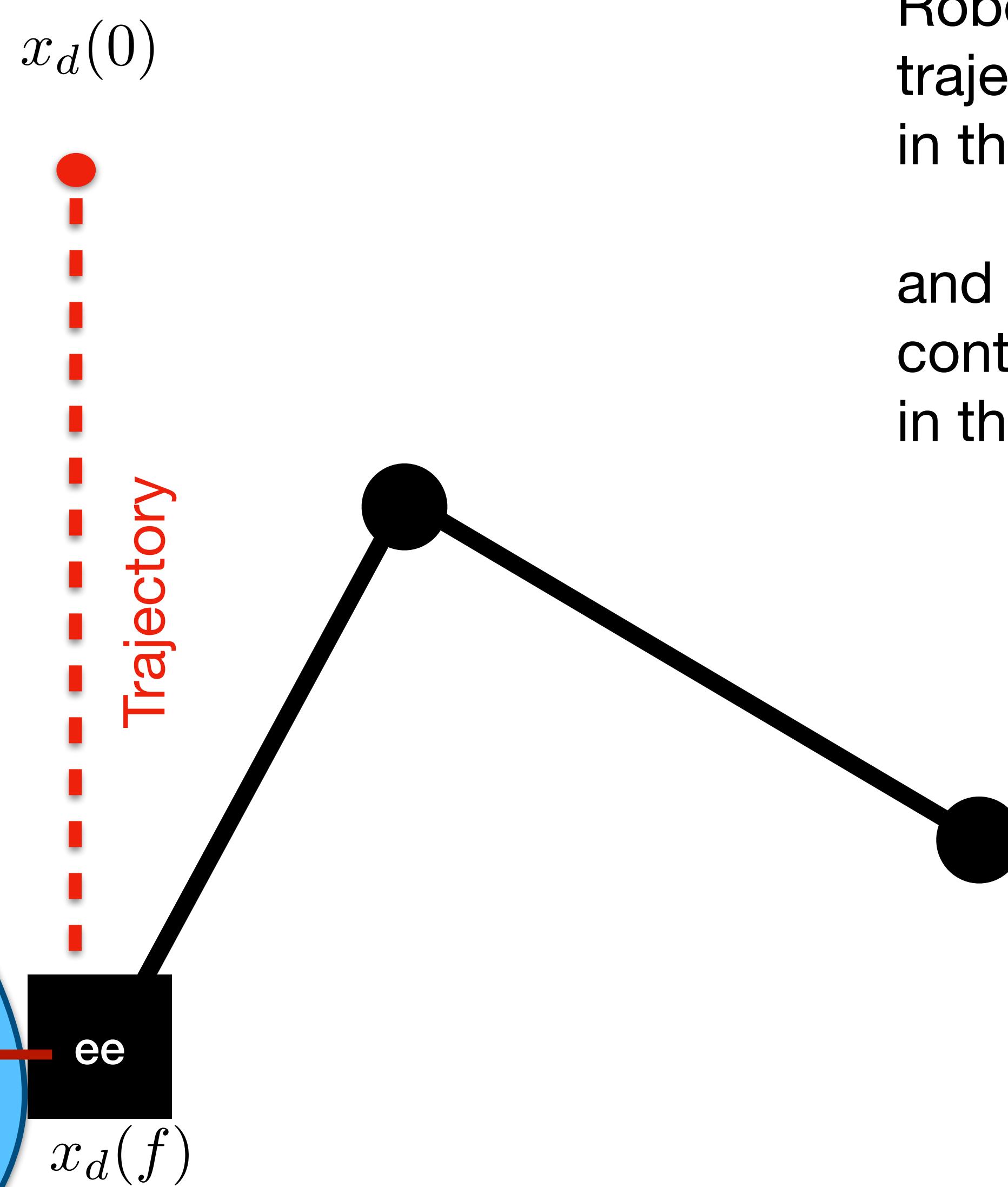
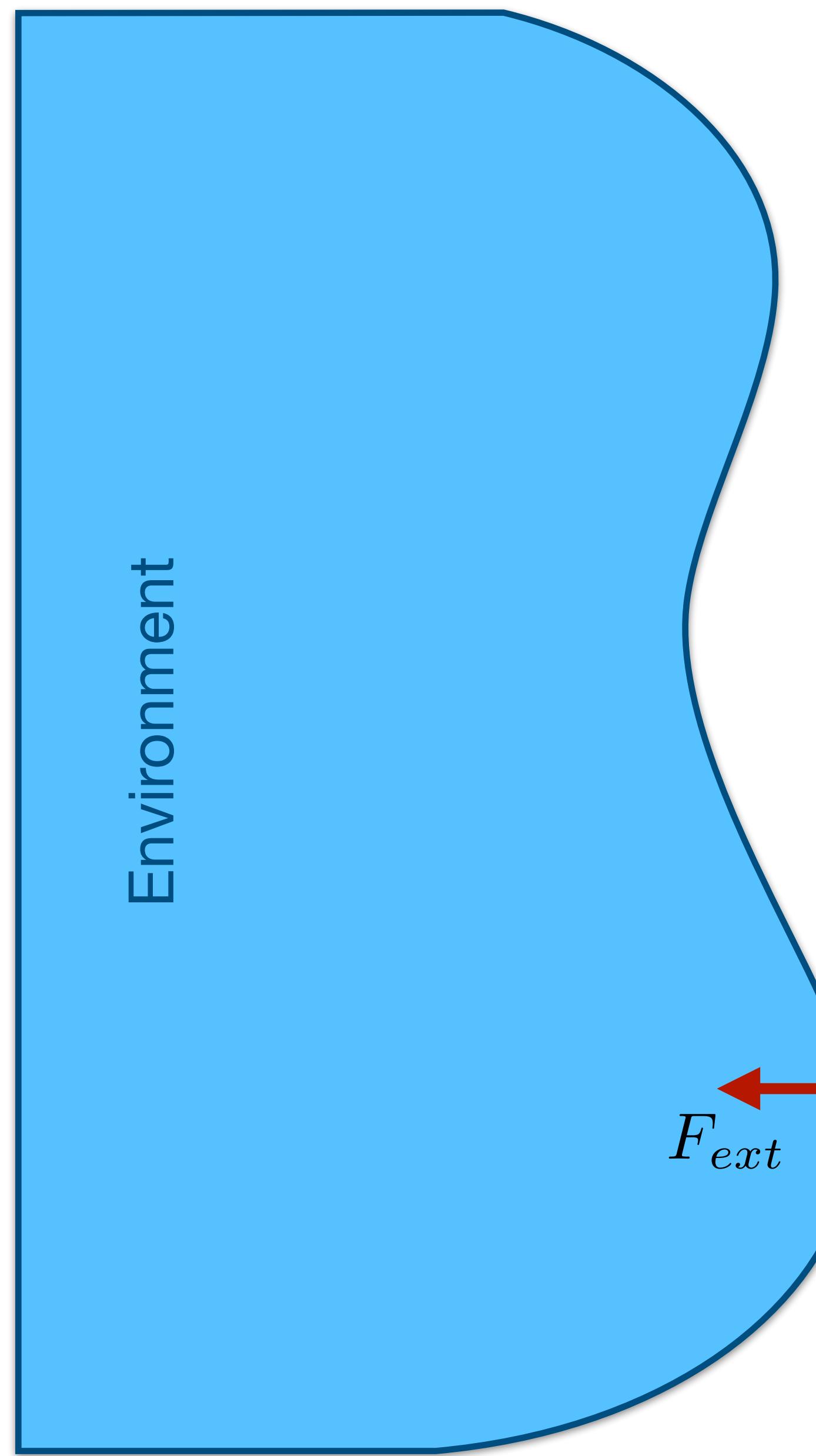
Environment

## The Problem with Hybrid Position/Force Control



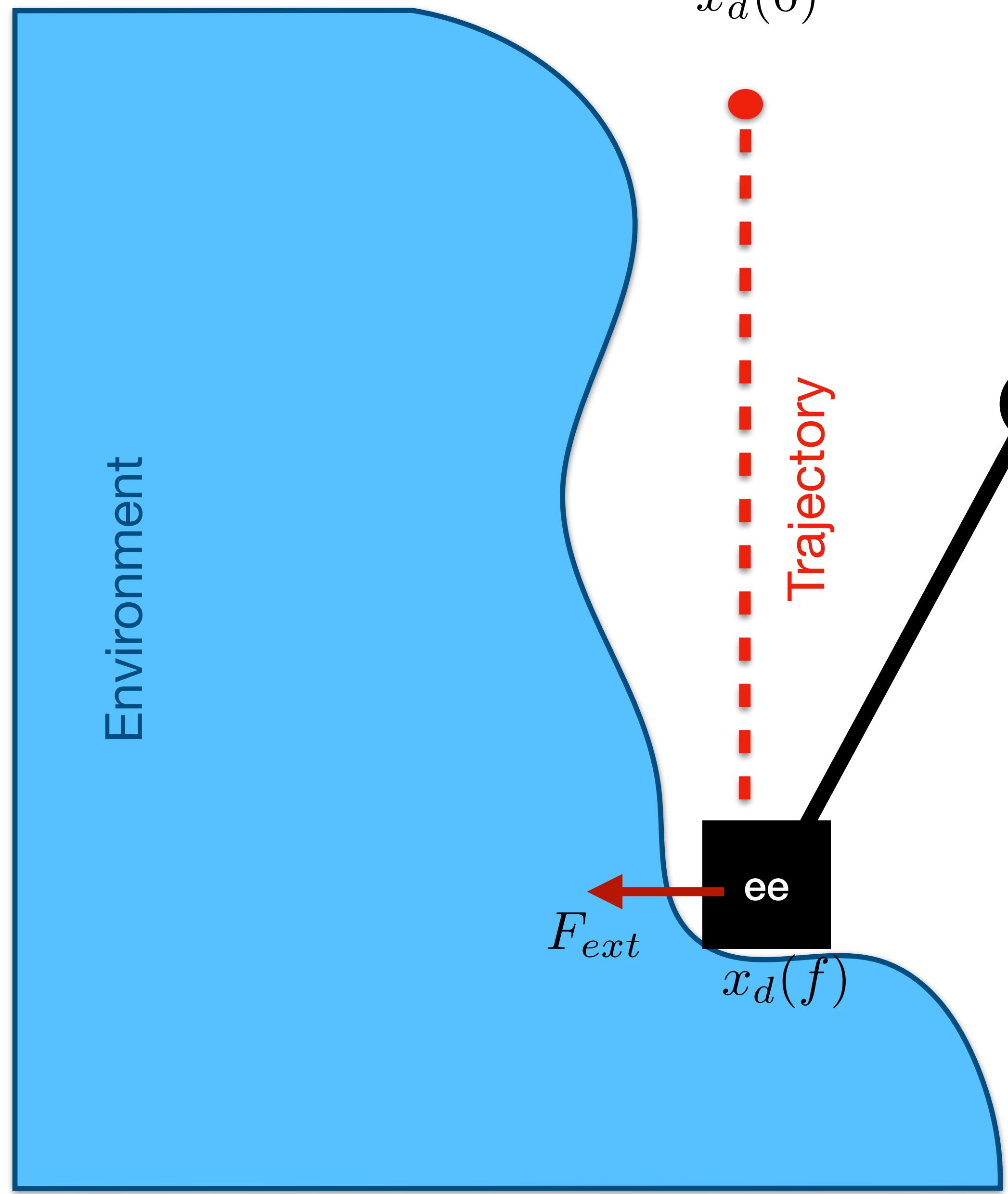
Only works when contact has  
already been established!

