EE120 Signals and Systems

Chap1. Linear Time invariant systems

Definition 1 Signals
Signals is a function that depicts physical phenomena with usually one variable(often refers to time t), we denote a continous signal as $x(t)$ and a discrete signal as $x[n]$. Here are some examples of special discrete signals:

1. Unit pulse $\delta[n] = \delta_{n=0}$;
2. Unit step $u[n] = I_{n>0}$;

where $\delta_{n=0}$ represents the Kronecker's delta and $I_{n>0}$ represents the indicator function.

Notice that a useful proposition:

$$\boxed{\delta[n] = u[n] - u[n-1] \Longleftrightarrow u[n] = \sum_{k=0}^{\infty} \delta[n-k]}$$

Proof
The left side of the proposition is quite trivial, if we sum up the term $u[n] - u[n-1]$ we have

$$\sum u[n] - u[n-1] = \sum \delta[n]$$

Definition 2 System
A working function that transform signal $x(t) \mapsto y(t)$. A system is called **memoryless** if its outputs depend solely on the input given at that time. For the case of output only depends on the input before the given time, we call it a **causal** system. For system giving bounded outputs with bounded imputs, we say it is **stable.**

Example 1
An example for memoryless system is that consider a resistor, with input signal current $i(t)$ and output voltage $v(t)$ we have $v(t) = i(t)R$. However in the case of moving average filter $y[n] = 1/3\{x[n+1] + x[n] + x[n-1]\}$, since we require register to store information before the n-th sample, we know it has memory; and $y[n]$ requires the infomation of the $n+1$-th sample, hence it is not causal.
The model of accomulator is some time unstable since it sum up all signal from $x[n]$, may result in divergence.

Definition 3 Linearity and time invariance
We are very familiar with the definition of a linear map, just apply citeria to a given system. For time-invariant property, we have

$$x(t-T) \mapsto y(x-T)$$

for any **shift** T.

In a LTI system, we denote $h[n]$ as the output of unit impulse $\delta[n]$, we call $h[n]$ as unit impulse. Since for signal $x[n]$ we can rewrite it into the form of

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k]$$

applying linearity we have

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] \stackrel{\text{def}}{=} (x*h)[n]$$

where $(x*h)[n]$ denotes the **convolution** of signal $x[n]$ and $h[n]$.
Properties of convolutions

1. Unit pulse as identity, i.e.

$$(x*\delta)[n] = x[n].$$

Proof: Consider that

$$(x*\delta)[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k] = x[n]$$

Remarks: The proof might look like I am performing circular reasoning, since we do use the formula $x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k]$ to derive the convolution operation! However, you should get the operation first from some observations of normal example to derive its generalized form, without the definition of convolution we can get $x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k]$ as well! So there is no worry about circular reasoning.

2. Shift impulse property, i.e

$$x[n]*\delta[n-T] = x[n-T]$$

3. Commutativity

$$x*h = h*x$$

4. Distributivity

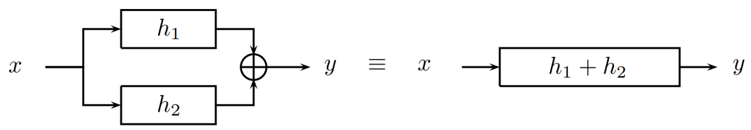
$$x*(h_1+h_2) = x*h_1 + x*h_2$$

5. Associativity

$$x*(h_1*h_2) = (x*h_1)*h_2$$

From convolution to LTI systems

First example we need to check is the parallel combination of two LTI, by distributivity we yield that two LTI in parallel is equal to sum two maps up.



And then the deduction of two LTI in series may be easy to check since

$$(x*h_1)*h_2 = x*(h_1*h_2)$$

due to the associativity.