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300205

October, 2020

B.Tech. (CSE/CE/IT) II SEMESTER

Maths-II (Probability & Statistics) (BSC 106E)

Time : 3 Hours]   [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*
4. *Use of Calculator is Allowed.*
5. *Normal distribution table is required*

PART-A

1. (a) Calculate the quartile coefficient of skewness from the given data :

Weight (lbs)	70-80	80-90	90-100	100-110
No. of persons		12	18	35 42
Weight (lbs)	110-120	120-130	130-140	140-150
No. of persons		50	45	20 8

(1.5)

(b) Tbe first four moments about the working mean 28.5 of a distribution are 0.294, 7.144, 42.409 and 454.98. Calculate the moments about the mean. Also evaluate $\beta_1 \beta_2$. (1.5)

(c) If the events A and B are such that $P(A) \neq 0$, $P(B) \neq 0$ and A is independent of B, then prove that B is independent of A. (1.5)

(d) For a certain normal distribution, the first moment about 10 is 40 and the fourth moment about 50 is 48. What is arithmetic mean and standard deviation of the distribution ? (1.5)

(e) A speaks truth in 60% cases and B in 70% cases. In what percentage of cases are they likely to contradict each other? (1.5)

(f) If X represents the outcome when a pair of dice is thrown find the expectation E(X) and variance V(X). (1.5)

(g) During testing a sample of 300 chips 10 have been found to be defective. Can the manufacturers claim that 2% of the chips are defective may be accepted ? (1.5)

(h) Calculate the rank correlation coefficient from the following data showing ranks of 10 students in two subjects :

Maths	3	8	9	2	7
Physics	5	9	10	1	8
Maths	10	4	6	1	5
Physics	7	3	4	2	6

(1.5)

(i) If F is the pull required to lift a load W by means of pulley, fit a linear law $F = a + bW$ against the following data :

W	50	70	100	120
F	12	15	21	25

(1.5)

(j) Prove that the coefficient of correlation is independent of the change of scale and change of origin for the variables x and y. (1.5)

PART-B

2. (a) If $X_1, X_2, \ldots\ldots X_K$ are k independent Poisson variates with parameters $\lambda_1, \lambda_2, \ldots\ldots, \lambda_k$ respectively, prove that the conditional distribution :

$$P((X_1 \cap X_2 \cap \ldots\ldots\ldots \cap X_K \mid X),$$

where $X = X_1 + X_2 + \ldots + X_K$ is fixed, is multinomial. (7)

(b) State and prove Chebyshev inequality. (8)

3. (a) Show that the exponential distribution 'lacks memory', i.e., if X has an exponential distribution, then for every constant $a \geq 0$, one has $P(Y \leq x \mid X \geq a) = P(X \leq x)$ for all x, where $Y = X - a$. (8)

(b) The daily consumption of milk in a city, in excess of 20,000 litres, is approximately distributed as a Gamma variate with parameters $a = 1/10000$ and $\lambda = 2$. The city has a daily stock of 30,000 litres. What is the probability that the stock is insufficient on a particular day? (7)

4. Let X and Y have bivariate normal distribution with parameters :

$\mu_x = 5$, $\mu_y = 10$, $\sigma_x^2 = 1$, $\sigma_y^2 = 25$ and Corr $(X, Y) = \rho$.

(a) If $\rho > 0$, find ρ when $P(4 < Y < 16 \mid X = 5) = 0.954$.

(b) If $\rho = 0$, find $P(X + Y) \leq 16$. (15)

5. (a) Fit a binomial distribution to the following frequency distribution :

x	0	1	2	3	4	5	6
f	13	25	52	58	32	16	4

(8)

(b) Find the correlation coefficient between x and y for the given value. Find also the two regression lines.

x	1	2	3	4	5
y	10	12	16	28	25

x	6	7	8	9	10
y	36	41	49	40	50

(7)

6. (a) Determine the constants a, b and c by the method of least squares such that $y = ax^2 + bx + c$ fits the given data :

x	2	4	6	8	10
y	4.01	11.08	30.12	81.89	222.62

(8)

(b) A normally distributed population has mean 6.8 and standard deviation 1.5. A sample of size 400 has been 6.75. Is the difference between the population mean and the sample mean significant. (7)

7. (a) In a school the heights of six randomly choosen girls are 63, 65, 68, 69, 71 and 72 inches and those of nine randomly choosen boys 61, 62, 65, 66, 69, 70, 71, 72 and 73 inches. Test if the girls are taller than the boys. (8)

7. Let the function $f(z)$ be analytic inside a circle $|z-a| = R$, where R is the radius and a is the centre. Then $f(z)$ has the power series representation denotes factorial

$$f(z) = f(a) + (z-a)f'(a) + \frac{(z-a)^2}{2\,|}f''(a) + \dots + \frac{(z-a)^n}{n\,|}f^{(n)}(a) + \dots,\ |z-a| < R,$$

$n\,|$ denotes factorial n. (15)