Task: Slide over surface  
Approach: Force Control



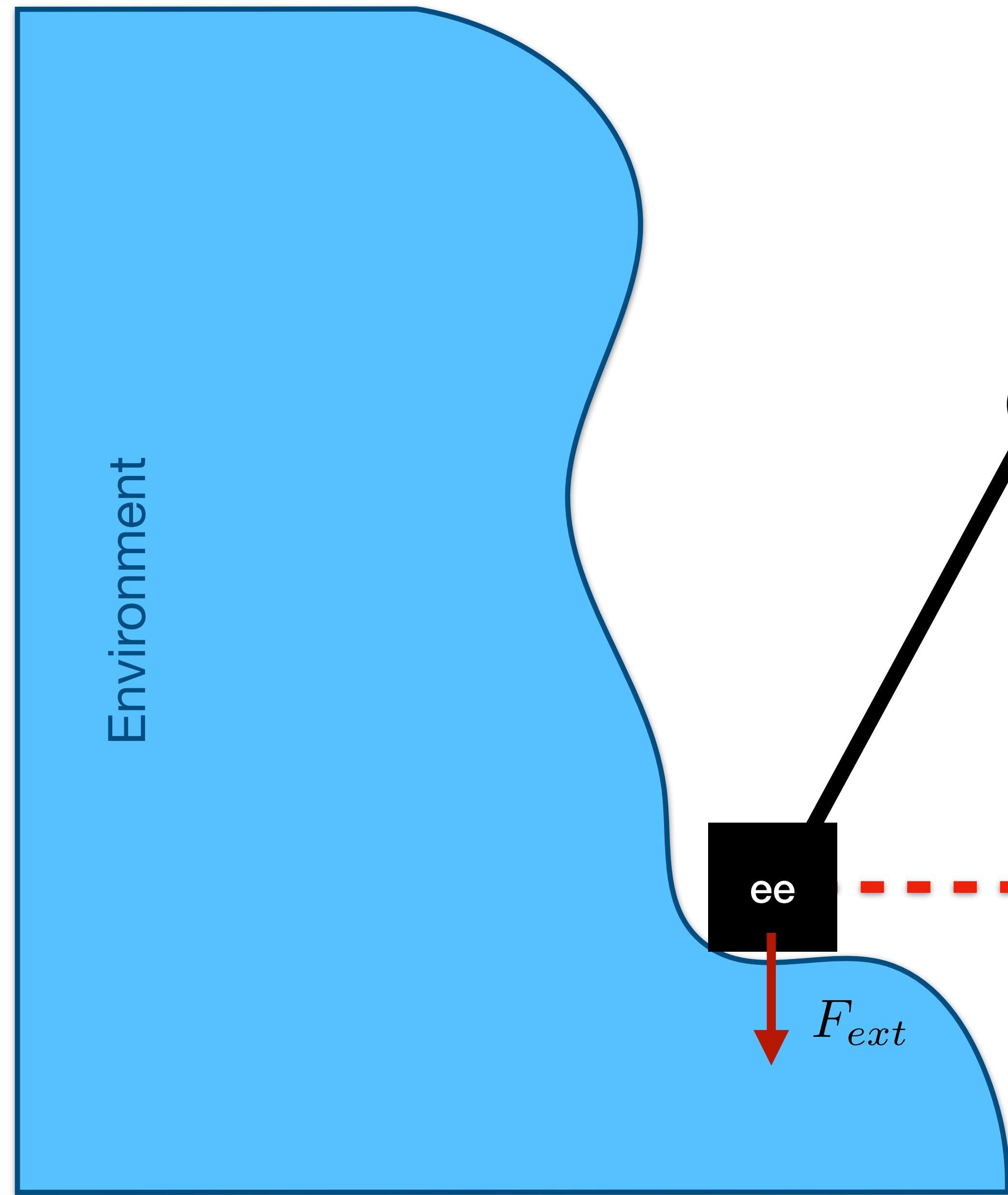
Robot moves along desired trajectory with high precision in the one subspace and tries to maintain desired contact force between EE and env in the other subspace.

Only works when contact has already been established!



Robot moves along desired trajectory with high precision in the one subspace and tries to maintain desired contact force between EE and env in the other subspace.

Only works when contact has already been established!



Robot moves along desired trajectory with high precision in the one subspace and tries to maintain desired contact force between EE and env in the other subspace.

Only works when contact has already been established!

## **Approach IV: Impedance Control**

# Impedance Control

# Impedance Control

(Impedance: Measure of how much a structure resists motion)

# Impedance Control

(Impedance: Measure of how much a structure resists motion)

$$Z = \frac{F}{\Delta X}$$

# Impedance Control

(Impedance: Measure of how much a structure resists motion)

Impedance 

$$Z = \frac{F}{\Delta X}$$

# Impedance Control

(Impedance: Measure of how much a structure resists motion)

$$Z = \frac{F}{\Delta X}$$

Impedance →

Input: Displacement ←

# Impedance Control

(Impedance: Measure of how much a structure resists motion)

Output: Force

Impedance

$$Z = \frac{F}{\Delta X}$$

Input: Displacement

The diagram illustrates the formula for impedance,  $Z = \frac{F}{\Delta X}$ . Handwritten annotations include "Output: Force" above the formula, "Input: Displacement" below it, and "Impedance" to the left of the fraction. Red arrows point from each word to its corresponding term in the equation: one arrow points from "Output: Force" to the force term  $F$ , another from "Input: Displacement" to the displacement term  $\Delta X$ , and a third from "Impedance" to the fraction itself.

# Impedance Control

(Impedance: Measure of how much a structure resists motion)

Output: Force

Impedance

$$Z = \frac{F}{\Delta X}$$

Input: Displacement

The diagram illustrates the relationship between force, displacement, and impedance. At the top, the text "Output: Force" is positioned above the equation  $Z = \frac{F}{\Delta X}$ . To the left of the equation, the word "Impedance" is written with a red curved arrow pointing from it to the symbol  $Z$ . Below the equation, the text "Input: Displacement" is written with a red curved arrow pointing from it to the symbol  $\Delta X$ .

$$Z^{-1} = \frac{\Delta X}{F}$$

# Impedance Control

(Impedance: Measure of how much a structure resists motion)

Output: Force

$$Z = \frac{F}{\Delta X}$$

Impedance

Input: Displacement

Admittance

$$Z^{-1} = \frac{\Delta X}{F}$$

The diagram illustrates the mathematical relationship between impedance and admittance. It features two equations side-by-side. The left equation,  $Z = \frac{F}{\Delta X}$ , represents impedance, with 'Impedance' written above it and a red curved arrow pointing to the variable  $Z$ . The right equation,  $Z^{-1} = \frac{\Delta X}{F}$ , represents admittance, with 'Admittance' written above it and a red curved arrow pointing to the variable  $Z^{-1}$ . The text 'Output: Force' is positioned above the first equation, and 'Input: Displacement' is positioned below the second equation.

# Impedance Control

(Impedance: Measure of how much a structure resists motion)

The diagram illustrates the relationship between Impedance and Admittance. It features two equations side-by-side. The left equation, labeled "Output: Force" at the top and "Input: Displacement" at the bottom, shows Impedance ( $Z$ ) as the ratio of Force ( $F$ ) to Displacement ( $\Delta X$ ). The right equation, labeled "Output: Displacement" at the top and "Admittance" at the bottom, shows Admittance ( $Z^{-1}$ ) as the ratio of Displacement ( $\Delta X$ ) to Force ( $F$ ). Red arrows point from the labels "Impedance" and "Admittance" to their respective equations.

$$Z = \frac{F}{\Delta X}$$
$$Z^{-1} = \frac{\Delta X}{F}$$

# Impedance Control

(Impedance: Measure of how much a structure resists motion)

The diagram illustrates the relationship between Impedance and Admittance. It consists of two main parts. On the left, labeled "Output: Force" at the top and "Input: Displacement" at the bottom, is the equation  $Z = \frac{F}{\Delta X}$ . Red arrows point from "Impedance" to the top of the fraction and from "Input: Displacement" to the bottom of the fraction. On the right, labeled "Output: Displacement" at the top and "Input: Force" at the bottom, is the equation  $Z^{-1} = \frac{\Delta X}{F}$ . Red arrows point from "Admittance" to the top of the fraction and from "Input: Force" to the bottom of the fraction.

$$Z = \frac{F}{\Delta X}$$
$$Z^{-1} = \frac{\Delta X}{F}$$

# Impedance Control

(Impedance: Measure of how much a structure resists motion)

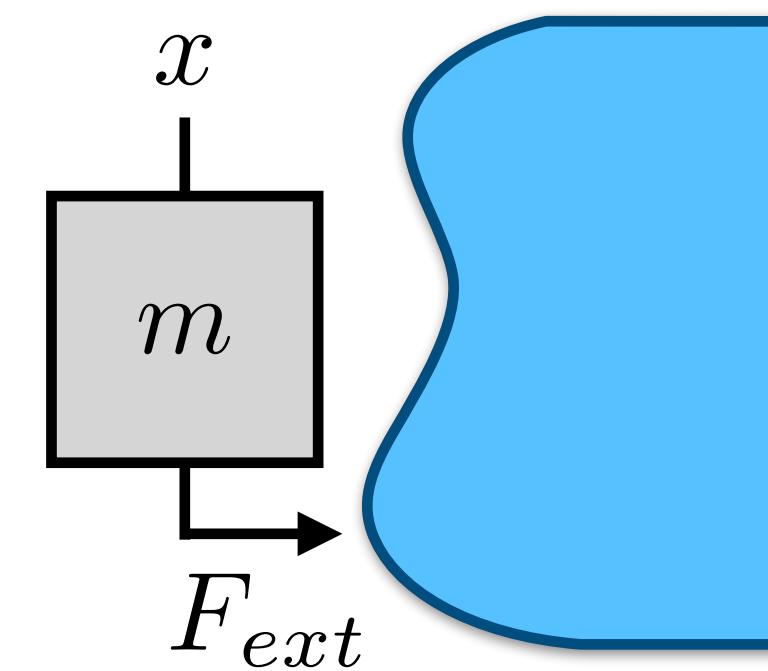
$$Z = \frac{F}{\Delta X}$$

Output: Force      Input: Displacement

Impedance      Admittance

$$Z^{-1} = \frac{\Delta X}{F}$$

Output: Displacement      Input: Force



# Impedance Control

(Impedance: Measure of how much a structure resists motion)

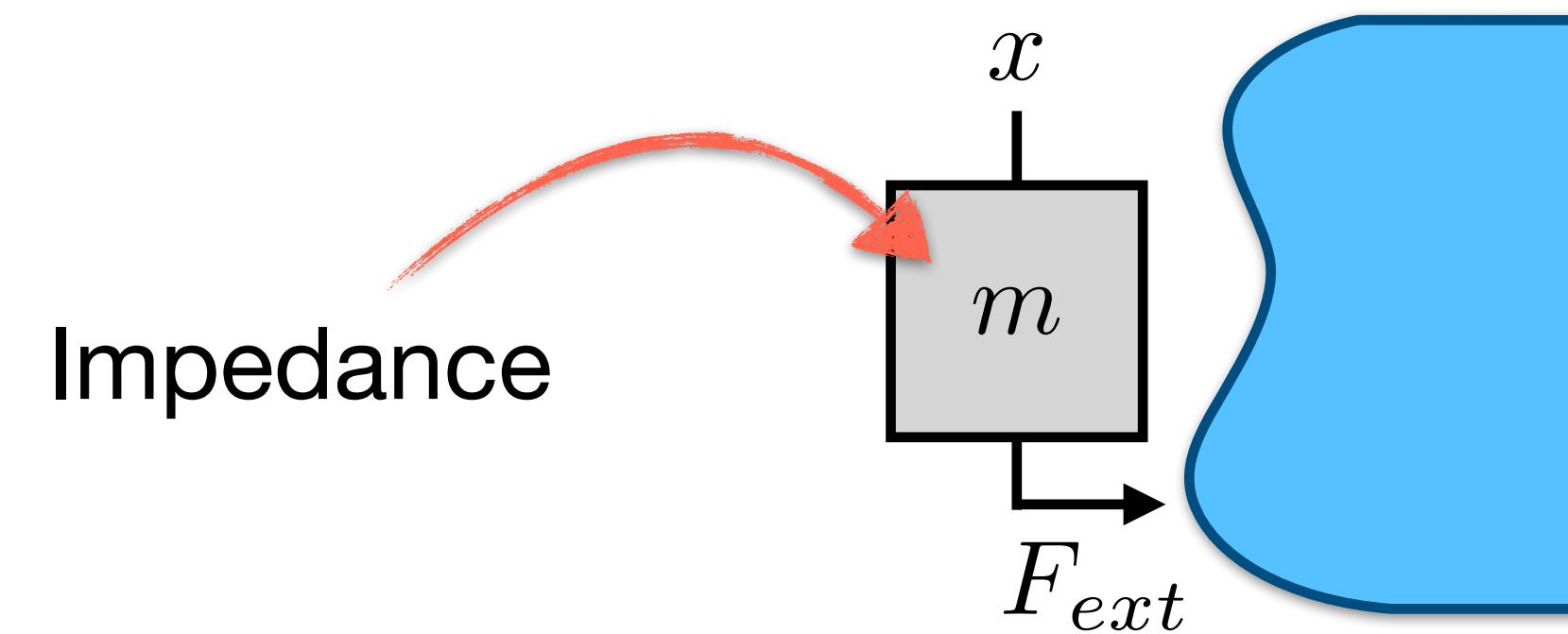
$$Z = \frac{F}{\Delta X}$$

Output: Force      Input: Displacement

Impedance      Admittance

$$Z^{-1} = \frac{\Delta X}{F}$$

Output: Displacement      Input: Force



# Impedance Control

(Impedance: Measure of how much a structure resists motion)

Output: Force

$$Z = \frac{F}{\Delta X}$$

Input: Displacement

Impedance

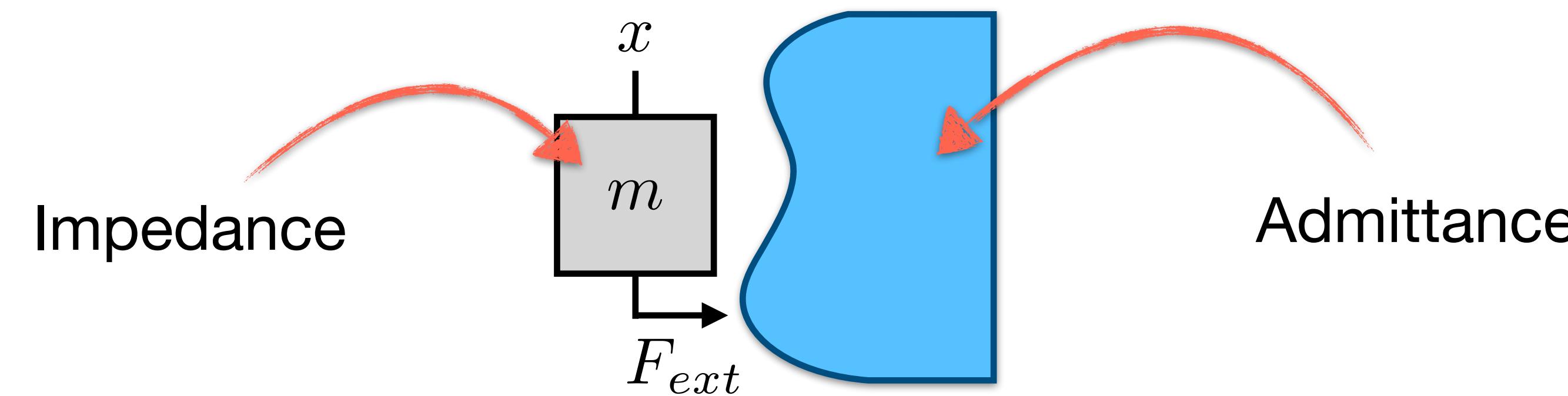
Admittance

Output: Displacement

$$Z^{-1} = \frac{\Delta X}{F}$$

Input: Force

The diagram illustrates the mathematical relationship between impedance and admittance. On the left, impedance is defined as force  $F$  divided by displacement  $\Delta X$ . Red arrows indicate the flow of information: one arrow from  $F$  to the numerator, another from  $\Delta X$  to the denominator, and a third from the result back to  $F$ . Below this, the input is labeled 'Displacement'. On the right, admittance is defined as displacement  $\Delta X$  divided by force  $F$ . Red arrows show the flow from  $\Delta X$  to the numerator, from  $F$  to the denominator, and from the result back to  $\Delta X$ . Below this, the input is labeled 'Force'.



