CPE 381: Fundamentals of Signals and Systems for Computer Engineers

Homework #4 Solution

1. Consider the following filters with the given poles and zeros and DC constant.

$H_1(s)$: $K = 1$; $poles\ p_1 = -1,\ p_{2,3} = -0.5 \pm j2\pi$; $zeros\ z_{1,2} = \pm j2\pi$;

$H_2(s)$: $K = 1$; $poles\ p_1 = -1,\ p_{2,3} = -1 \pm j2\pi$; $zeros\ z_1 = 1,\ z_{2,3} = 1 \pm j2\pi$;

$H_1(s)$: $K = 1$; $poles\ p_1 = -1,\ p_{2,3} = -1 \pm j2\pi$; $zeros\ z_1 = 1$

Use MATLAB to plot the magnitude response of these filters and indicated the type of filters they represent.

See section 5.7.3 in the textbook:

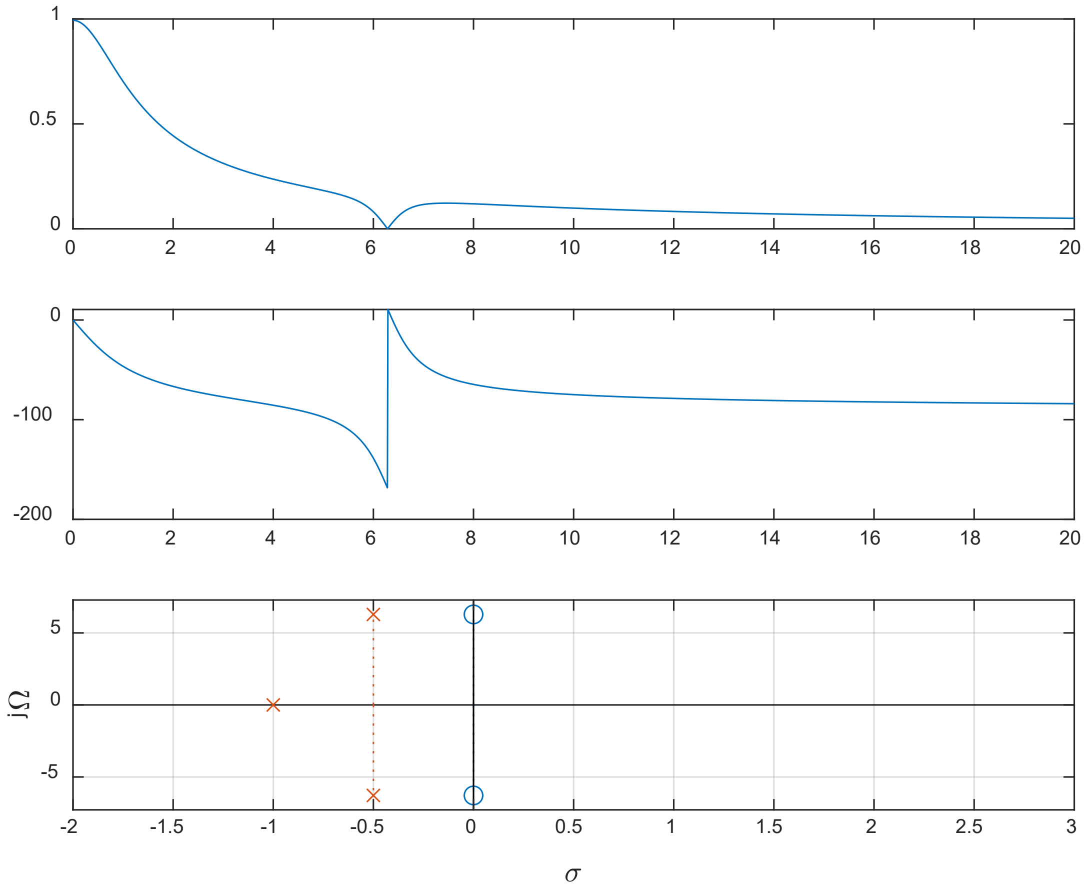
$$H(s) = \frac{\prod_i (s - z_i)}{\prod_k (s - p_k)}$$

$$H_1(s) = \frac{(s - j2\pi)(s + j2\pi)}{(s+1)(s + 0.5 - j2\pi)(s + 0.5 + j2\pi)} = \frac{s^2 + 4\pi^2}{s^3 + 2s^2 + (1.25 + 4\pi^2)s + (0.25 + 4\pi^2)}$$

