CPE381 Fundamentals of Signals & Systems for Computer Engineers.

Homework 1 Solutions

1

$$\int_0^t dV(x)\, dx = \int_0^t \frac{i_c(x)}{C}\, dx$$

$$V(x)\Big|_0^t = \frac{1}{C}\int_0^t i_c(x)\, dx$$

$$V(t) - V(0) = \frac{1}{C}\int_0^t i_c(x)\, dx$$

@ $t = 2\cancel{s}$; $C = 10F$; $V(0) = 0.2V$; $i_c(x) = \sin(x)$

$$V(2) - V(0) = \frac{1}{10}\int_0^2 \sin(x)\, dx$$

$$V(2) - 0.2 = \frac{1}{10}\left[-\cos(x)\right]\Big|_0^2$$

$$V(2) = 0.2 + \frac{1}{10}\left[\cos(0) - \cos(2)\right]$$

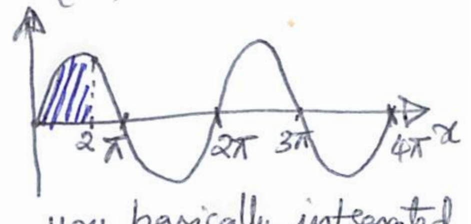
$$= 0.2 + 0.1\left[1 - (-0.42)\right]$$

∵ 2 is in radians
$\cos(2) = -0.416$

$$= 0.2 + 0.1\left[1.42\right]$$

$$= 0.2 + 0.142 = 0.342$$

$$\boxed{V(2) = 0.342\,V}$$



you basically integrated a portion of the sine wave.

```
% Homework 1 - Question 1 parts (b) through (d)
clear; % clears the variable space

C = 10; % this is capacitance C = 10F

V0 = 0.2; % this is V(0)

Ts = 0.2;
% this is the sampling period (for part (b) Ts = 0.2;
% (c) Ts = 0.1 and for part (d) Ts=0.01. You will manually
% change the Ts appropriately in this line for each part.

x = 0:Ts:2; % defining the range of variable x

V2 = 0; % initializing V(2) to zero

for i = 1:length(x)
    % performing the summation in this loop
    V2 = V2 + ((1/C)*Ts*sin(x(i)));
end

V2 = V0 + V2;
%adding the value of V(0) to the summation result from loop

V2

% notice that if you do not end a line with semicolon (;)
% then it will print the output to command window
```

Results:

Part (b): Ts = 0.2 will give the following output in command window

V2 =

0.3502

Part (c): Ts = 0.1 will give the following output in command window

V2 =

0.3460

Part (d): Ts = 0.01 will give the following output in command window

V2 =

0.3421

Part (e): Comparison of parts (b) through (d) with part (a).

Notice that as we make Ts small, the value for V(2) is very close to the result in part (a) i.e. V(2)=0.342V

This shows that fine resolution (small value of Ts) helps in getting close to analog computation.

Extra alternative code:

You can also write the code using in-built MATLAB 'sum' function as shown below:

```
% Alternate code
% Homework 1 - Question 1 parts (b) through (d)
clear;

C = 10; % this is capacitance C = 10F
V0 = 0.2; % this is V(0)
Ts = 0.2; % Ts = 0.2 or 0.1 or 0.01
x = 0:Ts:2;
y = sin(x);
V2 = (1/C)*Ts*sum(y); % performing the summation using
inbuilt MATLAB sum() function
V2 = V0 + V2;
% adding the value of V(0) to the summation result from
loop
V2
```

② $z^7 + 1 = 0$

Solving for $z^7 = -1$

Assuming the general case $z^n = -1$ if 'n' is odd then the solutions are of the form

$$z_k = e^{\frac{j(2k+1)\pi}{n}} \quad ; k = 0, 1, 2, \dots (n-1)$$

For $n = 7$:-

$$z^7 = -1$$

$$z_k = e^{\frac{j(2k+1)\pi}{7}} \quad ; \boxed{k = 0, 1, 2, 3, 4, 5, 6}$$

$k = 0$; $z_0 = e^{j(0+1)\pi/7} = e^{j\pi/7}$

$$z_0 = \cos\frac{\pi}{7} + j\sin\frac{\pi}{7} = \boxed{0.901 + j0.434}$$

$k = 1$; $z_1 = e^{j(2 \cdot 1+1)\pi/7} = e^{j\frac{3\pi}{7}}$

$$z_1 = \cos\frac{3\pi}{7} + j\sin\frac{3\pi}{7}$$

$$\boxed{z_1 \simeq 0.226 + j0.975}$$

$k = 2$; $z_2 = e^{j(2 \cdot 2+1)\pi/7} = e^{j5\pi/7} = \cos\frac{5\pi}{7} + j\sin\frac{5\pi}{7}$