# Impedance Control

(Impedance: Measure of how much a structure resists motion)

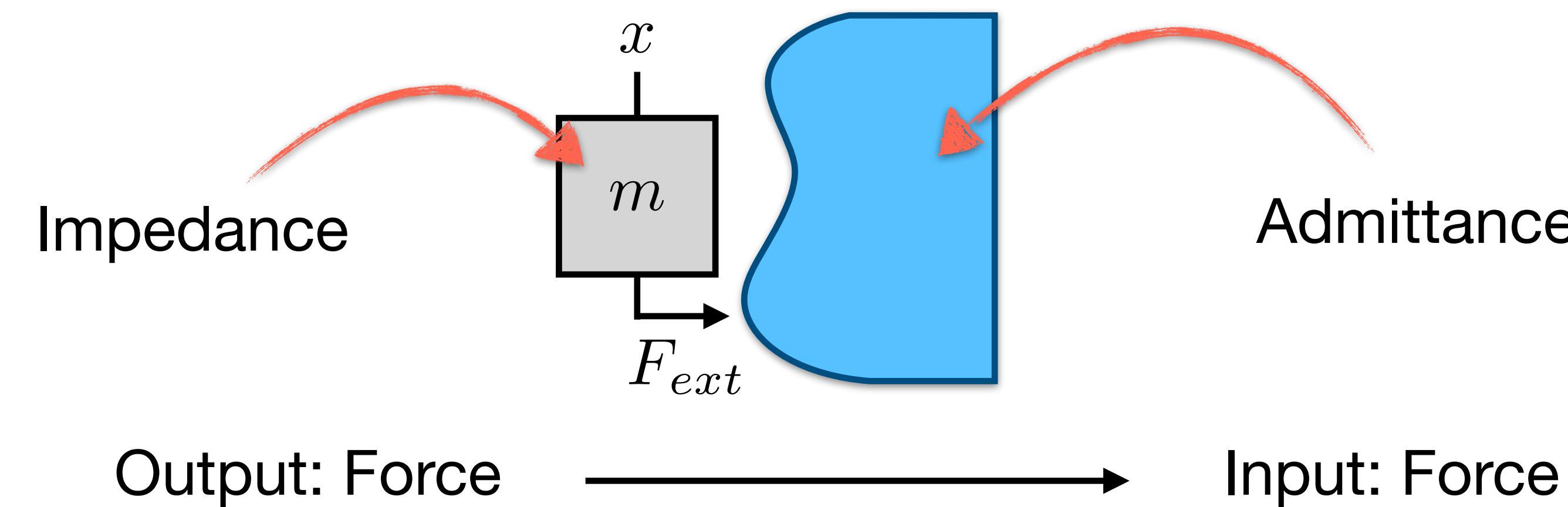
$$Z = \frac{F}{\Delta X}$$

Output: Force      Input: Displacement

Impedance      Admittance

$$Z^{-1} = \frac{\Delta X}{F}$$

Output: Displacement      Input: Force



# Impedance Control

(Impedance: Measure of how much a structure resists motion)

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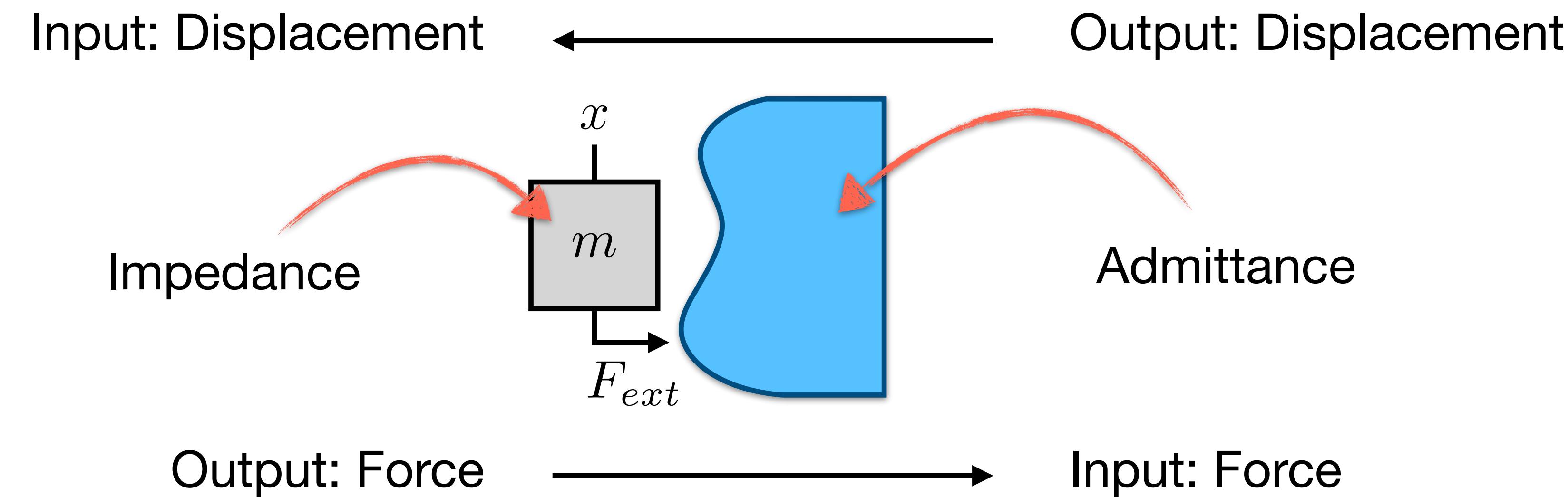
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Impedance

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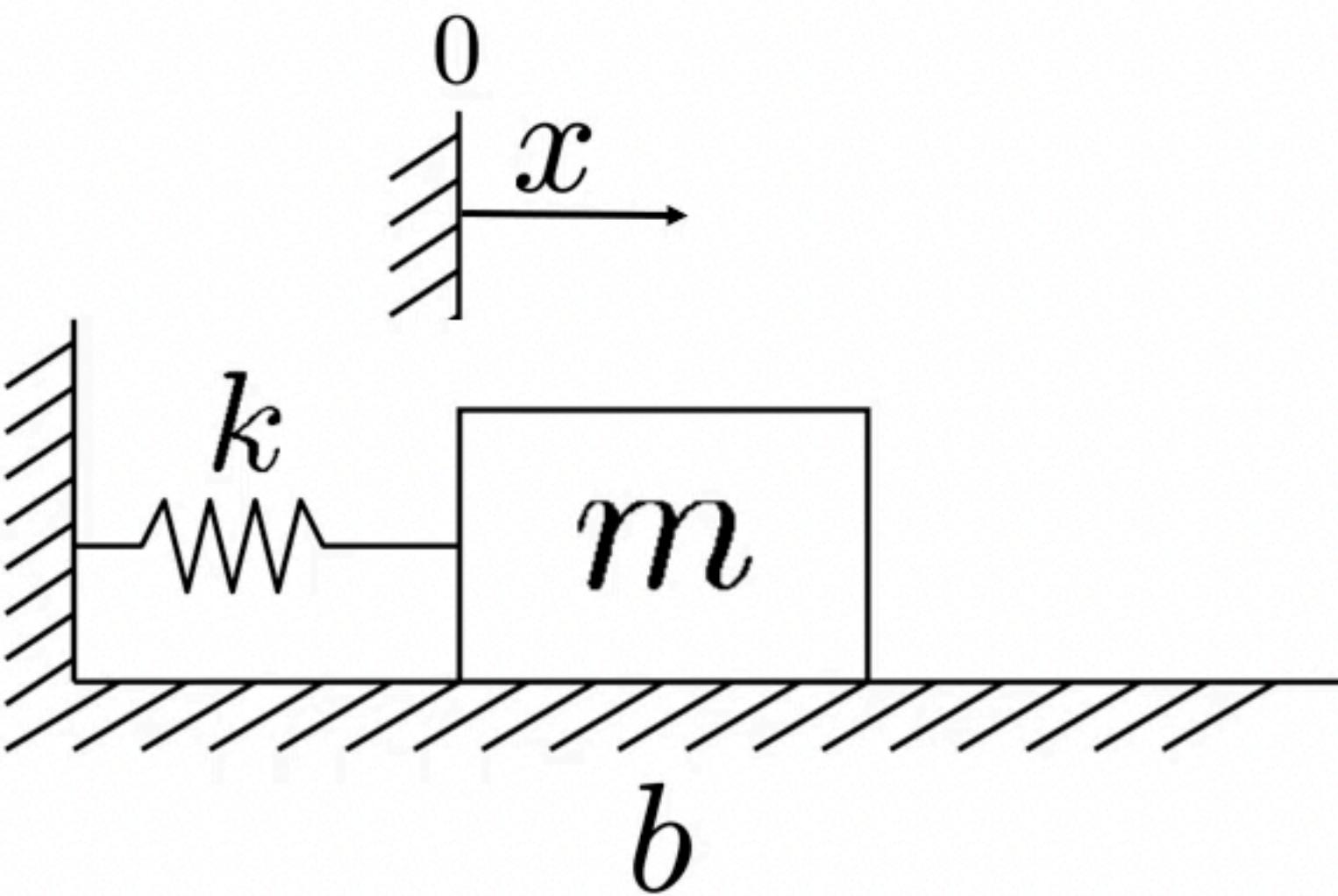
# How to model impedance?

