YMCA UNIVERSITY OF SCIENCE AND TECHNOLOGY, FARIDABAD
B.TECH EXAMINATION (Under CBS) , May-2018
MATHEMATICS I (HAS-103)

Time: 3hrs **M.Marks:60**

Note: PART-I is compulsory and attempt any four questions from PART-II.

PART - I

Q.1

a) Find the rank of the matrix

$$A = \begin{bmatrix} 3 & 1 & 2 & 4 \\ -1 & 0 & 4 & 9 \end{bmatrix}$$

b) Find the eigen values of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

c) Is the matrix $\begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix}$ orthogonal ?

d) Expand tanx by Maclaurin's series.

e) What is formula of radius of curvature for explicit equation (y=f(x)) and implicit equation f(x,y)= 0.

f) Show that the asymptotes of the curve $x^2y^2 = a^2(x^2 + y^2)$ form a square of side 2a.

g) If $u = \frac{y^2}{2x}$, $v = \frac{x^2+y^2}{2x}$.Find $\frac{\partial(u,v)}{\partial(x,y)}$

h) Test the convergence of the series

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots\ldots\ldots\ldots\ldots + \infty$$

i) Change the order of integration $\int_0^a \int_y^a \frac{xdxdy}{x^2+y^2}$

j) Find the value of $\Gamma\left(\frac{1}{2}\right)$ $2\times10 = 20$

PART – II

Q.2(a) Using Gauss –jordan method , find the inverse of the matrix

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & 4 \end{bmatrix}$$ (5)

(b) Use Cayley –Hamilton theorem to find the matrix

$$A^8 - 5A^7 + 7A^6 - 3A^5 + 8A^4 - 5A^3 + 8A^2 - 2A + I$$

If the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ (5)

Q.3(a) Evaluate $\int_0^{\frac{\pi}{2}} sin^5\theta \, d\theta$,using beta function. (5)

(b) Find the radius of curvature at y = 2a on the curve $y^2 = 4ax$ (5)

Q.4(a)Find all the asymptotes of the curve $x^2y^2 - x^2y - xy^2 + x + y + 1 = 0$ (5)

(b) Using Taylor's series , expand e^x in powers of (x-2) (5)

Q.5(a)Find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ (5

(b)If $u=sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=tanu$ (5

Q.6(a)Find the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, using double integration

(b) Find the volume of the region bounded by the paraboloid $z=x^2+y^2$ and the plane $z=4$

Q.7(a)Test the convergence of the series

$$x+\frac{1}{2}\frac{x^3}{3}+\frac{1.3}{2.4}\frac{x^5}{5}+\frac{1.3.5}{2.4.6}\frac{x^7}{7}+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\infty$$

(b) Discuss the convergence of the series

$$\frac{1}{1.2}-\frac{1}{3.4}+\frac{1}{5.6}-\frac{1}{7.8}+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\infty$$