$$\boxed{z_2 \simeq -0.624 + j0.782}$$

$k = 3$; $z_3 = e^{j(2 \cdot 3+1)\pi/7} = e^{j\pi} = \cos\pi + j\sin\pi = -1$

$$\boxed{z_3 = -1}$$

$k = 4$; $z_4 = e^{j(2 \cdot 4+1)\pi/7} = e^{j\frac{9\pi}{7}} = \cos\frac{9\pi}{7} + j\sin\frac{9\pi}{7}$

$$\boxed{z_4 = -0.624 - j0.782}$$

$$k=5; \quad z_5 = e^{j(2\cdot5+1)\pi/7} = e^{j\,11\pi/7} = e^{j(\pi + \frac{4\pi}{7})}$$

$$= e^{j\pi} \cdot e^{j\,4\pi/7}$$

$$= -e^{j4\pi/7}$$

$$= -\left[\cos\frac{4\pi}{7} + j\sin\frac{4\pi}{7}\right]$$

$$\boxed{z_5 = 0.223 - j0.975}$$

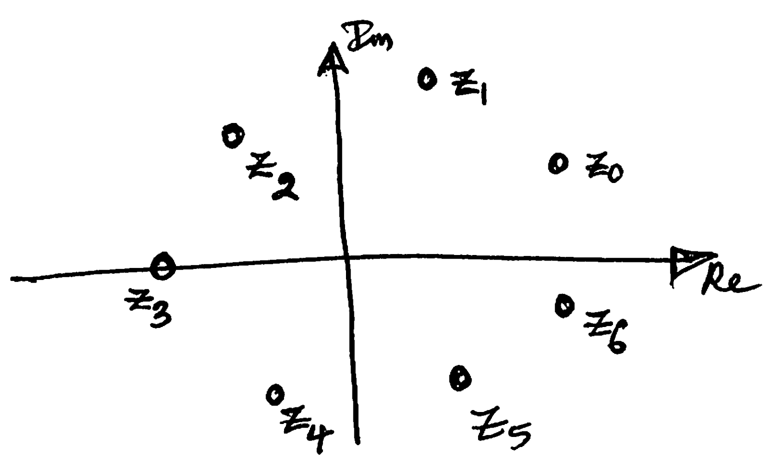
$$k=6; \quad z_6 = e^{j(2\cdot6+1)\pi/7} = e^{j\,13\pi/7} = e^{j(\pi + \frac{6\pi}{7})}$$