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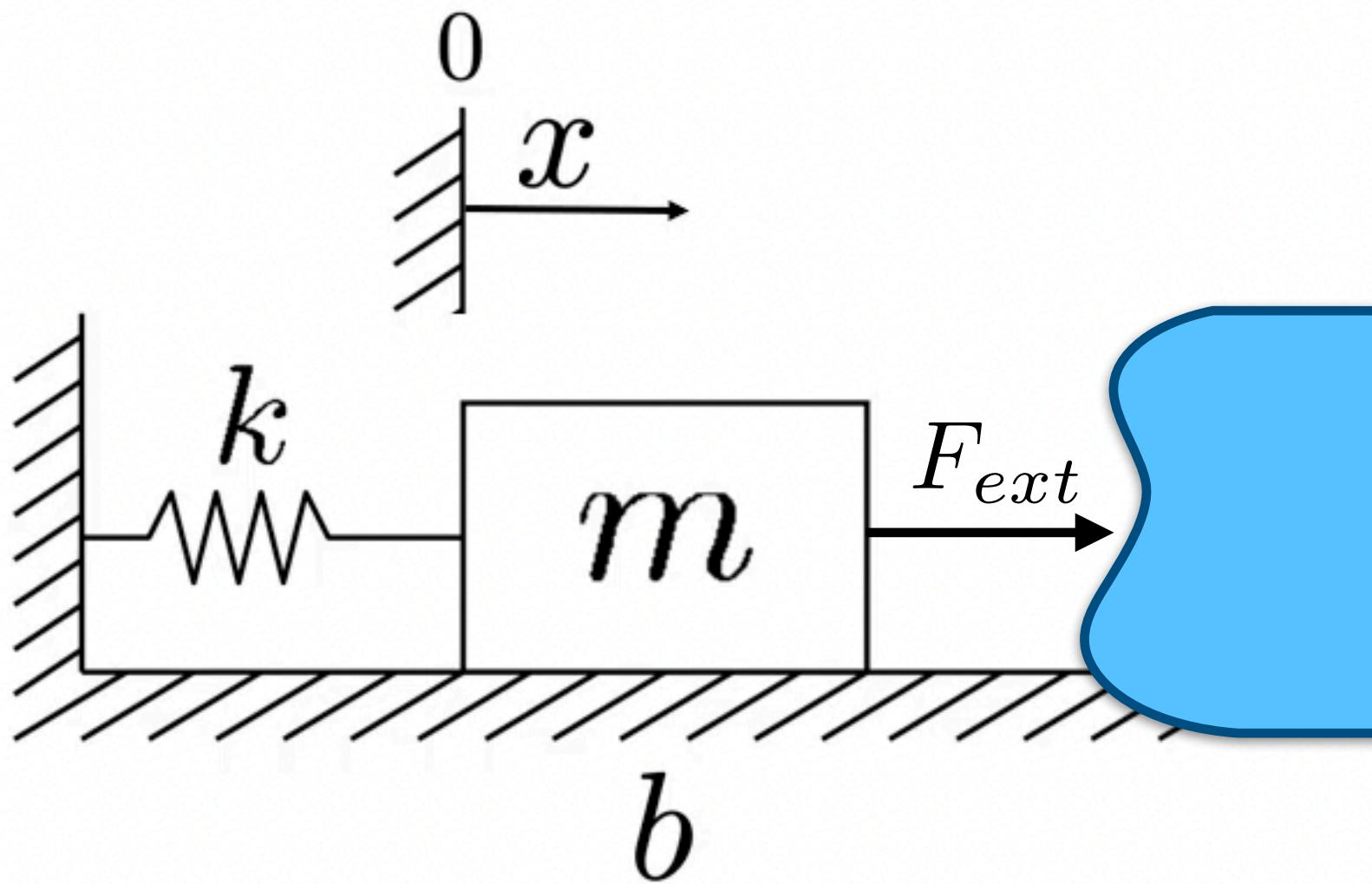
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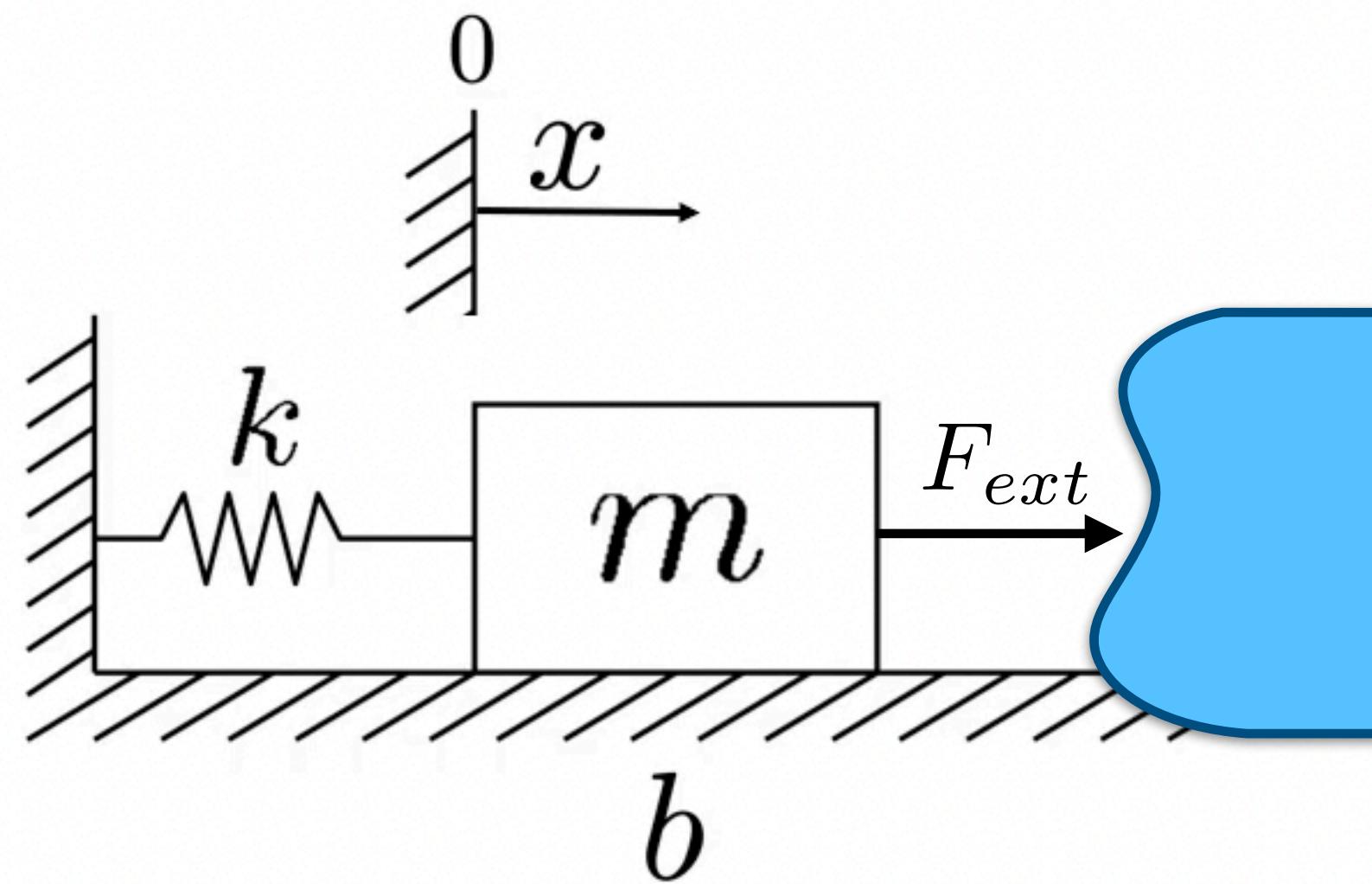
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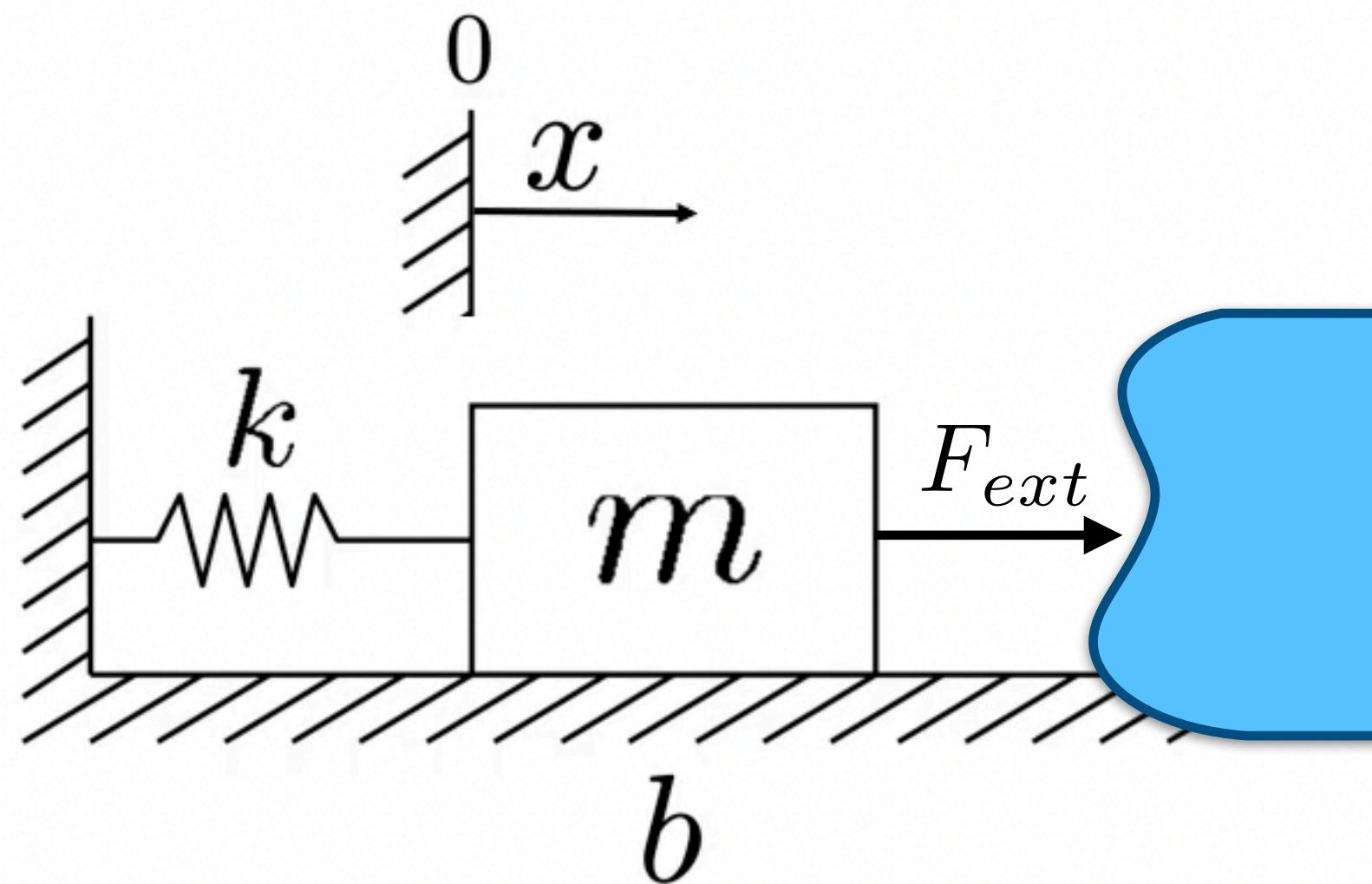
$f_{ext} = m\ddot{x} + b\dot{x} + kx$



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$$Z = \frac{F}{\Delta X}$$

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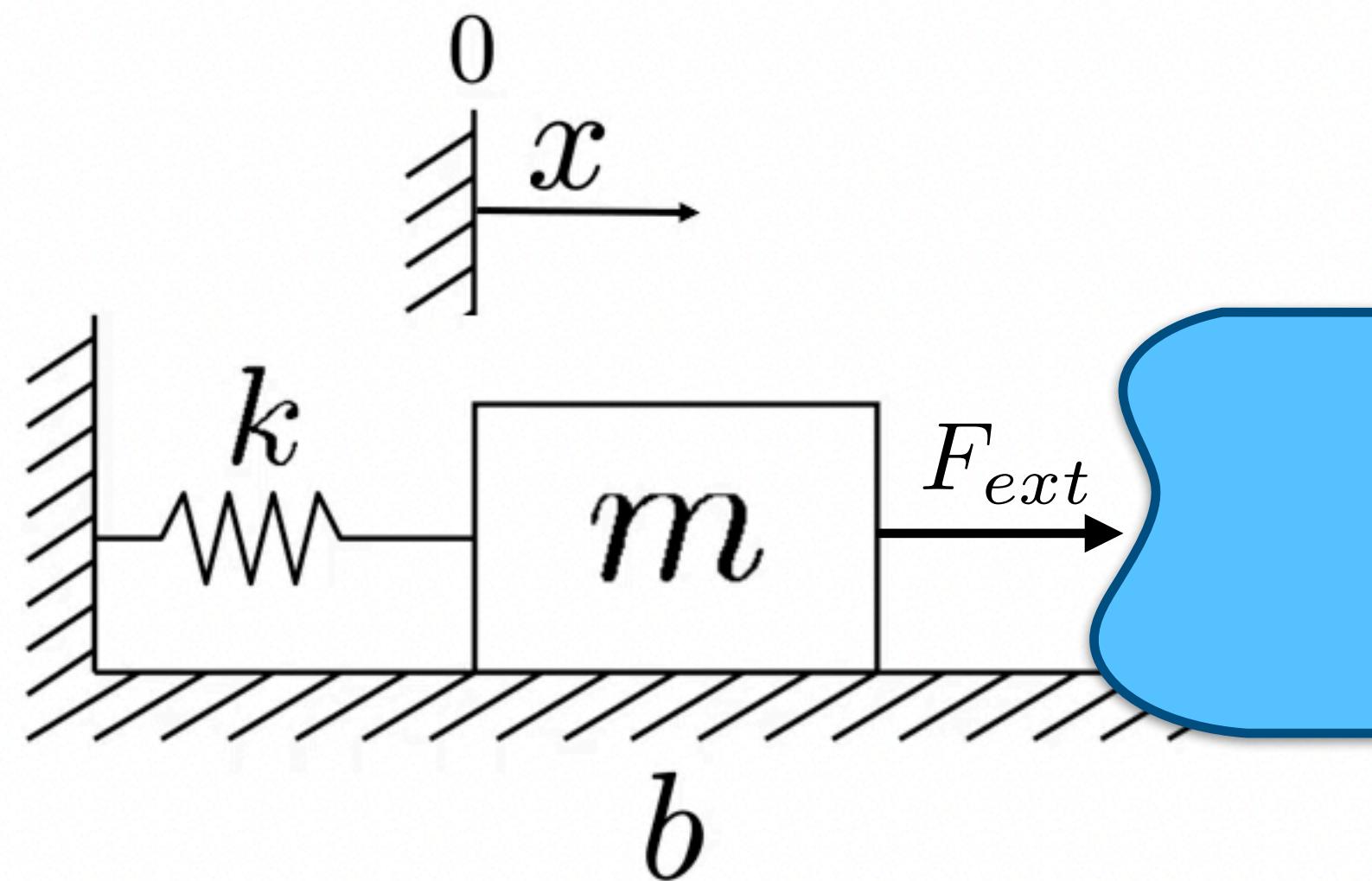


