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PDPM Indian Institute of Information Technology, Design and Manufacturing, Jabalpur

End-Term Examination

Digital Signal Processing (EC 2005)

Time: 03 Hours

Max. Marks: 40

Note: Attempt All Questions

Q.1 (a) Find the inverse DTFT of $X(e^{j\omega})$, for

$$X\left(e^{j\omega}\right) = \begin{cases} 0 & 0 \le |\omega| < \frac{\pi}{4} \\ 1 & \frac{\pi}{4} \le |\omega| \le \frac{3\pi}{4} \\ 0 & \frac{3\pi}{4} < |\omega| \le \pi \end{cases}$$

(01)

(b) Suppose that we want to design a discrete-time LTI system that has the property that if the input is $x(n) = \left(\frac{1}{2}\right)^n u(n) - \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)^{n-1} u(n-1)$ then the output is $y(n) = \left(\frac{1}{3}\right)^n u(n)$

(i) Find the frequency response of a discrete-time LTI system that has the foregoing property.

(01)

(ii) Find a difference equation relating x(n) and y(n) that characterizes the system. (02)

(c) If X(k) = {4, -j2, 0, j2} is the 4-point DFT of x(n), determine the DFT of $y(n) = x(\langle n-2 \rangle_4)$

(01)

Q2. (a) A signal $x_a(t)$ that is bandlimited to 10 kHz is sampled with a sampling frequency of 20 kHz. The DFT of N = 1000 samples of x(n) is then computed. What analog frequency does the index k = 150 correspond to?

(01)

(b) A continuous-time signal $x_a(t)$ is sampled with a frequency of f_s= 2 kHz. If a 1000-point DFT of 1000 samples is computed, what is the spacing between the frequency samples X(k) in terms of the analog frequency?

(01)

(c) Let $x(n) = \{\underset{\uparrow}{2}, 5, 0, 4\}$ and $h(n) = \{\underset{\uparrow}{4}, 1, 3\}$

Perform linear convolution using circular convolution. (02)

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(d) The even samples of the 11-point DFT of a length-11 real sequence are given by –

X(0) = 4, X(2) = -1 + j3, X(4) = 2 + j5, X(6) = 9-j6, X(8) = -5 -j8, and X(10) = $\sqrt{3} - j2$. Determine the missing odd samples of the DFT. (01)

Q 3. (a) Sampling a continuous-time signal $x_a(t)$ for 1 second generates a sequence of 4096 samples. (02)

(i) What is the highest frequency in $x_a(t)$ if it was sampled without aliasing?

(ii) If 4096-point DFT of the sampled signal is computed, what is the frequency spacing between the DFT coefficients?

(iii) Suppose that we are only interested in the range $200 \leq f \leq 300$ Hz. How many complex multiplies are required to evaluate these values computing the DFT directly, and how many are required if a decimation-in-time FFT is used (take $\log_2 4096 = 12$).

(b) Find the 4-point circular convolution of x(n) and h(n) given by

x(n) = { 1,1,1,1} and h(n) = {1,0,1,0} using radix-2 DIF-FFT algorithm. (03)

Q4. (a) Does $H(z) = 1 + 2z^{-1} + 2z^{-2} + z^{-3}$ describe a linear-phase filter? Justify your answer. (01)

(b) Consider a discrete-time system with frequency response $H(e^{j\omega})$ and real impulse response h(n). Suppose that.for this system, $|H(e^{j\pi}/2)| = 2$, and Group Delay $[\tau_g(\pi/2)]$. Determine the output of the system when the input is $\cos(\pi n/2)$. (01)

(c) A digital notch filter is required to remove an undesirable f_o = 60 Hz hum associated with a power supply in an ECG recording application. The sampling frequency used is f_s = 500 samples/sec. Design a second-order FIR notch filter. (03)

Q5. A digital filter is characterized by the following properties: (05)

(i) It is high-pass and has one pole and one zero.

(ii) The pole is at a distance r = 0.9 from the origin of the z-plane.

(iii) Constant signals do not pass through the system

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Then,

(a) Plot the pole-zero pattern of the filter and determine its system function H(z).

(b) Compute the magnitude response and the phase response of the filter.

(c) Normalize the frequency response $H(e^{j\omega})$ so that , $|H(e^{j\pi})| = 1$.

(d) Determine the input-output relation (difference equation) of the filter in the time domain.

Q6. (a) Find the Kaiser window parameters, β and N, to design a low-pass filter with cut-of frequency $\omega_c = \pi/2$, a stopband ripple $\delta_s = 0.002$, and a transition bandwidth no longer than 0.1 π. (02)

(b) Consider the following specifications for a bandpass filter: (03)

$$\begin{Bmatrix} & |H(e^{j\omega})| \leq 0.01 & 0 \leq |\omega| \leq 0.2\pi \\ 0.95 \leq & |H(e^{j\omega})| \leq 1.05 & 0.3\pi \leq |\omega| \leq 0.7\pi \\ & H(e^{j\omega}) \leq 0.02 & 0.8\pi < |\omega| \leq \pi \end{Bmatrix}$$

For these specifications a linear phase BP FIR filter is to be designed using a Blackman window. For this calculate-

(i) Minimum stopband attenuation in dB (A_s)

(ii) Filter Order (N)

(iii) The causal impulse response of the filter that is to be windowed.

Q7. Design Lowpass Butterworth filter using impulse invariant method for satisfying the following constraints: Passband digital frequency: 0.162 rad; Stopband digital frequency: 1.63 rad; Passband ripple: 3 dB; Stopband attenuation: 30 dB; Sampling Frequency: 8 kHz. (05)

Q8. With the help of a block diagram differentiate between Von Neumann, Harvard, and Super Harvard architectures. Explain the features of Super Harvard Architecture that improves its throughput. (05)