YMCA UNIVERSITY OF SCIENCE AND TECHNOLOGY, FARIDABAD
B.TECH EXAMINATION (Under CBS) , May-2018
MATHEMATICS I (HAS-103)

Time: 3hrs **M.Marks:60**

Note: PART-I is compulsory and attempt any four questions from PART-II.

PART - I

Q.1

a) Find the rank of the matrix

$$A = \begin{bmatrix} 3 & 1 & 2 & 4 \\ -1 & 0 & 4 & 9 \end{bmatrix}$$

b) Find the eigen values of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

c) Is the matrix $\begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix}$ orthogonal ?

d) Expand tanx by Maclaurin's series.

e) What is formula of radius of curvature for explicit equation (y=f(x)) and implicit equation f(x,y)= 0.

f) Show that the asymptotes of the curve $x^2y^2 = a^2(x^2 + y^2)$ form a square of side 2a.

g) If $u = \frac{y^2}{2x}$, $v = \frac{x^2+y^2}{2x}$.Find $\frac{\partial(u,v)}{\partial(x,y)}$

h) Test the convergence of the series

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots\dots\dots\dots\dots + \infty$$

i) Change the order of integration $\int_0^a \int_y^a \frac{xdxdy}{x^2+y^2}$

j) Find the value of $\Gamma\left(\frac{1}{2}\right)$ $2\times 10 = 20$

PART – II

Q.2(a) Using Gauss –jordan method , find the inverse of the matrix

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & 4 \end{bmatrix}$$ (5)

(b) Use Cayley –Hamilton theorem to find the matrix

$$A^8 - 5A^7 + 7A^6 - 3A^5 + 8A^4 - 5A^3 + 8A^2 - 2A + I$$

If the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ (5)

Q.3(a) Evaluate $\int_0^{\frac{\pi}{2}} sin^5\theta \, d\theta$,using beta function. (5)

(b) Find the radius of curvature at y = 2a on the curve $y^2 = 4ax$ (5)

Q.4(a)Find all the asymptotes of the curve $x^2y^2 - x^2y - xy^2 + x + y + 1 = 0$ (5)

(b) Using Taylor's series , expand e^x in powers of (x-2) (5)

Q.5(a)Find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ (5

(b)If $u=sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=tanu$ (5

Q.6(a)Find the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, using double integration

(b) Find the volume of the region bounded by the paraboloid $z=x^2+y^2$ and the plane $z=4$

Q.7(a)Test the convergence of the series

$$x+\frac{1}{2}\frac{x^3}{3}+\frac{1.3}{2.4}\frac{x^5}{5}+\frac{1.3.5}{2.4.6}\frac{x^7}{7}+\cdots\ldots\ldots\ldots\ldots\ldots\infty$$

(b) Discuss the convergence of the series

$$\frac{1}{1.2}-\frac{1}{3.4}+\frac{1}{5.6}-\frac{1}{7.8}+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\infty$$

Sr. No....................................

May 2019

B.Tech (All Branches), I SEMESTER (Reappear)

Mathematics-I(HAS-103)

Time: 3 Hours

Max. Marks:60

Instructions:
1. *It is compulsory to answer all the questions (2 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) Find the Rank of the matrix $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$ (2)

(b) Prove that product of eigen values of a matrix A is equal to the determinent of A. (2)

(c) Using Maclaurin's Theorem ,expand $\log secx$. (2)

(d) If $\sin u = \frac{x^2y^2}{x+y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\ tanu$ (2)

(e) If $z = \log(e^x + e^y)$, show that $rt - s^2 = 0$ (2)

(f) Evaluate $\int\int_R (x + y)dy\ dx$, R is the region bounded by x=0, x=2, y=x, y=x+2 (2)

(g) Change the order of Integration $\int_0^\infty \int_0^x e^{-xy} y\ dy\ dx$ (2)

(h) Evaluate $\int_0^\infty x^6 e^{-3x} dx$ (2)

(i) Test the series $\sum_{n=1}^{\infty} \frac{1}{n+10}$ for convergence or divergence. (2)

(j) Discuss the convergence of the series $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$ (2)

PART -B

Q2 (a) Find the eigen values and eigen vectors of matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ (5)

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(b) Verify Cayley -Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$. Hence evaluate A^{-1} (5)

Q3 (a) Find all the asymptotes of the following curve (5)

$$(x-y)^2(x+2y-1) = 3x+y-7$$

(b) Show that radius of curvature at the ends of the major axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is equal to the semi-latus rectum of the ellipse. (5)

