Intoduction to Functions | Representation | Types | Examples

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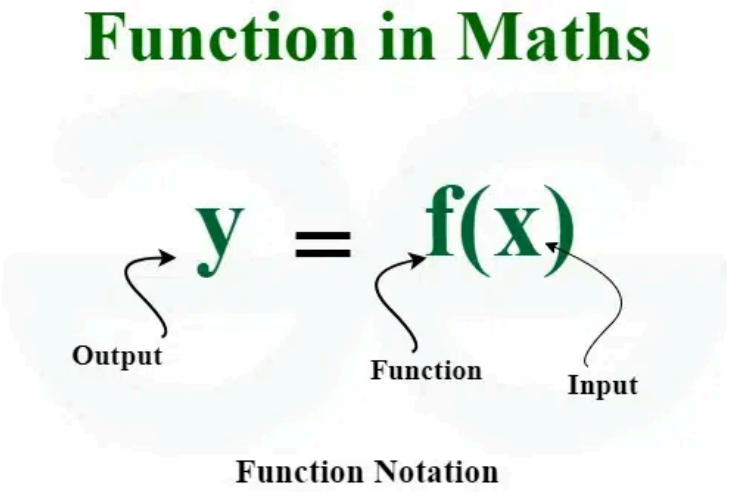
A function is a special relation or method connecting each member of set A to a unique member of set B via a defined relation. Set A is called the domain and set B is called the co-domain of the function. A function in mathematics from set A to set B is defined as,

$$f = \{(a,b) | \forall\ a \in A, b \in B\}$$

A function in mathematics is a relation between the input values (domain) and the output values (range) of the given sets such that no two variables from the domain sets are linked to the same variable in the range set.

A simple example of a function in math is f(x) = 2x, which is defined on R → R, here any variable in the domain is related to only one variable in the range.



Function Notation

Examples of Functions

A function in mathematics f is defined as, y = f(x) where **x** is the input value, and for each input value of x, we get a unique value of y. Various examples of the functions in math defined on R→R are,

Example 1: y = f(x) = 3x + 4

This is a **linear function**.

- If **x = 2**: f(2) = 3(2) + 4 = 6 + 4 = 10. So, the output is 10.
- **Domain**: All real numbers.
- **Range**: All real numbers.

Example 2: y = f(x) = sin(x) + 3

This is a **trigonometric function**.

- If **x = π/2**: f(π/2) = sin(π/2) + 3 = 1 + 3 = 4. The output is 4.
- **Domain**: All real numbers.
- **Range**: From 2 to 4.

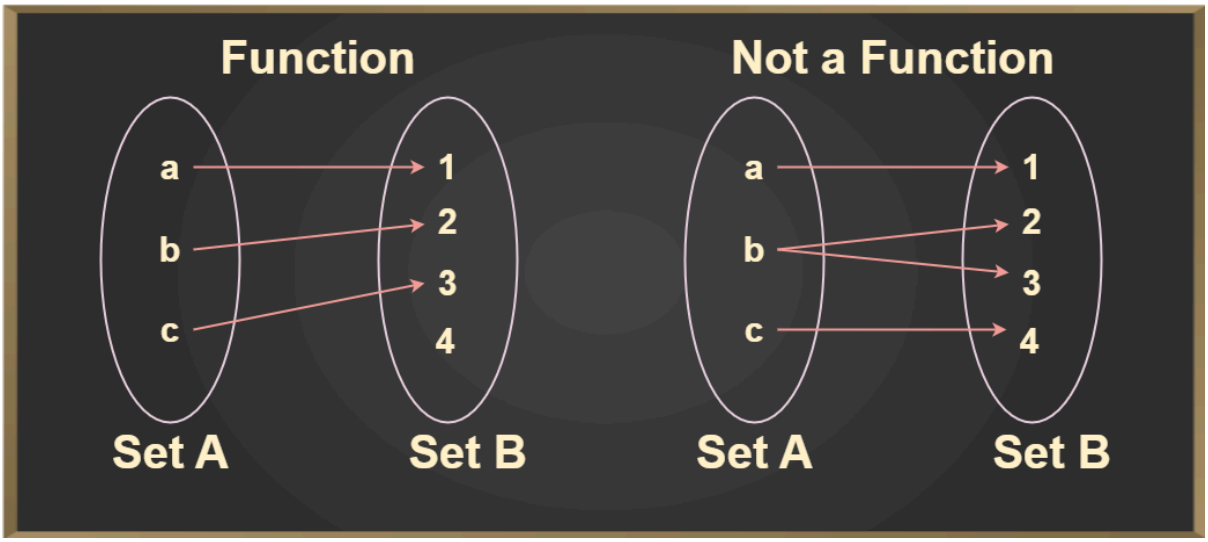
Example 3: y = f(x) = $-3x^2 + 3$

This is a **quadratic function**.