$$= e^{j\pi} \cdot e^{j6\pi/7} = -e^{j6\pi/7}$$

$$= -\left[\cos\frac{6\pi}{7} + j\sin\frac{6\pi}{7}\right]$$

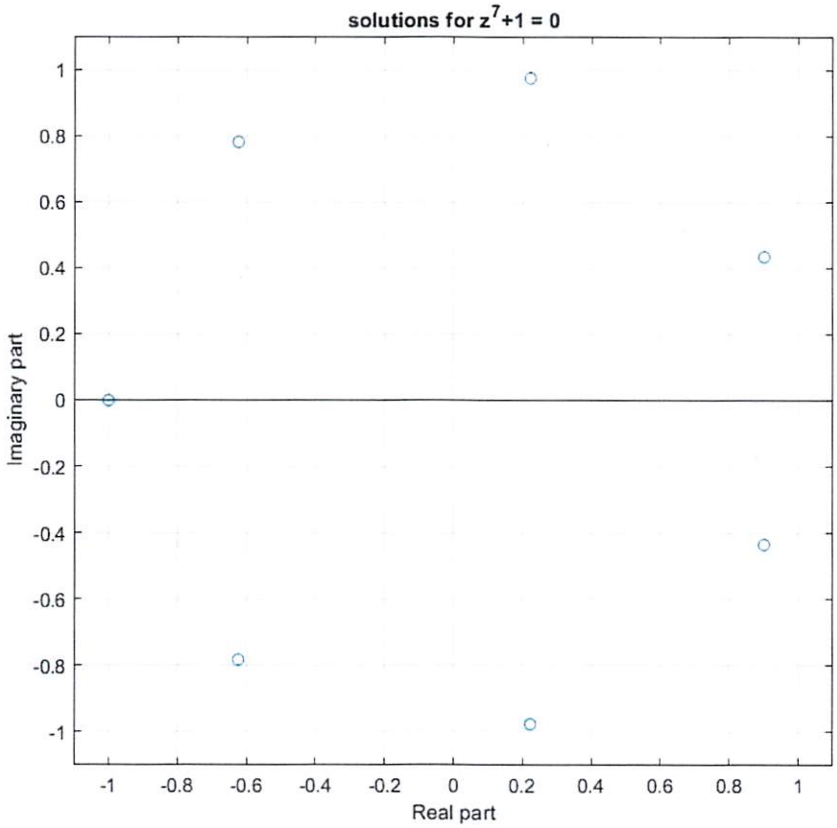
$$\boxed{z_6 = 0.901 - j0.434}$$

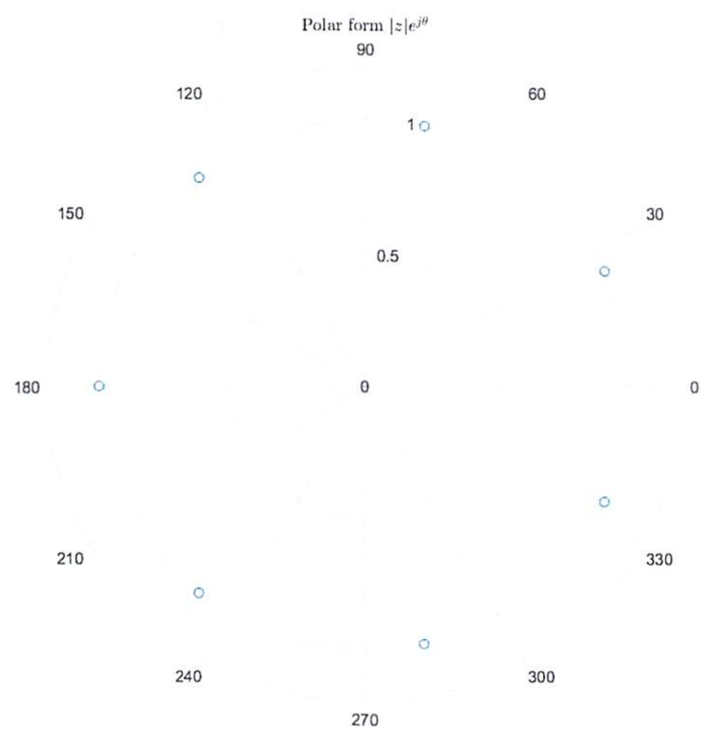
Notice that the solutions lie on a unit circle.



Using MATLAB to plot the roots of unity: (next page)

```
% Homework 1 - Question 2
clear;

k = 0:1:6;
z = exp(1i*pi.*(2*k+1)/7);

stem(real(z),imag(z),'linestyle','none');
title('solutions for z^7+1 = 0');
xlabel('Real part');
ylabel('Imaginary part');
grid on
axis([-1.1 1.1 -1.1 1.1]);

figure;
polarplot(z,'o');
title('Polar form $|z| e^{j\theta}$','interpreter','latex');
```


solutions for $z^7+1 = 0$
Imaginary part
Real part
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1


Polar form $|z|e^{j\theta}$
90
60
30
0
330
300
270
240
210
180
150
120
1
0.5
0

③ (a) $A = -1 + j2$

$|A| = \sqrt{(-1)^2 + (2)^2} = \sqrt{5} = 2.2361$

$\theta = \tan^{-1}\left(\frac{2}{-1}\right) = \pi - \tan^{-1}(2) = 2.034$ radians (or)

$\frac{2.034 * 180}{\pi}$ degrees $\simeq 116.57^\circ \simeq 116.6^\circ$

$\boxed{\text{polar form: } 2.236\, e^{j116.6^\circ} \text{ or } 2.236 \angle 116.6^\circ}$

