DAN OTIENO - EE 307 - Homework 8.

8.11 Two infinitely long parallel wires are separated by a distance of 20 cm. If the wires carry current of 10 A in opposite directions, calculate the force on the wires.

$$F = \frac{\mu_0 I_1 I_2}{2\pi\rho} \; ; \quad \mu_0 = 4\pi \times 10^{-7}, \quad I_1 = I_2 = 10A.$$
$$\rho = (20 \times 10^{-2})\,m$$

$$F = \frac{\cancel{4\pi}^{2} \times \cancel{10^{-7}}^{10^{-5}} \times \cancel{10 \times 10}^{5}}{\cancel{2\pi}_{1} \times \cancel{20}_{1} \times \cancel{10^{-2}}_{1}} = 10 \times 10^{-5}\,N = \boxed{100\,\mu N}$$

8.25 In a magnetic material, with $\chi_m = 6.5$, the magnetization is $\mathbf{M} = 24y^2\mathbf{a}_z$ A/m. Find μ_r, $\mathbf{H}$, and $\mathbf{J}$ at $y = 2$ cm.

$$\mu_r = \chi_m + 1 = 6.5 + 1 = \boxed{7.5}$$

$$H = \frac{M}{\chi_m} = \frac{24y^2 a_z}{6.5} \; ; \text{ at } y = 2\,cm, \; H = \frac{24(2^2 \times 10^{-4})}{6.5} a_z$$

$$\therefore \boxed{H \approx 1.48\, a_z\, mA/m}$$

$$J = \nabla \times H = \begin{vmatrix} \delta/\delta x & \delta/\delta y & \delta/\delta z \\ 0 & 0 & 24y^2/6.5 \end{vmatrix}$$

$$= \frac{48y}{6.5} a_x \; ; \text{ at } y = 2, \; J = \frac{48(2 \times 10^{-2})}{6.5} a_x$$

$$\therefore \boxed{J \approx 0.148\, a_x\, A/m^2}$$

8.30 Region 1, for which $\mu_1 = 2.5\mu_o$, is defined by $z < 0$, while region 2, for which $\mu_2 = 4\mu_o$, is defined by $z > 0$. If $\mathbf{B}_1 = \underbrace{6\mathbf{a}_x - 4.2\mathbf{a}_y}_{T} + \underbrace{1.8\mathbf{a}_z}_{N}$ mWb/m^2, find $\mathbf{H}_2$ and the angle $\mathbf{H}_2$ makes with the interface.

$$B_{1N} = 1.8\,a_z = B_{2N}\;;\quad H_{1T} = H_{2T}\;;\quad \frac{B_{1T}}{\mu_1} = \frac{B_{2T}}{\mu_2}$$

$$(B_{2T})(\mu_1) = (B_{1T})(\mu_2) \quad \therefore\; B_{2T} = \frac{(B_{1T})(\mu_2)}{\mu_1}$$

$$B_{2T} = (6a_x - 4.2a_y)\left(\frac{\overset{1.6}{\cancel{4\mu_0}}}{\cancel{2.5\mu_0}}\right) = 9.6\,a_x - 6.72\,a_y$$

$$B_2 = B_{2T} + B_{2N} = 9.6\,a_x - 6.72\,a_y + 1.8\,a_z \text{ mWb/m}^2$$

$$H_2 = \frac{B_2}{\mu_2} = \frac{(9.6,\,-6.72,\,1.8)\times 10^{-3}}{4\times 4\pi\times 10^{-7}}$$

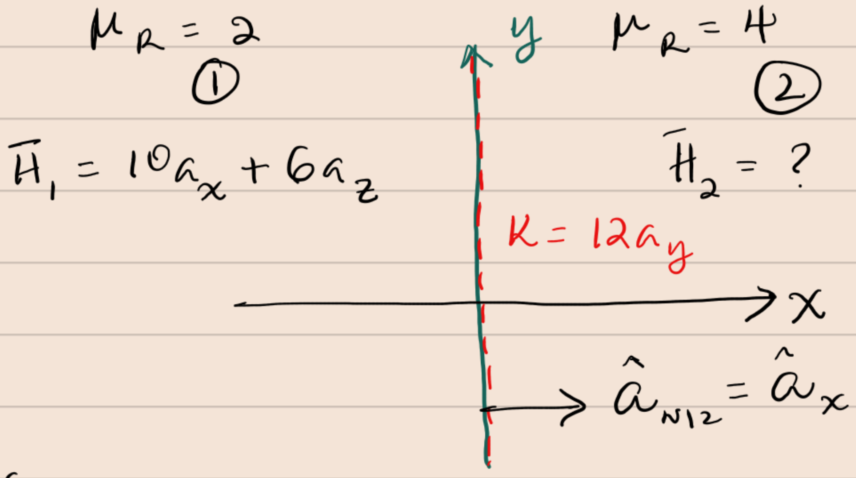
$$\therefore\; \boxed{H_2 \approx 1{,}909.9\,a_x - 1{,}336.9\,a_y + 358.1\,a_z \text{ A/m}}$$

