

August/September-2022

Sr. No 015201

B.Tech.(ECE/ENC/EEIOT)- II SEMESTER

(Calculus, Ordinary Differential Equation and Complex Variable)(BSC-106D)

Time: 3 Hours

Instructions:

1. It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.
2. Answer any four questions from Part -B in detail.
3. Different sub-parts of a question are to be attempted adjacent to each other.

Max. Marks:75

PART-A

Que.1(a) Evaluate  $\int_0^1 \int_y^{y^3+1} x^2 y dx dy$

(b) Find the area lying between the parabola  $y = 4x - x^2$  and the line  $y = x$ . (1.5)

(c) Solve  $(xy^3 + y)dx + (2x^2y^2 + x + y^4)dy = 0$ . (1.5)

(d) Solve the differential equation:  $y = 2px + p^4x^2$  (solvable for y). (1.5)

(e) Solve  $(D^4 + 6D^2 + 9)y = 0$ , where  $D = d/dx$ . (1.5)

(f) Write the Bessel's differential equation of order n. (1.5)

(g) State C-R Equations. (1.5)

(h) Define conformal mapping. (1.5)

(i) State Cauchy's integral theorem and Cauchy's integral formula. (1.5)

(j) State Cauchy's Residue Theorem. (1.5)

PART-B

Que.2 (a) Change the order of integration in the given integral and then evaluate  $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy dx$

(b) Verify the Green's theorem in the plane for  $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ , where C is the boundary of the region defined by  $x = 0, y = 0, x + y = 1$ . (7)

Que.3 (a) Solve the differential equation  $(2y \sin x + 3y^4 \sin x \cos x)dx - (4y^3 \cos^2 x + \cos x)dy = 0$ . (8)

(b) Solve the differential equation:  $y = 2px + y^2 p^3$  (Solvable for x). (7)

Que.4 (a) Using variation of parameter, solve  $(D^2 - 6D + 9)y = \frac{e^{3x}}{x^2}$ , where  $D = d/dx$ . (8)

(b) Express  $4x^3 - 2x^2 - 3x + 8$  in terms of Legendre's polynomial. (7)

Que.5 (a) Show that the function  $u = e^{-2xy} \sin(x^2 - y^2)$  is harmonic. Find the conjugate function 'v' and express  $u + iv$  as an analytic function of z. (7)

(b) Under the transformation  $w = 1/z$ , find the image of the given curve:  $|z - 2i| = 2$ . (8)

Que.6 (a) Expand  $\frac{e^{2z}}{(z-1)^3}$  about the singularity  $z = 1$  in Laurent's series. (7)

(b) Evaluate  $\int_0^{2\pi} \frac{d\theta}{2 + \cos \theta}$  using Residue theorem. (8)

Que.7 (a) Find the volume bounded by the cylinder  $x^2 + y^2 = 4$  and the planes  $y + z = 4$  and  $z = 0$ . (7)

(b) Find the sum of the residues of the function  $f(z) = \frac{z \sin z}{z \cos z}$  at its poles inside the circle  $|z| = 2$ . (8)