$$m\ddot{x} = -kx - b\dot{x} + f_{ext}$$

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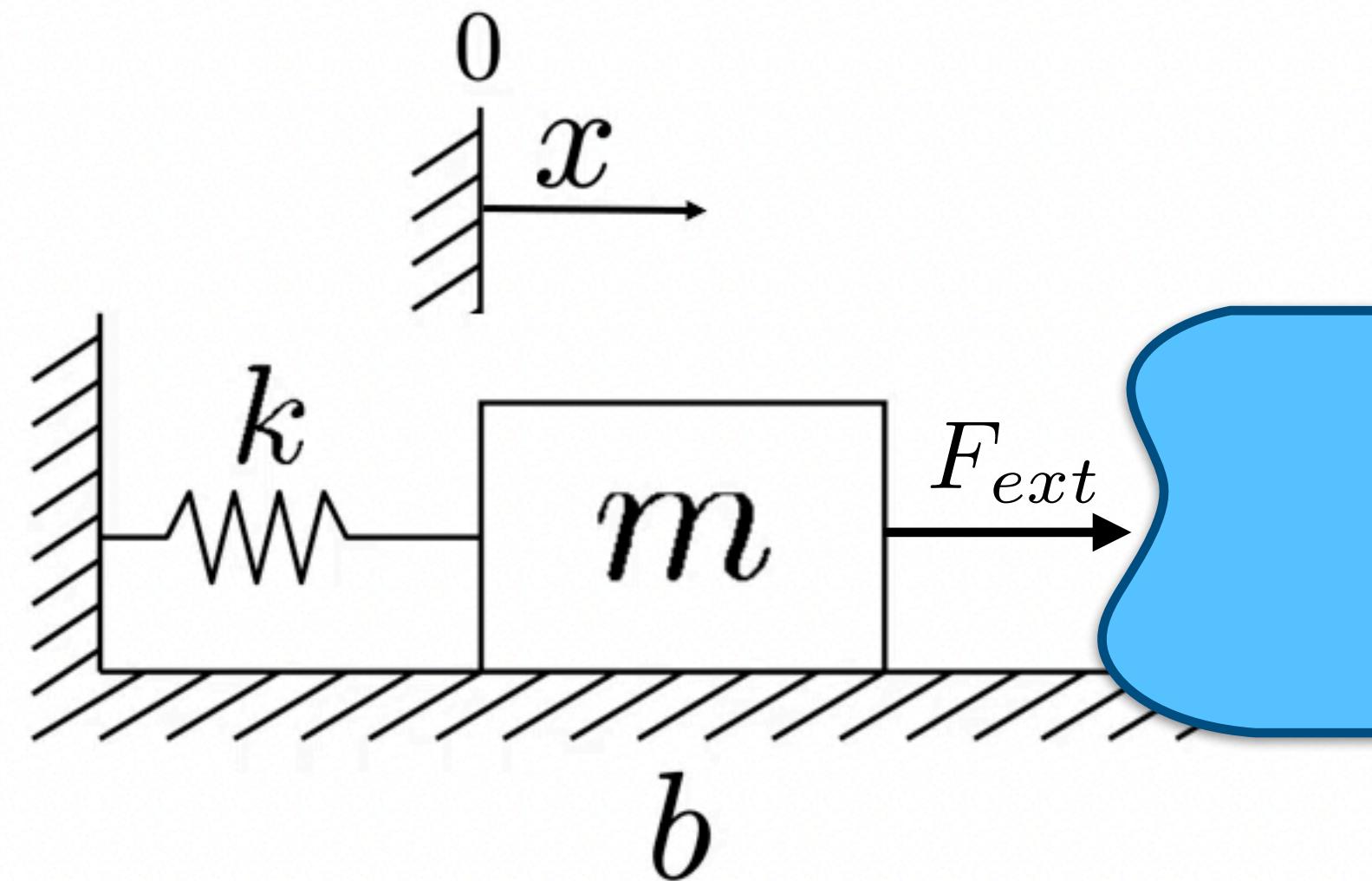
$$m\ddot{x} = -kx - b\dot{x} + f_{ext}$$

$$m(\ddot{x} - \ddot{x}_d) = -k(x - x_d) - b(\dot{x} - \dot{x}_d) + f_{ext}$$

# How to model impedance?

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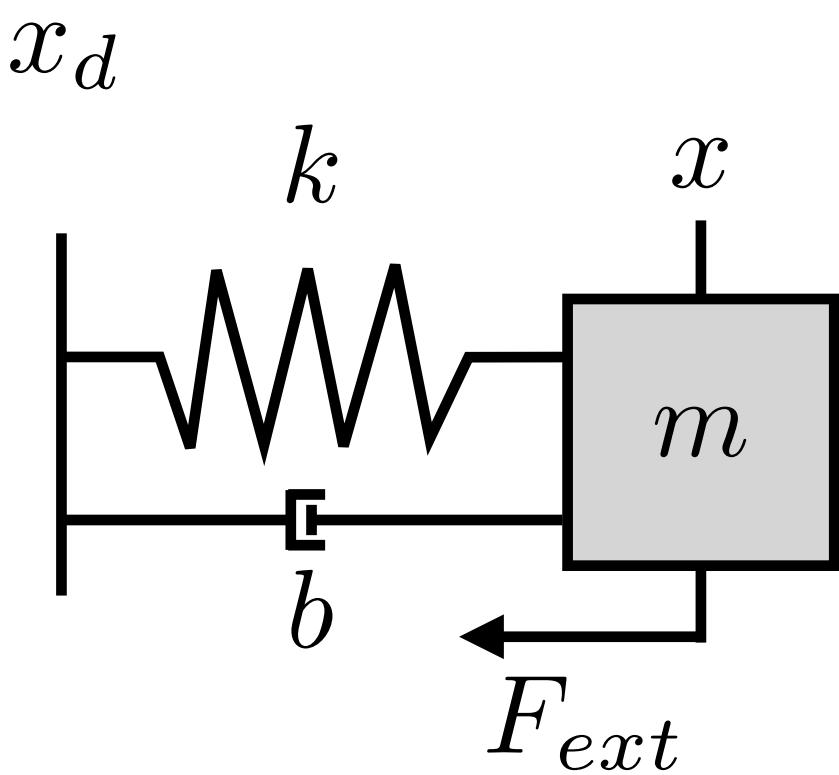


$$m\ddot{x} = -kx - b\dot{x} + f_{ext}$$

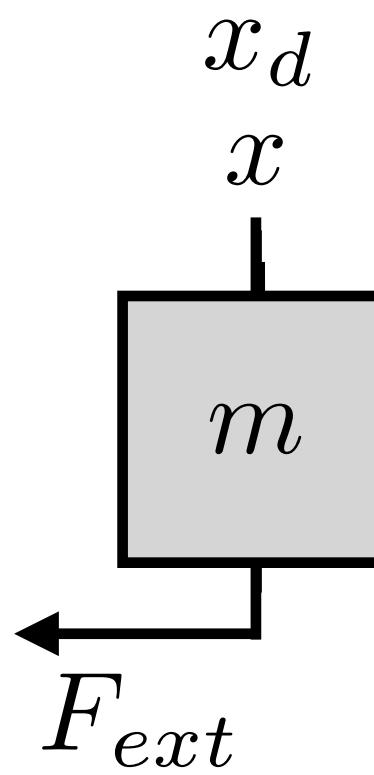
$$m(\ddot{x} - \ddot{x}_d) = -k(x - x_d) - b(\dot{x} - \dot{x}_d) + f_{ext}$$

