Assignment - 2
ME 639

Q1. To show that the columns of R_0^1 are orthogonal

~~We can do~~

The columns of a matrix A are orthogonal if $A^T A = B$ is a diagonal

(i.e if $\forall_{i,j}\ B(i,j) = 0$)

$$R_0^1 = \begin{bmatrix} C_\theta & -S_\theta & 0 \\ S_\theta & C_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_0^{1T} = \begin{bmatrix} C_\theta & S_\theta & 0 \\ -S_\theta & C_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

let $B = R_0^{1T} R$

$$B = \begin{bmatrix} C_\theta & S_\theta & 0 \\ -S_\theta & C_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_\theta & -S_\theta & 0 \\ S_\theta & C_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We can see clearly that B is a diagonal matrix,

Thus, columns of Rotation matrix R_0^1 are orthogonal.

Q2. $$R_0^1 = \begin{bmatrix} C_\theta & -S_\theta & 0 \\ S_\theta & C_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \det(R_0')$
$= \cos\theta(\cos\theta - 0) - (-\sin\theta)(\sin\theta) + 0(0-0)$
$= \cos^2\theta + \sin^2\theta$
$= 1$

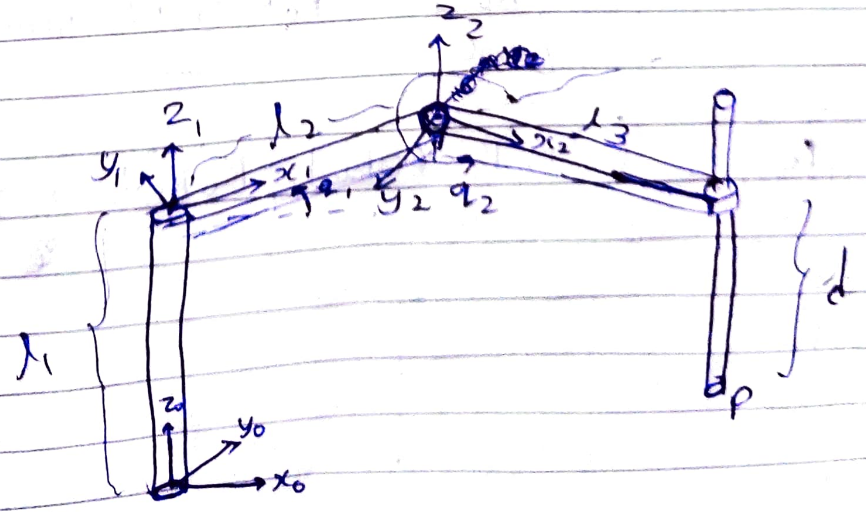
Thus $\det(R_0') = 1$.

Q5. To show $R S(a) R^T = S(Ra)$

Here $R \in SO(3)$ and $a \in \mathbb{R}^3$

Now, let there be $b \in \mathbb{R}^3$ be an arbitrary vector.
Then
$R S(a) R^T b = R(a \times R^T b)$
This follows from the property that
For any vectors a and p belonging to $\mathbb{R}^3$,
$S(a) p = a \times p$
Thus, $R S(a) R^T b = R(a \times R^T b)$
$= (Ra) \times (R R^T b) \quad (\because R(a \times b) = Ra \times Rb)$
$= (Ra) \times b$
$= S(Ra) b$

Q6



We know that,

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

Here,

$$P_3 = \begin{bmatrix} 0 \\ 0 \\ d \end{bmatrix} ; \quad R_1^2 = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad d_2^3 = \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix}$$

$$d_1^2 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix} \quad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$$

$$R_2^3 = I$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} R_0^1 R_1^2 & R_0^1 d_1^2 + d_0^1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I & d_2^3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} R_0^1 R_1^2 & R_0^1 R_1^2 d_2^3 + R_0^1 d_1^2 + d_0^1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

