Sheet No.1

NATIONAL INSTITUTE OF TECHNOLOGY, KURUKSHETRA

THEORY EXAMINATION

Question Paper

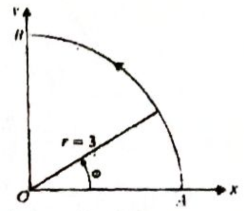
Roll No. ____________

Month and Year of Examination : **DEC. 2022**
Programme : **B.Tech** Semester: **3rd**
Subject: **Field and Waves**
Course No: **ECPC-31** Maximum Marks: **50**
Number of Questions to be Attempted: **5** Time Allowed: **3 Hrs.**
Total No. of Questions: **6** Total No. of Pages Used: **3**

Unless stated otherwise, the Symbols have their usual meanings in context with the Subject. Assume suitably and state, additional data required, if any.

The Candidates, before starting to write the solutions, should please check the Question Paper for any discrepancy, and also ensure that they have been delivered the question paper of **right course no.** and right subject title.

Note: Attempt any five questions. All questions carry equal marks.

1a.	Given $\mathbf{F} = \mathbf{a}_x xy - \mathbf{a}_y 2x$, evaluate the scalar line integral $\int_A^B \mathbf{F}.dl$ along the quarter circle shown in Fig. 1 using Cartesian and Cylindrical coordinate systems. Comment.  Fig. 1	2
1b.	Two infinitely long coaxial cylindrical surfaces, $r = a$ and $r = b$ $(b > a)$, carry surface charge densities ρ_{sa} and ρ_{sb}, respectively. i. Determine $\mathbf{E}$ everywhere. ii. What must be the relation between a and b in order that $\mathbf{E}$ vanishes for $r > b$	3
1c.	Determine the $\mathbf{E}$ field both inside and outside a spherical cloud of electrons with a uniform volume charge density $\rho = -\rho_0$ (where ρ_0 is a positive quantity) for $0 \le R \le b$ and $\rho = 0$ for $R > b$ by solving Poisson's and Laplace's equation for V.	5
2a.	Determine the capacitance per unit length of a two wire transmission line with parallel conducting cylinders of different radii a_1 and a_2, their axes being separated by a distance D (where $D > a_1 + a_2$).	3
2b.	Derive equation of continuity for electric charge. Explain its significance and consequences with the help of a suitable example.	3
2c.	A very long, thin conducting strip of width w lies in the xz-plane between $x = \pm w/2$. A surface current $J_s = \mathbf{a}_z J_{s0}$ flows in the strip. Find the magnetic flux density at an arbitrary point outside the strip.	4

NATIONAL INSTITUTE OF TECHNOLOGY, KURUKSHETRA

3a. A conducting circular loop of a radius $0.1m$ is situated in the neighborhood of a very long power line carrying a 60-Hz current as shown in Fig. 2 with $d = 0.15m$. An ac millimeter inserted in the loop reads $0.3mA$. Assume the total impedance of the loop including the milliammeter to be 0.01Ω.

i. Find the magnitude of the current in the power line

ii. To what angle about the horizontal axis should the circular loop be rotated in order to reduce the milliammeter reading to $0.2mA$.

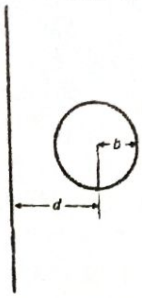


Fig. 2

6

3b. Describe briefly Faraday disk generator and the principle behind its working. Draw its neat labeled sketch and derive the expression for generated emf. 4

4a. Derive the general wave equations for $\mathbf{E}$ and $\mathbf{H}$ in a non conducting simple medium where a charge distribution ρ and a current distribution $\mathbf{J}$ exist. Convert the wave equations to Helmholtz's equations for sinusoidal time dependence. Write the general solutions for $\mathbf{E}(R,t)$ and $\mathbf{H}(R,t)$ in terms of ρ and $\mathbf{J}$. 5

4b. It is known that $\mathbf{E}$ and $\mathbf{B}$ can be determined from the potentials V and $\mathbf{A}$. The vector potential $\mathbf{A}$ was introduced through the relation $\mathbf{B} = \nabla \times \mathbf{A}$ because of the solenoidal nature of $\mathbf{B}$. In a source free region, $\nabla \cdot \mathbf{E} = 0$, we can define another type of vector potential $\mathbf{A}_e$, such that $\mathbf{E} = \nabla \times \mathbf{A}_e$. Assuming harmonic time dependence

i. Express $\mathbf{H}$ in terms of $\mathbf{A}_e$

ii. Show that $\mathbf{A}_e$ is a solution of a homogeneous Helmholtz's equation. 5

5a. For a harmonic, uniform plane wave propagating in a simple medium, both $\mathbf{E}$ and $\mathbf{H}$ vary in accordance with the factor $e^{-j\mathbf{k}\cdot\mathbf{R}}$. Show that the four Maxwell's equations for uniform plane wave in a source-free region reduce to the following:

i. $\mathbf{k} \times \mathbf{E} = \omega\mu\mathbf{H}$

ii. $\mathbf{k} \times \mathbf{H} = -\omega\varepsilon\mathbf{E}$

iii. $\mathbf{k} \cdot \mathbf{E} = 0$

iv. $\mathbf{k} \cdot \mathbf{H} = 0$

6

5b. State and prove Poynting theorem. Explain its significance with the help of a suitable numerical example. 4

OR

5a. Show that a plane wave with an instantaneous expression for the electric field $\mathbf{E}(z,t) = \mathbf{a}_x E_{10}\sin(\omega t - kz) + \mathbf{a}_y E_{20}\sin(\omega t - kz + \psi)$ is elliptically polarized. Find the polarization ellipse. 4

NATIONAL INSTITUTE OF TECHNOLOGY, KURUKSHETRA

5b. A uniform plane wave of angular frequency ω in medium 1 having a refractive index n_1 is incident on a plane interface at $z = 0$ with medium 2 having a refractive index n_2 $(< n_1)$ at the critical angle. Let E_{i0} and E_{t0} denote the amplitudes of the incident and refracted electric filed intensities, respectively.

i. Find the ratio E_{t0} / E_{i0} for perpendicular polarization.

ii. Find the ratio E_{t0} / E_{i0} for parallel polarization

iii. Write the instantaneous expressions of $\mathbf{E}_i(x, z; t)$ and $\mathbf{E}_t(x, z; t)$ for perpendicular polarization in terms of the parameters $\omega, n_1, n_2, \theta_i$ and E_{i0}. 6

6a. Demonstrate mathematically that the field quantities in free space manifest themselves as circuit quantities when bounded by infinite parallel conductors. 5

6b. Derive **TE** mode field expressions in a rectangular waveguide. Also give expressions for cut-off frequency and corresponding wavelength. Explain which mode is the dominant mode and reason thereof. 5

OR

6a. The characteristic impedance of a given lossless transmission line is $75\,\Omega$. Use a Smith chart to find the input impedance at 200 MHz of such a line that is

i. 1 m long and open circuited

ii. 0.8 m long and short circuited

iii. Determine the corresponding input admittances also. 5

6b. Write a short note on cavity resonator and explain its practical significance with the help of an example. 5