$$m(\ddot{x} - \ddot{x}_d) + b(\dot{x} - \dot{x}_d) + k(x - x_d) = +f_{ext}$$

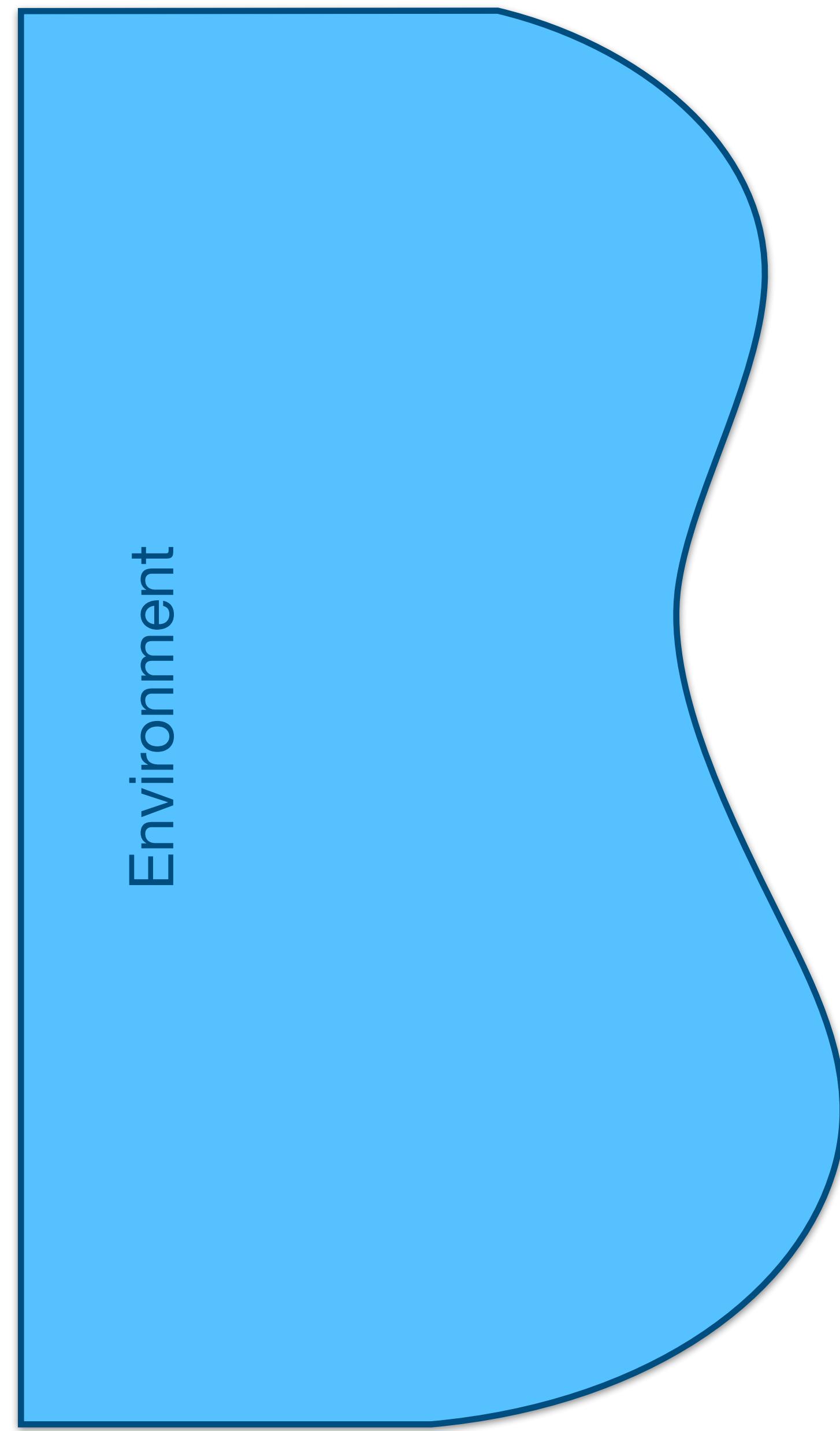
# Spring-Mass-Damper System revisited



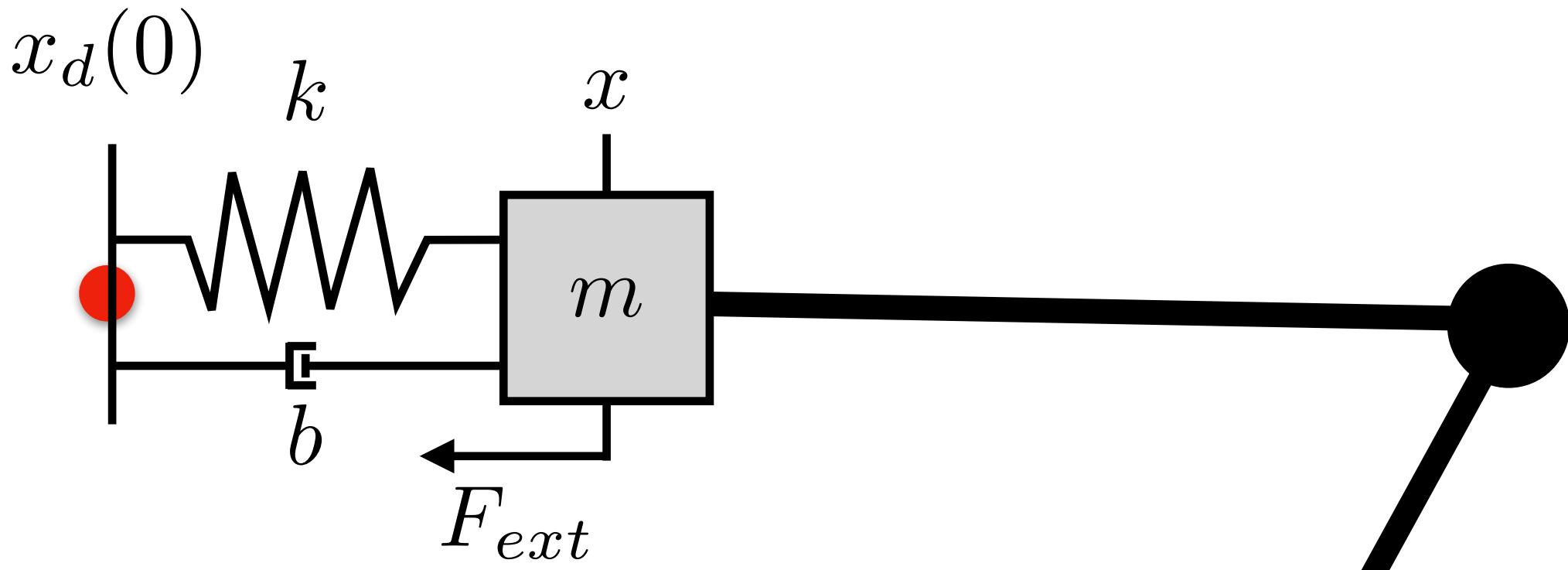
# Spring-Mass-Damper System revisited

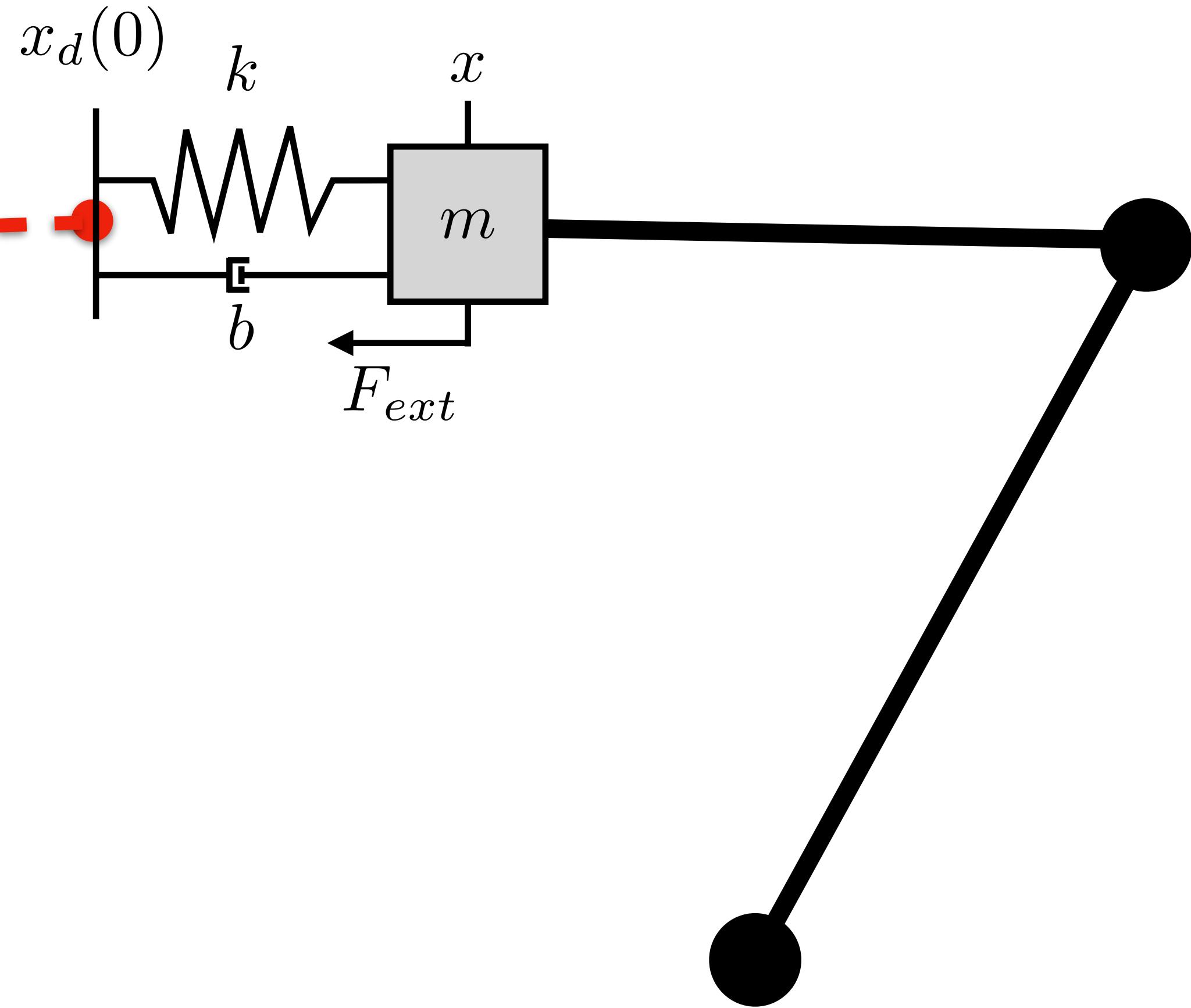
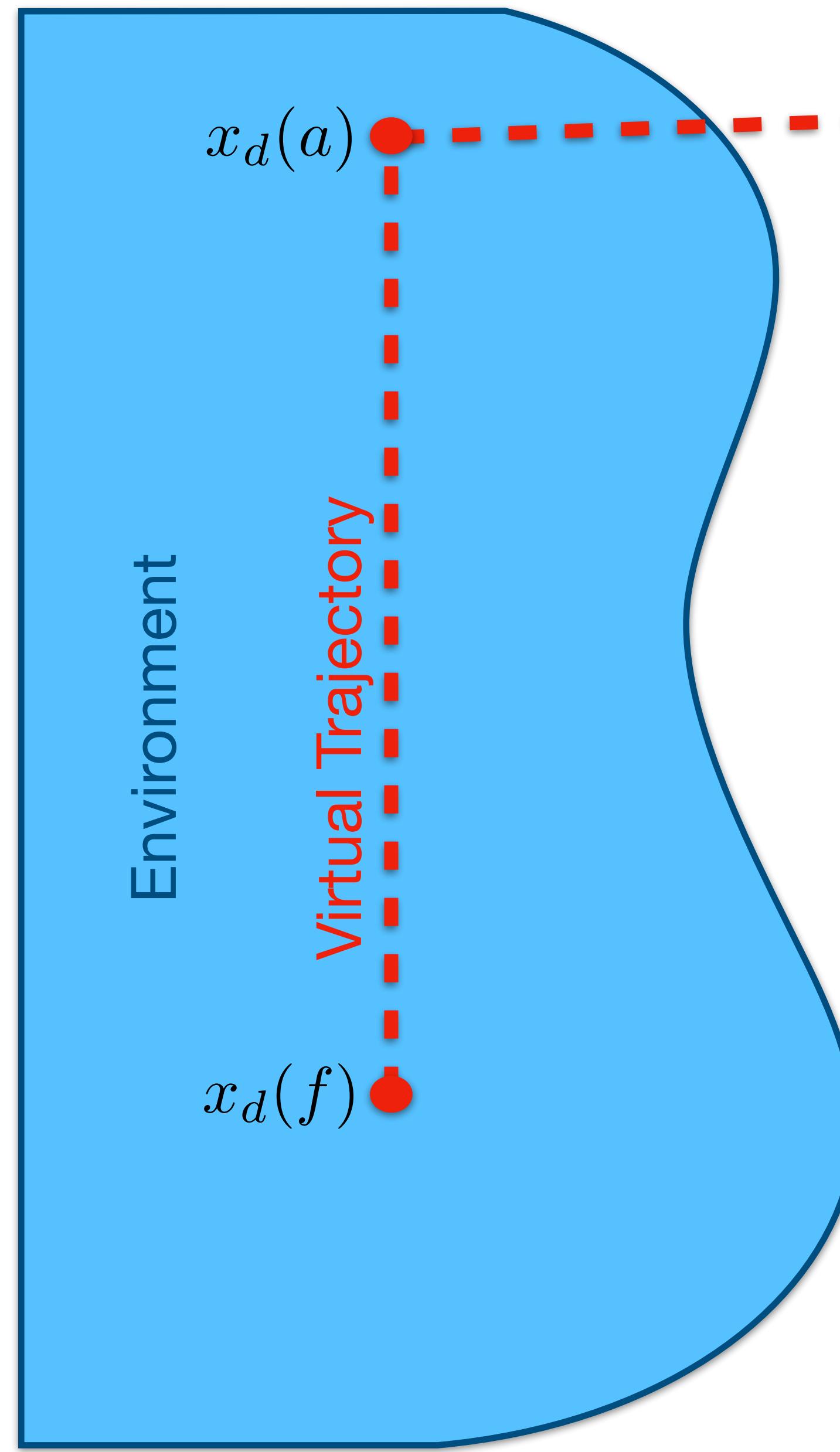


In free space (without contact), the end effector reaches the desired position and no forces are exerted

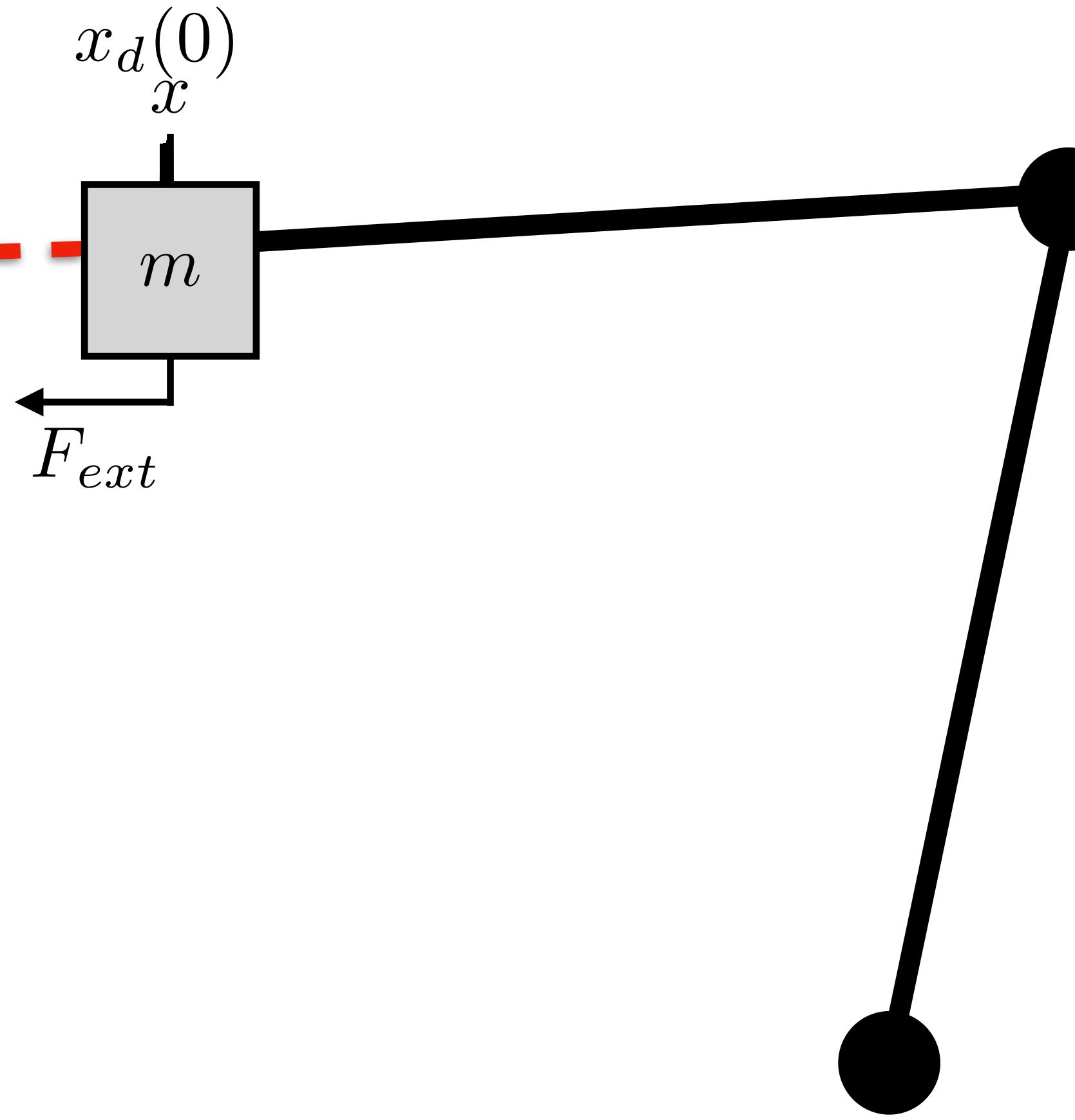
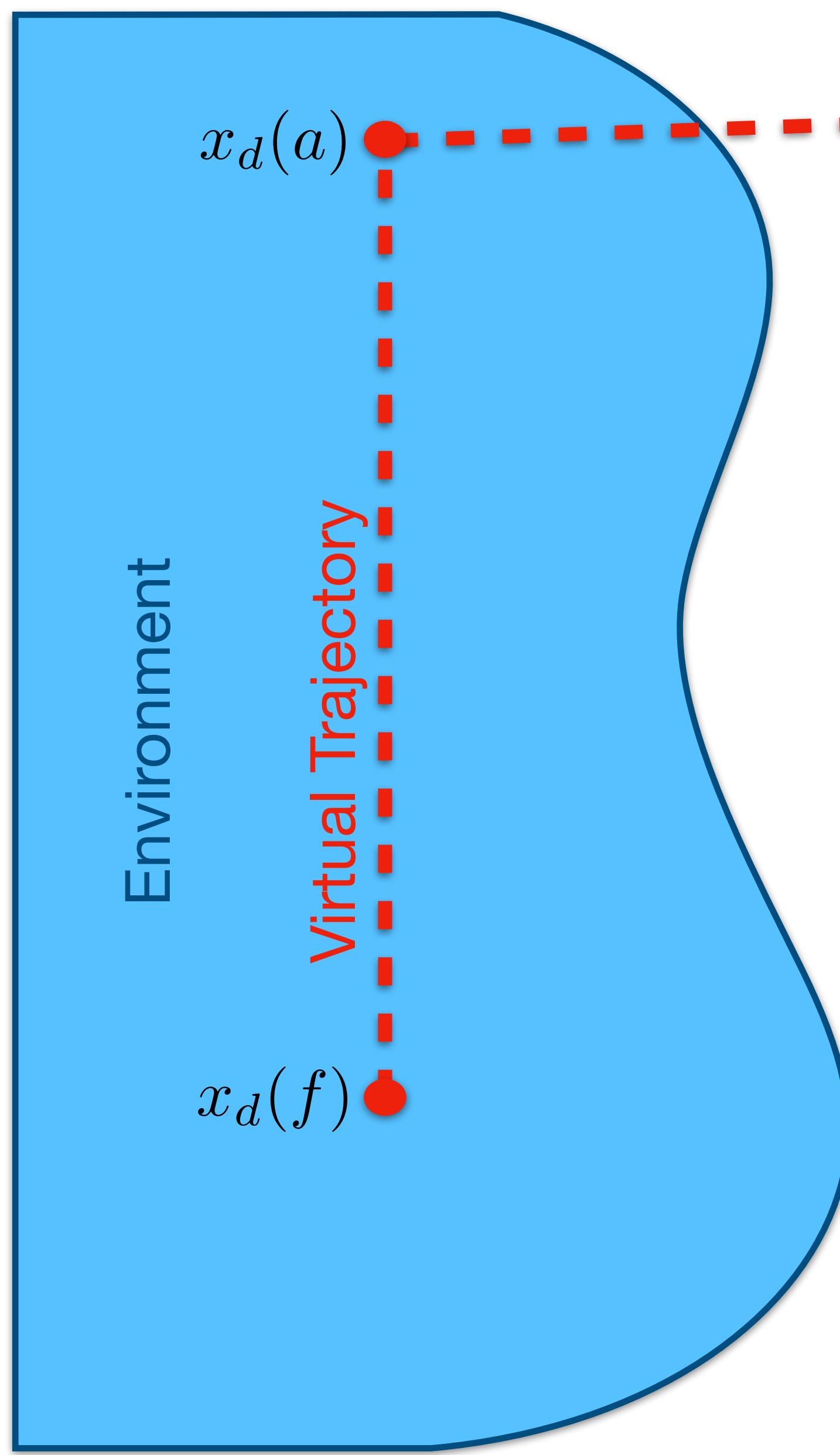


