1E3101

Roll No. ____________ Total No. of Pages: 3

1E3101

B. Tech. I - Sem. (Main / Back) Exam., - 2023

1FY2 – 01 Engineering Mathematics - I

Time: 3 Hours **Maximum Marks: 70**

Instructions to Candidates:

Attempt all ten questions from Part A, five questions out of seven questions from Part B and three questions out of five from Part C.

Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used /calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. NIL ________________ 2. NIL ________________

PART – A [10×2=20]

(Answer should be given up to 25 words only)

All questions are compulsory

Q.1 Find the limit of the sequence $<x_n>$, where $x_n = \frac{5n-3}{7n+8}$.

Q.2 Write the power series expansion of logarithm function.

Q.3 Evaluate a_n in the Fourier series of the function $f(x) = x + x^2, -\pi < x < \pi$.

Q.4 Define Cauchy's $(E - \delta)$ definition of continuity.

Q.5 Write Euler's theorem on homogeneous function.

Q.6 Evaluate: $\int_0^\infty x^6 e^{-2x} dx$ by using beta – gamma function.

Q.7 Evaluate: $\iint xy\,dx\,dy$, where the region of integration is $x + y < 1$ in the positive quadrant.

Q.8 Change the order of integration of the following double integration:

$$\int_0^4 \int_x^{2\sqrt{x}} f(x,y)\,dx\,dy$$

Q.9 If $\vec{f} = x^2y\hat{\imath} - 2xy^2z\,\hat{\jmath} + 3x^2z\hat{k}$, find div f at the point (3, –1, –2).

Q.10 State Stokes theorem.

PART – B

[5×4=20]

(Analytical/Problem solving questions)

Attempt any five questions

Q.1 Prove that –

$$B(m,n) = \frac{\Gamma m\,\Gamma n}{\Gamma(m+n)}$$

Q.2 Test the convergence of the following series –

$$\frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + \text{-------}$$

Q.3 Find a Fourier series for the function $f(x) = x^2$ in the interval $-\pi < x < \pi$ and deduce the following: $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$

Q.4 Find the equations of the tangent plane and normal to the surface –

$x^3 + y^3 + 3xyz = 3$ at the point (1, 2, –1).

Q.5 Evaluate the point where the function –

$x^3y^2(1 - x - y)$

Will have maxima. Also find the maximum value.

Q.6 Evaluate the integral –

$$\int_0^1 \int_0^x \frac{x^3\,dx\,dy}{\sqrt{x^2+y^2}}$$

by changing into polar coordinates.

Q.7 If $\vec{a}$ and $\vec{b}$ are differentiable vector point functions, then prove that –

$$\text{div}\left(\vec{a} + \vec{b}\right) = \vec{b}.\text{curl}\,\vec{a} - \vec{a}.\text{curl}\,\vec{b}$$

PART – C [3×10=30]

(Descriptive/Analytical/Problem Solving/Design Questions)

Attempt any three questions

Q.1 Find the volume of spindle shaped solid generated by revolving the Astroid about the x – axis –

$$x^{2/3} + y^{2/3} = a^{2/3}$$

Q.2 If u = log $x^3 + y^3 + z^3 - 3xyz$, then prove that –

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$$

Q.3 Find half range cosine series for the function –

$f(x) = 2x - 1, 0 < x < 1$

hence deduce that –

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \text{-------}$$

Q.4 Find the volume of the tetrahedron bounded by the co – ordinate planes and the plane –

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Q.5 State Gauss's divergence theorem. Verify Gauss's divergence theorem for $\vec{F} = xy\,\hat{i} + z^2\hat{j} + 2yz\hat{k}$ on the tetrahedron $x = y = z = 0$ and $x + y + z = 1$.