Q4 (a) If $\theta = t^n e^{r^2/4t}$, what value of n will make $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right) = \frac{\partial\theta}{\partial t}$? (5)

(b) If xyz=8, Find the values of x,y for which $u = \frac{5xyz}{x+2y+4z}$ is a maximum. (5)

Q5 (a) Transform the equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} = 0$ into polar coordinates. (5)

(b) A pyramid is bounded by the three co-ordinate planes and the plane x+2y+3z=6. Compute this integral by double integration. (5)

Q6 (a) Evaluate $\iiint x^2yz\,dxdydz$ throughout the volume bounded by the planes $x=0, y=0, z=0, \frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ (5)

(b) Evaluate $\iint r\,sin\theta drd\theta$ over the area of the cardiod $r = a(1+cos\theta)$ above the initial line. (5)

Q7 (a) Discuss the convergence of the series : (5)

$$1+\frac{x}{2}+\frac{2!}{3^2}x^2+\frac{3!}{4^3}x^3+\frac{4!}{5^4}x^4+\cdots\ldots\ldots\infty$$

(b) Discuss the convergence of the series $x+\frac{2x^2}{2!}+\frac{3x^3}{3!}+\frac{4x^4}{4!}+\cdots\ldots\ldots\infty$ (5)

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Roll No. 19001013032

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300106

December, 2019
B.Tech. (ME/MA/AE) 1st Semester
MATHEMATICS: Calculus and Linear Algebra
(BSC103A)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART - A

1. (a) Define evolutes and involutes with example. (1.5)

(b) Evaluate $\int_0^{\pi/2} \sqrt{\tan \theta}\, d\theta$. (1.5)

(c) State Lagrange's Mean Value theorem. (1.5)

(d) Evaluate $\lim_{x \to \infty} \frac{x^n}{e^x}$. (1.5)

(e) If $<a_n>$ is bounded and $b_n \to 0$, then $a_n b_n \to 0$.

(1.5)

(f) Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^n . x^{2n}$. (1.5)

(g) Show that $\lim_{(x,y)\to(o,o)}\left(\frac{x^2}{x^4+y^2}\right)$ does not exist. (1.5)

(h) Show that the vector

$$\vec{F}=(6xy+z^3)\,\vec{i}+(3x^2-z)\,\vec{j}+(3xz^2-y)\,\vec{k}$$

is irrotational. (1.5)

(i) If 2 and 3 are eigen values of

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix},$$

find the eigen values of A^{-1} and A^3. (1.5)

(j) Find rank of matrix

1	−7	3	−3
7	20	−2	25
5	−2	4	7

by using determinant. (1.5)

PART - B

2. (a) Find the evolute of the rectangular hyperbola $xy = c^2$. (8)

(b) A sphere of radius a is divided into two parts by a plane at a distance $\frac{a}{2}$ from the centre. Show that the ratio of the volume of two parts is 5 : 27. (7)

3. (a) Expand $\sin x$ as a finite series in powers of x, with remainder in Lagrange's form. Hence, find the series for $\sin x$. (7)

(b) Using Rolle's theorem, prove that there is no real a for which the equation $x^2 - 3x + a$ has two different roots in $[-1, 1]$. (8)

4. (a) Test the convergence of the series given below :

$$\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-1}+\ldots\ldots\infty \qquad (8)$$

(b) Find the half-range cosine series for $f(x)=x$ in the interval $[0, \pi]$ and deduce that

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots=\frac{\pi^2}{8}. \qquad (7)$$

5. (a) Test the continuity of the function $f(x,y)=\dfrac{xy}{\sqrt{x^2+y^2}}$, if $(x, y)\neq(0,0)$ and $f(0,0)=0$ at the origin. (10)

(b) Discuss the maxima and minima of

$$f(x,y)=x^3y^2(1-x-y). \qquad (5)$$

6. (a) For what value of k, the equations $x+y+z=1$, $2x+y+4z=k$ and $4x+y+10z=k^2$ have (i) unique solution, (ii) infinite number of solutions, (iii) no solution and solve them completely in each case of consistency. (10)

(b) Find the eigen values of the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$.