- If **x = 1**: f(1) = $-3(1)^2 + 3 = -3 + 3 = 0$. The output is 0.
- **Domain**: All real numbers.
- **Range**: y ≤ 3.

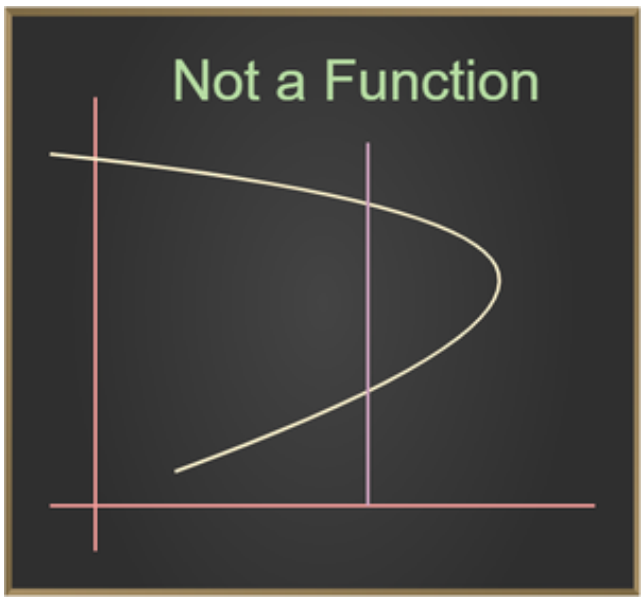
For any two non-empty sets A and B, a function **f: A→B** denotes that **f** is a function from A to B, where **A** is a domain and **B** is a co-domain.

For any element, a ∈ A, a unique element, b ∈ B is there such that (a,b) ∈ f. The unique element b which is related to a is denoted by f(a) and is read as f of a. This can be better understood from the image below:



Vertical Line Test

line test is used to determine whether a curve is a function or not. If any curve cuts a vertical line at more than one point then the curve is not a function.



Domain and Range of a Function

- **Domain:** The set of all possible input values (x-values) for which the function is defined.
- **Range:** The set of all possible output values (y-values) that the function can produce.

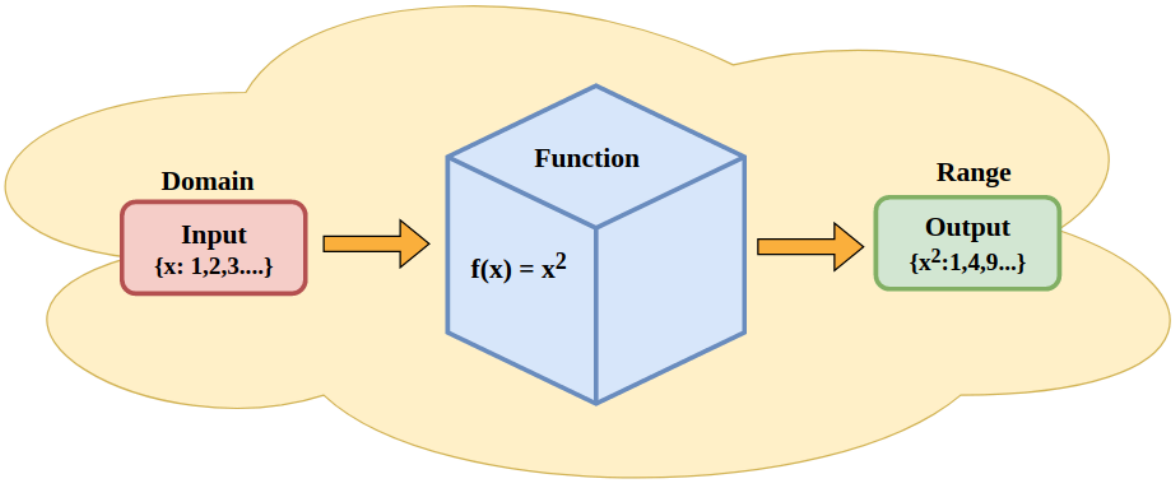
Domain and Range of a function are the input and output value of a function respectively.

For example: $f(x) = x^2$

- The **domain** is all real numbers (R) because any real number can be squared.
- The **range** is all non-negative real numbers ([0, ∞)) because squaring any number always gives a positive or zero result.

