Assignement 4
COSC 4P61, Theory of Computation
Fall, 2019

Due: Dec. 5, Thursday, 5:00 PM.

1. (40) Construct a Turing machine for $L = \{a^n b^n c^{2n} | n \geq 1\}$. Please first describe the idea behind your construction in English.

2. (bonus 30) Construct a Turing machine for adding 1 to a binary number n with no leading 0's (i.e. $n \in \{0\} \bigcup \{1\}\{0+1\}^*$). Initially, the binary number is on the tape enclosed by a pair of #'s with state q_0 and head pointing to the left #. For convenience, we can assume that the tape is a two way infinite tape. For examples, for input #11111#, the output should be #100000#; for input #10111#, the output should be #11000#; while for input #100#, the output should be #101#.

3. (20) Let G be an unrestricted grammar. Then the problem of determiningg whether or not $L(G) = \emptyset$ is undecidable. Let M_1 and M_2 be two arbitrary Turing machines. Show that the problem $L(M_1) \subseteq L(M_2)$? is undecidable.

4. (20) Given $A = \{< M > | M$ is a finite automaton and $L(M) = \emptyset\}$ where $< M >$ is some encoding of the machine M. Is A Turing-decidable? Prove your answer.

5. (20) Let M be a deterministic Turing machine that accepts a nonrecursive language. Prove that the halting problem for M is undecidable. That is, there is no TM that takes input w and determines whether the computation of M halts with input w.