MA 3251 – STATISTICS AND NUMERICAL METHODS

(Common to : All Branches (Except Marine Engineering))

(Regulations 2021)

Time : Three hours Maximum : 100 marks

(Statistical table need to be provided)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Define Type I error and Type II error in testing of hypothesis.

2. State the uses of Chi-square test.

3. Define the terms experimental and extraneous variables in design of experiments.

4. Complete the following ANOVA table

Sources of variation	Sum of squares	Degrees of freedom	Mean squares	F-ratio
Between brands	45,226	3	_____	_____
Within brands	_____	22	6,811	
	1,95,068	_____		

5. What is need for pivoting in Gauss elimination method?

6. State the advantage of Gauss Seidel method over Gauss Jacobi method while solving system of linear equations.

7. List the merits and demerits of Lagrange interpolation method.

8. Interpret geometrically Trapezoidal rule.

9. Why Euler method is known as point – slope method?

10. Give an example for single step and multi-step method for solving ordinary differential equations.

PART B — (5 × 16 = 80 marks)

11. (a) (i) The dynamic modulus of concrete is obtained for two different concrete mixes. For the first mix, $n_1 = 33$, $\bar{x}_1 = 115.1$, and $s_1 = 0.47$ psi. For the second mix, $n_2 = 31$, $\bar{x}_2 = 114.6$ and $s_2 = 0.38$. Test, with $\alpha = 0.05$, the null hypothesis of equality of mean dynamic modulus versus the two-sided alternative. (8)

(ii) In an air-pollution study performed at an experiment station, the following amount of suspended benzene soluble organic matter (in micrograms per cubic meter) was obtained for eight different samples of air: 2.2, 1.8, 3.1, 2.0, 2.4, 2.0. 2.1, 1.2. Assuming that the population sampled is normal. Test the null hypothesis $\mu = 2$ against the alternative hypothesis $\mu > 2$ at the 0.05 level of significance. (8)

Or

(b) (i) A manufacturer of machine bearings claims that 90% of the heavy machine bearings have a work life of more than 5 years. You doubt this claim and want to refute it on the basis of a sample of 200 bearings where 170 did work for more than 5 years. Conduct a test of hypotheses using $\alpha = 0.10$. (8)

(ii) Mechanical engineers, testing a new arc-welding technique, classified welds both with respect to appearance and an X-ray inspection. (8)

		Appearance		
		Bad	Normal	Good
X-ray	Bad	20	7	3
	Normal	13	51	16
	Good	7	12	21

Using Chi-square statistic, test for independence using $\alpha = 0.05$.

2. (a) Four different, though supposedly equivalent, forms of a standardized reading achievement test were given to each of 5 students, and the following are the scores which they obtained:

	Student 1	Student 2	Student 3	Student 4	Student 5
Form A	75	73	59	69	84
Form B	83	72	56	70	92
Form C	86	61	53	72	88
Form D	73	67	62	79	95

Treating students as blocks, perform an analysis of variance to test at the level of significance $\alpha = 0.01$ whether it is reasonable to treat the 4 forms as equivalent. (16)

Or

(b) Analyse the variance in the following Latin square of yields (in kgs) of paddy where A, B, C, D denote the different methods of cultivation :

D:122	A:121	C:123	B:122
B:124	C:123	A:122	D:125
A:120	B:119	D:120	C:121
C:122	D:123	B:121	A:122

Examine whether the different methods of cultivation have given significantly different yields at 5% L.O.S. (16)

3. (a) (i) Using Newton-Raphson method, find a root of the equation $e^x - x^3 - \cos 25x = 0$ nearer to $x = 4.5$ (correct to three decimal places). (8)

(ii) Apply Gauss elimination method, to solve (8)

$2x - y + 3z = 9;\ x + y + z = 6;\ x - y + z = 2.$

Or

(b) (i) Apply Gauss-Seidel method to solve equations $27x + 6y - z = 85;$ $x + y + 54z = 110;\ 6x + 15y + 2z = 72.$ (Perform 5 iterations) (8)

(ii) Using power method, find the largest eigenvalue and the corresponding eigenvector of the Matrix $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$

Take $[1, 0, 0]^T$ as the initial eigenvector. (8)

14. (a) (i) The following table gives the velocity v of a particle at time t. Find its velocities when t = 3, t = 11. Also determine the acceleration at t = 0. (8)

t	0	2	4	6	8	10	12
v	4	6	16	34	60	94	136

(ii) Apply Lagrange's method, to find the value of $f(x)$ at $x = 9$ from the given data. (8)

x	5	7	11	13	17
$f(x)$	150	392	1452	2366	5202

Or

(b) (i) The highway patrol uses a radar gun to clock the speed of a motorist. The gun is equipped with a device that records the speed at 4-second intervals as given in the table below.

Time	0	4	8	12	16	20	24	28	32	36	40
Speed	64	68	71	74	76	72	64	63	68	73	72

Apply both Trapezoidal and Simson's 1/3 rule, to find the total distance traveled by the car. (8)

(ii) Evaluate $\int_0^1 \int_0^1 \frac{dxdy}{x+y+1}$, by using Trapezoidal rule and taking $h = 0.5$, $k = 0.25$. (8)

15. (a) (i) Given $\frac{dy}{dx} = x^2 + y^2$, $y(1) = 2$, find $y(1.1)$ using Taylor's series method of the fourth order, $y(1.2)$ using Euler's method. (8)

(ii) Solve $\frac{dy}{dx} = \frac{1}{x+y}$, $y(0) = 1$, find $y(0.1)$ using Runge–Kutta method of fourth order. (8)

Or

(b) Given $\frac{dy}{dx} = x - y^2$, $y(0) = 0$, find $y(0.2)$, $y(0.4)$, $y(0.6)$ using Taylor's series method of the third order. Also find $y(0.8)$ using Milne's method. (16)