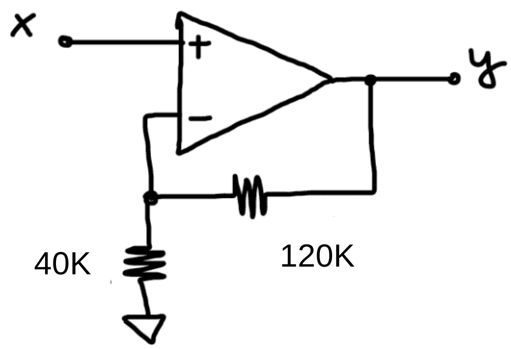
Department of Electrical and Computer Engineering
The University of Alabama in Huntsville

CPE 381: Fundamentals of Signals and Systems for Computer Engineers

Homework #2 Solution

1. What is the transfer function of the following circuits



$$x = \frac{40K}{40K + 120K} y \ \rightarrow y = 4x$$

Transfer function is therefore:

$$\frac{y}{x} = 4$$

2. Simulate the effect of multipath in wireless communication. Generate dumped sine wave $x(t)$ with amplitude A=1 and frequency f=400Hz sampled at F_s= 11,025Hz with time constant 0.5 seconds (i.e. e^{-t}). Assume that the signal is transmitted over three paths, so that the received signal is

$$y(t) = x(t) + 0.6x(t\text{-}0.2) + 0.2x(t\text{-}0.5)$$

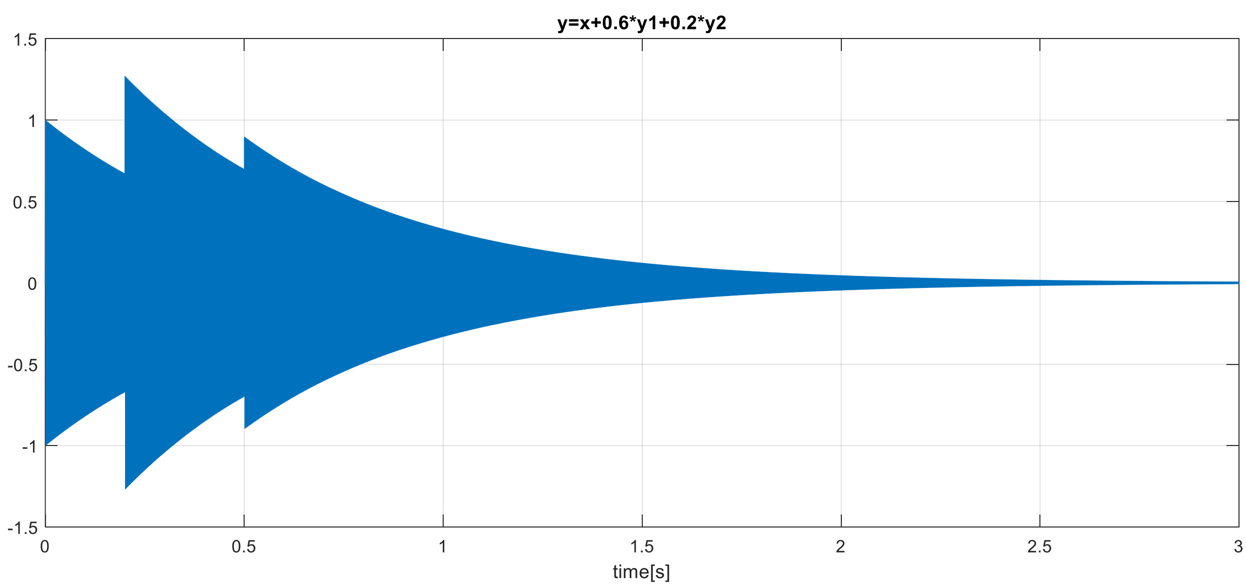
Determine the number of samples corresponding to delay using sampling frequency Fs from the file. Plot the function x(t) and output y(t) and use *sound* function in Matlab to listen to original and received signals.

```
%% Multipath
Fs=11025;  % sampling frequency
Ts=1/Fs;    % sampling interval
t=0:Ts:3;   % time
x=exp(-2*t).*sin(2*pi*400.*t);    % original signal
N=length(t);    % length of vector

td1=0.2*Fs;   % time delay
i1=round(td1)   % integer delay
i2=round(0.5*Fs)   % integer delay
y1=[zeros(1,i1) x(1:N-i1)]; % delayed signal
y2=[zeros(1,i2) x(1:N-i2)]; % second delayed signal
y=x+0.6*y1+0.2*y2;  % received signal

% plot the function and listen the result
plot(t,y),title('y=x+0.6*y1+0.2*y2'),xlabel('time[s]')
sound(y,Fs);

% save as audio file
Audiowrite('HW2.WAV',y,Fs),title('Multipath example')
```