Hence, find the matrix whose eigen values are $\frac{1}{6}$ and -1. (5)

7. (a) Evaluate $\int_{-\infty}^{\infty} xe^{-x^2}\,dx$, if it exists. (7)

(b) Find the radius of convergence of the series $\sum_{n=0}^{\infty}\left(\frac{(-1)^n}{8^n}x^{3n}\right)$ and the interval of convergence. (8)

300109

December, 2019

B.Tech. (ECE/EIC/ECC/FAE) 1st SEMESTER

Mathematics-I (Calculus and Linear Algebra)

(BSC-103D)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART-A

1. (a) Evaluate $\int x^2 \sin 2x dx$.

(b) Using property of Beta function, Evaluate

$$\int_0^1 x^{11}(1-x)^5 dx.$$

$x^5 - 5x^4 + 5x^3 - 1$.

(d) Find the Taylor's series expansion of $\sin x$ about $x = \pi/4$.

(e) Find the half range cosine series for

$f(x) = x,\ 0 \le x \le \pi$.

(f) Prove that the exponential series

$$1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\dots\dots + \frac{x^n}{n!} + \dots\dots\infty.$$

(g) Show that the vector

$$\vec{f} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$$

is irrotational.

(h) If $\phi = x^2y + xy^2 + z^2$, then find grad ϕ at (1, 1, 1).

(i) Find the inverse of $A = \begin{bmatrix} 1 & 6 & 2 \\ 0 & -2 & 4 \\ 3 & 1 & 2 \end{bmatrix}$.

(j) Find the Eigen value of the given matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$ (1.5×10=15)

2. (a) Show that the evolute of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ is another cycloid. (8)

(b) The area bounded by $y^2 = 4x$ and the line $x = 4$ is revolved about the line $x = 4$. Find the volume of the solid of revolution. (7)

3. (a) Using L'Hospital rule, solve the indeterminant form $\lim_{x\to 0}\left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$. (8)

(b) Using Rolle's theorem, prove that there is no real 'a' for which the equation $x^2 - 3x + a = 0$ has two different roots in [–1, 1]. (7)

4. (a) Test the convergence and divergence of the series :

$$\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \dots\dots\dots\infty,\quad x > 0.$$ (8)

(b) Find the radius of convergence and interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{n!}{n^n} x^n$. (7)

5. (a) Find the shortest and longest distance from the point (1, 2, –1) to the sphere $x^2 + y^2 + z^2 = 24$ using Lagrange's method of undetermined multipliers. (8)

(b) If $r^2 = x^2 + y^2 + z^2$, then prove that

$$\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} + \frac{\partial^2 r}{\partial z^2} = \frac{2}{r}.$$ (7)

6. (a) For what values of k, the equations $x + y + z = 1$, $2x + y + 4z = k$ and $4x + y + 10z = k^2$ have

(i) Unique solution

(ii) infinite number of solutions

(iii) no solution. (8)

(b) Verify the Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$ Also find A^{-1}. (7)

7. (a) Diagonalize the given matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}.$$ (8)

(b) Find the equation of the tangent plane and the normal to the surface of $z^2 = 4(1 + x^2 + y^2)$ at (2, 2, 6). (7)

December, 2019

B.TECH. (Civil Engg) - 1st Semester,

Mathematics-I (BSC103B)

Time : 3 Hours]

[Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART - A

1. (a) Prove that $\dfrac{B(m+1, n)}{B(m, n+1)} = \dfrac{m}{n}$. (1.5)

(b) Evaluate $\displaystyle\int_0^\infty \frac{dx}{a^2 + x^2}$, $a > 0$, if it exists. (1.5)

(c) Evaluate $\displaystyle\lim_{x \to \frac{\pi}{2}} (\cos x)^{\frac{\pi}{2} - x}$. (1.5)

(d) Test the convergence of the series

$$\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\text{.......}\infty$$ (1.5)

(e) If $z = f(x + ct) + g(x - ct)$, prove that $\frac{\partial^2 z}{\partial t^2}=\frac{\partial^2 z}{\partial x^2}$ (1.5)

(f) Find div $\vec{F}$ where $\vec{F}$ = grad $(x^3 + y^3 + z^3 - 3xyz)$ (1.5)

(g) Find the rank of the matrix.

$$A = \begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$$ (1.5)

(h) Let $V = R^2 = \{(a_1, a_2)a_1, a_2 \in R\}$. Test whether $W = \{(x, y) \mid 5x + 9y = 0; x, y \in R\}$ is its subspace or not ? (1.5)

(i) State Rank-Nullity Theorem of Vector Space. (1.5)

(j) Check the Linear Dependence or Independence of the vectors (2, –1, 4), (0, 1, 2), (6, –1, 16). (1.5)

PART - B

2. (a) Find the evolute of the rectangular hyperbola $xy = c^2$. (8)

