(Please write your Exam Roll No.) Exam Roll No. ....147........

END TERM EXAMINATION

THIRD SEMESTER [B.TECH] JANUARY 2024

Paper Code: CIC-205 **Subject: Discrete Mathematics**

Time: 3 Hours **Maximum Marks: 75**

Note: Attempt five questions in all including Q. No.1 which is compulsory. Select one question from each unit. Assume missing data, if any.

Q1 Answer the following short answer type questions (Compulsory):-

(a) Define a power set and give an example. (2)

(b) State De Morgan's Laws in propositional logic. (2)

(c) What is the principle of mathematical induction? (2)

(d) Give an example of an equivalence relation. (2)

(e) Define a Boolean Algebra. (2)

(f) Explain the difference between a group and a monoid. (2)

(g) What is Euler's formula in graph theory? (2)

(h) Define a Hamiltonian circuit in a graph. (1)

UNIT-I

Q2 (a) Construct a Venn diagram for three sets A, B, C with the following conditions: $A\cap B=\emptyset, B\cap C\neq\emptyset, A\cap C\neq\emptyset$. (5)

(b) Create a truth table for the proposition $p\rightarrow(q\wedge\neg r)$ and determine its logical equivalence. (5)

(c) Define a binary relation R on the set {1, 2, 3, 4} where R contains (a, b) if and only if a divides b. Determine if R is reflexive, symmetric, and transitive, providing justification for each property. (5)

Q3 (a) Using the principle of inclusion and exclusion, find the number of integers between 1 and 100 that are divisible by 2, 3, or 5. Provide step-by-step calculations. (5)

(b) Prove that in a group of six people, at least three people either all know each other or none of them knows the others. Use the pigeonhole principle in your proof. (5)

(c) Given a set S of n elements, derive an expression for the number of ways to choose a non-empty subset of S. Validate your expression with n = 5. (5)

UNIT-II

Q4 (a) Let f(x) = 2x - 1 and g(x) = x/2 + 1. Find the compositions f(g(x)) and g(f(x)) and determine their domains. Illustrate each step and clarify the reasoning behind the determination of their domains. (5)

(b) Show that every finite distributive lattice can be represented as a lattice of sets, using the lattice formed by the subsets of the set {a, b, c} as an example. Provide a detailed explanation. (5)

(c) Design a logic circuit for the Boolean expression $\overline{(A+\overline{B})C+\overline{A}C}$ and simplify it using Karnaugh maps. Include a step-by-step process for the circuit and a detailed explanation of the Karnaugh map simplification. (5)

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Q5 (a) Define Euler's totient function φ(n). Calculate φ(30) and explain the significance of each step in your calculation process. (5)

(b) Demonstrate how generating functions can be used to solve the recurrence relation $a_n = a_{n-1} + n$ with $a_0 = 1$, providing a specific example. Illustrate your solution with a clear explanation. (5)

(c) Apply the Master's theorem to analyze the time complexity of the recursive algorithm T(n) = 2T(n/2) + nlogn. (5)

UNIT-III

Q6 (a) Prove that every element of a finite group has an order. Use this to show that every group of prime order is cyclic. Your proof should be detailed, covering all necessary steps and reasoning. (5)

(b) Given the group G = {1, 2, 3, 4, 5, 6} under multiplication modulo 7, demonstrate how cosets of a subgroup H in G partition G, using H = {1, 2, 4}. Clearly demonstrate and provide a rationale for each step. (5)

(c) Discuss the significance of normal subgroups in group theory and provide an example of a non-trivial normal subgroup in the symmetric group S4. Provide a clear explanation with an illustrative example from S4. (5)

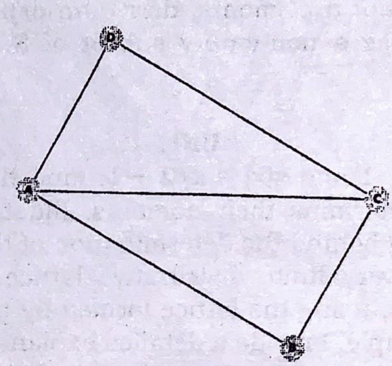
Q7 (a) Explain the concept of group homomorphism and prove that the kernel of a homomorphism is a subgroup of the domain. Include definitions and a thorough proof with clear logical steps. (5)

(b) Consider a group G = {1,2,3,4,5,6} under the multiplication modulo 7. (5)
- (i) Find the multiplication table of G
- (ii) Find 2^{-1}, 3^{-1} and 6^{-1}
- (iii) Is G Cyclic?

