8. (a) Express β function in terms of Γ function. Hence find the value of $\beta\left(3, \frac{-1}{2}\right)$. [4]

(b) A tank contains 500 gal of brine in which 150 lb of salt are dissolved. Fresh water runs into the tank at a rate of 5 gal/min. The mixture, kept practically uniform by stirring, runs out at the same rate. How much salt will there be in the tank at the end of 2 hours? [4]

*****

Qn. Set Code-1

Semester 1st
Programme B Tech
Branch All Branches

AUTUMN END SEMESTER EXAMINATION-2023

1st Semester B.Tech

DIFFERENTIAL EQUATIONS AND LINEAR ALGEBRA

MA11001

(For 2023 & Previous Admitted Batches)

Time: 3 Hours Full Marks: 50

Answer any SIX questions.
Question paper consists of four SECTIONS i.e. A, B, C and D.
Section A is compulsory.
Attempt minimum one question each from Sections B, C, D.
The figures in the margin indicate full marks.
All parts of a question should be answered at one place only

SECTION-A

1. Answer the following questions: [1 × 10]

(a) Find a second-order differential equation for the given basis of solutions $\{1, e^{4x}\}$.

(b) If $A = \begin{pmatrix} 2 & 3 \\ 0 & -1 \end{pmatrix}$, then find all eigenvalues of the matrix $A^2 - A$.

(c) Obtain the general solution to $x^2y'' + xy' - 4y = 0$.

(d) Is the following set of vectors linearly independent or dependent?

$$\{[2 \quad -3 \quad 7], [-2 \quad 1 \quad -3], [-2 \quad -1 \quad 1]\}$$

(e) Write the nature of the eigenvalues of Hermitian and skew-Hermitian matrices.

(f) The decay rate of the number of insects at any time t is proportional to the number present at that time and becomes one-third in 5 days. Write the mathematical model for this physical problem.

(g) Find the symmetric matrix from the quadratic form

$$Q = (x_1 + 2x_2)^2 + 3x_1x_3$$

(h) Find the differential equation corresponding to the orthogonal trajectories for the family of curves

$$x = \sqrt{y-k}\ , k \text{ is a constant.}$$

(i) Evaluate $\int_0^{\infty} \sqrt{x}e^{-x}\, dx$.

(j) Calculate the radius of convergence of the series

$$\sum_{m=0}^{\infty} \frac{(-1)^m}{k^m}(x+3)^{2m}\ , k \neq 0$$

SECTION-B

2. (a) Solve the initial value problem [4]

$$2xyy' - 3y^2 = x^2,\ \ y(1) = 2$$

(b) Find the general solution to the ordinary differential equation [4]

$$y'' + 7y' + 12y = \frac{1}{3}e^{-4x}$$

3. (a) Diagonalize the matrix [4]

$$A = \begin{bmatrix} 3 & 0 \\ 7 & -2 \end{bmatrix}.$$

(b) Using reduction of order, find the second independent solution to the equation $x^2y'' - 4xy' + 6y = 0$ using a known solution $y_1 = x^2$ and hence solve the initial value problem for $y(1) = 3, y'(1) = 5$. [4]

SECTION-C

4. (a) Find the transient current in the RLC circuit with [4]

$R = 6\,\Omega,\ L = 0.2$ H, $C = 0.025$ F,
E $= 110\ sin10t\ volts.$

(b) Test for exactness. If not exact, use an integrating factor and hence solve the ordinary differential equation [4]

$$(xe^{x+y} + 1)dy + (e^{x+y} - y)dx = 0.$$

5. (a) Test the orthogonality of the matrix $\begin{bmatrix} 0.96 & -0.28 \\ 0.28 & 0.96 \end{bmatrix}$ and find its eigenvalues. [4]

(b) Find eigenvalues and eigenvectors of the following matrix: [4]

$$A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & -2 \\ 0 & 0 & 2 \end{bmatrix}$$

6. (a) If $A = \begin{bmatrix} 1 & 0 & 3 \\ 4 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$, then find the inverse of the matrix using Gauss-Jordan method. [4]

(b) Use the formula of Legendre's polynomial $P_n(x)$ to show $P_3(x) = \frac{1}{2}(5x^3 - 3x)$. [4]

SECTION-D

7. (a) Using the variation of parameter method, find the solution to the differential equation [4]

$$(x^2D^2 + xD - 0.25I)y = 3x$$

(b) Solve the system of equations by Gauss elimination method: [4]

$$3x - y = 1,$$

$$2x + y + z = 2, \text{ and}$$

$$4y + 3z = 2.$$