1E3101

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B.Tech. I Sem. (Main) Examination, April/May - 2022

1FY2-01 Engineering Mathematics-I

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

Attempt all ten questions From Part A, five Questions out of seven questions from Part B and three questions out of five questions from Part C .

Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.

Use of following supporting material is permitted during examination (As mentioned in form No. 205).

Part - A

(Answers should be given up to 25 words only)

All questions are compulsory. **(10×2=20)**

1. Define Beta function.

2. Write Euler's formula for a Fourier series.

3. Let $f = y^x$. What is $\frac{\partial^2 f}{\partial x \partial y}$ at $x=2, y=1$?

4. Consider a spatial curve in three-dimensional space given in parametric form by $x(t)=\cos t, y(t)=\sin t, z(t)=\frac{2}{\pi}t, 0 \le t \le \pi/2$. The length of the curve is......

5. In the Taylor series expansion of e^x about $x=2$, the coefficient of $(x-2)^4$ is ....

6. Define the convergence of a power series.

7. The Directional derivative of the scalar function $f(x,y,z)=x^2+2y^2+z$ at the point $P=(1,1,2)$ in the direction of the vector $\vec{a}=3\hat{i}-4\hat{j}$ is ......

8. Curl of vector $\vec{V}(x,y,z)=2x^2\hat{i}+3z^2\hat{j}+y^3\hat{k}$ at $x=y=z=1$ is ....

9. Velocity vector of a flow field is given as $\vec{V}(x,y,z)=2xy\hat{i}-3x^2z\hat{j}$. The vorticity vector at (1, 1, 1) is .....

10. The area enclosed between the curves $y^2=4x$ and $x^2=4y$ is ....

Part - B

(Analytical/Problem solving questions)

Attempt any five questions: **(5×4=20)**

1. Evaluate the following integrals:

i) $\int_0^{\infty} x^4 e^{-x^4}\, dx$

ii) $\int_0^{\pi/2} \sin^6\theta \cos^7\theta \, d\theta$.

2. The region in the first quadrant enclosed by the y-axis and the graphs of $y=\cos x$ and $y=\sin x$ is revolved about the x-axis to form a solid. Find its volume.

3. Test the convergence/divergence of the series.

$$\frac{1.2}{3^2.4^2}+\frac{3.4}{5^2.6^2}+\frac{5.6}{7^2.8^2}+....$$

4. Find the Fourier series expansion of the following periodic function with period 2π:

$$f(x)=\begin{cases}\pi+x, & if\ -\pi<x<0 \\ 0, & if\ 0\le x<\pi\end{cases}$$

5. Consider the function:

$f(x;y)=\sqrt{\dfrac{e^{\sin(x)}}{x^{2014}+\sqrt{x^{2012}}+1}}+\cos(xy)$. Find the second partial derivative $\dfrac{\partial}{\partial y}\left(\dfrac{\partial f}{\partial x}\right)$.

6. A scalar potential ϕ has the gradient $\nabla\phi=yz\hat{i}+xz\hat{j}+zy\hat{k}$. Evaluate the integral $\int_C \nabla\phi.d\vec{r}$ on the Curve $C:\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, if the curve C is parameterised as follows : $x=t, y=t^2, z=3t^2, 1\le t\le 3$.

7. Find the area of the region R in the xy-plane enclosed by the circle $x^2+y^2=4$, above the line y=1. and below the line $y=\sqrt{3x}$.

Part - C

(Descriptive/Analytical/Problem solving/Design Questions)

Attempt any three questions. **(3×10=30)**

1. If $f(x)=\begin{cases}\pi x, & 0<x<1\\ \pi(2-x), & 1<x<2\end{cases}$ using half range cosine series, show that $\frac{\pi^4}{96}=1+\frac{1}{3^4}+\frac{1}{5^4}+....$

2. Show that $div\left(grad\, r^n\right)=n(n+1)r^{n-2}$, where $r=\sqrt{x^2+y^2+z^2}$. Hence , show that $\nabla^2\left(\frac{1}{r}\right)=0$.

3. The pressure P at any point (x,y,z) in space is $P=400xyz^2$. Find the highest pressure at the surface of a unit sphere $x^2+y^2+z^2=1$.

4. Find the work done by a force $\overline{F}=\left(y^2\cos x+z^3\right)\hat{i}+\left(2y\sin x-4\right)\hat{j}+\left(3xz^2+z\right)\hat{k}$ in moving a particle from $P(0,1,-1)$ to $Q\left(\frac{\pi}{2},-1,2\right)$.

5. Apply stoke's theorem to find the value of $\int_C (ydx+zdy+xdz)$. Where C is the curve of intersection of $x^2+y^2+z^2=a^2$ and $x+z=a$.