Notes on Quantum Field Theory

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Abstract

These notes are based on [1] and are for personal study purposes only.

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Contents

1 Classical Field Theory

Quantum field theory is just quantum mechanics with an infinite number of oscillators

1.1 Hamiltonians and Lagrangians

A classical field theory is just a mechanical system with a continuous set of degrees of freedom. Field theories can be defined in terms of either a Hamiltonian or a Lagrangian, which we often write as integrals over all space of Hamiltonian or Lagrangian densities:

$$H = \int \mathrm{d}^3x \mathcal{H}, \quad L = \int \mathrm{d}^3x \mathcal{L}. \tag{1}$$

Formally, the *Hamiltonian* (density) is a functional of fields and their conjugate momenta $\mathcal{H}[\phi, \pi]$. The *Lagrangian* (density) is the Legendre transform of the Hamiltonian (density). Formally, it is defined as

$$\mathcal{L}[\phi, \dot{\phi}] = \pi[\phi, \dot{\phi}]\dot{\phi} - \mathcal{H}[\phi, \pi[\phi\dot{\phi}]], \tag{2}$$

where $\dot{\phi} = \partial_t \phi$ and $\pi[\phi, \dot{\phi}]$ is implicitly defined by $\partial\mathcal{H}[\phi, \pi]/\partial\pi = \dot{\phi}$. The inverse transform is

$$\mathcal{H}[\phi, \pi] = \pi\dot{\phi}[\phi, \pi] - \mathcal{L}[\phi, \dot{\phi}[\phi, \pi]], \tag{3}$$

where $\dot{\phi}[\phi, \pi]$ is implicitly defined by $\partial\mathcal{L}[\phi, \dot{\phi}]/\partial\dot{\phi} = \pi$.

References

[1] L. Ciambelli, *From Asymptotic Symmetries to the Corner Proposal*, *PoS* **Modave2022** (2023) 002 [2212.13644].