1.1 Determine the unit vector along the direction OP, where O is the origin and P is point $(4, -5, 1)$.

$$V = (4, -5, 1) = 4a_x - 5a_y + a_z.$$

$$a = \frac{V}{|V|} = \frac{4}{\sqrt{42}} a_x - \frac{5}{\sqrt{42}} a_y + \frac{1}{\sqrt{42}} a_z$$

$$= \boxed{0.6172 a_x - 0.7715 a_y + 0.1543 a_z}$$

1.3 If $\mathbf{A} = 4\mathbf{a}_x - 2\mathbf{a}_y + 6\mathbf{a}_z$ and $\mathbf{B} = 12\mathbf{a}_x + 18\mathbf{a}_y - 8\mathbf{a}_z$, determine:

(a) $\mathbf{A} - 3\mathbf{B}$

(b) $(2\mathbf{A} + 5\mathbf{B})/|\mathbf{B}|$

(c) $\mathbf{a}_x \times \mathbf{A}$

(d) $(\mathbf{B} \times \mathbf{a}_x) \cdot \mathbf{a}_y$

a:

$$A - 3B = (4, -2, 6) - 3(12, 18, -8)$$
$$= (4, -2, 6) - (36, 54, -24)$$
$$= [(4-36), (-2-54), (6-(-24))]$$
$$= \boxed{(-32, -56, 30)}$$

b:

$$\frac{(2A + 5B)}{|B|} = \frac{2(4, -2, 6) + 5(12, 18, -8)}{\sqrt{12^2 + 18^2 + 8^2}}$$

$$2A + 5B = [(8 + 60), (-4 + 90), (12 - 40)]$$

$$\frac{2A + 5B}{|B|} = \frac{(68, 86, -28)}{\sqrt{532}}$$

$$= \boxed{2.948 a_x + 3.728 a_y - 1.214 a_z}$$

c:

$a_x = (1,0,0);\ a_x \times A = \begin{vmatrix} 1 & 0 & 0 \\ 4 & -2 & 6 \end{vmatrix}$

$= 0a_x + [(0\times4)-(1\times6)]a_y + [(0\times4)-(-2\times1)]a_z$

$= 0a_x - 6a_y + 2a_z = \boxed{-6a_y + 2a_z}$

d:

$(B\times a_x) = \begin{vmatrix} 12 & 18 & -8 \\ 1 & 0 & 0 \end{vmatrix} = 0a_x - 8a_y - 18a_z$

$(0,-8,-18)\cdot(0,1,0) = \boxed{-8}$

1.5 Let $\mathbf{A} = -2\mathbf{a}_x + 5\mathbf{a}_y + \mathbf{a}_z$, $\mathbf{B} = \mathbf{a}_x + 3\mathbf{a}_z$, and $\mathbf{C} = 4\mathbf{a}_x - 6\mathbf{a}_y + 10\mathbf{a}_z$.

(a) Determine $\mathbf{A} - \mathbf{B} + \mathbf{C}$

(b) Find $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$

(c) Calculate the angle between $\mathbf{A}$ and $\mathbf{B}$

a:

$A - B + C = (-2,5,1) - (1,0,3) + (4,-6,10)$

$= (-3,5,-2) + (4,-6,10)$

$= \boxed{(1,-1,8)}$

b:

$A\cdot(B\times C)$:

$(B\times C) = \begin{vmatrix} 1 & 0 & 3 \\ 4 & -6 & 10 \end{vmatrix} = (0+18),(12-10),(-6-0)$

$= (18,2,-6)$

$A\cdot(B\times C) = (-2,5,1)\cdot(18,2,-6)$

$= -36 + 10 - 6$

$= \boxed{-32}$

c:

Angle between A & B:

$$\cos\theta_{AB} = \frac{A\cdot B}{|A||B|} = \frac{(-2,5,1)\cdot(1,0,3)}{\sqrt{4+25+1}\cdot\sqrt{1+9}}$$

$$= \frac{-2+0+3}{\sqrt{30}\cdot\sqrt{10}} = \frac{1}{17.321}$$

$$\approx 0.05773$$

$$\theta_{AB} = \cos^{-1}(0.05773) = \boxed{86.69^\circ}$$

1.13 Determine the dot product, cross product, and angle between

$$\mathbf{P} = 2\mathbf{a}_x - 6\mathbf{a}_y + 5\mathbf{a}_z \quad \text{and} \quad \mathbf{Q} = 3\mathbf{a}_y + \mathbf{a}_z$$

dot product:

$$(2,-6,5)\cdot(0,3,1) = 0-18+5 = \boxed{-13}$$

cross product:

$$P\times Q = \begin{vmatrix} 2 & -6 & 5 \\ 0 & 3 & 1 \end{vmatrix} = (-6-15)a_x + (0-2)a_y + (6-0)a_z$$

$$= \boxed{-21a_x - 2a_y + 6a_z}$$

Angle:

$$\cos\theta_{PQ} = \frac{P\cdot Q}{|P||Q|} = \frac{-13}{\sqrt{4+36+25}\cdot\sqrt{9+1}} = \frac{-13}{\sqrt{65}\cdot\sqrt{10}}$$

$$\approx -0.51$$

$$\theta_{PQ} = \cos^{-1}(-0.51) \approx \boxed{120.664^\circ}$$

1.27 If $\mathbf{H} = 2xy\mathbf{a}_x - (x+z)\mathbf{a}_y + z^2\mathbf{a}_z$, find:

(a) A unit vector parallel to $\mathbf{H}$ at $P(1, 3, -2)$

(b) The equation of the surface on which $|\mathbf{H}| = 10$

a:

$$H(1,3,-2) = (2\cdot1\cdot3)a_x - (1-2)a_y + (-2^2)a_z$$
$$= 6a_x + a_y + 4a_z$$

$$|H| = \sqrt{36+1+16} = \sqrt{53}$$

$$a_H = \frac{H}{|H|} = \frac{6}{\sqrt{53}}a_x + \frac{1}{\sqrt{53}}a_y + \frac{4}{\sqrt{53}}a_z$$

$$= \boxed{0.8242a_x + 0.1374a_y + 0.5494a_z}$$

b:

$$|H| = 10 = \sqrt{(2xy)^2 + (x+z)^2 + (z^2)^2}$$

$$10 = \sqrt{4x^2y^2 + (x+z)^2 + z^4}$$

$$\therefore \boxed{100 = 4x^2y^2 + x^2 + 2xz + z^2 + z^4}$$

1.31 **E** and **F** are vector fields given by $\mathbf{E} = 2x\mathbf{a}_x + \mathbf{a}_y + yz\mathbf{a}_z$ and $\mathbf{F} = xy\mathbf{a}_x - y^2\mathbf{a}_y + xyz\mathbf{a}_z$. Determine:

(a) $|\mathbf{E}|$ at (1, 2, 3)

(b) The component of **E** along **F** at (1, 2, 3)

(c) A vector perpendicular to both **E** and **F** at (0, 1, −3) whose magnitude is unity

a:

$$\text{At } (1,2,3),\ E = (2\cdot1)a_x + (1)a_y + (2\cdot3)a_z$$
$$= (2,1,6)$$

$$\therefore |E| = \sqrt{2^2+1^2+6^2} = \sqrt{41} \approx \boxed{6.403}$$

b:

$$F = (2,-4,6);\ E_F = \frac{(E\cdot F)F}{|F|^2} = \frac{36}{56}(2,-4,6)$$
$$= \boxed{1.286\,a_x - 2.571\,a_y + 3.857\,a_z}$$

c:

$$\text{At } (0,1,-3):\ E = (0,1,-3) \ \&\ F = (0,-1,0)$$

$$E \times F = \begin{vmatrix} 0 & 1 & -3 \\ 0 & -1 & 0 \end{vmatrix} = (0-3),\ (0),\ (0) = (-3,0,0).$$

$$a_{E\times F} = \pm\frac{E\times F}{|E\times F|} = \frac{-3,0,0}{\sqrt{-3^2}} = \boxed{\pm\, a_x}$$

2.1 Convert the following Cartesian points to cylindrical and spherical coordinates:

(a) $P(2, 5, 1)$

(b) $Q(-3, 4, 0)$

(c) $R(6, 2, -4)$

a:

$\rho = \sqrt{x^2+y^2} = \sqrt{2^2+5^2} = \sqrt{4+25} = \sqrt{29}$

≈ 5.385

$\phi = \arctan(y/x) = \arctan\left(\frac{5}{2}\right) \approx 68.2^\circ.$

$r = \sqrt{x^2+y^2+z^2} = \sqrt{30} \approx 5.477$

$\theta = \arctan\left(\frac{\sqrt{x^2+y^2}}{z}\right) = \arctan\left(\frac{5.385}{1}\right) \approx 79.48^\circ$

$P(\rho, \phi, z) = \boxed{P(5.385, 68.2^\circ, 1)}$

$P(r, \theta, \phi) = \boxed{P(5.477, 79.48^\circ, 68.2^\circ)}$

b:

$\rho = \sqrt{-3^2+4^2} = \sqrt{25} = 5$

$\phi = \tan^{-1}(4/-3) \approx -53.13 = 306.87^\circ$

$r = \sqrt{-3^2+4^2+0} = \sqrt{25} = 5$; $\theta = \tan^{-1}(\infty) = 90^\circ$

$Q(\rho, \phi, z) = \boxed{Q(5, 306.87^\circ, 0)}$

$Q(r, \theta, \phi) = \boxed{Q(5, 90^\circ, 306.87^\circ)}$

C:

$\rho = \sqrt{6^2+2^2} = \sqrt{40} \approx 6.325.$

$\phi = \tan^{-1}(2/6) = 18.43^\circ$

$r = \sqrt{36+4+16} = \sqrt{56} \approx 7.483.$

$\theta = \tan^{-1}(6.325/-4) \approx -57.69 = 122.31^\circ$

$R(\rho, \phi, z) = \boxed{R(6.325, 18.43^\circ, -4)}$

$R(r, \theta, \phi) = \boxed{R(7.483, 122.31^\circ, 18.43^\circ)}$

2.5 Given point $T(10, 60°, 30°)$ in spherical coordinates, express T in Cartesian and cylindrical coordinates.

T is given in spherical coordinates, $\therefore T(r, \theta, \phi)$

$r = 10, \ \theta = 60^\circ, \ \phi = 30^\circ$

$x = r \sin\theta \cos\phi = 10 \sin(60^\circ)\cos(30^\circ)$

$= (10)(0.866)(0.866) \approx \underline{7.5}$

$y = r\sin\theta \sin\phi = 10 \sin(60^\circ) \sin(30^\circ)$

$= (10)(0.866)(0.5) = \underline{4.33}$

$z = r\cos\theta = 10\cos(60^\circ) = (10)(0.5) = \underline{5}$

$\rho = r\sin\theta = 10\sin(60^\circ) = (10)(0.866) = \underline{8.66}.$

Cartesian coordinates : $T(x, y, z)$

$= \boxed{T(7.5, 4.33, 5)}$

Cylindrical coordinates: $T(\rho, \phi, z)$:

$= \boxed{T(8.66, 30^\circ, 5)}$

3.5 Calculate the area of the surface defined by $r = 5, 0 < \theta < \pi/4, 0 < \phi < \pi/2$.

$$dS = r^2 \sin\theta \, d\theta \, d\phi$$

$$S = r^2 \int_0^{\pi/2} d\phi \int_0^{\pi/4} \sin\theta \, d\theta \Big|_{r=5} = 25\left(\frac{\pi}{2}\right)\left(-\cos\theta\right)\Big|_0^{\pi/4}$$

$$= \frac{25\pi}{2}\left(-0.707 + 1\right) \approx \boxed{11.502}$$

3.6 Calculate the volume defined by $2 < \rho < 5, 0 < \phi < 30°, 0 < z < 10$.

$$dV = \rho \, d\rho \, d\phi \, dz$$

$$V = \int_{z=0}^{10} dz \int_{\phi=0}^{30} d\phi \int_{\rho=2}^{5} \rho \, d\rho$$

$$= 10\left(\pi/6\right)\left[\frac{\rho^2}{2}\Big|_2^5\right] = \frac{5\pi}{6}\left[\rho^2\Big|_2^5\right]$$

$$= 5\pi/6 \left[5^2 - 2^2\right] = \frac{5\pi}{6}\left[25 - 4\right]$$

$$= 5\pi/6 \, (21) = \frac{5\pi}{2}(7) \approx \boxed{54.98}$$