Environment



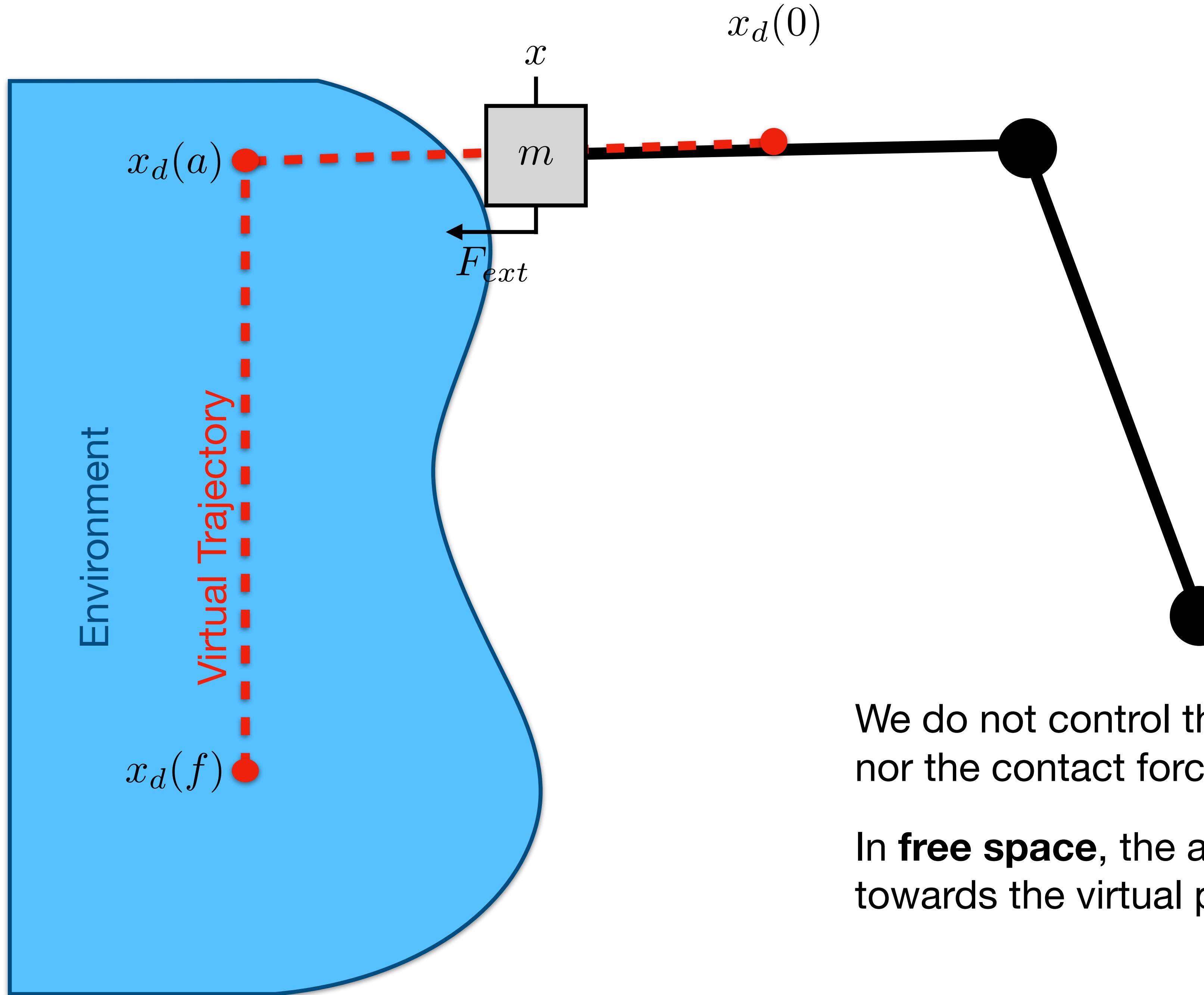


We do not control the robot's end effector pose,  
nor the contact force, but a **virtual ee pose**.



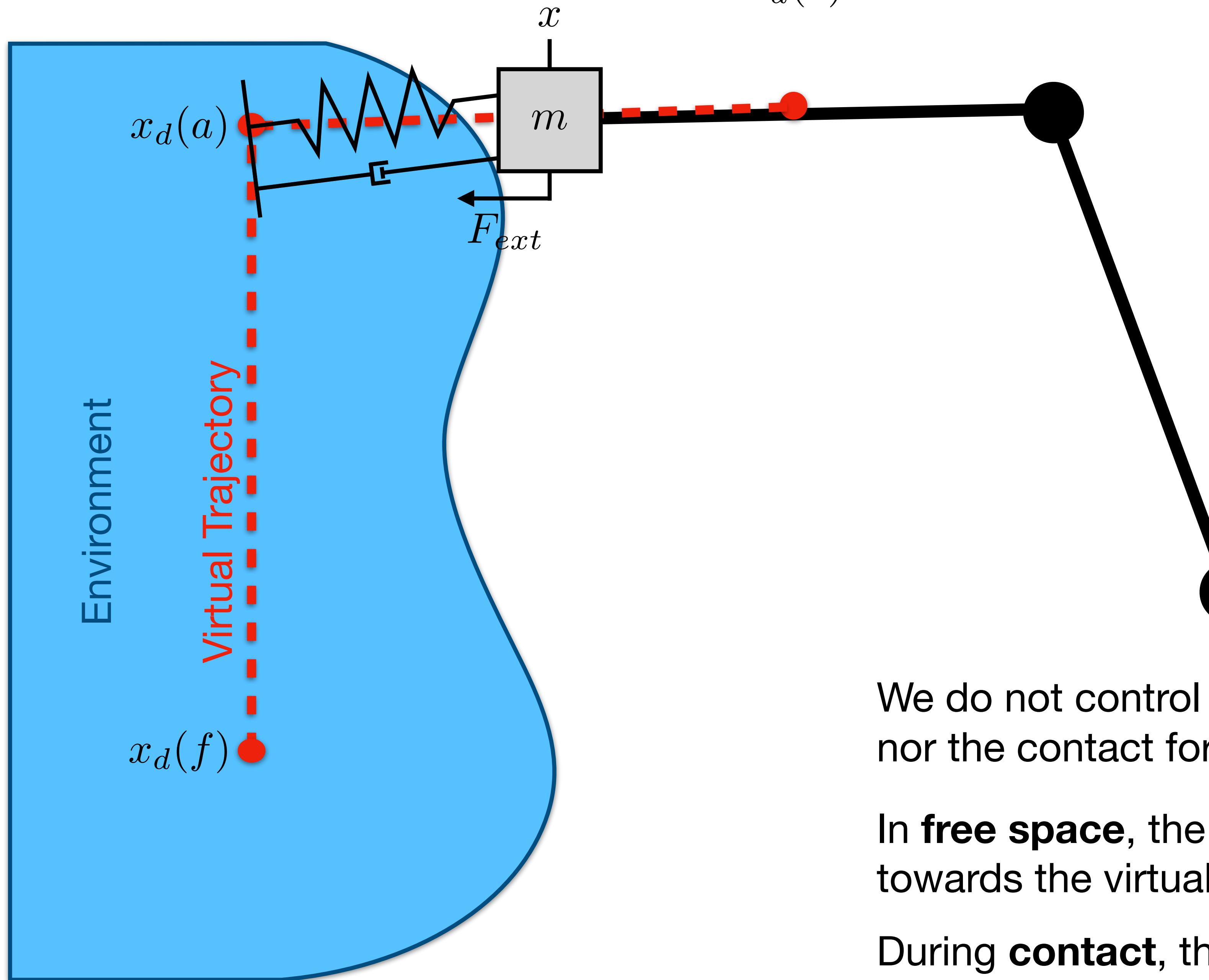
We do not control the robot's end effector pose, nor the contact force, but a **virtual ee pose**.

In **free space**, the actual ee pose converges towards the virtual pose.