(b) $B = -1 - j2$

$|B| = \sqrt{(-1)^2 + (-2)^2} = \sqrt{5} = 2.236$

$\theta = \tan^{-1}\left(\frac{-2}{-1}\right) = \pi + \tan^{-1}(2) = 4.249$ radians

(or) $\frac{4.249 * 180}{\pi} \simeq 243.4^\circ$

$\boxed{\text{polar form: } 2.236 \angle 243.4^\circ}$

(c) $C = 3e^{j240}$

$C = 3\left[\cos 240 + j \sin 240\right] = 3\cos(240) + j3\sin(240)$

$C = (3)(-\frac{1}{2}) + j(3)(-\frac{\sqrt{3}}{2}) = \boxed{-1.5 - j2.598}$

(d) $D = 5e^{-j150} = 5\left[\cos(-150) + j\sin(-150)\right]$

$= 5\cos(150) - j5\sin(150) = \boxed{-4.33 - j2.5}$

(e) $E = \frac{C}{D} = \frac{3e^{j240}}{5e^{-j150}} = \frac{3}{5}e^{j(240-(-150))}$

$= \frac{3}{5}e^{j390} = \frac{3}{5}e^{j(360+30)} = \frac{3}{5}e^{j360} \cdot e^{j30} = \frac{3}{5}(1)e^{j30}$

$= \frac{3}{5}(\cos 30 + j\sin 30) = \frac{3}{5}\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right) = \boxed{0.52 + j0.3}$

4 (a) $\sin(\alpha-\beta)$

$$\sin(\alpha-\beta) = \frac{e^{j(\alpha-\beta)} - e^{-j(\alpha-\beta)}}{2j} = \frac{e^{j\alpha}e^{-j\beta} - e^{-j\alpha}e^{j\beta}}{2j}$$

$$= \frac{(\cos\alpha + j\sin\alpha)(\cos\beta - j\sin\beta) - (\cos\alpha - j\sin\alpha)(\cos\beta + j\sin\beta)}{2j}$$

$$= \frac{\cos\alpha\cos\beta + j\sin\alpha\cos\beta - j\cos\alpha\sin\beta + \sin\alpha\sin\beta - \cos\alpha\cos\beta - j\cos\alpha\sin\beta + j\sin\alpha\cos\beta - \sin\alpha\sin\beta}{2j}$$

$$= \frac{2j\sin\alpha\cos\beta - 2j\cos\alpha\sin\beta}{2j} = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\boxed{\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta}$$

(b) $$\cos(\alpha+\beta) = \frac{e^{j(\alpha+\beta)} + e^{-j(\alpha+\beta)}}{2} = \frac{e^{j\alpha}e^{j\beta} + e^{-j\alpha}e^{-j\beta}}{2}$$

$$= \frac{(\cos\alpha + j\sin\alpha)(\cos\beta + j\sin\beta) + (\cos\alpha - j\sin\alpha)(\cos\beta - j\sin\beta)}{2}$$

$$= \frac{\cos\alpha\cos\beta + j\cos\alpha\sin\beta + j\sin\alpha\cos\beta - \sin\alpha\sin\beta + \cos\alpha\cos\beta - j\cos\alpha\sin\beta - j\sin\alpha\cos\beta - \sin\alpha\sin\beta}{2}$$

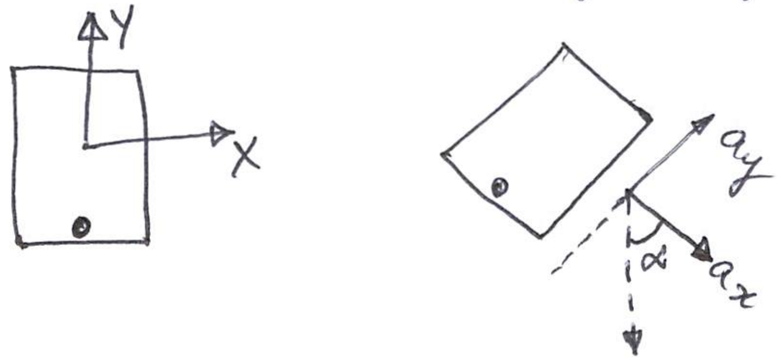
$$= \frac{2\cos\alpha\cos\beta - 2\sin\alpha\sin\beta}{2} = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\boxed{\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta}$$

