8. (a) If $P_m(x)$ is the Legendre polynomial of degree m, then show that $(m+1)P_{m+1}(x) = (2m+1)xP_m(x) - mP_{m-1}(x)$. [4]

(b) Test the consistency and hence solve the system [4]

$$3x + 2y = 5$$
$$2x - z = 2$$
$$4y + 5z = 8$$

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AUTUMN END SEMESTER EXAMINATION-2022

1st Semester B.Tech

DIFFERENTIAL EQUATION AND LINEAR ALGEBRA

MA11001

(For 2022 Admitted Batch)

Time: 3 Hours Full Marks: 50

Answer any SIX questions.
Question paper consists of four SECTIONS i.e. A, B, C and D.
Section A is compulsory.
Attempt minimum one question each from Sections B, C, D.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

SECTION-A

1. Answer the following questions. [1 × 10]

(a) If the growth rate of the number of bacteria at any time t is proportional to the number present at t and becomes 1.5 times in 1 week. Write the mathematical modeling of this physical problem.

(b) Find a general solution of $x^2y'' - 2y = 0$.

(c) Find a 2nd order homogeneous ordinary differential equation for the given basis of solutions $\{e^{4x},\ xe^{4x}\}$.

(d) Is the given set of vectors linearly independent?

$\{[0.4 \quad -0.2 \quad 0.2], [0.0 \quad 0.0 \quad 0.0], [3.0 \quad -0.6 \quad 1.5]\}$

(e) Find the coefficient matrix from the system $y_1 = 2x_1 + 3x_2,\ y_2 = -x_1 + 4x_2$.

(f) Find the value of $\int_0^\infty x^{-\frac{1}{2}} e^{-2x}\, dx$.

(g) What is the radius of convergence for the power series $\sum_{m=0}^{\infty} \frac{(-1)^m}{3^m}(x-2)^{2m}$.

(h) Find the differential equation of the orthogonal trajectories to the family of curves $y = \sqrt{x+c}$. c =constant.

(i) What is the absolute value of the eigen value of a unitary matrix?

(j) If $A = \begin{pmatrix} -2 & 1 \\ 0 & 3 \end{pmatrix}$, then find all eigen values of the matrix A^3.

SECTION-B

2. (a) Find the second independent solution to the ordinary differential equation $x^2y'' - xy' + y = 0$ using a known solution $y_1 = x$ and hence solve the initial value problem for $y(1) = 2, y'(1) = 3$. [4]

(b) Solve the initial value problem [4]

$2xyy' + (x-1)y^2 = x^2e^x, y(1) = 0.$

3. (a) Find the general solution to the ordinary differential equation $y'' + 5y' + 4y = \frac{1}{2}e^{-4x}$. [4]

(b) Find an eigen basis and diagonalize the matrix [4]

$A = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$.

SECTION-C

4. (a) Find the inverse of the matrix [4]

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 2 \\ 2 & 0 & 3 \end{bmatrix}$ using Gauss-Jordan method.

(b) Test for exactness. If not exact, use an integrating factor and hence solve the ordinary differential equation [4]

$$(ye^x + 1)dx + (e^{x+y} + xe^x)dy = 0.$$

5. (a) Prove that the matrix $\begin{bmatrix} \frac{1}{2} & \frac{i\sqrt{3}}{2} \\ \frac{i\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ is unitary and hence find its eigen values. [4]

(b) Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$.

6. (a) Find the transient current in the RLC − circuit with $R = 4\Omega, L = 0.5\,H, C = 0.1\,F, E = 500\sin 2t\ Volts$.

(b) Find eigenvalues and eigenvectors of the following matrix.

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{bmatrix}$$

SECTION-D

7. (a) Solve the ordinary differential equation

$$(D^2 + 2D + 2I)y = e^{-x}\sec^3 x.$$

(b) A tank contains 400 gal of brine in which 100 lb of salt are dissolved. Fresh water runs into the tank at a rate of 4 gal/min. The mixture, kept practically uniform by stirring, runs out at the same rate. How much salt will there be in the tank at the end of 1 hour?