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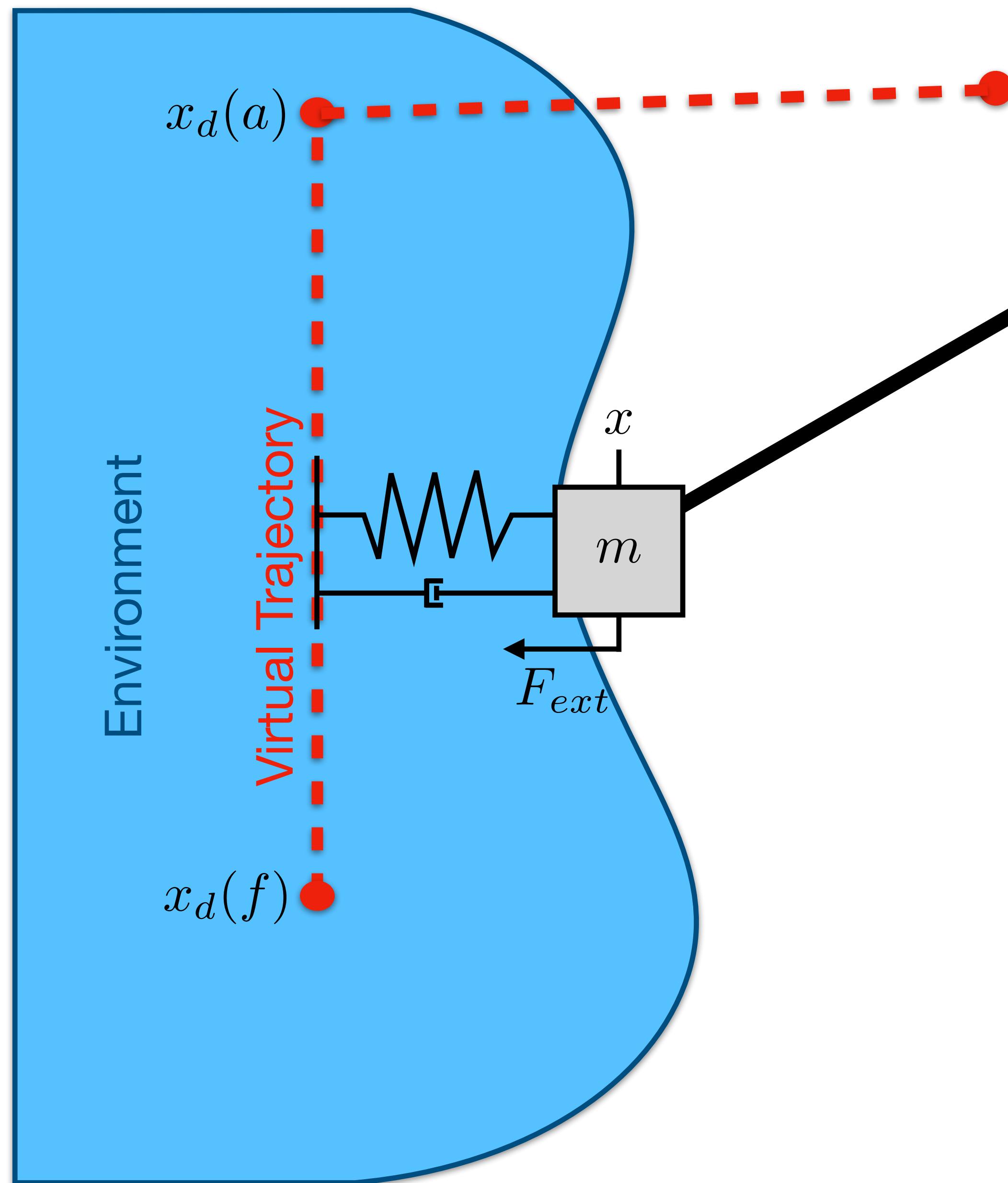
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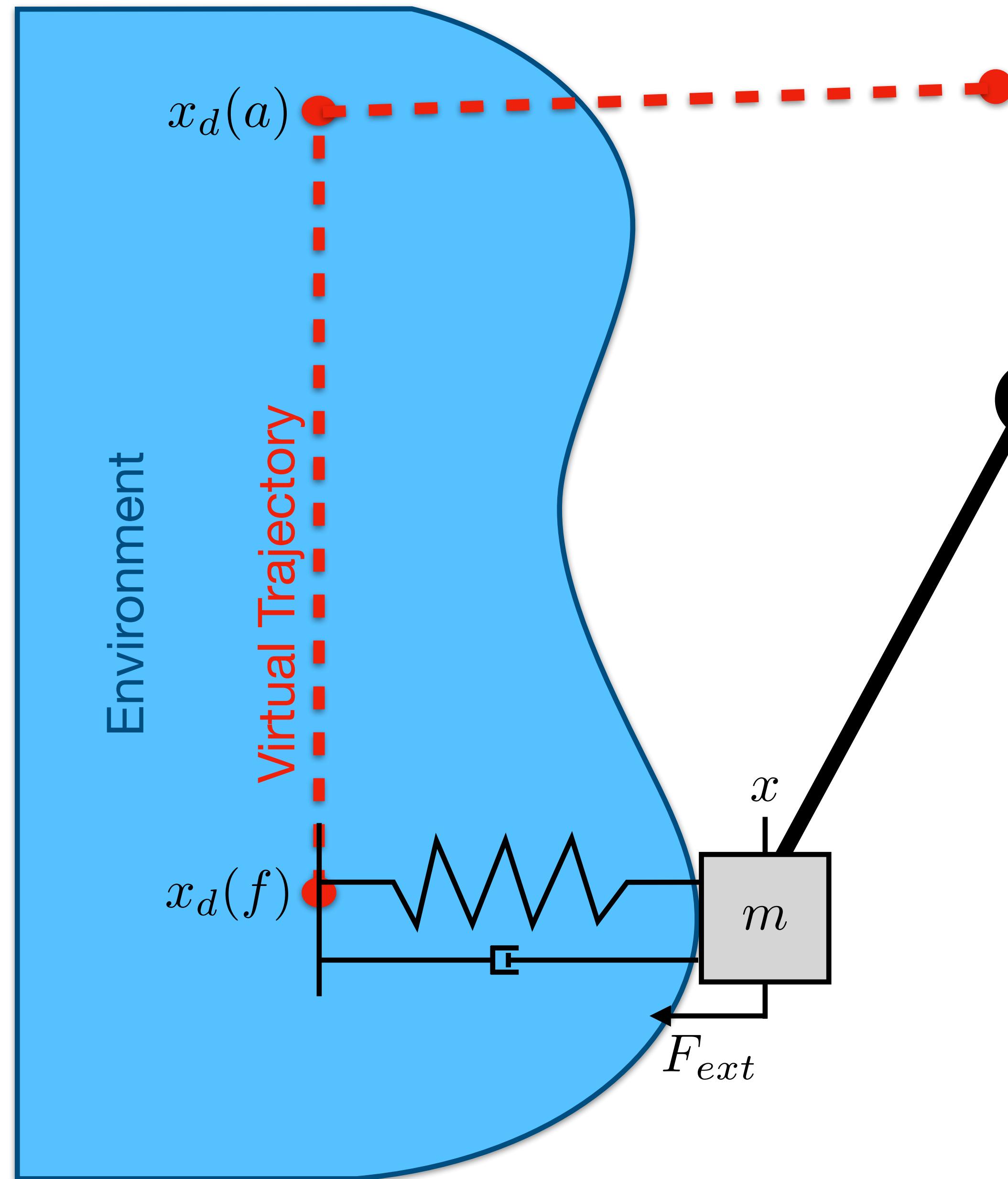
During **contact**, the actual ee pose and forces result from the end effector's impedance (SMD)



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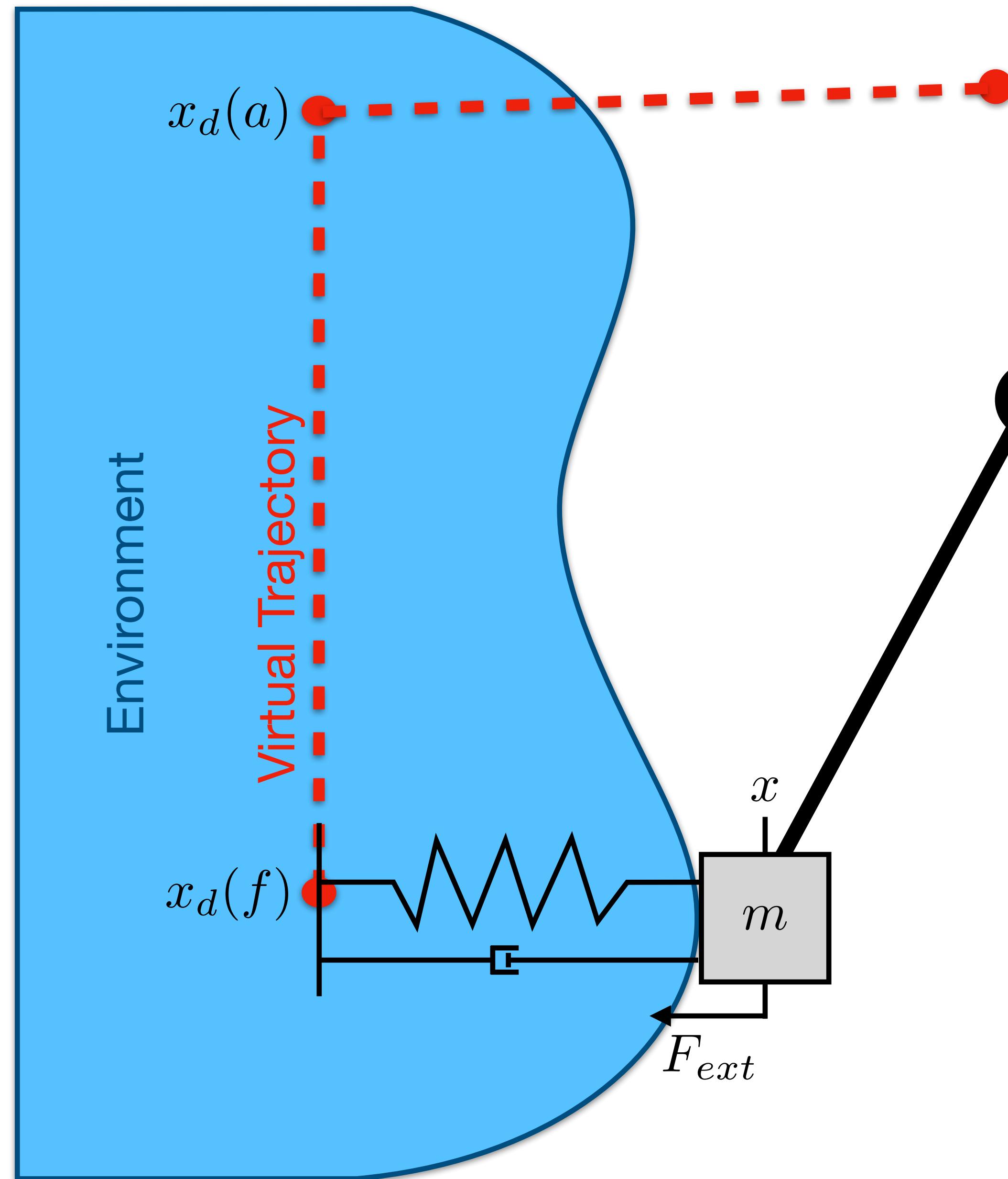
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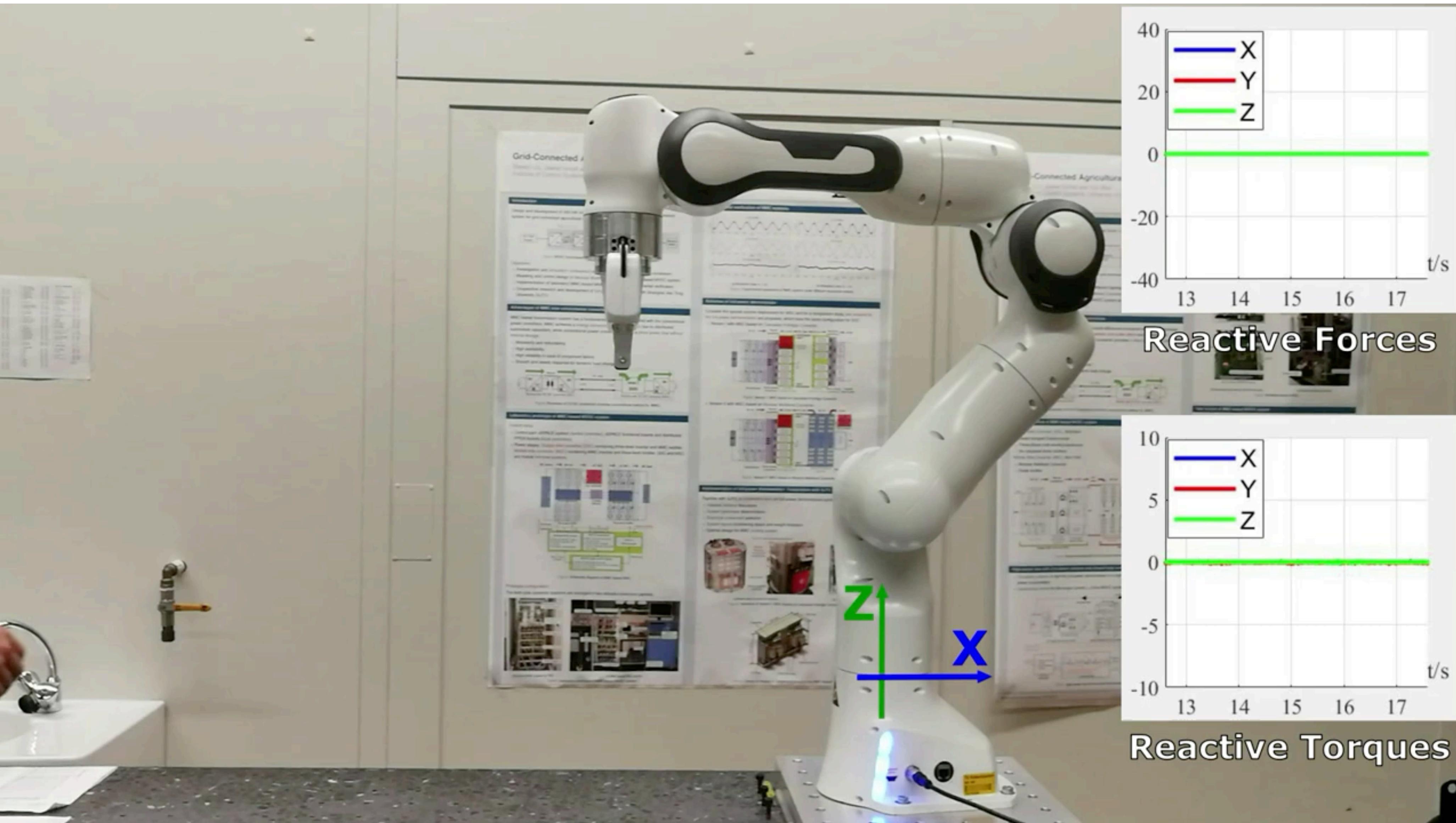


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# Impedance Control



# Comparison of Control Strategies

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# Comparison of Control Strategies

Position Control



Control  
only position

Control  
only force

# Comparison of Control Strategies

Position Control

Force Control



Control  
only position

Control  
only force

# Comparison of Control Strategies

$$Z = \frac{F}{\Delta X}$$

Position Control

Impedance Control

Force Control



Control  
only position

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# Comparison of Control Strategies

$$Z = \frac{F}{\Delta X}$$

Position Control

Impedance Control

Force Control



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# Comparison of Control Strategies

$$Z = \frac{F}{\Delta X}$$

Position Control

Impedance Control

Force Control



Control  
only position

Control  
only force

High Impedance

# Comparison of Control Strategies

$$Z = \frac{F}{\Delta X}$$

Position Control

Impedance Control

Force Control



Control  
only position

High Impedance

Control  
only force

Low Impedance

# Impedance Control: The great integration of what you have learned

