SPRING MID SEMETER EXAMINATION-2023
Subject: Probability and Statistics
Code: MA-2011

B.Tech
4th Semester (Regula
SAS-2023

Full Marks: 20   **Time: 1.5 Hrs**

Answer any FOUR QUESTIONS including question No. 1 which is
compulsory. The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable and all
parts of a question should be answered at one place only

---

1. Answer the following questions   [5×1=5]

   (a) Explain the random variable with an example.

   (b) If $A_1, A_2, \cdots$ are mutually exclusive events with values $P(A_k) = \frac{m}{k}$ for some m. These values represent the legitimate probability values or not. Explain.

   (c) If the random variable X has the pmf $p(x) = kx^2, (x = 1, 2, 3, 4, 5)$, then what is $P(Y \geq 3)$ where $Y = 4X - 5$.

   (d) If the random variable X has a geometric distribution process with probability of getting the success is $p = 0.35$. What is the variance of $Y = -2X + 1$?

   (e) There are n candidates facing a interview for a job. The random variable X counts the rank of the candidates from 1 to n. If X has a uniform distribution with variance 4, then compute n.

2. A certain system can experience three different type defects. Let A_i $(i = 1,2,3)$ denote the event that the system has a defect of type i. Suppose that   [1.5+1.5+2=5]

$P(A_1) = 0.12$	$P(A_2) = 0.07$	$P(A_3) = 0.05$
$P(A_1 \cup A_2) = 0.13$	$P(A_1 \cup A_3) = 0.14$	$P(A_2 \cup A_3) = 0.10$
$P(A_1 \cup A_2 \cup A_3) = 0.16$		

   (a). What is the probability that the system have all type of defects?

   (b). What is the probability that the system has both type 2 and type 3 defects but not a type 1 defect?

   (c). Given that the system does not have a type 1 defect, what is the probability that the system has both type 2 and type 3 defects?

3. Consider the system of components connected as in the accompanying picture. Components 1 and 2 are connected in parallel, so that subsystem works iff either 1 or 2 works; since 3 and 4 are connected in series, that subsystem works iff both 3 and 4 work. If components work independently of one another and P(component works)= p.   [3+2=5]
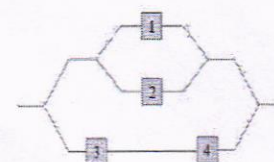
   (a). Calculate the value of p if P(system works) $= 0.99$.

   (b). Given that the component 2 doesn't work, what is the probability that the system works?

4. (a) A consumer organization that evaluates ne automobiles customarily reports the number of major defects in each car examined. Let X denote the number of major defects in a randomly selected car of a certain type. The cdf of X is as follows: [3+2=5]

   Compute mean and variance of X.

$$F(x) = \begin{cases} 0 & x < 0 \\ .06 & 0 \leq x < 1 \\ .19 & 1 \leq x < 2 \\ .39 & 2 \leq x < 3 \\ .67 & 3 \leq x < 4 \\ .92 & 4 \leq x < 5 \\ .97 & 5 \leq x < 6 \\ 1 & 6 \leq x \end{cases}$$

   (b) The rv X has Poisson distribution with parameter $\theta = 3$. What is the value of $P(X \geq 2\sigma)$

5. Find the mean and variance of the random variable $X \sim Bin(n, p)$.   [5]

****