Department of Electrical and Computer Engineering
The University of Alabama in Huntsville

CPE 381: Fundamentals of Signals and Systems for Computer Engineers

Homework #1 Solution

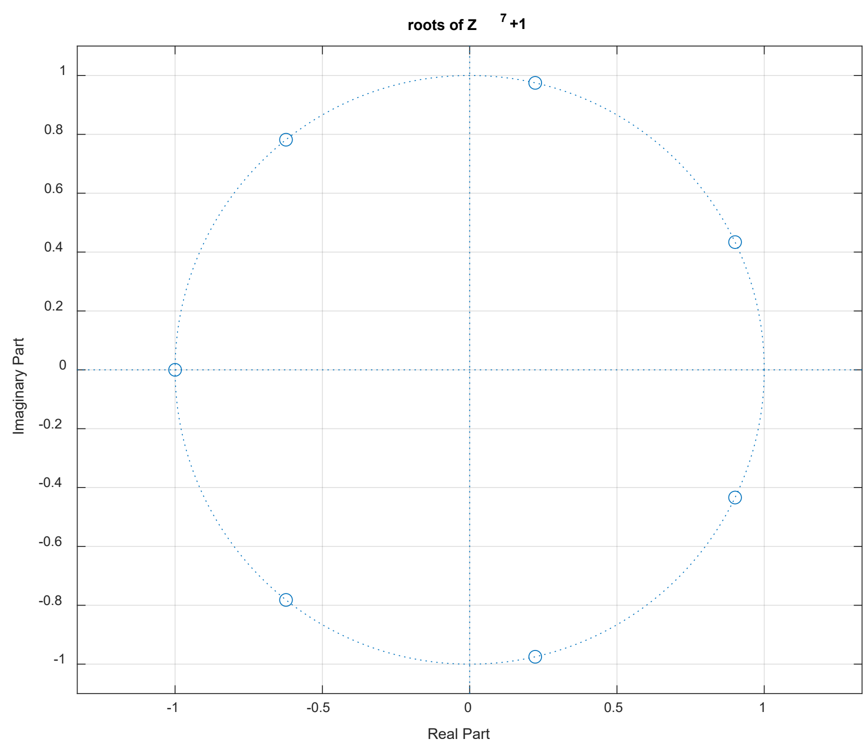
1. (10 points)

Find and plot the roots of

$$Z^7 + 1 = 0$$

$z^7 = -1 \rightarrow$

$$z_k = e^{j(2k+1)\pi/7}, k = 0,1, \dots 6$$

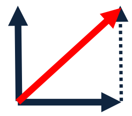


2. (15 points)

Represent the following complex numbers in alternative form (polar ←→ {Re,Im} z=x +*j*y)

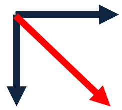
a) $1 + j$

$\sqrt{2}e^{j\pi/4}$



b) $1 - j$

$\sqrt{2}e^{-j\pi/4}$



c) $5\ e^{j210°}$

$5e^{j\,210} = 5e^{j(180+30)} = 5e^{j180}\ e^{j30}$

$e^{j180°} = \cos(180°) + j\sin(180°) = -1 + j\,0 = -1$

$\Rightarrow -\ 5e^{j30} = -5\cos(30) - j5\sin(30) = -4.33 - j\,2.5$

d) $5\ e^{-j210°}$

$5e^{-j\,210} = 5e^{-j(180+30)} = 5e^{j180}\ e^{-j30} = -\ 5e^{-j30} = -5\cos(-30) - j5\sin(-30) = -4.33 + j\,2.5$

e) $Z{\cdot}Z^*$

$Z{\cdot}Z^* = (X+jY)(X-jY) = X^2 + Y^2 = |Z|^2$

3. (20 points)

Use Euler's identity to find trigonometric identities in terms of sin(α), sin(β), cos(α), and cos(β):

a) cos(α + β)

$$\cos(\alpha + \beta) = \frac{e^{j(\alpha+\beta)} + e^{-j(\alpha+\beta)}}{2} = \frac{(\cos(\alpha) + j\sin(\alpha))(\cos(\beta) + j\sin(\beta)) + (\cos(\alpha) - j\sin(\alpha))(\cos(\beta) - j\sin(\beta))}{2}$$
$$= \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

b) sin(α + β)

