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Exercise Sheet 6

Exercise 1: Dual formulation of the Soft-Margin SVM ($\mathbf{5+20+10+5}$ P)

The primal program for the linear soft-margin SVM is

$$\min_{\boldsymbol{w},b,\boldsymbol{\xi}} \frac{1}{2}\|\boldsymbol{w}\|^2 + C\sum_{i=1}^{N}\xi_i$$

subject to

$$\forall_{i=1}^{N}: \; y_i \cdot (\boldsymbol{w}^\top \phi(\boldsymbol{x}_i) + b) \geq 1 - \xi_i \quad \text{and} \quad \xi_i \geq 0$$

where $\|.\|$ denotes the Euclidean norm, ϕ is a feature map, $\boldsymbol{w} \in \mathbb{R}^d, b \in \mathbb{R}$ are the parameter to optimize, and $\boldsymbol{x}_i \in \mathbb{R}^d, y_i \in \{-1, 1\}$ are the labeled data points regarded as fixed constants. Once the hard-margin SVM has been learned, prediction for any data point $\boldsymbol{x} \in \mathbb{R}^d$ is given by the function

$$f(\boldsymbol{x}) = \text{sign}(\boldsymbol{w}^\top \phi(\boldsymbol{x}) + b).$$

(a) *State* the conditions on the data under which a solution to this program can be found from the Lagrange dual formulation *(Hint: verify the Slater's conditions)*.

(b) *Derive* the Lagrange dual and show that it reduces to a constrained quadratic optimization problem. State both the objective function and the constraints of this optimization problem.

(c) *Describe* how the solution $(\boldsymbol{w}, b)$ of the primal program can be obtained from a solution of the dual program.

(d) *Write* a kernelized version of the dual program and of the learned decision function.

Exercise 2: SVMs and Quadratic Programming (10 P)

We consider the `CVXOPT` Python software for convex optimization. The method `cvxopt.solvers.qp` solves quadratic optimization problems given in the matrix form:

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & \frac{1}{2}\boldsymbol{x}^\top P \boldsymbol{x} + \boldsymbol{q}^\top \boldsymbol{x} \\ \text{subject to} \quad & G\boldsymbol{x} \preceq \boldsymbol{h} \\ \text{and} \quad & A\boldsymbol{x} = \boldsymbol{b}. \end{aligned}$$

Here, $\preceq$ denotes the element-wise inequality: $(\boldsymbol{h} \preceq \boldsymbol{h}') \Leftrightarrow (\forall_i : h_i \leq h'_i)$. Note that the meaning of the variables $\boldsymbol{x}$ and $\boldsymbol{b}$ is different from that of the same variables in the previous exercise.

(a) *Express* the matrices and vectors $P, \boldsymbol{q}, G, \boldsymbol{h}, A, \boldsymbol{b}$ in terms of the variables of Exercise 1, such that this quadratic minimization problem corresponds to the kernel dual SVM derived above.

Exercise 3: Programming (50 P)

Download the programming files on ISIS and follow the instructions.