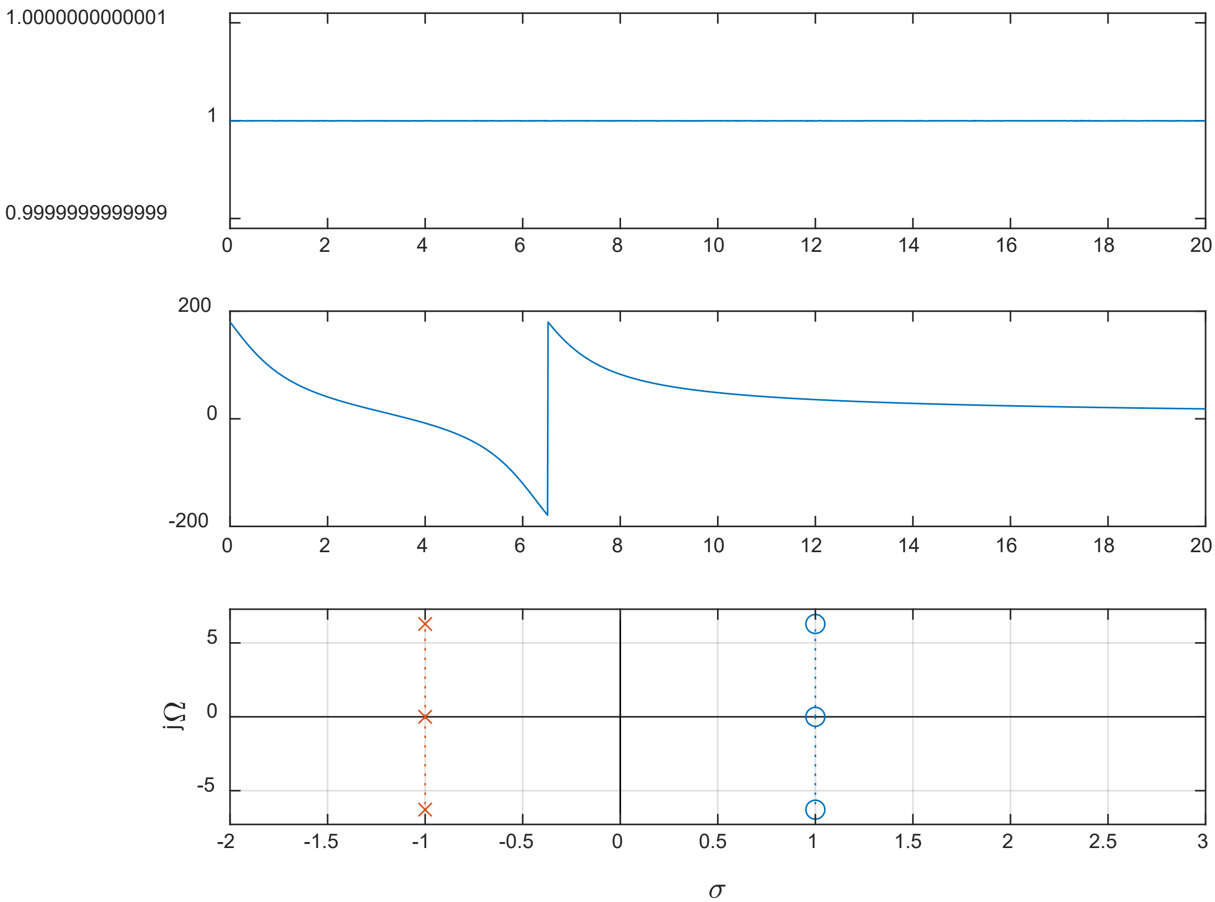
$$H_2(s) = \frac{(s-1)(s - 1 - j2\pi)(s - 1 + j2\pi)}{(s+1)(s + 1 - j2\pi)(s + 1 + j2\pi)} = \frac{s^3 - 3s^2 + (3 + 4\pi^2)s - (1 + 4\pi^2)}{s^3 + 3s^2 + (3 + 4\pi^2)s + (1 + 4\pi^2)}$$

$$H_3(s) = \frac{(s-1)}{(s+1)(s + 1 - j2\pi)(s + 1 + j2\pi)} = \frac{s^3 - 3s^2 + (3 + 4\pi^2)s - (1 + 4\pi^2)}{s^3 + 3s^2 + (3 + 4\pi^2)s + (1 + 4\pi^2)}$$