3. Consider the signal x(t) = cos(0.4π·*t*) + 4·cos(2π·*t* /7), −∞ < t < ∞.

Is *x(t)* periodic? If it is, what is the period T_0 of $x(t)$?

$$x_1 = \cos(0.4\pi t) = \cos\left(\frac{2\pi t}{5}\right), \qquad T_1 = 5s$$

and for

$$x_2 = \cos\left(\frac{2\pi t}{7}\right), \qquad T_2 = 7s$$

Since T_1/T_2 is rational number, x(t) is periodic, and

x_1 will repeat after 5, 10, 15, 20, 25, 30, **35**, 40, …

x_2 will repeat after 7, 14, 21, 28, **35**, 42, … seconds

Period of the sum is **the least common multiple** of 5 and 7:

$$T_0 = 35 \text{ s}$$

What is the average power of $x(t)$?

Verify that the power P_x is the sum of powers of two components P_1(cos(0.4π·*t*)) and P_2(4·cos(2π·*t* /7)).
Note: you have to prove that the power of the sum is equal to the sum of powers; therefore, the power of x(t) must be calculated, not substituted.

$$P_x = \frac{1}{T_0}\int_0^{T_0} (x_1 + x_2)^2 dt = \frac{1}{T_0}\int_0^{T_0} \left(x_1^2 + 2x_1\ x_2\ + x_2^2\right)\ dt$$

and

$$P_x = \frac{1}{35}\int_0^{35} \left(x_1^2 + 2x_1\ x_2\ + x_2^2\right)\ dt$$

$$P_{x1} = \frac{1}{35}7\int_0^{5} x_1^2 dt = \frac{1}{5}\int_0^{5} x_1^2 dt = P_1 = \frac{1}{2}1 = 0.5$$

$$P_{x2} = \frac{1}{35}5\int_0^{7} x_2^2 dt = \frac{1}{7}\int_0^{7} x_2^2 dt = P_2 = \frac{1}{2}4^2 = 8$$

$$\frac{1}{35}\int_0^{35} 2x_1 x_2 dt = 0$$

Since

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\cos(2\pi t/5t)\cos(2\pi t/7t) = \cos\left(2\pi t\frac{7-5}{35}\right) + \cos(2\pi t\frac{7+5}{35}) = \cos\left(2\pi t\frac{1}{35/2}\right) + \cos\left(2\pi t\frac{1}{35/12}\right)$$

Therefore, their integral is equal to 0 (integral of the multiple full periods of cos function), and

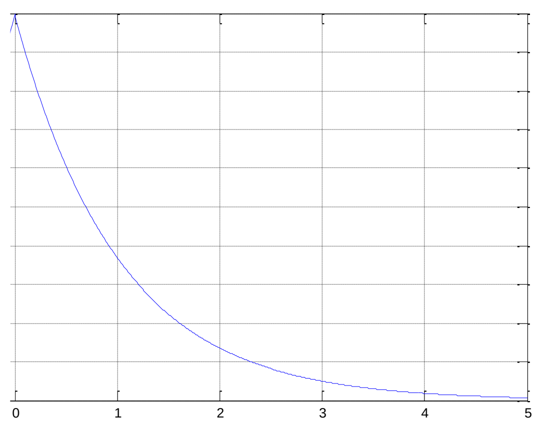
$$P_x = P_1 + P_2 = \frac{1}{2}1 + \frac{1}{2}16 = 8.5$$

