Sr. No……80012

YMCA UNIVERSITY OF SCIENCE AND TECHNOLOGY, FARIDABAD

B.Tech., I SEMESTER

MATHEMATICS I (HAS-103-C)

Time: 3 Hours | Max. Marks: 75

Instructions:
1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART –A

Q1 (a) Find the rank of $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$ (1.5)

(b) Show that the radius of curvature at the point $a/4, a/4$ on the curve $\sqrt{x}+\sqrt{y} = \sqrt{a}$ is $\frac{a}{\sqrt{2}}$. (1.5)

(c) Prove that $B(m,n) = B(n,m)$ (1.5)

(d) Evaluate $\int_0^1 \int_0^{x^2} e^{y/x} dy\, dx$ (1.5)

(e) Find curl of a vector point function $(x^2 + xy^2)\hat{i} + (y^2 + x^2y)\hat{j}$. (1.5)

(f) Evaluate $\int_1^2 \int_{-\sqrt{2-y}}^{\sqrt{2-y}} 2x^2y^2\, dxdy$ (1.5)

(g) Show that the function $u = x + y - z$, $v = x - y + z$, $w = x^2 + y^2 + z^2 - 2yz$ are not independent of one another . Also find the relation between them. (1.5)

(h) Expand e^{xy} at (1, 1) up to three terms. (1.5)

(i) Find grad ϕ when $\phi = 3x^2y - y^3z^2$ at (1,-2,-1). (1.5)

(j) Examine whether $u(x,y) = \frac{x^2}{y}$, is homogeneous. if yes, find the degree (1.5)

PART –B

Q2 (a) Evaluate $\int_C \vec{F}.\vec{dr}$, where $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ *and the curve C is rectangle in the xy – plane bounded by* $y = 0, x = a, y = b, x = 0$. (7)

(b) Use Green's theorem to evaluate $\oint_c [(y - sinx)dx + cosx\, dy]$, where C is the plane triangle enclosed by the lines $y = 0, x = \frac{\pi}{2}$ *and* $y = \frac{2}{\pi}x$. (8)

Q3 (a) Evaluate $\int_v (2x + y)dV$, where V is closed region bounded by the cylinder $z= 4-x^2$ and the planes x=0,y=0,y=2 and z=0. (7)

(b) Find the area included between the curves $y^2 = 4a(x + a)$ and $y^2 = 4b(b - x)$ (8)

Q4 (a) Solve by rank method the system of equations x+2y-5z= -9,3x-y+2z= 5, 2x+3y-z=3, (7)

(b) 4x-5y+z= -3

Find the characteristic equation of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$ and show that A satisfies the equation. Hence evaluate $A^{-1} and A^4$. (8)

Q5 (a) Find the Asymptotes of (7)

$$(y - x)(y - 2x)^2 + (y + 3x)(y - 2x) + 2x + 2y - 1 = 0$$

(b) Find the radius of curvature of $x = a\cos^3 t,\ y = a\sin^3 t$ on the curve $x^{2/3} + y^{2/3} = a^{2/3}$. (8)

Q6 (a) Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube. (7)

(b) Prove that $\int_0^1 x^\alpha (\log x)^n \, dx = \frac{(-1)^n n!}{(\alpha+1)^{n+1}}$ (8)

Q7 (a) Verify Euler's Theorem for the functions $f(x, y) = \frac{x^2 (x^2-y^2)^3}{(x^2+y^2)^3}$ (7)

(b) Investigate the value of λ and μ so that the equations 2x+3y+5z =9 , 7x +3y -2z =8,2x+3y+λz =μ have (i) no solution (ii) unique solution and (iii) infinite number of solutions. (8)

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