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Roll No. 21001015064 Total Pages : 4

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B.Tech. (ECE/FAE/ECO/EEE) 1st SEMESTER

Mathematics-I

(Calculus and Linear Algebra) (BSC-103D)

Time : 3 Hours] [Max. Marks : 75

Instructions:

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART-A

1. (a) Evaluate $\int x^2 \sin 2x dx$.

(b) Find the equation of the tangent to the curve $y = x^2 + 2$ at $x = 3$.

(c) Verify Rolle's theorem for $f(x) = x^3 - 6x^2 + 11x - 6$ in [1, 3].

(d) Using L'Hospital rule, solve the indeterminant form

$$\lim_{x \to 0}\left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$$

(e) Define Even and odd functions. Also give two example of each.

(f) Expand log $(1 + x)$ using Maclaurin's series for one variable upto third degree

(g) Show that $\lim_{x \to \infty} (x^2 - x^2) = \infty$

(h) Find dy/dx given that $y = x^2 + \log \sin x$

(i) Find the rank of the given matrix

$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 3 & 4 \\ 0 & 5 & 7 \end{bmatrix}$$

(j) Find the sum and product of the eigen values of the given matrix $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$. (1.5×10=15)

PART-B

2. (a) Find the center of curvature of the parabola $y^2 = 4ax$ at the point (x, y). Also find its evolute. (8)

(b) State and Prove relation between Beta and Gamma functions. (7)

3. (a) Using Taylor's series expansion, expand $\tan^{-1} x$ in powers of $(x - 1)$ upto four terms. (8)

(b) Find the maximum and minimum values of $f(x) = \sin 2x + 5$. (7)

4. (a) Discuss the convergence of the given series :

$$\frac{x}{1}+\frac{1}{2}\cdot\frac{x^3}{3}+\frac{1.3}{2.4}\cdot\frac{x^5}{5}+\frac{1.3.5}{2.4.6}\cdot\frac{x^7}{7}+\ldots\ldots\ldots \quad (8)$$

(b) Find the fourier series expansion for the function $f(x) = x - x^3$ in the interval $-1 < x < 1$. (7)

5. (a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}. \quad (8)$$

(b) Prove that the rectangular solid of maximum volume which can be inscribed in a sphere is a cube. (7)

6. (a) Check the consistency of the given system of linear equation :

$x + y + z = -3,\ 3x + y - 2z = -2,\ 2x + 5y + 7z = 7.$

(8)

(b) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}.$$ Also find A^{-1}. (7)

7. (a) Diagonalize the given matrix :

$$A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}, \text{ also find } A^4. \quad (8)$$

(b) Test the convergence and absolute convergence :

$$1-\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}-\frac{1}{4\sqrt{4}}+\ldots\ldots.. \quad (7)$$