Roll No. 24101303 1

Total Pages : 3

013101

December 2024

B.Tech (Mechanical Engineering) - I SEMESTER

MATHEMATICS - I **(Calculus and Linear Algebra)**

(BSC-103A)

Time: 3 Hours **Max. Marks:75**

Instructions:

1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) Find the volume of the solid generated by revolving the ellipse (1.5)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

a > b be the major axis.

(b) Evaluate $\int_0^\infty e^{-x} \sin x \, dx$, *if it exists.* (1.5)

(c) Evaluate (1.5)

$$\lim_{x \to 0} \frac{e^{ax} - e^{bx}}{x}$$

(d) State Rolle's Theorem. (1.5)

(e) State Parseval's Identity (1.5)

(f) Test the convergence of the following sequence: (1.5)

$$\{\frac{1}{3}, \frac{-2}{3^2}, \frac{3}{3^3}, \frac{-4}{3^4}, ----\}$$

(g) Prove that (1.5)

$$\nabla \times \nabla \emptyset = 0$$

Where $\emptyset$ is a scalar point function.

(h) If (1.5)

$$r^2 = x^2 + y^2 + z^2$$

Then prove that,

$$\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} + \frac{\partial^2 r}{\partial z^2} = \frac{2}{r}$$

(i) Find the eigenvalues of the following matrix: (1.5)

$$A = \begin{bmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{bmatrix}$$

(j) State Rank-Nullity Theorem. (1.5)

PART -B

Q2 (a) Find the Evolute of the rectangular hyperbola (8)

$$x\,y = c^2$$

(b) Prove that, (7)

$$\beta\,(m\,,n) = \frac{\Gamma m\,.\,\Gamma n}{\Gamma\,(m+n)} \quad , \quad m > 0\ , \quad n > 0$$

Q3 (a) Using Taylor's theorem, prove that (8)

$$x - \frac{x^3}{6} < \sin x < x - \frac{x^3}{6} + \frac{x^5}{120} \qquad for\, x > 0$$

(b) Using Mean Value Theorem, show that (7)

$$x \;> log_e\,(1+x) > x - \frac{x^2}{2} \qquad ;\ if\ \ x > 0$$

Q4 (a) Find the half range sine series for (8)

$$f(x) = x\,(\pi - x)$$

in the interval (0, π) and hence deduce that

$$\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + ------ = \frac{\pi}{32}$$

(b) Discuss the convergence of the p-series (7)

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \quad , \quad p > 0$$

Q5 (a) A rectangular box, open at the top, is to have a volume of 32 cc. Find dimensions of the box which requires least amount of material for its construction. (8)
(Use Lagrange's method of multiplier)

(b) Find the maximum and minimum value of the following function: (7)

$$\sin x\ \sin y\ \sin(x+y) \quad , \ 0 < x\,,y < \pi$$

Q6 (a) Diagonalise the matrix A by means of an orthogonal transformation: (8)

6

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

(b) Find all non-trivial solutions of the following system of linear equations: (7)

5

$$7x + y - 2z = 0$$
$$x + 5y - 4z = 0$$
$$3x - 2y + z = 0$$

Q7 (a) Prove that, (3)

$$\Gamma m . \Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma(2m)$$

(b) Find the maxima and minima of the function (3)

3

$$10x^6 - 24x^5 + 15x^4 - 40x^3 + 108$$

(c) Find the radius of convergence of the series:

$$\sum_{n=0}^{\infty} \frac{n!}{n^n} x^n$$ (3)

(d) Find the divergence and curl of the following vector at the point (2, -1, 1)

$$\vec{v} = xyz\,\vec{i} + 3x^2y\,\vec{j} + (xz^2 - y^2 z)\,\vec{k}$$ (3)

(e) Find the rank of the following matrix:

3

$$A = \begin{bmatrix} 1 & 2 & -1 & -5 & 4 \\ 1 & 3 & -2 & -7 & 5 \\ 2 & -1 & 3 & 0 & 3 \end{bmatrix}$$ (3)