

Exercise Sheet 6

Exercise 1: Rotational equivariance (40 P)

In this exercise, we will derive an equivariant function given a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ that is invariant to the rotation group, i.e.

$$f(\mathbf{x}) = f(g\mathbf{x})$$

for any $g \in \text{SO}(n)$.

Hint: Use the fact that g is orthogonal.

- (a) Show that $f(\mathbf{x}) = \|\mathbf{x}\|$ is invariant under $\text{SO}(n)$.
- (b) Show that the derivative of any f is equivariant under $\text{SO}(n)$:

$$\nabla f(\mathbf{x}) = \nabla f(g\mathbf{x})$$

for any $g \in \text{SO}(n)$.

- (c) Calculate the gradient $u(\mathbf{x}) = \nabla f(\mathbf{x}) = \nabla \|\mathbf{x}\|$ to obtain an equivariant function $u : \mathbb{R}^n \rightarrow \mathbb{R}^n$.
- (d) Analog to (c), derive an equivariant function $v : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$

Exercise 2: Programming (60 P)

Download the programming files on ISIS and follow the instructions.