September 22, 2024

Conformally coupled scalar fields

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Abstract: Personal compilation of some calculations related to zero-weight conformal scalar fields.

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1 Conformally coupled scalar fields

Let's consider the D-dimensional action of a self-gravitating conformal scalar field ϕ

$$S[g_{\mu\nu},\phi] = \int \mathrm{d}^D x\sqrt{-g}\left(\frac{R}{2\kappa} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{2}\xi_D R\phi^2\right) . \tag{1.1}$$

Here, R stands for the scalar curvature, $\kappa = 8\pi G_N$ is a coupling related with Newton's gravitational constant and ξ_D is the conformal coupling given by

$$\xi_D = \frac{D-2}{4(D-1)} . \tag{1.2}$$

This is the precise value of the nonminimal coupling to gravity that ensures that the scalar contribution to the action (1.1) is invariant, up to a boundary term, under a conformal transformation

$$g_{\mu\nu} \mapsto \tilde{g}_{\mu\nu}\Omega(x)^2 g_{\mu\nu}, \qquad \phi \mapsto \bar{\phi} = \Omega(x)^{-\frac{D-2}{2}}\phi\,, \tag{1.3}$$

where $\Omega(x)$ is an arbitrary local function.

The action (1.1) can be rewritten as

$$S[g_{\mu\nu},\phi] = S_{\mathrm{EH}}[g_{\mu\nu}] + S_{\mathrm{M}}[g_{\mu\nu},\phi] \tag{1.4}$$

where

$$S_{\mathrm{EH}}[g_{\mu\nu}] = \frac{1}{2\kappa}\int \mathrm{d}^D x\sqrt{-g}R \tag{1.5}$$

and

$$S_{\mathrm{M}}[g_{\mu\nu},\phi] = -\frac{1}{2}\int \mathrm{d}^D x\sqrt{-g}\left(\nabla_\mu\phi\nabla^\mu\phi + \xi_D R\phi^2\right) . \tag{1.6}$$

We can define an auxiliary metric

$$\tilde{g}_{\mu\nu} = (\sqrt{\kappa\xi_D}\phi)^{\frac{4}{D-2}} g_{\mu\nu} \tag{1.7}$$

References