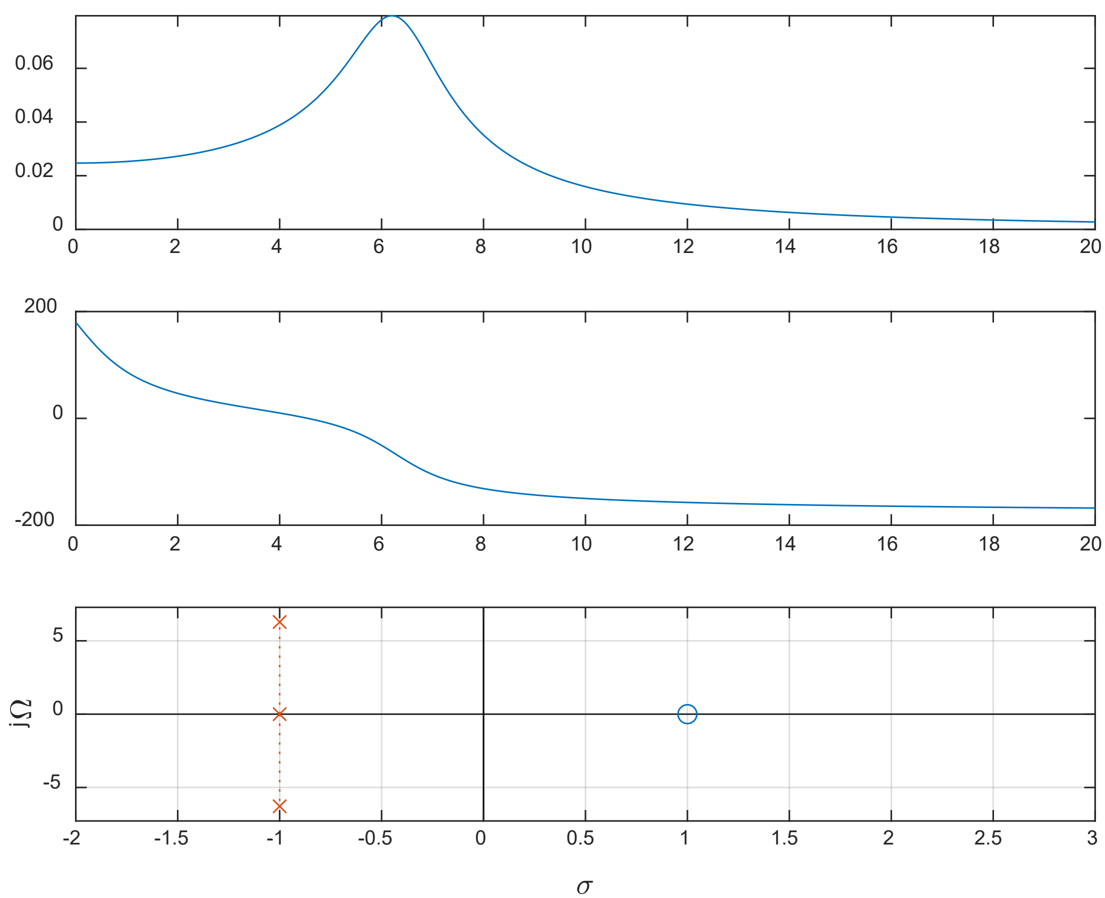
`` % CPE381 HW4_1 clear all; clf % 1 n1=[1 0 4*pi^2]; d1= [1 2 (1.25+4*pi^2) (0.25+4*pi^2)]; figure(1) wmax=20; freqresp_s(n1,d1,wmax) % 2 n2=[1 -3 3+4*pi^2 -(1+4*pi^2)]; d2=[1 3 3+4*pi^2 (1+4*pi^2)]; figure(2) freqresp_s(n2,d2,wmax) % 3 n3=[1 -1]; d3=d2; figure(3) freqresp_s(n3,d3,wmax) function [w,Hm,Ha]=freqresp_s(b,a,wmax) w=0:0.01:wmax; H=freqs(b,a,w); Hm=abs(H); Ha=angle(H)*180/pi; figure subplot(311) plot(w,Hm) subplot(312) plot(w,Ha) subplot(313) splane(b,a) ```	``` function splane(num,den) % function splane % input: coefficients of numerator (num) and denominator (den) in % decreasing order % output: pole/zero plot % use: splane(num,den) % z=roots(num); p=roots(den); A1=[min(imag(z)) min(imag(p))];A1=min(A1)-1; B1=[max(imag(z)) max(imag(p))];B1=max(B1)+1; N=20; D=(abs(A1)+abs(B1))/N; im=A1:D:B1; Nq=length(im); re=zeros(1,Nq); A=[min(real(z)) min(real(p))];A=min(A)-1; B=[max(real(z)) max(real(p))];B=max(B)+1; stem(real(z),imag(z),'o:') hold on stem(real(p),imag(p),'x:') hold on %plot(re,im,'k');xlabel('\sigma');ylabel('j\Om ega') grid % axis([A -A min(im) max(im)]) axis([min(im) max(im) min(im) max(im)]); hold off ```



