

Exercise Sheet **Density Estimation**

Exercise 1: Deriving the Evidence Lower Bound (25 + 25 P)

In this exercise we want to derive the evidence lower bound mentioned in the lecture.

(a) Therefore you first show:

$$D_{KL}[Q(z|X)||P(z|X)] = \mathbb{E}_{z \sim Q(z|X)} [\log Q(z|X) - \log P(X|z) - \log P(z)] + \log P(X) \quad (1)$$

where D_{KL} is the KL divergence.

(b) Show:

$$\log P(X) - D_{KL}[Q(z|X)||P(z|X)] = \mathbb{E}_{z \sim Q(z|X)} [\log P(X|z)] - D_{KL}[Q(z|X)||P(z)] \quad (2)$$

Now you can choose suitable tractable functions $P(X|z)$, $Q(z|X)$ (e.g. as neural networks) and $P(z)$ (e.g. as isotropic gaussian) and one can train the variational autoencoder by maximizing the ELBO.

Exercise 2: Programming (50 P)

Download the programming files on ISIS and follow the instructions.

Fill in the gaps in the GPT implementation.