DAN OTIENO - HW 1.

1. (10 points) Write the formula and plot the roots of

$$z^7 + 1 = 0$$

$z^7 = -1$

following the form $z_k = e^{j(2k+1)\pi/n}$, for $n = 7$

$$\boxed{z_k = e^{j(2k+1)\pi/7}\ ;\ k = (0, 1, 2, 3, 4, 5, 6).}$$

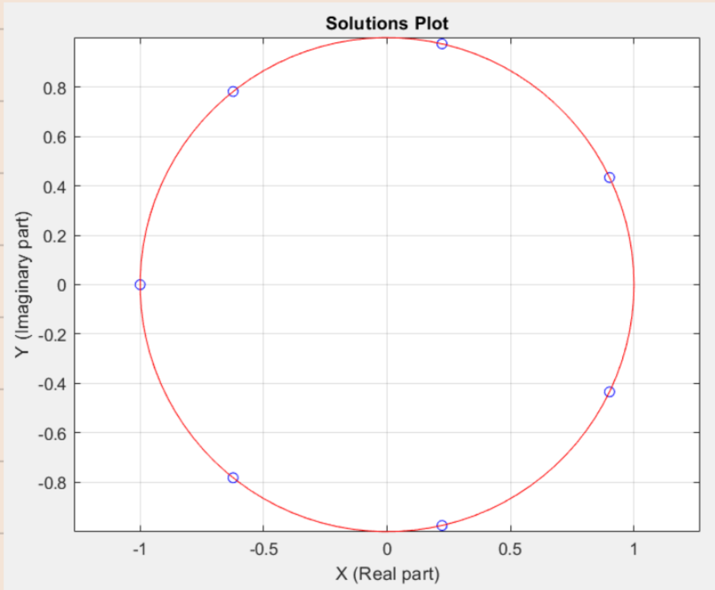
Solutions plotted using Matlab:

hw1.m

```
% Dan Otieno.
% CPE 381-01.
% Homework 1 - Q1.
% 01/27/2023.

k = 0:1:6;
z = exp(1i*pi.*(2*k+1)/7);

plot(real(z),imag(z),'bo')
title('Solutions Plot');
axis equal
grid on
ylabel('Y (Imaginary part)');
xlabel('X (Real part)');
z1 = abs(z(1));
c = linspace(0,2*pi,100);
hold on
plot(z1*cos(c),z1*sin(c),'r-')
```



2. (15 points) Represent the following complex numbers in alternative form (polar ←→ {Re,Im} z=x +jy)

a) $1+j$

b) $1-j$

c) $5\,e^{j210°}$

d) $5\,e^{-j210°}$

e) $z\,z^*$

(a) $1+j$; $x=1$, $y=1$

Magnitude $= \sqrt{1^2+1^2} = \sqrt{2}$.

$\theta = \tan^{-1}(y/x) = \tan^{-1}(1/1) = \tan^{-1}(1) \approx 0.785$

$\therefore \boxed{z = \sqrt{2}\,e^{j0.785}}$

(b) $1-j$; $x=1$, $y=-1$

Magnitude $= \sqrt{1^2+(-1)^2} = \sqrt{2}$

$\theta = \tan^{-1}(-1/1) = \tan^{-1}(-1) \approx -0.785$

$\therefore \boxed{z = \sqrt{2}\,e^{-j0.785}}$

(c) $5e^{j210°}$

$= 5e^{j(180°+30°)} = 5e^{j180°}\cdot e^{j30°}$

$* \; e^{j180°} = \cos(180°) + j\sin(180°) = -1 + j0 = -1$

$\therefore 5e^{j180°}\cdot e^{j30°} = 5(-1)\cdot e^{j30°} = -5e^{j30°}$

$= -5\cos(30°) - j5\sin(30°) = \boxed{-4.33 - j2.5}$

(d) $5e^{-j210^\circ}$

$= 5e^{-j(180^\circ+30^\circ)} = 5e^{j180^\circ} \cdot e^{-j30^\circ}$

$e^{j180^\circ} = -1 \quad \therefore$ we have $-5e^{-j30^\circ}$

$= -5\cos(-30^\circ) - j5\sin(-30^\circ) = \boxed{-4.33 + j2.5}$

(e) zz^*

$= z \cdot z^*$

$= (x+jy)(x-jy)$

$= x^2 + y^2 = \boxed{|z|^2}$

---

3. (20 points) Use Euler's identity to find trigonometric identities in terms of sin(α), sin(β), cos(α), and cos(β):

   a) sin(α + β)

   b) cos(α + β)

   Demonstrate all the steps in formula evaluation.