(5) 12-bit accuracy $\Rightarrow$ digital output range : 0 to $2^{12}-1$

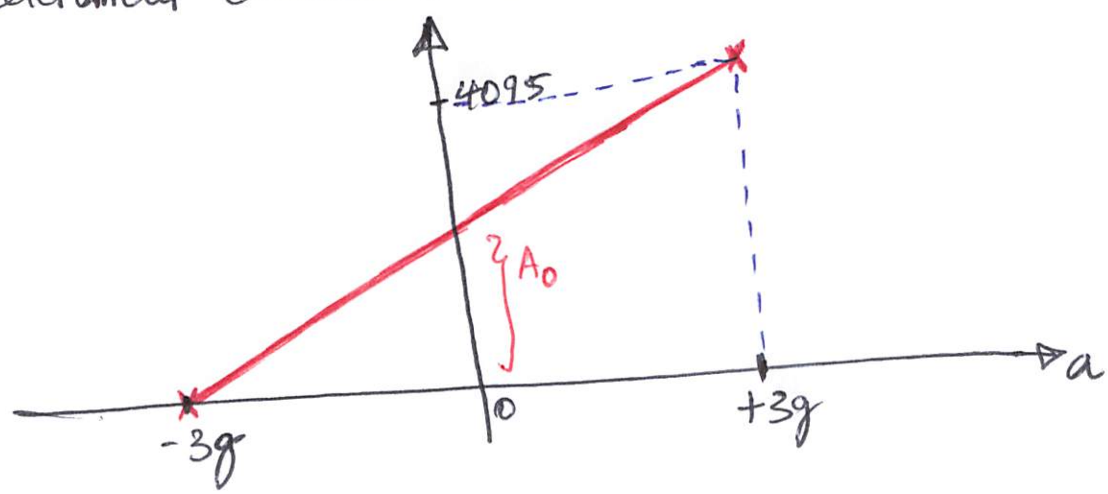
0 to 4095

Sensitivity range : $-3g$ to $+3g$



X and Y value (in counts) is the same as A_x and A_y (in counts)

Accelerometer Characteristics



y-intercept : $A_0 = \dfrac{4095-0}{2} \simeq 2048$

Sensitivity : $S = \dfrac{4095-0}{3g-(-3g)} = \dfrac{4095}{6g} \simeq 682.5 \left[\dfrac{\text{counts}}{g}\right]$

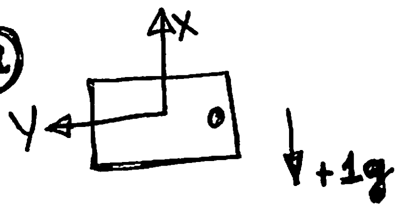
Acceleration output :

$$A = A_0 + Sa$$

For x- and y- components : $X = A_x = A_0 + S \cdot a_x$

$Y = A_y = A_0 + S \cdot a_y$

(a)



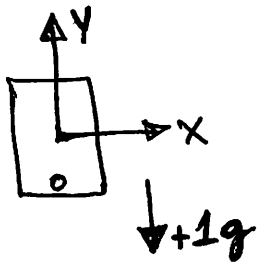
$$a_x = -1g$$
$$a_y = 0g$$

$$X = A_0 + s.a_x = 2048 + (682.5)\left[\frac{\text{counts}}{\cancel{g}}\right] \times (-1\cancel{g})$$
$$= 2048 - 682.5$$
$$\boxed{X \simeq 1366}$$

$$Y = A_0 + s.a_y = 2048 + (682.5)\left[\frac{\text{counts}}{g}\right] \times (0g)$$

$$\boxed{Y = 2048}$$

(b)

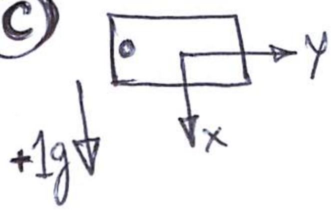


$$a_x = 0g$$
$$a_y = -1g$$

$$X = A_0 + s.a_x = 2048 + (682.5)\left[\frac{\text{counts}}{g}\right] \times (0g)$$
$$\boxed{X = 2048}$$

$$Y = A_0 + s.a_y = 2048 + (682.5)\left[\frac{\text{counts}}{\cancel{g}}\right] \times (-1\cancel{g})$$
$$\boxed{Y \simeq 1366}$$

(c)



