ABV- Indian Institute of Information Technology & Management, Gwalior

Semester I (2023-24)

Major

Course Title: Engineering Mathematics-I

Course Code: ES102

MM: $60 - 65$ **Duration: 180 minutes**

Note:

1. All parts of a question should be answered consecutively.
2. The question paper has six questions in two pages.
3. Questions no. 1 b) and 5 b) are open questions, and the marks will be purely based on the justification and Mathematical explanations.
4. Question No. 3 c), d), 4 a): Marks are only for the proper justification.
5. Question no. 3 b): you must show that the example is a commutative ring with identity but not a field

1. a) Solve the following system using LU-Decomposition
    $2x_1 + 3x_2 + x_3 = 9; \ x_1 + 2x_2 + 3x_3 = 6; \ 3x_1 + x_2 + 2x_3 = 8$ (5)
    b) How can we construct symmetric and skew symmetric matrix from any random real square matrix   (2-5)
    c) State Cayley Hamilton Theorem (CHT)   (1)
    d) verify CHT for the following matrix using CHT and also find its inverse using CHT
    $$\begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}$$ (6)

2. a) Transform the following quadratic for to canonical form
    $3x_1^2 - 2x_2^2 - x_3^2 - 4x_1x_2 + 12x_2x_3 + 8x_1x_3$ (7)
    b) Fill in the blanks and prove: The _____ vectors of a _____ matrix forms a unitary system.   (3)

3. a) Define Isomorphism between two linear transformation.   (2)
    b) Give an example of a finite commutative ring with identity but not a field and
    prove your claim.   (2.5)
    c) Is the vector $(0,4,-4,2)$ in the subspace of R^4 spanned by the vectors
    $(1,1,1,1), (1,1,1,-1), (1,1,-1,1), (1,-1,1,1)$?   (2.5)
    d) Find out the vector in R^4 which is not in the subspace spanned by the vectors
    $(1,1,1,1), (1,1,1,-1), (1,1,-1,1), (1,-1,1,1)$ (2.5)

4. a) Which of the following are linear transformations from R^2 to R^2 (2)
    i) $T(x,y) = (x + 11, y + 23)$, ii) $T(x,y) = (x + 11y, 0)$, iii) $T(x,y) = (x^2, y)$

b) Let F be a subfield of the complex numbers and let T be the function from F^3 to F^3 defined by
$T(x, y, z) = (x - y + 2z, 2x + y, -x - 2y + 2z)$
i) check whether T is a Linear Transformation
ii) If (a, b, c) is a vector in F^3, what are the conditions on a, b, c that the vector be in the range of T?
iii) what are the conditions on a, b, c that the vector (a, b, c) be in the nullspace of T?   (2+1.5+1.5)

5. a) Find the spanning set for the null space of the matrix
$$\begin{pmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{pmatrix}$$
(3)
b) Define Vector space. Construct your own example of a Vector space and discuss any one subspace of the identified vector space.   (4.5-6.5)

6. a) Find $\lim_{(x,y) \to (1,2)} (x^2 y^3) - x^3 y^2 + 3x + 2y)$ (2)
b) Find out $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ for the following function using limiting conditions (limit definition of derivative) $f(x, y) = 17x^2 + 21xy - 4y^2 + 13x - 23y + 51$ (1.5+1.5)
c) Find out the absolute maximum and absolute minimum for the following,
$f(x, y) = x^2 - 2xy + 4y^2 - 4x - 2y + 24$, where $0 \le x \le 5$ and $0 \le y \le 3$ (7)

End of Question Paper
Best of Luck

$-3 + 2$

$1 - 3$

$2 - 16$