H1 is a notch filter; it behaves like low-pass filter at low frequencies.



H2 is an all pass filter



H3 is a low-pass filter.

2. An ideal low pass filter *H(s)* with zero phase and magnitude response:

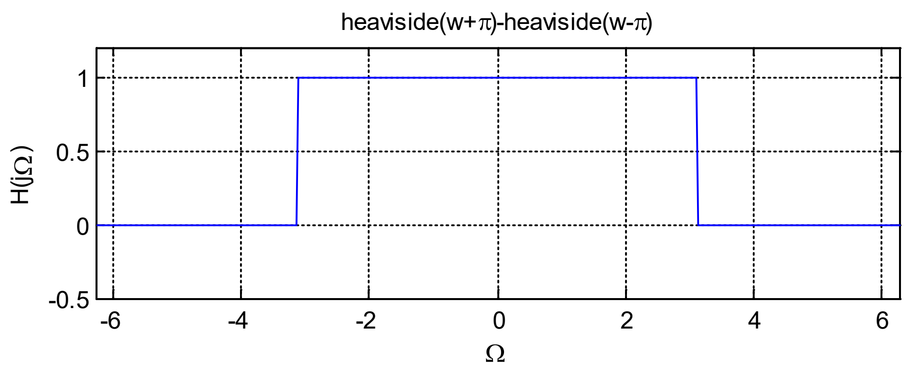
$$|H(j\Omega)| = \begin{cases} 1 & -\pi \le \Omega \le \pi \\ 0 & otherwise \end{cases}$$

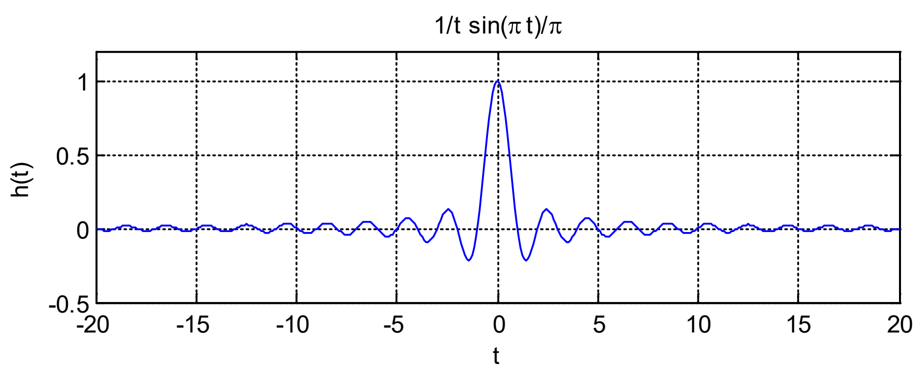
a) The impulse response is h(t) = sin(πt)/ πt , which is non-causal since h(t) $\neq$ 0 for t<0. (textbook 5.7.2.)