(a) $\sin(\alpha+\beta)$:

$$= \frac{e^{j(\alpha+\beta)} - e^{-j(\alpha+\beta)}}{2j} = \frac{e^{j\alpha}\cdot e^{j\beta} - e^{-j\alpha}\cdot e^{-j\beta}}{2j}$$

$$= \frac{\left[(\cos\alpha + j\sin\alpha)(\cos\beta + j\sin\beta)\right] - \left[(\cos\alpha - j\sin\alpha)(\cos\beta - j\sin\beta)\right]}{2j}$$

$*$ $(\cos\alpha + j\sin\alpha)(\cos\beta + j\sin\beta)$

$= \cos\alpha\cos\beta + j\sin\beta\cos\alpha + j\sin\alpha\cos\beta + j\sin\alpha\, j\sin\beta$

$$* \ (\cos\alpha - j\sin\alpha)(\cos\beta - j\sin\beta)$$
$$= \cos\alpha\cos\beta - j\sin\beta\cos\alpha - j\sin\alpha\cos\beta + j\sin\alpha\, j\sin\beta$$

Subtract:

$$\left(\cancel{\cos\alpha\cos\beta} + j\sin\beta\cos\alpha + j\sin\alpha\cos\beta + \cancel{j\sin\alpha\, j\sin\beta}\right)$$
$$-$$
$$\left(\cancel{\cos\alpha\cos\beta} - j\sin\beta\cos\alpha - j\sin\alpha\cos\beta + \cancel{j\sin\alpha\, j\sin\beta}\right)$$

$$= \frac{j\sin\beta\cos\alpha + j\sin\alpha\cos\beta + j\sin\beta\cos\alpha + j\sin\alpha\cos\beta}{2j}$$

$$= \frac{2j\sin\beta\cos\alpha + 2j\sin\alpha\cos\beta}{2j}$$

$$= \frac{\cancel{2j}(\sin\beta\cos\alpha + \sin\alpha\cos\beta)}{\cancel{2j}}$$

$$= \boxed{\sin\beta\cos\alpha + \sin\alpha\cos\beta}$$

(b) $\cos(\alpha+\beta)$

$$= \frac{e^{j(\alpha+\beta)} + e^{-j(\alpha+\beta)}}{2}$$

$$= \frac{e^{j\alpha} \cdot e^{j\beta} + e^{-j\alpha} \cdot e^{-j\beta}}{2}$$

$$= \frac{\left(\begin{array}{l} [(\cos\alpha + j\sin\alpha)(\cos\beta + j\sin\beta)] + \\ [(\cos\alpha - j\sin\alpha)(\cos\beta - j\sin\beta)] \end{array} \right)}{2}$$

$*\ (\cos\alpha + j\sin\alpha)(\cos\beta + j\sin\beta) =$
$\cos\alpha\cos\beta + j\sin\beta\cos\alpha + j\sin\alpha\cos\beta$
$+ j\sin\alpha\, j\sin\beta.$

$*\ (\cos\alpha - j\sin\alpha)(\cos\beta - j\sin\beta) =$
$\cos\alpha\cos\beta - j\sin\beta\cos\alpha - j\sin\alpha\cos\beta$
$+ j\sin\alpha\, j\sin\beta.$

Add:

$\cos\alpha\cos\beta + \cancel{j\sin\beta\cos\alpha} + \cancel{j\sin\alpha\cos\beta} + j\sin\alpha\, j\sin\beta$

$+$

$\cos\alpha\cos\beta - \cancel{j\sin\beta\cos\alpha} - \cancel{j\sin\alpha\cos\beta} + j\sin\alpha\, j\sin\beta$

$$= \frac{2\cos\alpha\cos\beta - 2(\sin\alpha\sin\beta)}{2}$$

$$= \frac{\cancel{2}((\cos\alpha\cos\beta) - (\sin\alpha\sin\beta))}{\cancel{2}}$$

$$= \boxed{(\cos\alpha\cos\beta) - (\sin\alpha\sin\beta)}$$

6. (30 points)

Accelerometer with analog output, sensitivity ±2g, and power supply of +3V is used in smartphone to determine orientation of the smartphone according to the figure below.

What are the values of X and Y components [in Volts] for the following positions

a)	b)	c)	d)
X =	X =	X =	X =
Y =	Y =	Y =	Y =

What is the angle of the smartphone if:

e) X = 1.875 V, Y = 0.8505 V → α =

f) X = 2.1495 V, Y = 1.875V → α =

Please draw a phone as a part of the solution to avoid confusion.