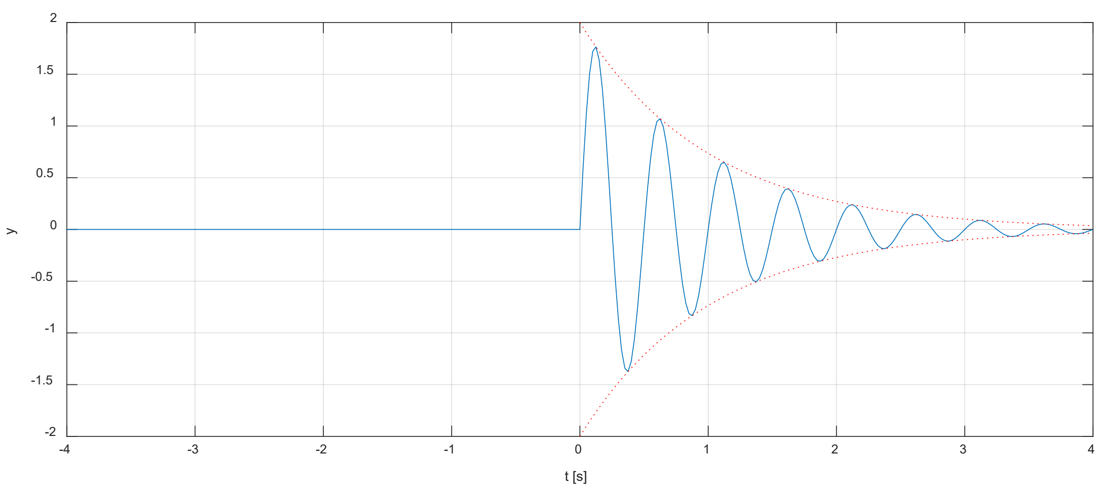
$$\sin(\alpha + \beta) = \frac{e^{j(\alpha+\beta)} - e^{-j(\alpha+\beta)}}{2j} = \cos(\alpha)\sin(\beta) + \sin(\alpha)\cos(\beta)$$

4. (10 points)

```matlab
%% Initialization
Fs=20;  % sampling frequency
Ts=1/Fs; % sampling interval
f=2; % signal frequency 2 Hz
tmax=4; % maximum time
t=-tmax:Ts:tmax; % time [s]
N=length(t); % number of elements in the vector
i0=round(4*Fs)+1; % index of time 0 (4 seconds after -4 seconds)
t1=0:Ts:4; % time > 0 [s]

%% #4: Signal
A=2;    % Amplitude
xenv=A*exp(-t1); % envelope A*e^(-t)
x=xenv.*sin(2*pi*f*t1); % causal signal for t>0
% create samples for t<0 (just zeros)
y=zeros(1,N); % initialize all elements to zero
y(i0:N)=x; % add calculated values from time zero - equivalent to index i0

% plot signal with envelope, labels, and grid
figure
plot(t,y,t1,xenv,'r:',t1,-xenv,'r:'),xlabel('t [s]'),ylabel('y'),grid
% the same signal with the stem plot if necessary
% figure
% stem(t,y),xlabel('t [s]'),ylabel('y'),grid
```

