23207

Velammal College of Engineering and Technology
Viraganoor, Madurai – 625 009
(Autonomous)

B.E. End Semester Examinations November 2023

First Semester
Time : 3 Hours

Regulation 2021
Max. Marks 100

21MA101 - Matrices and Calculus

(Common to All Branches)

Answer ALL Questions
PART-A (10 x 2 = 20 Marks)

1. Find the Sum and Product of the Eigen values of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.

2. Find the nature of the quadratic form $f(x_1, x_2, x_3) = 2x_1^2 - x_2^2$.

3. Interpret the given function $f(x) = x^2$ is an even or odd.

4. Demonstrate $\dfrac{dy}{dx}$ if $x^2 + y^2 = 16$

5. Demonstrate the Jacobian $\dfrac{\partial(u, v)}{\partial(x, y)}$ if $u = \dfrac{2x - y}{2}, v = \dfrac{y}{2}$

6. Compute $\dfrac{du}{dt}$ if given $u = x^2 + y^2, x = at^2, y = 2at$

7. Evaluate $\int (e^{2x} + 3x - 7)dx.$.

8. Demonstrate the value of $\displaystyle\int_{-10}^{10} x^2 \sin x\, dx$.

9. Evaluate $\displaystyle\int_{1}^{a}\int_{1}^{b} xy\, dx\, dy.$

10. Evaluate $\displaystyle\int_{0}^{\pi}\int_{0}^{\cos\theta} r\, dr\, d\theta.$

Part – B (5 x 16 = 80)

11. a) Apply Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$, find A^{-1} and A^4.

OR

b) Reduce the quadratic form $x_1^2 + x_2^2 + x_3^2 - 2x_1x_2 - 2x_2x_3 - 2x_1x_3$ to canonical form through an orthogonal transformation and also find rank. Index, signature and nature.

12. a) (i) Compute the domain, range for $f(x) = \sqrt{5x+10}$

   (ii) Compute the infinite limit value of $\lim_{x \to -3^+} \frac{x+2}{x+3}$.   (8 + 8 Marks)

OR

b) (i) Demonstrate $\frac{dy}{dx}$ if $y = (\sin x)^x$.   (8 + 8 Marks)

   (ii) Obtain the local maximum and minimum value of the function $f(x) = x + 2\sin x$ in $[0, 2\pi]$

13. a) (i) Apply Taylor's series, expand $e^x \cos y$ in powers of x and y up to second degree.

   (ii) A rectangular box, open at the top, is to have a volume of 32cc. Make use of the information, find the dimensions of the box that requires the least material for its construction.   (8 + 8 Marks)

OR

b) (i) If $z = f(x, y)$, where $x = u^2 - v^2$ and $y = 2uv$, Develop that

$$\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} = 4(u^2 + v^2)\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right).$$

   (ii) Obtain the absolute maximum and minimum value of the function
$f(x) = 3x^4 - 4x^3 - 12x^2 + 1$ in $[-2, 3]$ (8 + 8 Marks)

14. a) (i) Apply the integral property, evaluate $\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx$

   (ii) Compute $\int e^{ax} \sin bx\, dx$. by using integration by parts.   (8 + 8 Marks)

OR

b) Apply the integral concepts, evaluate $\int \tan^{-1} x\, dx$ and hence deduce the value of $\int_0^1 \tan^{-1} x\, dx$.

15. a) (i) Apply the concept of Change the order of integration, evaluate $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy\,dy\,dx$.

   (ii) Calculate the area between the parabolas $x^2 = 4ay$ and $y^2 = 4ax$ (8 + 8 Marks)

OR

b) Apply triple integral concept, find the volume of the ellipsoid.