Sensitivity:

$1g \Rightarrow \quad S = V_{cc} / Range = 3V / 4g$

$= 0.75\ [V/g]$

Acceleration:

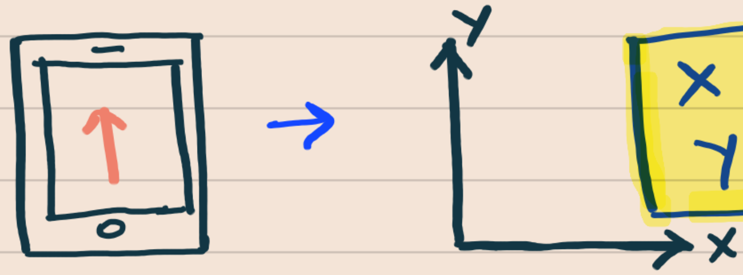
$A = A_0 + S \cdot a$

$A_0 = 0g = 1.5V$

$A_1 = (+1g) = 1.5V + 0.75\ [V/g] \cdot 1g = 2.25V$

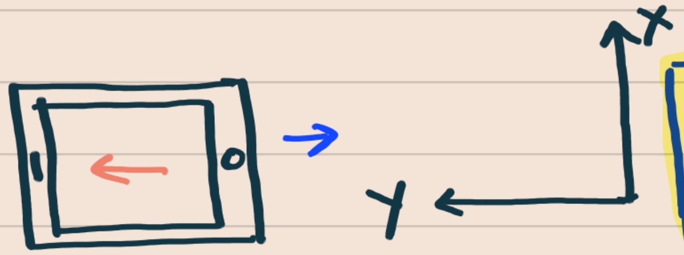
$A_{-1} = (-1g) = 1.5V + 0.75\ [V/g] \cdot (-1g) = 0.75V$

ⓐ



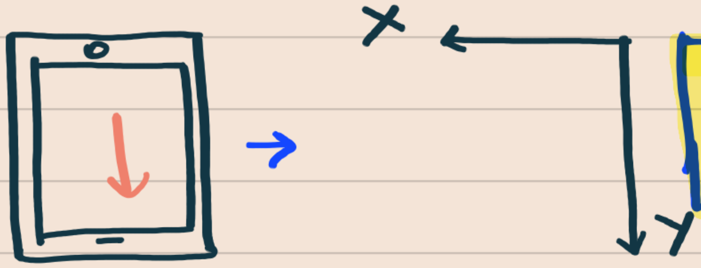
$X = 0g = 1.5V$
$Y = -1g = 0.75V$

ⓑ



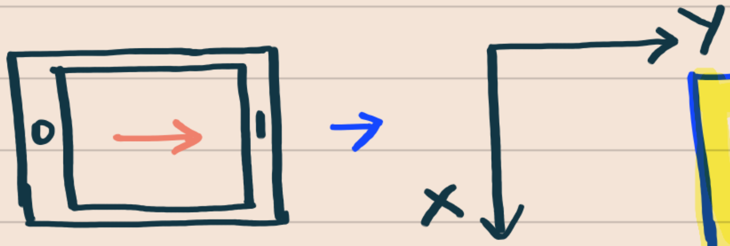
$X = -1g = 0.75V$
$Y = 0g = 1.5V$

ⓒ



$X = 0g = 1.5V$
$Y = 1g = 2.25V$

ⓓ



$X = 1g = 2.25V$
$Y = 0g = 1.5V$

- Angle of smartphone is:

ⓔ $X = 1.875V$, $Y = 0.8505V$

$$\text{acceleration } (a_x) = \frac{A - A_0}{S}$$

$$= \frac{(1.875V - 1.5V)}{0.75\,[V/g]} = 0.5g$$

$$(a_y) = \frac{(0.8505V - 1.5V)}{0.75\,[V/g]} = -0.866g$$

$$\alpha = \tan^{-1}(a_y/a_x) = \tan^{-1}(-0.866/0.5)$$

$$\approx \boxed{-\pi/3 \approx -60^\circ}$$

ⓕ $x = 2.1495\,V$, $y = 1.875\,V$

$$(a_x) = \frac{(2.1495\,V - 1.5\,V)}{0.75\,[V/g]} = 0.866\,g$$

$$(a_y) = \frac{(1.875\,V - 1.5\,V)}{0.75\,[V/g]} = 0.5\,g$$

$$\alpha = \tan^{-1}(0.5/0.866) = \boxed{\pi/6 = 30^\circ}$$