$a_x = 1g$

$a_y = 0g$

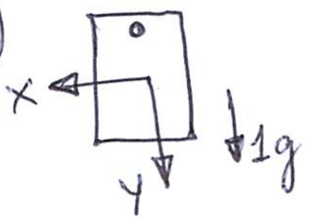
$X = A_0 + S \cdot a_x = 2048 + (682.5)\left[\frac{\text{counts}}{\cancel{g}}\right] * 1\cancel{g}$

$$\boxed{X \simeq 2731}$$

$Y = A_0 + S \cdot a_y = 2048 + \left(\frac{682.5}{g}\right) \times 0g$

$$\boxed{Y = 2048}$$

(d)



$a_x = 0g$

$a_y = 1g$

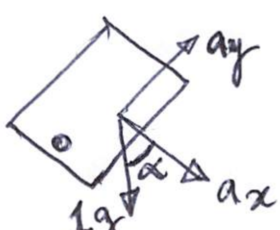
$X = A_0 + S \cdot a_x = 2048 + (682.5)\left[\frac{\text{counts}}{g}\right] * 0g$

$$\boxed{X = 2048}$$

$Y = A_0 + S \cdot a_y = 2048 + (682.5)\left[\frac{\text{counts}}{\cancel{g}}\right] * 1\cancel{g}$

$$\boxed{Y \simeq 2731}$$

(e)



$a_x = (1g)\cos(\alpha)$

$a_y = (-1g)\sin(\alpha)$

$\alpha = 24°$ $\Rightarrow a_x = (1g)\cos(24°) = 0.9135g$

$a_y = (-1g)\sin(24°) = -0.4067g$

$$X = A_0 + S \cdot a_x = 2048 + (682.5)\left[\frac{counts}{\cancel{g}}\right] * 0.9135\cancel{g}$$

$$\boxed{X \simeq 2671}$$

$$Y = A_0 + S \cdot a_y = 2048 + (682.5)\left[\frac{counts}{\cancel{g}}\right] * (-0.4067\cancel{g})$$

$$\boxed{Y \simeq 1770}$$

(f) $X = 622$

$Y = 2600$

$$X = A_0 + S \cdot a_x$$

$$\Rightarrow a_x = \frac{X - A_0}{S} = \frac{622 - 2048}{682.5\left[\frac{counts}{g}\right]}$$

$$a_x \simeq -2.1g$$

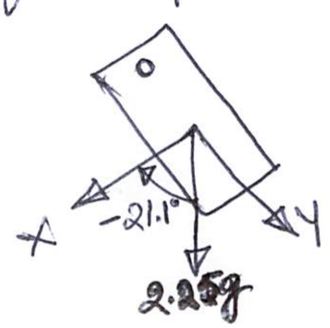
$$Y = A_0 + S \cdot a_y \Rightarrow a_y = \frac{Y - A_0}{S} = \frac{2600 - 2048}{682.5\left[\frac{counts}{g}\right]}$$

$$a_y \simeq 0.81g$$

$$\alpha = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{0.81g}{-2.1g}\right)$$