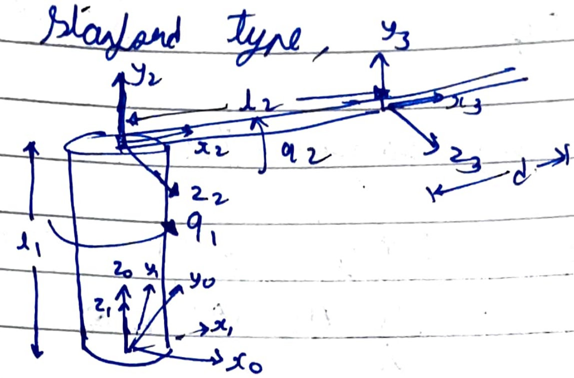
$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} R_0^1 R_1^2 P_3 + R_0^1 R_1^2 d_2^3 + R_0^1 d_1^2 + d_0^1 \\ 1 \end{bmatrix}$$

~~$P_0 = [\;]$~~

Doing calculations we get,

$$P_0 = \begin{bmatrix} l_2 cq_1 + l_3 c(q_1+q_2) \\ l_2 sq_1 + l_3 s(q_1+q_2) \\ l_1 + d \end{bmatrix}$$

Q8. For RRP Stanford type,



$$P_3 = \begin{bmatrix} d \\ 0 \\ 0 \end{bmatrix} ; \quad R_2^3 = I \qquad d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\pi/2 & -s\pi/2 \\ 0 & s\pi/2 & c\pi/2 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ 0 & 0 & -1 \\ sq_2 & cq_2 & 0 \end{bmatrix} ; \quad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$$

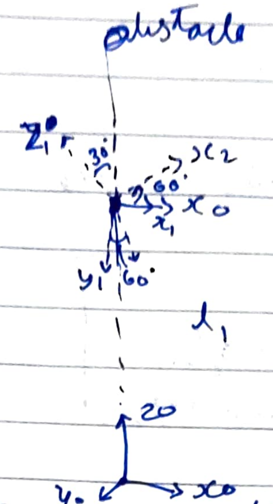
$$R_0^1 = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

calculating we get

$$P_0 = \begin{bmatrix} (d+l_2)\cos q_1 \cos q_2 \\ (d+l_2)\sin q_1 \cos q_2 \\ (d+l_2)\sin q_2 + l_1 \end{bmatrix}$$

Q9.



Here it is given that,

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\cancel{d_2^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}} \qquad R_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(30°) & -\sin(30°) \\ 0 & \sin(30°) & \cos(30°) \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} \cos(60°) & -\sin(60°) & 0 \\ \sin(60°) & \cos(60°) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

The position vector of obstacle P_0 can be given by,

$$P_0 = H_0^1 \, H_1^2 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$\cancel{\begin{Bmatrix} P_0 \\ 1 \end{Bmatrix} = \begin{bmatrix} \\ \end{bmatrix}}$$

On solving we get

$$P_0 = \begin{bmatrix} 0 \\ -3/2 \\ 10 + 3\sqrt{3}/2 \end{bmatrix}$$

Q11.

The manipulator Jacobian for RRP ~~SCA~~ SCARA configuration is given by.

$$\begin{bmatrix} V_0^n \\ W_0^n \end{bmatrix} = J_0^n \dot{q} = \begin{bmatrix} J_v \\ J_w \end{bmatrix} \dot{q}$$

Jacobian

Here there are 3 links

$\rho_1 = 1$ & $z_0 = \hat{K_0} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\rho_2 = 1$ & $z_1 = R_1^2 \cdot \hat{K_2} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\rho_3 = 0$