Representation of Functions in Math

We can define a function in mathematics as a machine that takes some input and gives a unique output. The function $f(x) = x^2$ is defined below as,



For the above function: $f(x) = x^2$:

- **Input**: x: {1, 2, 3, ... }
- **Function**: Squares each input.
- **Output**: {1, 4, 9, ... } which consists of perfect squares.

We can represent a function in math by the **three methods** as,

For instance, for a function, "$f(x) = x^3$ "
The **set of ordered pairs** is: f = {(1, 1), (2, 8), (3, 27)}
Each pair follows the rule $f(x) = x^3$, meaning each x-value has a unique y-value.

- **Table Form**

A function can also be represented in tabular format, listing input values (x) and their corresponding function values f(x).
Lists x-values and their corresponding f(x)-values in a structured format.

Read more - Function Table

- **Graphical Form**

A function can be represented visually using a **graph** on a coordinate plane. The graph shows the relationship between x and y.
Represents the function visually on a coordinate plane, showing how x and y relate.
For example:

- The function $f(x) = x^2$ produces a **parabola**.
- The function $f(x) = x^3$ results in a **cubic curve**.

Read more - Graphing of Function

How to Identify a Function?

The function is classified as a special type of relation in math. There are the following rules which can be used to identify a function:

- A relation in which each input is mapped to a unique output is a Function. This is called one to one function.
- A relation in which two inputs (preimage) are mapped to a single output is also a Function. This is a many-to-one function.
- A relation in which one input is mapped to two different output is not a function.
- A relation in which many inputs are mapped to many outputs

Different Types of Functions are used to solve various types of mathematical problems especially related to curves and equations. Three major types functions in mathematics are based on the element mapping from set A to set B.

- **Injective or One-to-One Function**
- **Surjective or Onto Function**
- **Bijective Function**
- **Many One Function**

Also, Read Even and Odd Functions.

Algebra of Functions

Algebra of Functions involves the algebraic operations performed between two functions. The algebraic operation for two functions f(x) and g(x) defined on the real value of x are mentioned below:

- (f + g) (x) = f(x) + g(x)
- (f - g) (x) = f(x) - g(x)
- (f.g) (x) = f(x).g(x)
- (k f(x)) = k (f(x)); {For, k is a real number}
- (f/g)(x) = f(x) /g(x); {For g(x) ≠ 0}

Composition of Functions

If f: A → B and g: B→ C are two functions. Then the composition of f and g is denoted as f(g) and it is defined as the function **fog = f(g(x)) for x ∈ A.**

Let's take two functions f(x) = x + 3 and g(x) = $2x^2$

fog = f(g(x))
⇒ fog = f($2x^2$)
⇒ fog = $2x^2$ + 3

Learn More, Composition of Function

mapped for its input and corresponding output values.

To plot a function on a first find some points that lies on the function and then join these points according to the locus of the function. For example to graph the function (straight line) f(x) = y = 5x - 2 we need some point on the graph. To find the point the point on the graph we first take the random values of x and then find their corresponding values of y, as,

f(x) = y = 5x- 2

- if x = 0, y = 5(0) - 2 = -2 ⇒ (x, y) = (0, -2)
- if x = 1, y = 5(1) - 2 = 3 ⇒ (x, y) = (1, 3)
- if x = 2, y = 5(2) - 2 = 8 ⇒ (x, y) = (2, 8)

Now joining these points we can get the graph of the function y = 5x - 2

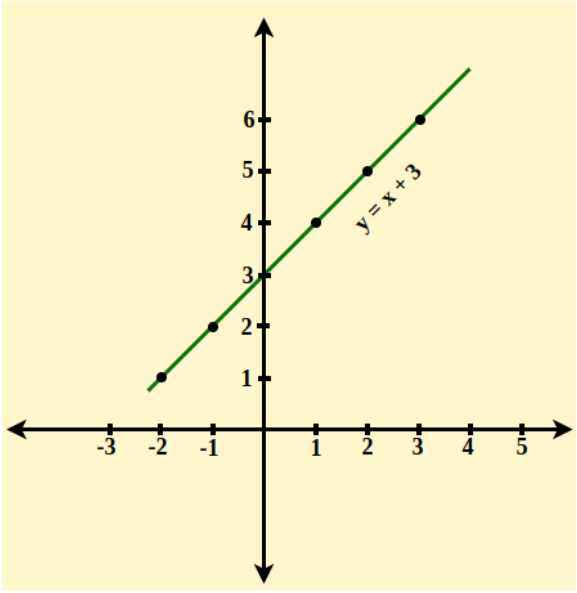
Graphing Functions

Knowing the values of x allows a function f(x) to be represented on a graph. Because y = f(x), we can find the associated value for y by starting with the values of x. As a result, we can plot a graph in a coordinate plane using x and y values. Consider the following scenario:
Assume y = x + 3
When x = 0, y = 3