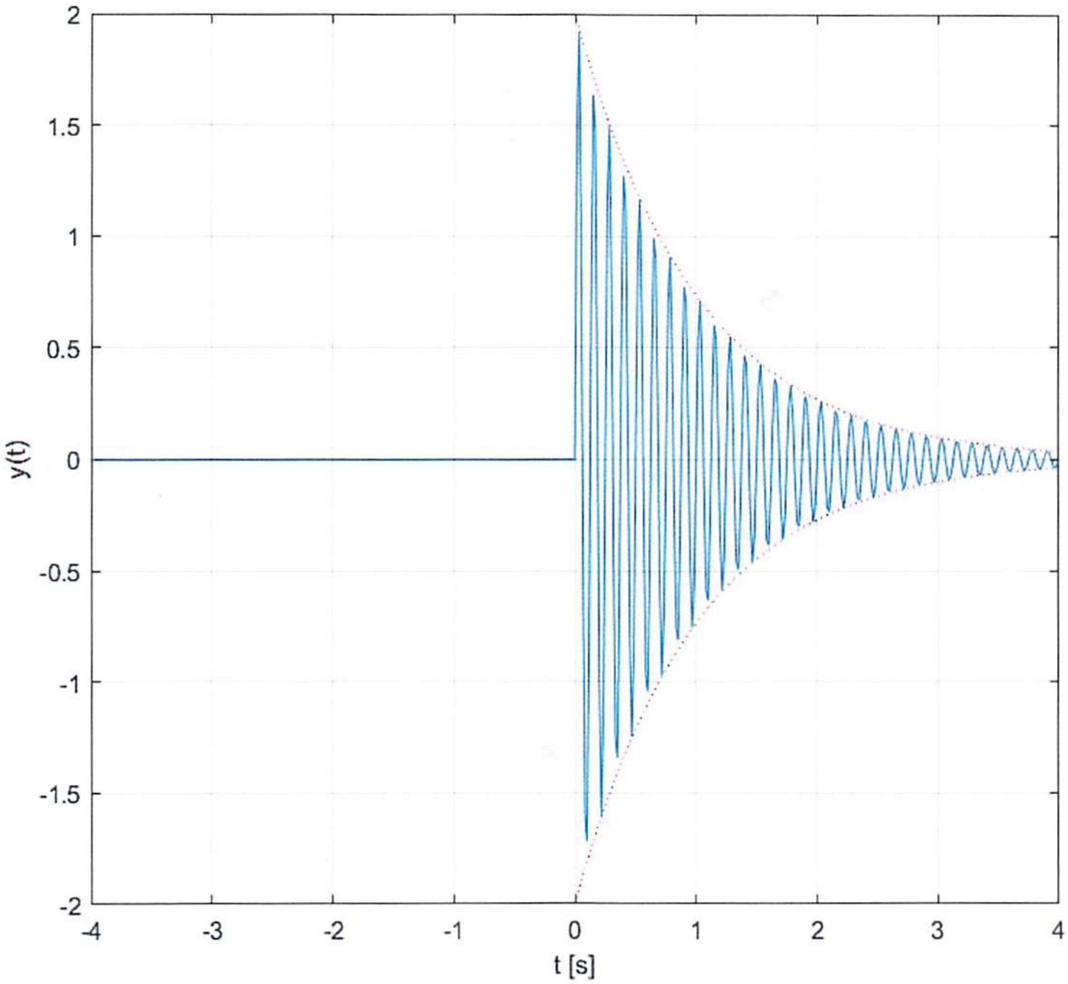
$$\boxed{\alpha \simeq -21.1^\circ}$$

orientation of the phone:-



Question 6

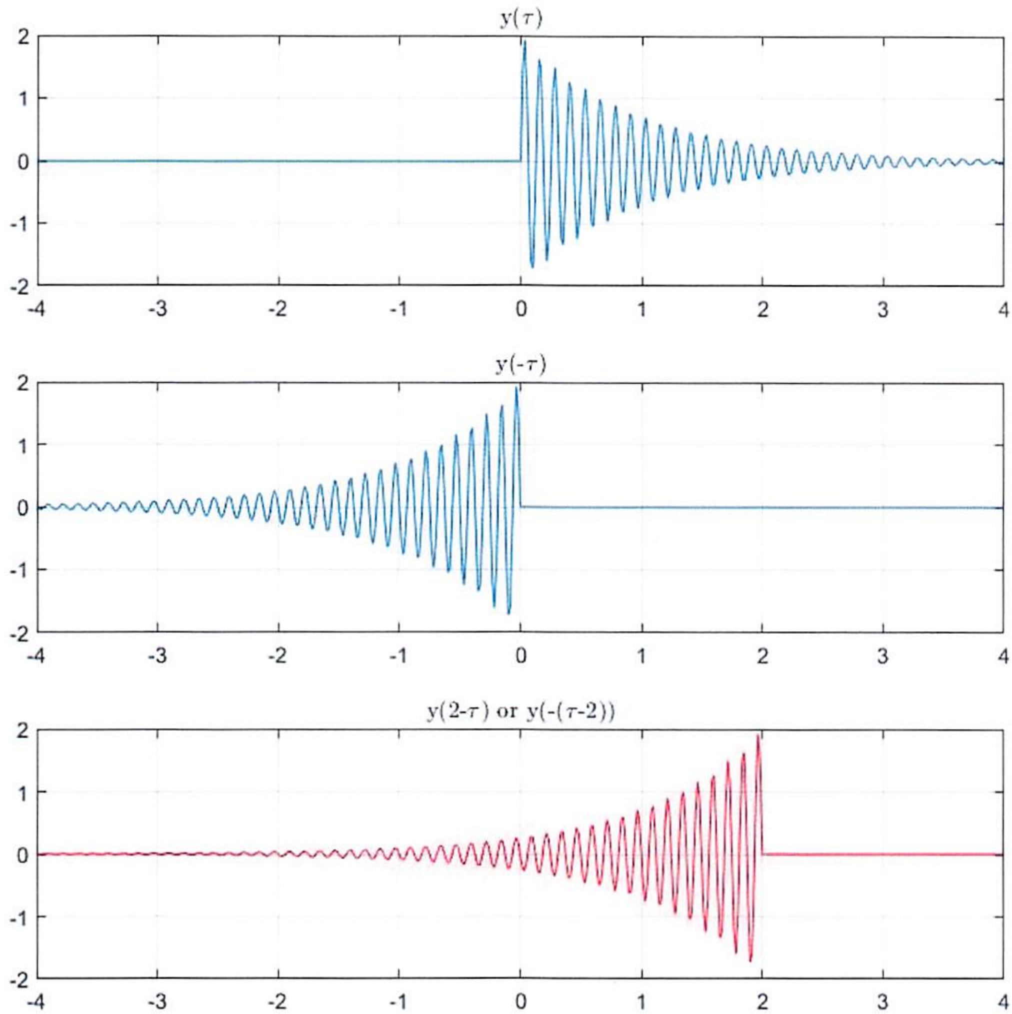
```
% Homework 1 - Question 6

%% Initialization
Fs=60; % sampling frequency
Ts=1/Fs; % sampling interval
f=8; % signal frequency 8 Hz
tmax=4; % maximum time
t=-tmax:Ts:tmax; % time [s]
N=length(t); % number of elements in the vector
i0=round(4*Fs)+1; % index of time 0 (4 seconds after -4
seconds)
t1=0:Ts:4; % time > 0 [s]

%% Signal
A=2; % Amplitude
xenv=A*exp(-t1); % envelope A*e^(-t)
x=xenv.*sin(2*pi*f*t1); % causal signal for t>0
% create samples for t<0 (just zeros)
y=zeros(1,N); % initialize all elements to zero
y(i0:N)=x; % add calculated values from time zero -
equivalent to index i0
% plot signal with envelope, labels, and grid
plot(t,y,t1,xenv,'r:',t1,-xenv,'r:'),xlabel('t
[s]'),ylabel('y(t)'),grid
```


2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
y(t)
-4
-3
-2
-1
0
1
2
3
4
t [s]

Question 7

```
% Homework 1 - Question 7
%% Initialization
Fs=60; % sampling frequency
Ts=1/Fs; % sampling interval
f=8; % signal frequency 8 Hz
tmax=4; % maximum time
t=-tmax:Ts:tmax; % time [s]