Thus,

$$J_w = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

For the linear velocity,

The prismatic joint ρ is

$$\frac{\partial d_0^n}{\partial x} = z_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$\frac{\partial d_0^n}{\partial q_1} = z_0 \times (R_0^0 \cdot d_0^n)$$

$$= \cancel{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_2 s q_1 + l_3 \sin(q_2 - q_1) \\ l_2 \cos \end{bmatrix}}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_2 c q_1 + l_3 c(q_1 + q_2) \\ l_2 s q_1 + l_3 s(q_1 + q_2) \\ l_1 + d \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} l_2 c q_1 + l_3 c(q_1 + q_2) \\ l_2 s q_1 + l_3 s(q_1 + q_2) \\ l_1 + d \end{bmatrix}$$

$$\frac{\partial d_0^n}{\partial q_1} = \begin{bmatrix} -l_2 s q_1 - l_3 s(q_1 + q_2) \\ l_2 c q_1 + l_3 c(q_1 + q_2) \\ 0 \end{bmatrix}$$

$$\frac{\partial d_0^n}{\partial q_2} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_3 c(q_1 + q_2) \\ l_3 s(q_1 + q_2) \\ d \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} l_3 c(q_1 + q_2) \\ l_3 s(q_1 + q_2) \\ d \end{bmatrix}$$

$$\frac{\partial d_0^n}{\partial q_2} = \begin{bmatrix} -l_3 s(q_1 + q_2) \\ l_3 c(q_1 + q_2) \\ 0 \end{bmatrix}$$

Final Jacobian J,

$$J = \begin{bmatrix} -l_2 s q_1 - l_3 s(q_1+q_2) & -l_3 s(q_1+q_2) & 0 \\ l_2 c q_1 + l_3 c(q_1+q_2) & l_3 c(q_1+q_2) & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Q13. For the RRR configuration

$$J_w = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$J_1 = \begin{bmatrix} z_0 \times (o_n - o_0) \\ z_0 \end{bmatrix}$$

Here $z_0 = \hat{k}_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$o_n - o_0 = R_0^0 d_0^n$$

Here $o_0 = 0$,

$$= R_0^1 d_1^n$$

$$= R_0^1 (d_1^2 + R_1^2 d_2^n)$$

$$= R_0^1 d_1^2 + R_0^1 R_1^2 d_2^3 + R_0^1 R_1^2 R_2^3 d_3^n$$

Calculating

$$J_1 = \begin{bmatrix} -l_1 s q_1 + l_2 s(q_1+q_2) + l_3 s(q_1+q_2+q_3) \\ l_1 c q_1 + l_2 c(q_1+q_2) + l_3 c(q_1+q_2+q_3) \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} z_2 \times (O_n - O_2) \\ \\ \\ z_2 \end{bmatrix}$$

$$z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$O_n - O_2 = R_0^2 d_2^n = R_0^2 (d_2^3 + R_2^3 d_3^n)$$
$$= R_0^2 d_2^3 + R_0^3 d_3^n$$

Calculating

$$J_2 = \begin{bmatrix} -(l_2 s(q_1+q_2) + l_3 s(q_1+q_2+q_3)) \\ l_2 c(q_1+q_2) + l_3 c(q_1+q_2+q_3) \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_3 = \begin{bmatrix} z_3 \times (O_n - O_3) \\ z_3 \end{bmatrix}$$

$$O_n - O_3 = R_0^3 d_3^n$$

$$J_3 = \begin{bmatrix} -l_3\, s(q_1+q_2+q_3) \\ l_3\, c(q_1+q_2+q_3) \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J = \begin{bmatrix} J_1 & J_2 & J_3 \end{bmatrix}$$

$$J = \begin{bmatrix} -(l_1 s q_1 + l_2 s(q_1+q_2) + l_3 c(q_1+q_2+q_3) & -(l_2 s(q_1+q_2) + l_3 s(q_1+q_2+q_3)) & -l_3 s(q_1+q_2+q_3) \\ l_1 c q_1 + l_2 c(q_1+q_2) + l_3 c(q_1+q_2+q_3) & l_2 c(q_1+q_2) + l_3 c(q_1+q_2+q_3) & l_3 c(q_1+q_2+q_3) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$