AUTUMN END SEMESTER EXAMINATION-2018

1st Semester B.Tech

MATHEMATICS-I

MA-1003

[For 2018 Admitted Batch]

Time: 3 Hours Full Marks: 50

Answer any SIX questions including question No.1 which is compulsory.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

1. (a) Find the wronskian of the functions $\cos 5x$, $sin5x$. [1 × 10]

   (b) Find the general solution of $x^2y'' - 2y = 0$.

   (c) Solve $yy' + 36x = 0$.

   (d) Find an Integrating factor of $(x^2 + y^2)dx - 2xy\,dy = 0$.

   (e) Are the given vectors[1 0 1], [1 1 0], [0 1 0] Linearly independent?

   (f) Is the set of vectors in R^2 where $v_1 \leq v_2$ is a vector space? Justify your answer.

   (g) Find the Algebraic multiplicity and Geometric multiplicity of $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.

   (h) Expand $\sin x$ in power of $\left(x - \frac{\pi}{2}\right)$ using Taylor's series method.

(i) Find the appropriate form of particular solution of $y'' - 2y' + y = e^x$ by method of undermined co-efficient method.

(j) Using Euler' s method find $y(0.1)$ taking $h = 0.1$ from

$y' = 1 + xy,\ y(0) = 2.$

2. (a) A thermometer, reading 10°C, is brought into a room whose temperature is 23°C. Two minutes later the temperature reading is 18°C. How long will it take until the reading is practically 23°C, say, 22.8°C ? [4]

(b) Find the basis of solutions of the ODE by reducing to first order [4]

$(x^2 - x)y'' - xy' + y = 0$, where $y_1 = x$.

3. (a) Solve $(x^2D^2 - 4xD + 6I)y = 21x^{-4}$. [4]

(b) Solve the ODE $xy' = y + 2x^3 sin^2\left(\frac{y}{x}\right)$. [4]

4. (a) Find the eigen values and the corresponding eigenvectors of $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$. [4]

(b) Diagonalize the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$. [4]

5. (a) Solve $2xyy' + (x-1)y^2 = x^2e^x$. [4]

(b) Apply Picard's iteration method to solve [4]

$y' = x + y,\ y(0) = 0.$

Determine up to four steps.

6. (a) Find the inverse of the given matrix using Gauss-Jordan elimination method: $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 4 & 1 \end{bmatrix}$. [4]

(b) Using Gauss-Seidel Iteration method solve the system of equations [4]

$$5x + y + 2z = 19$$

$$x + 4y - 2z = -2$$

$$2x + 3y + 8z = 39$$

taking initial approximations as x = y = z =1 and proceed upto 3-steps.

7. (a) Find the rank of the matrix $A = \begin{bmatrix} 6 & 0 & -30 \\ 0 & -1 & 0\ 5 \\ 2 & 0 & -10 \end{bmatrix}$ [4]

(b) Find the maxima and minima of [4]

$$f(x,y) = 2x^3 + 6xy^2 - 3y^3 - 150x.$$

8. (a) Find out the type of conic section of the Quadratic form $x_1^2 - 12x_1x_2 + x_2^2 = 70$ and transform it to principal axes. [4]

(b) Solve $y' = 1 - y, y(0) = 0$ by Modified Euler method and obtain y correct up to 3 places of decimal at $x = 0.1$. [4]

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