N=length(t); % number of elements in the vector
i0=round(4*Fs)+1; % index of time 0 (4 seconds after -4
seconds)
t1=0:Ts:4; % time > 0 [s]
%% Signal
A=2; % Amplitude
xenv=A*exp(-t1); % envelope A*e^(-t)
x=xenv.*sin(2*pi*f*t1); % causal signal for t>0
% create samples for t<0 (just zeros)
y=zeros(1,N); % initialize all elements to zero
y(i0:N)=x; % add calculated values from time zero -
equivalent to index i0
%% Convolution ready signal
% for convolution we need signal y(2-tau)
d=2; % d = 2 seconds (delay)
t3=0:Ts:(tmax+d); % new time to fit in original plot
N3=length(t3);
y3=zeros(1,N);
xenv3=exp(-t3); % envelope
x3=A*xenv3.*sin(2*pi*f.*t3);
y3(1:N3)=fliplr(x3);
%% Educational plot
figure % create new figure
subplot(311)
plot(t,y),grid,title('y($\tau$)','interpreter','latex')
subplot(312)
plot(t,fliplr(y)),grid,title('y(-
$\tau$)','interpreter','latex')
subplot(313)
plot(t,y3,'r'),grid,title('y(2-$\tau$) or y(-($\tau$-
2))','interpreter','latex')
%% Q6 and Q7 comparison
figure
```

```
plot(t,y,'b',t,y3,'r')
xlabel('$\tau$ [s]','interpreter','latex'),ylabel('y'),grid
legend('y($\tau$)','y(2-$\tau$)','interpreter','latex')
```



Comparison of delayed signal with the original signal