Wheeler like polynomial in Loveclok theory

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Abstract

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Contents

1 Static ansatz

$$\mathcal{E}^{(p)r}{}_{r} = \frac{\alpha_p}{(D-2p-1)!}\left[2pR^{1A_1}_{1B_1}R^{A_2A_3}_{B_2B_3}\cdots R^{A_{2p-2}A_{2p-1}}_{B2p-2B_{2p-1}}\delta^{A_{2p}}_{B_{2p}} + (D-2p-1)R^{A_1A_2}_{B_1B_2}\cdots R^{A_{2p-1}A_{2p}}_{B2p-1B_{2p}}\right]\delta^{B_1\cdots B_{2p}}_{A_1\cdots A_{2p}} \tag{1}$$

where

$$R^{AB}_{CD} = \frac{1}{2r^2}\bar{R}^{AB}_{CD} - \frac{f}{r^2}\delta^{A}_{[C}\delta^{B}_{D]} \qquad \text{and} \qquad R^{1A}_{1B} = -\frac{f'}{2r}\delta^{A}_{B}\,. \tag{2}$$

Let's compute $R^{A_1A_2}_{B_1B_2}\cdots R^{A_{2p-1}A_{2p}}_{B2p-1B_{2p}}\delta^{B_1\cdots B_{2p}}_{A_1\cdots A_{2p}}$. Notice that we are dealing with a product of p Riemann tensors, which are a sort of binomial [cf. Eq. (2)] . On the other hand, we know from the binomial theorem formula that

$$(x+y)^p = \sum_{k=0}^{p}\binom{p}{k}x^k y^{p-k}\,. \tag{3}$$

Thus, we can use it as a pictoric way to compute the product of p Riemman tensors,

$$R^{A_1A_2}_{B_1B_2}\cdots R^{A_{2p-1}A_{2p}}_{B2p-1B_{2p}} = \sum_{k=0}^{p}\binom{p}{k}\left(\frac{1}{2r^2}\right)^k \bar{R}^{A_1A_2}_{B_1B_2}\cdots\bar{R}^{A_{2k-1}A_{2k}}_{B_{2k-1}B_{2k}}\left(-\frac{f}{r^2}\right)^{p-k}\delta^{A_{2k+1}}_{B_{2k+1}}\cdots\delta^{A_{2p}}_{B_{2p}} \tag{4}$$

$$= \sum_{k=0}^{p}\binom{p}{k}\frac{1}{r^{2k}}\frac{1}{2^k}\left(-\frac{f}{r^2}\right)^{p-k}\bar{R}^{A_1A_2}_{B_1B_2}\cdots\bar{R}^{A_{2k-1}A_{2k}}_{B_{2k-1}B_{2k}}\delta^{A_{2k+1}}_{B_{2k+1}}\cdots\delta^{A_{2p}}_{B_{2p}}\,. \tag{5}$$

Contraction with the $2p$ rank generalized Kronecker delta yields

$$R^{A_1A_2}_{B_1B_2}\cdots R^{A_{2p-1}A_{2p}}_{B2p-1B_{2p}}\delta^{(2p)} = \sum_{k=0}^{p}\binom{p}{k}\frac{1}{r^{2k}}\frac{1}{2^k}\left(-\frac{f}{r^2}\right)^{p-k}\bar{R}^{A_1A_2}_{B_1B_2}\cdots\bar{R}^{A_{2k-1}A_{2k}}_{B_{2k-1}B_{2k}}\underbrace{\delta^{A_{2k+1}}_{B_{2k+1}}\cdots\delta^{A_{2p}}_{B_{2p}}\delta^{A_1\cdots A_{2p}}_{B_1\cdots B2p}}_{\frac{(d-2k)!}{(d-2p)!}\delta^{A_1\cdots A_{2k}}_{B_1\cdots B_{2k}}} \tag{6}$$

$$= \sum_{k=0}^{p}\binom{p}{k}\frac{1}{r^{2k}}\frac{1}{2^k}\left(-\frac{f}{r^2}\right)^{p-k}\frac{(d-2k)!}{(d-2p)!}\bar{R}^{A_1A_2}_{B_1B_2}\cdots\bar{R}^{A_{2k-1}A_{2k}}_{B_{2k-1}B_{2k}}\delta^{A_1\cdots A_{2k}}_{B_1\cdots B_{2k}} \tag{7}$$

$$= \sum_{k=0}^{p}\binom{p}{k}\frac{1}{r^{2k}}\left(-\frac{f}{r^2}\right)^{p-k}\frac{(d-2k)!}{(d-2p)!}\bar{\mathscr{L}}^{(k)} \tag{8}$$

$$= \sum_{k=0}^{p}\binom{p}{k}\frac{1}{r^{2p}}(-f)^{p-k}\frac{(d-2k)!}{(d-2p)!}\bar{\mathscr{L}}^{(k)} \tag{9}$$

where d stands for the dimension of the base-manifold $d := D-2$ and we used the fact that

$$\bar{\mathscr{L}}^{(k)} = \frac{1}{2^k}\bar{R}^{A_1A_2}_{B_1B_2}\cdots\bar{R}^{A_{2k-1}A_{2k}}_{B_{2k-1}B_{2k}} \tag{10}$$

is the p-th Lagrangian density of the base-manifold.

Now, let's compute $R^{1A_1}_{1B_1}R^{A_2A_3}_{B_2B_3}\cdots R^{A_{2p-2}A_{2p-1}}_{B2p-2B_{2p-1}}\delta^{A_{2p}}_{B_{2p}}\delta^{B_1\cdots B_{2p}}_{A_1\cdots A_{2p}}$.

$$R^{1A_1}_{1B_1}R^{A_2A_3}_{B_2B_3}\cdots R^{A_{2p-2}A_{2p-1}}_{B2p-2B_{2p-1}}\delta^{A_{2p}}_{B_{2p}}\delta^{B_1\cdots B_{2p}}_{A_1\cdots A_{2p}} = \tag{11}$$

References