End-Term Examination (Regular and Re-appear)
(B.Tech CSE AI/ECE AI/ AI&ML) (Semester I)
(December, 2023) OFF LINE mode

Subject Code: BAS 109	Subject: Applied Mathematics
Time: 3 Hours	Maximum Marks: 60

Note: (a) Q 1 is compulsory. Attempt one question each from Units I, II, III and IV.

(b) Use of a calculator is not allowed.

Q1 (2.5*8=20)

(a) Check whether the following set of vectors is linearly independent or dependent.

$$\left\{\left(\frac{1}{2},1,1\right),\left(-1,-\frac{1}{2},1\right),(2,-2,1)\right\}.$$

(b) Find the eigenvalue of a Hermitian matrix $A=\begin{bmatrix}3 & 1-i\\ 1+i & 2\end{bmatrix}$.

(c) Determine whether the following functions are linear transformations $T:\mathbb{R}^2\to\mathbb{R}^2$. If they are, prove it; if not, provide a counterexample to one of the properties:

$$T\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}x^2\\ y^2\end{bmatrix}$$

(d) Find the Fourier coefficient a_n of the rectangular pulse function given by

$$f(x)=\begin{cases}0, & -\pi<x<-\pi/2\\ 1, & -\pi/2\le x\le \pi/2.\\ 0, & \pi/2<x<\pi\end{cases}$$

(e) Define $f:\mathbb{R}^2-\{(0,0)\}\to\mathbb{R}$ by

$$f(x,y)=\frac{x^{11}y}{x^{23}+y^2}.$$

Does $\lim_{(x,y)\to(0,0)} f(x,y)$ exist? If so, what is it?

(f) Find the quadratic Taylor's series expansion of $f(x,y)=x^2y+3y-2$ about the point $(-1,2)$.

(g) Find a unit normal vector to the surface $xy^2+2yx=8$ at the point (3,-2,1).

(h) If $\vec{F}=(x+3y)\vec{i}+(y-2z)\vec{j}+(x+\lambda z)\vec{k}$ is solenoidal, then find the value of λ.

UNIT-I

Q2 (10)

(a) Find the eigenvalues and any eigenvector of a matrix A given by

$$A=\begin{bmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{bmatrix}.$$

(b) Solve each of the following systems of linear equations:

$$\begin{aligned}4x_1-2x_2-7x_3&=5\\ -6x_1+5x_2+10x_3&=-11\\ -2x_1+3x_2+4x_3&=-3\\ -3x_1+2x_2+5x_3&=-5\end{aligned}$$

Q3 (10)

(a) Find inverse of matrix A using Cayley Hamilton method

$$A=\begin{bmatrix}3 & 1 & 1\\ -1 & 2 & 1\\ 1 & 1 & 1\end{bmatrix}.$$

(b) Find the rank of the matrix $\begin{pmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{pmatrix}$ by using Echelon or Normal form.

UNIT-II

Q4	(a) Let $T: \mathbb{R}^4 \to \mathbb{R}^3$ be a linear transformation defined by $$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 + 3x_3 - x_4 \\ 3x_1 + 5x_2 + 8x_3 - 2x_4 \\ x_1 + x_2 + 2x_3 \end{bmatrix}.$$ (i) Find a basis for the null space of T. (ii) Find the rank of the linear transformation T. (b) Apply Convolution theorem to find the inverse Laplace transforms of the function $f(s) = \frac{s}{(s^2+1)(s^2+4)}$.	(10)
Q5	(a) Find the Fourier series for the function $f(x) = \begin{cases} x, & -1 < x \le 0 \\ x+2, & 0 < x < 1, \end{cases}$ where, $f(x) = f(x+2)$. (b) Solve the following initial value problem using Laplace Transform $$y''(t) + 3y'(t) + 2y(t) = \sin(2t)$$ with $y(0) = 2$ and $y'(0) = -1$.	(10)

UNIT-III

Q6	(a) Find the nth derivative of $y = e^x(2x+3)^3$. (b) Find the minimum of $f(x,y) = y^2 + x^2y + x^4$.	(10)
Q7	If $z = x^n f\left(\frac{y}{x}\right) + y^{-n} g\left(\frac{x}{y}\right)$, then prove that $$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = n^2 z.$$	(10)

UNIT-IV

Q8	(a) Evaluate $I = \int_0^{\pi/2}\int_x^{\pi/2} \frac{\sin y}{y}\, dydx$ by changing the order of integration. (b) Evaluate $\int_C \vec{F}\cdot d\vec{r}$ where, $\vec{F}(x,y,z) = e^{2x}\hat{\imath} + z(y+1)\hat{\jmath} + z^3\hat{k}$. and C is given by $\vec{r}(t) = t^3\hat{\imath} + (1-3t)\hat{\jmath} + e^t\hat{k}$ for $0 \le t \le 2$.	(10)
Q9	(a) Evaluate $\iint \vec{F}.\hat{N}dS$ over S, where $\vec{F} = yz\hat{\imath} + zx\hat{\jmath} + xy\hat{k}$ and S is the portion of the surface of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant. (b) Find the work done in moving a particle in the force field $\vec{F} = 3x^2\hat{\imath} + (2xz - y)\hat{\jmath} + z\hat{k}$ along, the straight line from (0,0,0) to (2,1,3).	(10)