$$\tan\theta_2 = \frac{B_{2N}}{|B_{2T}|}\;;\quad \theta_2 = \tan^{-1}\left(\frac{B_{2N}}{|B_{2T}|}\right)$$

$$= \tan^{-1}\left(\frac{1.8}{\sqrt{9.6^2 + 6.72^2}}\right) = \tan^{-1}(0.1536)$$

$$\boxed{\theta_2 = 8.7324^\circ}$$

8.33 A current sheet with $\mathbf{K} = 12\mathbf{a}_y$ A/m is placed at $x = 0$, which separates region 1, $x < 0$, $\mu = 2\mu_o$ and region 2, $x > 0$, $\mu = 4\mu_o$. If $\mathbf{H}_1 = 10\mathbf{a}_x + 6\mathbf{a}_z$ A/m, find $\mathbf{H}_2$.



$$(\bar{H}_1 - \bar{H}_2) \times \hat{a}_{N12} = \bar{K} \, ;$$

$$\left[(10\,\hat{a}_x + 6\,\hat{a}_z) - (H_{2x}\hat{a}_x - H_{2z}\hat{a}_z) \right] \times \hat{a}_x = 12\,\hat{a}_y$$

$$\bar{H}_1 \times \hat{a}_x = 6\hat{a}_y$$

$$H_2 \times \hat{a}_x = (H_{2x}\hat{a}_x + H_{2y}\hat{a}_y + H_{2z}\hat{a}_z) \times \hat{a}_x$$

$$= -\bar{H}_{2y}\hat{a}_y + H_{2z}\hat{a}_z$$

$$6\hat{a}_y = 12\,\hat{a}_y + H_{2z}\hat{a}_y - H_{2y}\hat{a}_z$$

$$6 = 12 + H_{2z} \quad \therefore \; H_{2z} = -6$$

$$B_{1N} = B_{2N} \; ; \; \mu_1 H_{1N} = \mu_2 H_{2N}$$

$$H_{2N} = \frac{\mu_1 H_{1N}}{\mu_2} = \frac{2}{4}(10) = 5(\hat{a}_x)$$

$$\therefore \; \boxed{H_2 = 5\hat{a}_x - 6\hat{a}_z}$$

8.54 The magnetic circuit of Figure 8.42 has a current of 10 A in the coil of 2000 turns. Assume that all branches have the same cross section of 2 cm^2 and that the material of the core is iron with $\mu_r = 1500$. Calculate R, $\mathscr{F}$, and Ψ for

(a) The core

(b) The air gap

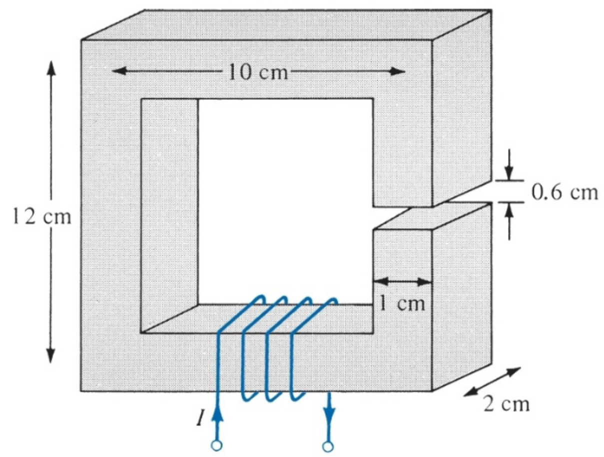


FIGURE 8.42 For Problem 8.54.

$$\mathscr{F} = NI = 2000(10) = \boxed{20{,}000\ \text{A.t}}$$

$$R_{core} = \frac{l_{core}}{\mu_o \mu_r S} = \frac{(24+20-6)\times 10^{-2}}{4\pi\times 10^{-7}\times 1500\times 2\times 10^{-4}} \approx \boxed{0.115\times 10^{7}\ \frac{\text{A.t}}{\text{m}}}$$

$$R_{air} = \frac{l_{air}}{\mu_o \mu_r S} = \frac{0.6\times 10^{-2}}{4\pi\times 10^{-7}\times 1\times 2\times 10^{-4}} \approx \boxed{2.387\times 10^{7}\ \text{A.t/m}}$$

$$R = R_{core} + R_{air} = (0.115 + 2.387)\times 10^{7}\ \text{A.t/m}$$
$$= 2.502\times 10^{7}\ \text{A.t/m}$$

$$\Psi = \frac{\mathscr{F}}{R};\ \Psi_{core} = \Psi_{air} = \frac{20{,}000}{2.502\times 10^{7}} \approx \boxed{8\times 10^{-4}\ \text{Wb/m}^2}$$

$$\mathscr{F}_{air} = \frac{R_{air}}{R_{air}+R_{core}}\mathscr{F} = \frac{2.387\times 20{,}000}{2.502} \approx \boxed{19{,}081\ \text{A.t}}$$

$$\mathscr{F}_{core} = \frac{R_{core}}{R_{air}+R_{core}}\mathscr{F} = \frac{0.115\times 20{,}000}{2.502} \approx \boxed{919\ \text{A.t}}$$