Reg.No. [redacted]

24189

Velammal College of Engineering and Technology

Viraganoor, Madurai – 625 009

(Autonomous)

B.E./B.Tech. End Semester Examinations November 2024

Third Semester **Regulation 2021**

Time : 3 Hours **Max. Marks 100**

21MA203 – Discrete Mathematics

(Common to CSE and IT and AI & DS branches)

Answer ALL Questions

PART-A (10 x 2 = 20 Marks)

1. Determine whether $(Q \rightarrow P) \wedge (\neg P \wedge Q)$ is a tautology or contradiction.
2. Write the following statement in symbolic form: If Avinash is not in a good mood or he is not busy, then he will go to Delhi.
3. In how many ways can letters of the word "MATHEMATICAL" be arranged?
4. State Pigeonhole principle.
5. Define Pseudo graph and give an example.
6. Draw the graph of the adjacency matrix $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$
7. Prove that in any group, identity element is the only idempotent element.
8. Define kernel of a homomorphism.
9. Draw the Hasse diagram of where X={2,4,5,10,12,20,25}and the relation $\leq$ be such that $x \leq y$ if x divides y.
10. Prove that in a Boolean algebra $(a')' = a$

Part – B (5 x 16 = 80 marks)

11. a) (i) Prove that $P \rightarrow (Q \rightarrow R) \Rightarrow (P \rightarrow Q) \rightarrow (P \rightarrow R)$. (8 Marks)

(ii) Obtain PDNF and PCNF for the formula $(\neg p \rightarrow r) \wedge (q \leftrightarrow p)$ without using truth table. (8 Marks)

OR

b) (i) Construct an argument to show that the following premises imply the conclusion "It rained". "If it does not rain or if there is no traffic dislocation, then the sports day will be held and the cultural programme will go on". "If the sports day is held, the trophy will be awarded" and "The trophy was not awarded". (8 Marks)

(ii) Prove the following implication:

$\forall x (P(x) \rightarrow (Q(x)), \forall x (R(x) \rightarrow \neg Q(x)) \Rightarrow \forall x (R(x) \rightarrow \neg P(x))$. (8 Marks)

12. a) (i) Prove that $n^3 + 2n$ is divisible by 3 for all $n \geq 1$ by mathematical induction. (8 Marks)

(ii) Among the first 1000 positive integers, determine the integers which are divisible by 5, 7 and 9 (8 Marks)

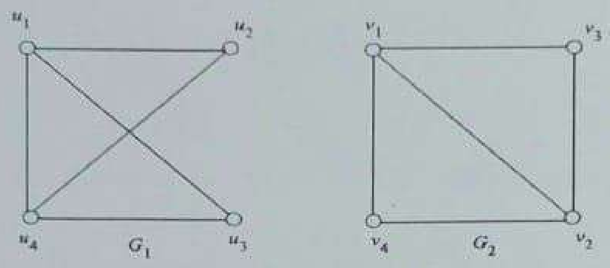
OR

b) (i) Solve the recurrence relation $a_{n+2} - 3a_{n+1} + 2a_n = 0$ by the method of generating function with the initial conditions $a_0 = 2, a_1 = 3$. (8 Marks)

(ii) How many permutations can be made out of the letters of the word 'COMPUTER'. How many of these (1) Begin with C, (2) End with R, (3) Begin with C and end with R, (4) C and R occupy the end places. (8 Marks)

13. a) (i) Prove that the maximum number of edges in a complete graph with 'n' vertices is $\frac{n(n-1)}{2}$. (8 Marks)

(ii) Show that the following graphs are isomorphic by considering their adjacency matrices. (8 Marks)



OR

b) (i) If G is a graph with n vertices and k components, then G has atmost $\frac{(n-k)(n-k+1)}{2}$ edges. (8 Marks)

(ii) Give an example for the following which is

- Eulerian but not Hamiltonian.
- Hamiltonian but not Eulerian.
- Both Hamiltonian and Eulerian. (8 Marks)

14. a) (i) If a and b are any two elements of a group $(G, *)$, then show that G is an abelian group if and only if $(a * b)^2 = a^2 * b^2$. (8 Marks)

(ii) A non-empty subset H of a group G is a subgroup of G if and only if $a, b \in H \Rightarrow a * b^{-1} \in H$. (8 Marks)

OR

b) State and prove Lagrange's theorem

15. a) (i) State and prove De morgans law in a lattice.

(ii) Prove that every chain is a Distributiva lattice.

OR

b) (i) In a Boolean algebra, prove that $a\bar{b} + b\bar{c} + c\bar{a} = \bar{a}b + \bar{b}c + \bar{c}a$ (8 Marks)

(ii) In a Boolean algebra, prove that following are equivalent.

(1) $a + b = b$, (2) $a \cdot b = a$, (3) $a' + b = 1$, (4) $a \cdot b' = 0$. (8 Marks)