(c) For the symmetric group S3, determine all subgroups and identify which are normal. Identify each subgroup and clearly explain why it is or is not a normal subgroup. (5)

UNIT-IV

Q8 (a) For a given graph G, show how to determine if it has an Eulerian path or circuit. Apply your method to the graph with vertices A, B, C, D, and edges AB, BC, CD, DA, AC. Provide a step-by-step methodology for your determination and apply it to the given graph. (5)



(b) Prove Euler's formula for planar graphs and use this formula to deduce the number of regions in a graph with 5 vertices and 7 edges. Include a detailed proof of Euler's formula and apply it step-by-step to the given graph scenario. (5)

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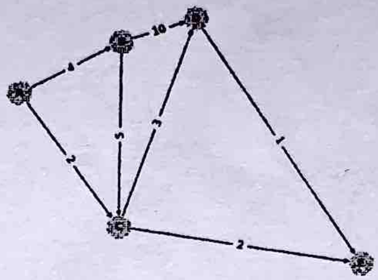
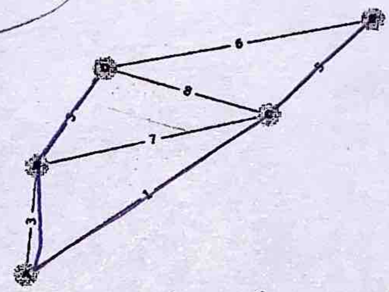
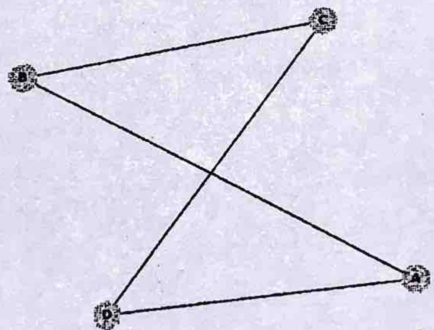
(c) Using Dijkstra's algorithm, find the shortest path in the given weighted graph from A to E. Detail each step in using Dijkstra's algorithm on the provided graph and discuss its time complexity with rationale. **(5)**



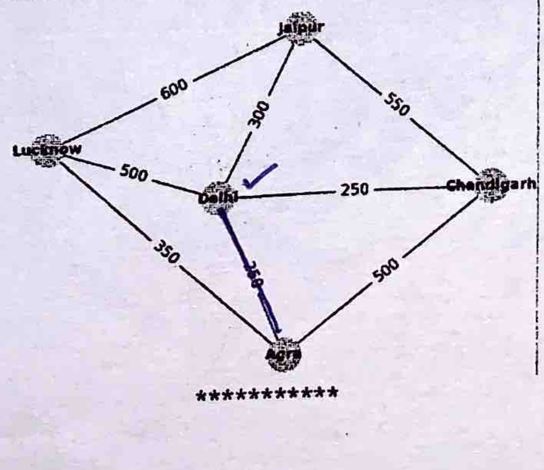
Q9 (a) Describe Kruskal's algorithm for finding a minimal spanning tree (MST) and apply it to a specific graph. Explain each step in Kruskal's algorithm and apply it to the provided graph, detailing the process. Discuss its time complexity. **(5)**



(b) Define the chromatic number of a graph. For the cycle graph composed of vertices A, B, C, D, with edges AB, BC, CD, DA, determine the chromatic number. Explain the reasoning behind your determination. **(5)**



(c) Discuss the traveling salesman problem and illustrate its solution for a set of cities (Delhi, Jaipur, Agra, Chandigarh, Lucknow) and beginning from Delhi. Discuss its effectiveness and limitations. **(5)**



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