Exercises for the course
Deep Learning 2
Summer Semester 2024

Machine Learning Group
Faculty IV – Electrical Engineering and Computer Science
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Exercise Sheet **Equivariant Neural Networks**

Exercise 1: Rotational equivariance (40 P)

In this excercise, we will derive an equivariant function given a function $f : \mathbb{R}^n \to \mathbb{R}$ that is invariant to the rotation group, i.e.

$$f(\mathbf{x}) = f(g\mathbf{x})$$

for any $g \in \mathrm{SO}(n)$.
Hint: Use the fact that g is orthogonal.

(a) Show that $f(\boldsymbol{x}) = \|\boldsymbol{x}\|$ is invariant under SO(n).

(b) Show that the derivative of any f is equivariant under SO(n):

$$g\nabla f(\boldsymbol{x}) = \nabla f(g\boldsymbol{x})$$

for any $g \in \mathrm{SO}(n)$.

(c) Calculate the gradient $u(\boldsymbol{x}) = \nabla f(\boldsymbol{x}) = \nabla\|\boldsymbol{x}\|$ to obtain an equivariant function $u : \mathbb{R}^n \to \mathbb{R}^n$.

(d) Analog to (c), derive an equivariant function $v : \mathbb{R}^n \to \mathbb{R}^{n\times n}$

Exercise 2: Programming (60 P)

Download the programming files on ISIS and follow the instructions.