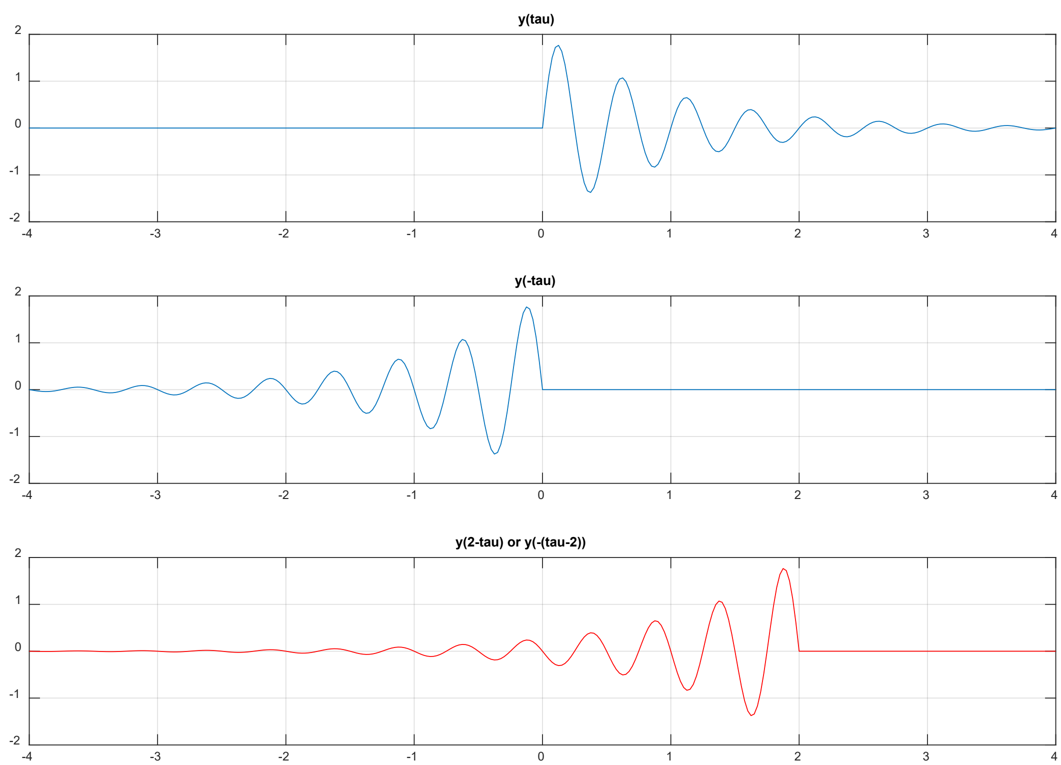


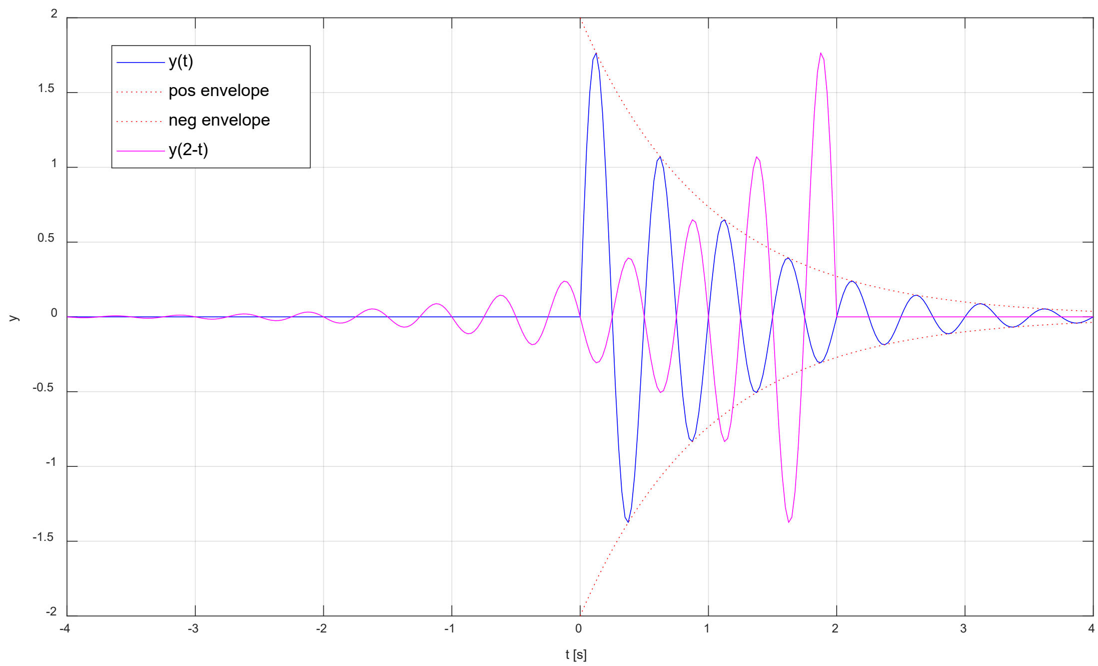
5. (15 points)

```
%% #5: Convolution ready signal
% for convolution we need signal y(2-tau)
tau=2; % tau = 2 seconds (delay)
t3=0:Ts:(tmax+tau); % new time to fit in original plot
N3=length(t3);
y3=zeros(1,N);
xenv3=exp(-t3); % envelope
x3=A*xenv3.*sin(2*pi*f.*t3);
y3(1:N3)=fliplr(x3);


%% Educational plot
figure  % create new figure
subplot(311)
plot(t,y),grid,title('y(tau)')
subplot(312)
plot(t,fliplr(y)),grid,title('y(-tau)')
subplot(313)
plot(t,y3,'r'),grid,title('y(2-tau) or y(-(tau-2))')

%% Both signals
figure  % create new figure
plot(t,y,'b',t1,xenv,'r:',t1,-xenv,'r:',t,y3,'m')
xlabel('t [s]'),ylabel('y'),grid
legend('y(t)','pos envelope','neg envelope','y(2-t)')
```





