Roll No. 1900101303 2

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# 300106

**December, 2019**

**B.Tech. (ME/MA/AE) 1st Semester**

**MATHEMATICS: Calculus and Linear Algebra**

**(BSC103A)**

Time : 3 Hours]

[Max. Marks : 75

*Instructions :*

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

## PART - A

**1.** (a) Define evolutes and involutes with example. (1.5)

(b) Evaluate $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta$. (1.5)

(c) State Lagrange's Mean Value theorem. (1.5)

(d) Evaluate $\lim_{x\to\infty} \frac{x^n}{e^x}$. (1.5)

(e) If $< a_n >$ is bounded and $b_n \to 0$, then $a_n b_n \to 0$.

(1.5)

(f) Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n .x^{2n}$. (1.5)

(g) Show that $\lim_{(x,y)\to(o,o)} \left(\frac{x^2}{x^4+y^2}\right)$ does not exist. (1.5)

(h) Show that the vector

$$\vec{F} = (6xy + z^3)\,\vec{i} + (3x^2 - z)\,\vec{j} + (3xz^2 - y)\,\vec{k}$$

is irrotational. (1.5)

(i) If 2 and 3 are eigen values of

$$A = \begin{array}{|c|c|c|} \hline 3 & 10 & 5 \\ \hline -2 & -3 & -4 \\ \hline 3 & 5 & 7 \\ \hline \end{array},$$

find the eigen values of $A^{-1}$ and $A^3$. (1.5)

(j) Find rank of matrix

| | | | |
|---|---|---|---|
| 1 | −7 | 3 | −3 |
| 7 | 20 | −2 | 25 |
| 5 | −2 | 4 | 7 |

by using determinant. (1.5)

## PART - B

**2.** (a) Find the evolute of the rectangular hyperbola $xy = c^2$.

(8)

(b) A sphere of radius $a$ is divided into two parts by a plane at a distance $\frac{a}{2}$ from the centre. Show that the ratio of the volume of two parts is 5 : 27. (7)

**3.** (a) Expand sin $x$ as a finite series in powers of $x$, with remainder in Lagrange's form. Hence, find the series for sin $x$. (7)

(b) Using Rolle's theorem, prove that there is no real $a$ for which the equation $x^2 - 3x + a$ has two different roots in [−1, 1]. (8)

4. (a) Test the convergence of the series given below :

$$\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-1}+\ldots\ldots\infty \qquad (8)$$

(b) Find the half-range cosine series for $f(x)=x$ in the interval $[0, \pi]$ and deduce that

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots=\frac{\pi^2}{8}. \qquad (7)$$

5. (a) Test the continuity of the function $f(x,y)=\dfrac{xy}{\sqrt{x^2+y^2}}$, if $(x, y)\neq(0,0)$ and $f(0,0)=0$ at the origin. (10)

(b) Discuss the maxima and minima of

$$f(x,y)=x^3y^2(1-x-y). \qquad (5)$$

6. (a) For what value of $k$, the equations $x+y+z=1$, $2x+y+4z=k$ and $4x+y+10z=k^2$ have (i) unique solution, (ii) infinite number of solutions, (iii) no solution and solve them completely in each case of consistency. (10)

(b) Find the eigen values of the matrix $A=\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$. Hence, find the matrix whose eigen values are $\frac{1}{6}$ and $-1$. (5)

7. (a) Evaluate $\int_{-\infty}^{\infty} xe^{-x^2}\,dx$, if it exists. (7)

(b) Find the radius of convergence of the series $\sum_{n=0}^{\infty}\left(\frac{(-1)^n}{8^n}x^{3n}\right)$ and the interval of convergence. (8)