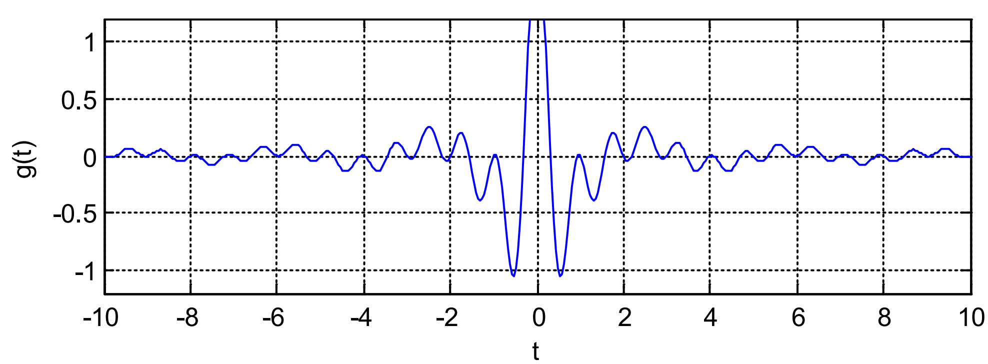
b) What is the effect of shifting the central frequency of the ideal filter for 7π ?

The bandpass filter. It can be implemented using ideal low-pass filter by shifting the central of the ideal low-pass filter

g(t) = 2*h(t)*cos(7*π*t)
and G(jΩ) = H(j(Ω - 7π)) + H(j(Ω + 7π))







3. A 12-bit AD converter is used to digitize signal with negative reference V_{R-} = 0.5V and positive reference V_{R+} = 2.5V.

a) (3 points) What is the quantization step?
b) (3 points) What is the output of the AD converter for V_{in} = 2.3 V ?
c) (2 points) What is the output of the AD converter for V_{in} = 0.35 V ?
d) (2 points) What is the output of the AD converter for V_{in} = 2.9 V ?

a) The quantization step is

$\Delta=(V_{r+} - V_{r-})/(2^{12}-1) = (2.5-0.5)/4095 = 0.4884$ mV

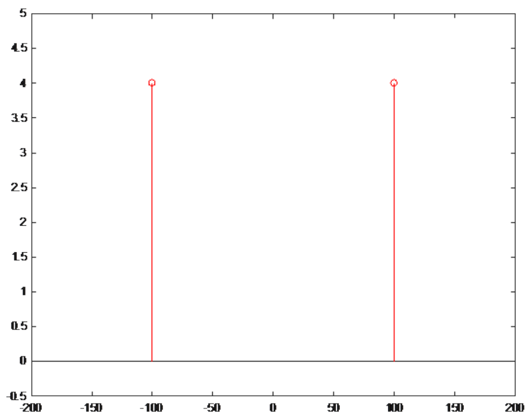
b) The output of the AD converter is

$ADout=(V_{in} - V_{r-})/ \Delta = (2.3 - 0.5) / \Delta = 3685$

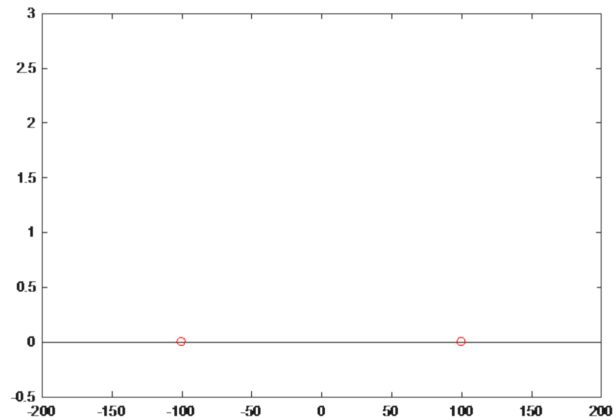
c) The output of the AD converter is 0

d) The output of the AD converter is 4095 (all ones)

4.



Magnitude



Phase