END TERM EXAMINATION

THIRD SEMESTER [B.TECH] JANUARY 2024

Paper Code: ES-201 Subject: Computational Methods

Time: 3 Hours Maximum Marks: 75

Note: Attempt five questions in all including Q.No.1 which is compulsory. Select one question from each unit. Assume missing data if any. Scientific calculator is allowed.

Q1	Attempt al	l questions:
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- The height of an observation tower was estimated to be 47m, whereas its actual height was 45m. Calculate the percentage relative error in the measurement. (1.5)
- b) Derive the formula for evaluating $\sqrt{12}$ by Newton's Raphson method. (1.5)
- c) Differentiate between partial and complete pivoting in solving linear system of algebraic equations. (1.5)

d)	Estima	Estimate the missing term in the following table:						
	x:	0	1	2	3	4	(1.5)	
134	f(x):	1	3	9	1_	81		

- e) Define cubic spline function and state the conditions required for cubic spline interpolation. (1.5)
- Find $\int_{0}^{\pi/2} \sqrt{\cos \theta} d\theta$ using Simpson's $1/3^{rd}$ rule for n=6. (2.5)
- Solve the IVP $\frac{dy}{dx} = xe^y$, y(0) = 0 using Picard's method and estimate y(0.2).
 - h) Determine which of the following equations are elliptic, parabolic, and hyperbolic. (2)
 - (i) $f_{xx} + 6f_{xy} + 9f_{yy} = 0$
 - (ii) $f_{xx} 2f_{xy} + 2f_{yy} = 2x + 5y$

UNIT-I

Find the root of the equation $x=e^{-x}$, correct to three decimal places by Secant method by performing six iterations. (8) Use Newton—Rapshon method to obtain a root of $\sin x = 1-x$ to three decimal places. (7)

- Q3 a) Perform four iterations of Golden section search method to minimize $f(x) = x^4 14x^3 + 60x^2 70x$, $x \in (0,2)$ with $\varepsilon = 10^{-3}$. (7.5)
 - b) Use steepest descent method for 3 iterations on $f(x_1, x_2, x_3) = (x_1 4)^4 + (x_2 3)^2 + 4(x_3 + 5)^4$ with initial point $x^{(0)} = [4, 2, -1]^T$ (7.5)

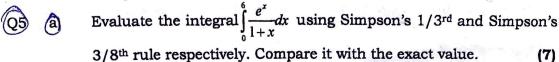
UNIT-II

- Q4 a) Prove that $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{(1+\delta^2/4)}$. (7)
 - b) The table gives the distance in nautical miles of the visible horizon for the given heights in feet above the earth's surface: (8)

 P.T.O.

x = height:	100	150	200	250	300	350	400
y = distance:	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find the value of y when x = 410 using Newton's backward interpolation formula.



Evaluate the integral $\int_{0}^{\sqrt{x}} \left(\frac{x}{\sin x}\right) dx$ using Romberg's method, correct to three decimal places. (8)

UNIT-III

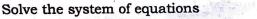
Q6 Investigate the values of λ and μ so that the system of equations (7)

$$2x+3y+5z=9$$

$$7x+3y-2z=8$$

$$2x + 3y + \lambda z = \mu$$

have (i) no solution, (ii) unique solution, (iii) an infinite number of solutions. (8)



 $\begin{cases} x_1 + 10x_2 - x_3 = 3\\ 2x_1 + 3x_2 + 20x_3 = 7\\ 10x_1 - x_2 + 2x_3 = 4 \end{cases}$

Using the Gauss elimination with partial pivoting.



Solve the system of equations using Dolittle factorisation (9) method

$$\begin{cases} 3x+2y+7z=4\\ 2x+3y+z=5\\ 3x+4y+z=7 \end{cases}$$

Determine the numerically dominant eigenvalue and eigenvector of the matrix $A = \begin{bmatrix} 15 & -4 & -3 \\ -10 & 12 & -6 \\ -20 & 4 & -2 \end{bmatrix}$ using Power method. Take the initial

vector
$$X^{(0)} = [1,1,1]^T$$
. (6)



UNIT-IV

Employ Taylor's method to obtain the approximate value of y at x = 0.2 for the differential equation $dy/dx = 2y + 3e^x$, y(0) = 0. Compare the numerical solution obtained with the exact solution. (7)

P.T.O.



Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with y(0) = 1 at x = 0.2, 0.4. (8)

Solve the initial value problem $y \frac{dy}{dx} = x$, y(0) = 1, using Euler's method in $0 \le x \le 0.8$, with h = 0.2. Compare the results with the a) Q9 exact solution at x = 0.8.

(7)Solve the partial differential equation b)

 $u_{xx} + u_{yy} = x + y + 1, \ 0 \le x \le 1, \ 0 \le y \le 1,$

u = 0 on the boundary

Numerically up to three iterations with h=1/3. Obtain the results correct to three decimal places.