

CPE 381: Fundamentals of Signals and Systems for Computer Engineers

**Homework #3**

Due: Monday, February 27, 2023 at 9:35 am

Please upload softcopy of your work and published Matlab scripts to Canvas

**Student name:**

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1 20	2 20	3 20	4 15	5 25	Total

1. (20 points) Write differential equation describing displacement  $x$  of suspended weight  $m$  on spring with elastic constant  $k$ .

Example: A 1 kg weight is hung on the end of a vertically suspended spring, thereby stretching the spring  $L=10$  cm. The weight is raised 5 cm above its equilibrium position and released from rest at time  $t=0$ . Find the displacement  $x$  of the weight from its equilibrium position at time  $t$ . Use  $g=10\text{m/s}^2$ .

2. (20 points) Use Matlab symbolic computation to find the Laplace transform of a real exponential

$$x(t) = 5e^{-2t} \cos(8t)u(t)$$

Plot the signal and the poles and zeros of their Laplace transform.

Repeat the analysis and plot the results for

$$x(t) = 5e^{-4t} \cos(8t)u(t)$$

Discuss the changes in the  $s$  plane and describe their effect on function in time domain.

3. (20 points) Consider a second order differential equation,

$$\frac{d^2 y(t)}{dt} + 3 \frac{dy(t)}{dt} + 2y(t) = x(t)$$

with initial conditions  $y(0) = 1$  and  $\frac{dy(t)}{dt}|_{t=0} = 0$  and  $x(t) = u(t)$ .

- Find the complete response  $y(t)$
- Find the steady state response and the transient response.

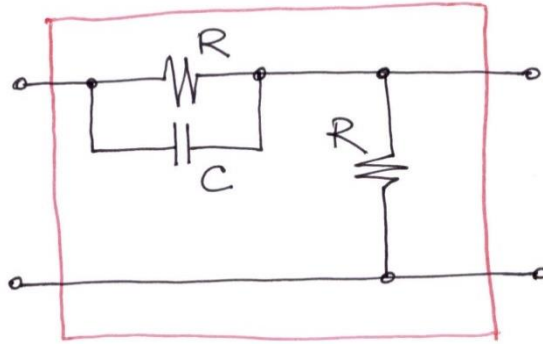
4. (15 points)

An unstable system can be stabilized by using negative feedback with a gain  $K$  in the feedback loop. Consider an unstable system with transfer function

$$H(s) = \frac{1}{s - 2}$$

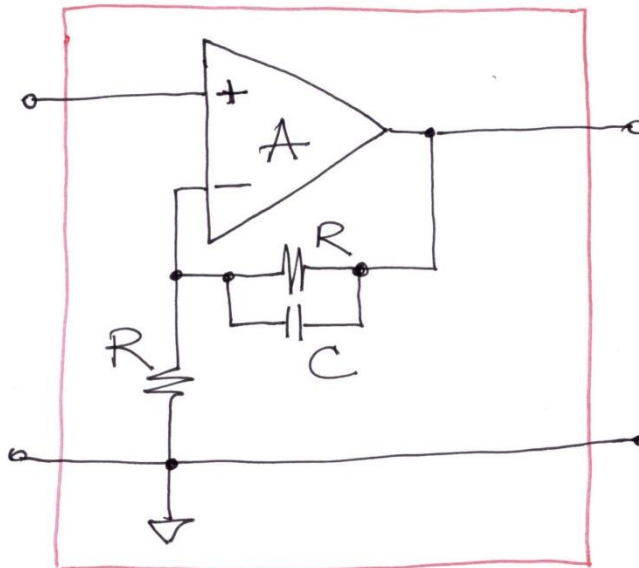
which has a pole in the right-hand  $s$ -plane, making the impulse response of the system  $h(t)$  grow as  $t$  increases. Use negative feedback with a gain  $K > 0$  in the feedback loop, and put  $H(s)$  in the forward loop. Draw a block diagram of the system. Obtain the transfer function  $G(s)$  of the feedback system and determine the value of  $K$  that makes the overall system BIBO stable (i.e. its poles in the open left-hand  $s$ -plane).

5. a) (5 points) What is the transfer function of the following circuit:



- b) (10 points) What is the transfer function of the following circuit?  
Hints:

- you can use solutions of problem #5 and #6a
- to simplify the result you can assume that  $A \rightarrow \infty$



- c) (10 points) Find and plot the unit-step response  $s(t)$  of the system?