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Nonlinear Electrodynamics

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Abstract: Personal notes on nonlinear electrodynamics.

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1 Nonlinear Electrodynamics

As is defined in the Plebanski book [XX], nonlinear electrodynamics are described by the following action principle

$$S_{\mathrm{E}}[g,A,P] = -\frac{1}{4\pi}\int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}F_{\mu\nu}P^{\mu\nu} - \mathscr{H}(\mathscr{P},\mathscr{Q})\right) \tag{1.1}$$

which depends on the metric $g_{\mu\nu}$, the gauge potential A_μ, and the antisymmetric tensor $P_{\mu\nu}$. Here the structural function $\mathscr{H}$ describes the precise nonlinear electrodynamics and depends, in general, on the two Lorentz scalars that can be constructed with $P^{\mu\nu}$ [XX]. As usual, the field strength is related to the gauge potential as $F = \mathrm{d}A$, ensuring the Faraday equations

$$\mathrm{d}F = 0. \tag{1.2}$$

On the other hand, the variation of action (1.1) with respect to the gauge potential leads to the Maxwell equations

$$\mathrm{d}\star P = 0, \tag{1.3}$$

where $\star$ stands for the Hodge dual, whereas varying (1.1) with respect to the antisymmetric tensor $P^{\mu\nu}$ yields the constitutive relations

$$F_{\mu\nu} = \frac{\partial\mathscr{H}}{\partial\mathscr{P}}P_{\mu\nu} + \frac{\partial\mathscr{H}}{\partial\mathscr{Q}}\star P_{\mu\nu}. \tag{1.4}$$

Notice that Maxwell electrodynamics is recovered for $\mathscr{H} = \mathscr{P}$, giving linear constitutive relations. Lastly, the corresponding energy-momentum tensor reads

$$4\pi T^{\mathrm{E}}_{\mu\nu} = F_{\mu\alpha}P_{\nu}{}^{\alpha} - g_{\mu\nu}\left(\frac{1}{2}F_{\alpha\beta}P^{\alpha\beta} - \mathscr{H}\right). \tag{1.5}$$

The main motivation for the action principle (1) is that now Maxwell equations (3) remain linear as the Fara- day ones (2) while the nonlinearity is encoded into the constitutive relations (4). Consequently, Maxwell equa- tions (3) can be now understood just like the Faraday ones (2), i.e., implying the local existence of a vector po- tential P = dA. Therefore, from the point of view of the action principle (1), a solution to nonlinear electro- dynamics can be understood as a pair of vector poten- tials A and A compatible with the constitutive relations (4). Additionally, since in four dimensions both Faraday (2) and Maxwell (3) equations define conservation laws,

References