DAN OTIENO - EE 307 - Homework 6.

6.1 Given $V = 5x^3y^2z$ and $\varepsilon = 2.25\varepsilon_o$, find (a) $\mathbf{E}$ at point $P(-3, 1, 2)$, (b) ρ_v at P.

a:

$$E = -\nabla V = -\left(\frac{\delta V}{\delta x}\hat{a}_x + \frac{\delta V}{\delta z}\hat{a}_y + \frac{\delta V}{\delta z}\hat{a}_z\right)$$

$$= -\left(15x^2y^2z\,\hat{a}_x + 10x^3yz\,\hat{a}_y + 5x^3y^2\,\hat{a}_z\right)$$

At point P:

$$E = -\left[15(-3)^2(1)^2(2)\,\hat{a}_x + 10(-3)^3(1)(2)\,\hat{a}_y + 5(-3)^3(1)^2\,\hat{a}_z\right]$$

$$= -\left[15(9)(1)(2)\,\hat{a}_x + 10(-27)(1)(2)\,\hat{a}_y + 5(-27)(1)\,\hat{a}_z\right]$$

$$= \boxed{270\,\hat{a}_x + 540\,\hat{a}_y + 135\,\hat{a}_z \text{ V/m}}$$

b:

$$\rho_v = \nabla \cdot D = -\varepsilon \nabla^2 V$$

$$\nabla^2 V = \frac{\delta^2 V}{\delta x^2} + \frac{\delta^2 V}{\delta y^2} + \frac{\delta^2 V}{\delta y^2}$$

$$= \frac{\delta}{\delta x}(15x^2y^2z) + \frac{\delta}{\delta y}(10x^3yz) + \frac{\delta}{\delta z}(5x^3y^2)$$

$$= 30xy^2z + 10x^3z$$

At point P, $\rho_v = -\varepsilon \nabla^2 V$

$$= -2.25\left(\frac{10^{-9}}{36\pi}\right)\left[30(-3)(1)(2) + 10(-27)(2)\right]$$

$$\approx \boxed{14.32 \text{ nC/m}^3}$$

6.10 Determine whether each of the following potentials satisfies Laplace's equation.

(a) $V_1 = 3xyz + y - z^2$ (per Canvas, do (a) only).

$$\nabla^2 V_1 = \frac{\delta^2 V}{\delta x^2} + \frac{\delta^2 V_1}{\delta y^2} + \frac{\delta^2 V_1}{\delta z^2}$$

1^{st} derivative $= 3yz + 1 - 2z$

2^{nd} derivative $= 0 + 0 - 2 = -2 \neq 0$.

∴ This potential does $\boxed{\text{NOT}}$ satisfy Laplace equation.

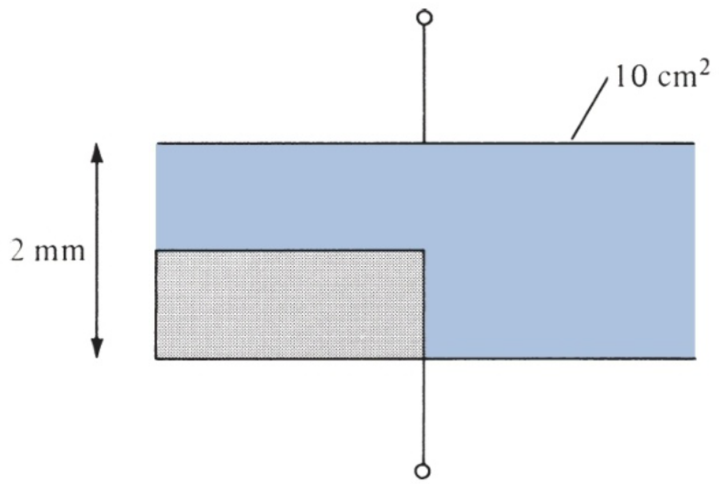

FIGURE 6.40 For Problem 6.39.

6.39 The parallel-plate capacitor of Figure 6.40 is quarter-filled with mica ($\varepsilon_r = 6$). Find the capacitance of the capacitor.

$$C_1 = \frac{\varepsilon_0 A}{d} \; ; \; C_2 = \frac{\varepsilon_0 \varepsilon_r A}{d} \; ; \; C_3 = \frac{\varepsilon_0 A}{2d} \; ; \; C = \frac{C_1 C_2}{C_1 + C_2} + C_3$$

$$C_1 C_2 = \left(\frac{\varepsilon_0 A}{d}\right)\left(\frac{\varepsilon_0 \varepsilon_r A}{d}\right) = \frac{\varepsilon_0^2 \varepsilon_r A^2}{d^2}$$

$$C_1 + C_2 = \frac{\varepsilon_0 A}{d} + \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{\varepsilon_0(\varepsilon_r + 1)A}{d}$$

$$\frac{\varepsilon_0^2 \varepsilon_r A^2}{d^2} \div \frac{\varepsilon_0(\varepsilon_r + 1)A}{d} = \frac{\overset{\varepsilon_0}{\cancel{\varepsilon_0^2}}\, \varepsilon_r \cancel{A}}{\underset{d}{\cancel{d^2}}} \times \frac{\cancel{d}}{\cancel{\varepsilon_0}(\varepsilon_r + 1)\cancel{A}}$$

$$\frac{C_1 C_2}{C_1 + C_2} + C_3 = \frac{\varepsilon_o \varepsilon_r}{d(\varepsilon_r + 1)} + \frac{\varepsilon_o A}{2d}$$

$$= \frac{\varepsilon_o A}{d}\left(\frac{1}{2} + \frac{\varepsilon_r}{\varepsilon_r + 1}\right)$$

$$C = \left(\frac{10^{-9}}{36\pi}\right)\left(\frac{10 \times 10^{-4}}{2 \times 10^{-3}}\right)\left(\frac{1}{2} + \frac{6}{7}\right) \approx \boxed{6\,pF}$$

6.51 A coaxial cable has inner radius of 5 mm and outer radius of 8 mm. If the cable is 3 km long, calculate its capacitance. Assume $\varepsilon = 2.5\varepsilon_o$.

$$C = \frac{2\pi \varepsilon L}{\ln(b/a)} = \frac{(2\pi)(2.5)\left(\frac{10^{-9}}{36\pi}\right)(3 \times 10^3)}{\ln\left(\frac{8}{5}\right)}$$

$$C \approx \boxed{0.867\,\mu F}$$

6.62 If the earth is regarded as a spherical capacitor, what is its capacitance? Assume the radius of the earth to be approximately 6370 km.

$$C = 4\pi\varepsilon_o a = \cancel{(4\pi)}\left(\frac{10^{-9}}{\underset{9}{\cancel{36\pi}}}\right)(6.37 \times 10^6)$$

$$C \approx \boxed{0.708\,mF}$$