4. Plot the signal

$$x(t) = e^{-t}\mathrm{u}(t)$$

What is the energy and the power of x(t)?

$$E_z = \int_0^\infty x^2(t)dt = \int_0^\infty (e^{-t})^2 dt = \left.\frac{e^{-2t}}{-2}\right|_0^\infty = 0.5$$



5. Section 3.3. (page 180)

The Laplace transform of the complex causal signal $e^{j(\Omega_0 t+\theta)}u(t)$ is found to be

$$\mathcal{L}[e^{j(\Omega_0 t+\theta)}u(t)] = \int_0^\infty e^{j(\Omega_0 t+\theta)}e^{-st}dt = e^{j\theta}\int_0^\infty e^{-(s-j\Omega_0)t}dt$$

$$= \frac{-e^{j\theta}}{s-j\Omega_0}e^{-\sigma t-j(\Omega-\Omega_0)t}\,|_{t=0}^{\infty} = \frac{e^{j\theta}}{s-j\Omega_0} \qquad \text{ROC: } \sigma > 0$$

According to Euler's identity

$$\cos(\Omega_0 t+\theta) = \frac{e^{j(\Omega_0 t+\theta)} + e^{-j(\Omega_0 t+\theta)}}{2}$$

by the linearity of the integral and using the above result, we get that

$$\begin{aligned}\mathcal{L}[\cos(\Omega_0 t+\theta)u(t)] &= 0.5\mathcal{L}[e^{j(\Omega_0 t+\theta)}u(t)] + 0.5\mathcal{L}[e^{-j(\Omega_0 t+\theta)}u(t)]\\ &= 0.5\frac{e^{j\theta}(s+j\Omega_0) + e^{-j\theta}(s-j\Omega_0)}{s^2+\Omega_0^2}\\ &= \frac{s\cos(\theta) - \Omega_0\sin(\theta)}{s^2+\Omega_0^2}\end{aligned}$$

and a region of convergence $\{(\sigma, \Omega) : \sigma > 0, -\infty < \Omega < \infty\}$.