Similarly,

- x = -2, y = -2 + 3 = 1
- x = -1, y = -1 + 3 = 2
- x = 1, y = 1 + 3 = 4
- x = 2, y = 2 + 3 = 5
- x = 3, y = 3 + 3 = 6

As a result, we may plot the graph for function x + 3 using these values.



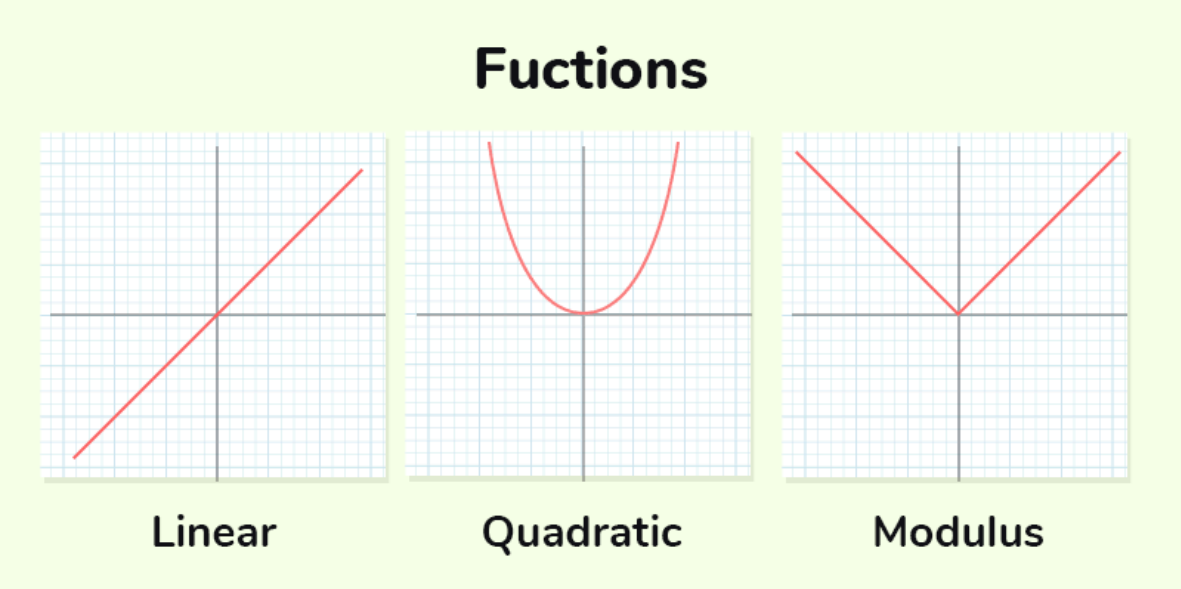
Read More about **Graphing Functions**.

Common Functions

Some Common Functions that commonly used in mathematics are discussed below:

Real Function

Real function in maths refers to a function whose domain and range are subsets of the real numbers (denoted as $\mathbb{R}$). In simpler terms, a real function is a mathematical rule or relationship that assigns a real number value to each real number input.



Real Functions

The function in which the exponents of algebraic variables are non-negative integers is called a Polynomial Function. If the power of the variable is 1 it is called a linear function, if the power is 2 it is called a quadratic function, and if the power is 3 it is called a cubic function. Some examples of polynomial functions are mentioned below:

- $y = x^2$
- $y = 2x + 3$
- $y = 3x^3$

Polynomial Function can further classified into the following types:

Linear function: Linear Functions is those in which maximum power of a variable is 1. The general Form of Linear Function is **y = mx + c**

Quadratic Function: Quadratic Function is those in which maximum power of variable is 2. General Form of quadratic function is, $\mathbf{ax^2 + bx + c = 0}$

Cubic Function: Cubic Function is those in which maximum power of variable is 3. General Form of cubic function is given as $\mathbf{ax^3 + bx^2 + cx + d = 0}$

Inverse Function: Inverse Function is the function containing the inverse of another function. Let's say we have a function y = f(x) then its inverse function will be $x = f^{-1}(y)$. In y = f(x), the domain is x and the range is y while in the case of $x = f^{-1}(y)$, the domain is y and the range is x. Thus we can say that the domain of the original function is the range of its inverse function and the range of the original function is the domain of the original function. Some examples of inverse functions are,

- $y = \tan^{-1}(x)$
- $y = x^{-1}$

Area Function

Area function typically refers to a mathematical function that calculates

Area of Circle Function: The Area of Circle (A) is a function of its radius(r) such that,

$$A = \pi r^2$$

Area of Triangle Function: The Area of Triangle (A) is a function of its base(b) and height(h) such that,

$$A = (bh)/2$$

Exponential Function

Exponential function is the one which is represented as $f(x) = e^x$. It is often used to show rapid growth or decay.