(b) Using Mean Value Theorem, show that

$$x > \log_e (1 + x) > x - \frac{x^3}{2} \text{ if } x > 0.$$ (7)

3. (a) A cone circumscribed a sphere of radius r. Prove that when the volume of the cone is minimum, its height is $4r$ and semi-vertical angle is $\sin^{-1}\left(\frac{1}{3}\right)$. (8)

(b) Discuss the convergence of the series

$$x+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+\frac{4^4x^4}{4!}+\text{.......}\infty.$$ (7)

4. (a) Find the volume of largest rectangular parellelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{x^2}{c^2}=1.$ (8)

(b) Find the Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$. (7)

5. (a) Verify Stokes's Theorem for $\vec{F}=(x^2-y^2)\hat{i}+2xy\hat{j}$ over the box bounded by the planes $x = 0$, $x = a$; $y = 0$, $y = b$; $z = 0$ and $z = c$. (8)

(b) Find the volume of the tetrahedron bounded by the co-ordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1.$ (7)

6. (a) Using Rank method, Solve the linear system of equations $x + y + z = 8$; $x - y + 2z = 6$; $3x + 5y - 7z = 14$. (8)

(b) Find the basis and dimension of the subspace spanned by the vectors $(1, -3, -2)$, $(-3, 1, 3)$, $(-2, -10, 2)$ in R^3. (7)

7. (a) Find the Eigen values and eigen vectors of the matix

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}.$$ (8)

(b) Find an orthonormal basis of the inner product space $R^3(R)$ with the standard inner product, given the basis $B = \{(1, -1, 0), (1, 2, 1), (0, 1, 1)\}$ using Gram-Schmidt orthogonalisation process. (7)

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April 2022

B.Tech. (RAI/ME)-I SEMESTER

Mathematics-I (Calculus and Linear Algebra) (BSC-103A)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART-A

1. (a) Describe rank of a matrix A with numerical example. (1.5)

   (b) State Rolle's Theorem. (1.5)

   (c) Expand the function log x using Taylor series. (1.5)

   (d) What is relation between Beta and Gamma function. (1.5)

   (e) Find the radius of convergence of the series

   $$\sum_{n=0}^{\infty} \frac{n!}{n^n} x^n .$$ (1.5)

(f) Explain Fourier series of a function. (1.5)

(g) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ then find the value of

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}.$$ (1.5)

(h) Find the divergence of the vector $\vec{V} = xyz$. (1.5)

(i) Explain Eigenvalues and Eigenvectors of square matrix A. (1.5)

(j) What are the Eigenvalues of the Hermitian matrix. (1.5)

PART-B

2. (a) For what values of k, the equations

$$x + y + z = 1,\ \ 2x + y + 4z = k$$

and $4x + y + 10z = k^2$ have

(i) a unique solution,

(ii) infinite number of solutions,

(iii) no solution,

and solve them completely in each case of consistency. (7)

(b) If $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$,

then find the Eigen values of $A^2 - 2A + I$. (8)

3. (a) Find the extreme values of the function

$$f(x, y) = x^3 + y^3 - 12x - 3y + 20.$$ (7)

(b) Find a unit normal to the surface $xy^3z^2 = 4$, at the point $(-1, -1, 2)$. (8)

4. (a) Find the Fourier series for the function $f(x) = x^2$, $-\pi < x < \pi$. Hence, show that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ... = \frac{\pi^2}{6}.$$ (7)

(b) Test the convergence of the following series

(i) $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{-n^2}$.

(ii) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$. (8)

5. (a) What will be the value of c of Lagrange's mean value theorem for the function $f(x) = x^3 + x$ in $[1, 2]$. (7)

(b) Evaluate $\lim_{x \to \frac{\pi}{2}} (\sec x)^{(\cot x)}$. (8)

6. (a) Will the improper integral $\int_1^{\infty} \frac{\log x}{x^2}$ be convergent or not? (7)

(b) (i) Find the value of $\int_0^1 x^7 (1-x)^6 \, dx$.

(ii) What will be the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$ along the major axis. (8)

7. (a) Let $f: R^2 \to R$ be defined by setting

$$f(x,y)=\frac{xy}{\sqrt{(x^2+y^2)}},$$

when $(x, y) \neq (0, 0)$, $f(0, 0) = 0$

Show that f_x and f_y exist at (0, 0), also, check that the continuity of the function f at origin. (7)

(b) Find the equation of the evolute of the parabola $y^2 = 4ax$. (8)