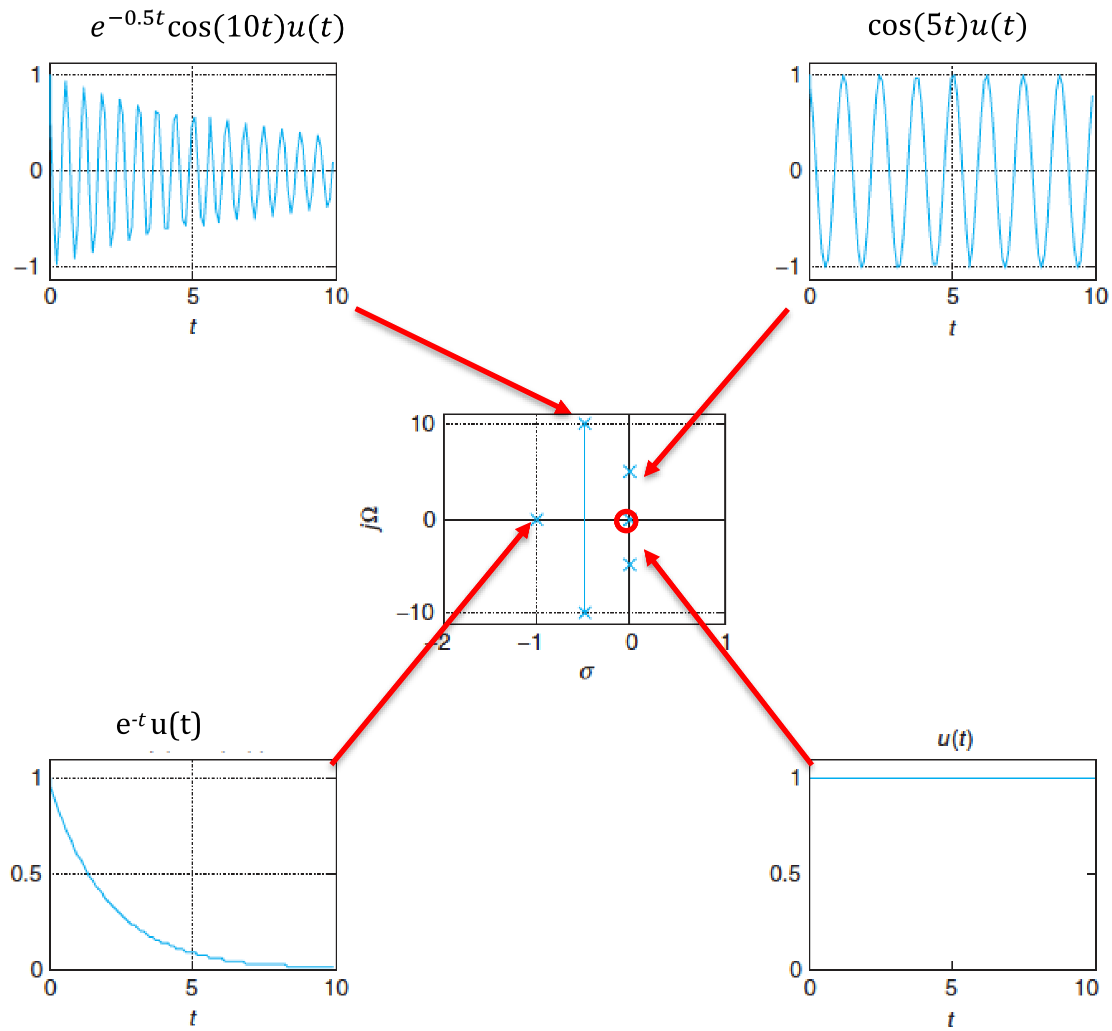
Now if we let $\theta = 0, -\pi/2$ in the above equation we have the following Laplace transforms:

$$\mathcal{L}[\cos(\Omega_0 t)u(t)] = \frac{s}{s^2+\Omega_0^2}$$

$$\mathcal{L}[\sin(\Omega_0 t)u(t)] = \frac{\Omega_0}{s^2+\Omega_0^2}$$

as $\cos(\Omega_0 t - \pi/2) = \sin(\Omega_0 t)$. The ROC of the above Laplace transforms is $\{(\sigma, \Omega) : \sigma > 0, -\infty < \Omega < \infty\}$, or the open right-hand s-plane (i.e., not including the $j\Omega$ axis). See Figure 3.6 for the pole-zero plots and the corresponding signals for $\theta = 0$, $\theta = \pi/4$, and $\Omega_0 = 2$. ■

6. Represent time domain signals corresponding to the poles in the s plane below:



7. Describe the basic properties of the one sided Laplace transform.

Causal functions and constants:	αf(t)	⇔	αF(s)
Linearity:	αf(t) + βg(t)	⇔	αF(s) + βG(s)
Time shifting:	f(t – α)	⇔	$e^{-\alpha s}$F(s)
Frequency shifting:	$e^{\alpha t}$f(t)	⇔	F(s – α)
Multiplication by t:	tf(t)	⇔	$-\frac{dF(s)}{ds}$
Derivative:	$\frac{df(t)}{dt}$	⇔	sF(s) – f(0-)
Second derivative:	$\frac{d^2f(t)}{dt^2}$	⇔	$s^2F(s) - sf(0-) - f^{(1)}(0)$
Integral:	$\int_{0-}^{t} f(t')dt$	⇔	$\frac{F(s)}{s}$
Expansion/Contraction:	f(αt)α ≠ 0	⇔	$\frac{1}{\lvert a \rvert} F(\frac{s}{\alpha})$