Logarithmic Function

Logarithmic function is a mathematical function that represents the inverse operation of exponentiation. It is represented as f(x) = log x.

Ceiling Function

Ceiling function denoted as $\lceil x \rceil$, rounds a real number x up to the nearest integer that is greater than or equal to x. In other words, it finds the smallest integer value that is greater than or equal to x.

Floor Function

Floor function denoted as $\lfloor x \rfloor$, rounds a real number x down to the nearest integer that is less than or equal to x. In other words, it finds the largest integer value that is less than or equal to x.

Modulus Function

Modulus function also known as the absolute value function, returns the

Signum Function

Signum function, also known as the sign function or signum function, is a mathematical function that returns the sign of a real number. It indicates whether the number is positive, negative, or zero.

Trigonometric Functions

Trigonometric functions are mathematical functions that relate the angles of a right triangle to the lengths of its sides. The six primary trigonometric functions are sine (sin), cosine (cos), tangent (tan), cosecant (cosec), secant (sec), and cotangent (cot).

Complex Functions

Any function in which the input variable are complex function are called the complex function. A complex number is a number that can be plotted on the complex plane. In a complex number, we have real number and imaginary number. A complex number(z) is represented as, z= x + iy and a complex function is represented as, f(z) = P(x, y) + iQ(x, y)

Applications of Functions

When we say that a variable quantity y is a function of a variable quantity x, we indicate that y is dependent on x and that y's value is determined by x's value. This dependency can be expressed as follows: f = y (x).

- The radius of a circle can be used to calculate the area of a circle. The radius r affects area A. We declare that A is a function of r in the mathematic language of functions. We can write A = f(r) =$\pi \times r^2$
- A sphere's volume V is a function of its radius. V = f(r) = $4/3 \times r^3$ denotes the dependence of V on r.
- Force is a function of the acceleration of a body of fixed mass m. F =

- Relation and Function
- Domain and Range of Trigonometric Functions
- Range of a Function

Solved Examples of Function

Example 1: For two functions f and g are defined as, $f(x) = x^2$ and $g(x) = \ln(2x)$. Find the composite function $(gof)(x)$

Solution:

Given:

- $f(x) = x^2$
- $g(x) = \ln(2x)$

$(gof)(x) = g(f(x))$

$[g(f(x)] = \ln(2f(x)) = \ln(2x^2) = 2\ln(\sqrt{2}x)$

Thus, $(gof)(x) = 2\ln(\sqrt{2}x)$

Example 2: Find the output of the function $g(t)= 6t^2 + 5$ at

- **(i) t = 0**
- **(ii) t = 2**

Solution:

Given Function: $g(t)= 6t^2 + 5t$

- *(i) t = 0*

$g(0) = 6(0)^2+5(0) = 0 + 0$
$\Rightarrow g(0) = 0$

- *(ii) t = 2*

$g(2) = 6(2)^2+5(2)$
$\Rightarrow g(2) = 24 + 10$
$\Rightarrow g(2) = 34$

Solution:

Let, length of the rectangle be l and the breadth of the rectangle is, b

Now,

- $b = l/5$

Area of Rectangle(A) = $l \times l/5 = l^2/5$

Thus, area of rectangle as the function of its length is,

$A(l) = l^2/5$

Practice Problems on Functions

1. Given the function f(x) = 3x + 5

- Find f(2)
- Find f(−1)
- Determine the domain and range of the function.

2. Given the function $g(x)=x^2 - 4x + 3$

- Find the roots of the function.
- Find g(3) and g(0).
- Determine the vertex of the function.

3. Given two functions f(x) = x + 2 and h(x) = 2x - 3

- Find the composite function (f ∘ h) (x)
- Evaluate (f ∘ h)(2)

Next Article

Set Theory

A relation R from set A to set B is a subset of the Cartesian product A × B. That is:R ⊆ A × BHere, A is called the domain, and B is called the co-...

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