y(t)
pos envelope
neg envelope
y(2-t)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
y
-4
-3
-2
-1
0
1
2
3
4
t [s]

3. (30 points)

Accelerometer (± 2g) with analog output and power supply of +3V is used in smartphone to determine orientation of the smartphone according to the figure below.

Sensitivity 1*g* → *s = 3V / 4 g* = 0. 75 [V / g]

Acceleration output for sensitivity *s*, acceleration *a, and DC offset (0g)* A_0:

A = A_0 + *s*a*

A_0 (0 g) = 1.5V

A_1 (+1 g) = 1.5V + 0. 75[V/g]*1[g] = 2.25V

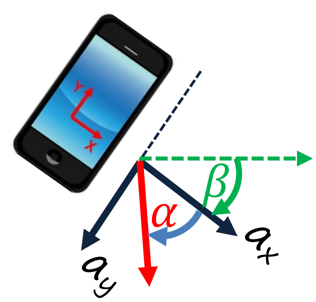
A_{-1} (-1 g) = 1.5V + 0. 75[V/g]*(-1[g]) = 0.75V

What are the values of X and Y components [in Volts] for the following positions

a)	b)	c)	d)
X = 1.5V (0 g)	X = 0.75V (-1g)	X = 1.5V (0 g)	X = 2.25V (1g)
Y = 0.75V (-1g)	Y = 1.5V (0 g)	Y = 2.25V (1g)	Y = 1.5V (0 g)

What is the angle of the smartphone if:

e) a_x = 1.875V, a_y = 0.8505V



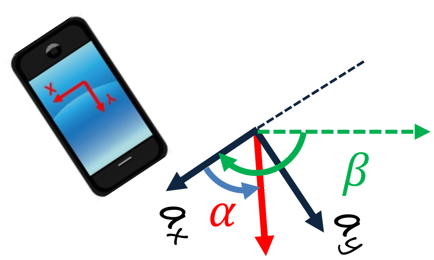
acceleration is:

$$a_x = \frac{A-A_0}{s} = \frac{1.875V-1.5V}{0.75\frac{V}{g}} = 0.5\,g, a_y = \frac{A-A_0}{s} = \frac{0.8505V-1.5V}{0.75\frac{V}{g}} = -0.866g,$$

and angle is

$$\alpha = \text{atan}(-0.866/0.5) = -60°, \beta = -30°$$

f) a_x = 2.1495V, a_y = 1.875V



$$a_x = 0.866\,g, a_y = 0.5g,$$

$$\alpha = \text{atan}\left(\frac{0.5}{0.87}\right) = 30°,\ \ \beta = -120°(between\ horizontal\ and\ X\ axis)$$