Roll No. .... Total Pages : 4

300204

May 2019

B.Tech. (ECE/EIC/EEE/FAE) IInd Semester

MATHEMATICS–II

(Calculus, Ordinary Differential Equations and Complex Variable)

(BSC106D)

Time : 3 Hours] [Max. Marks : 75

Instructions :

(i) It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.

(ii) Answer any four questions from Part-B in detail.

(iii) Different sub-parts of a question are to be attempted adjacent to each other.

PART–A

1. (a) Evaluate $\iint_R xy\,dx\,dy$ where R is the region in first quadrant bounded by x-axis, ordinate $x = 2a$ and the curve $x^2 = 4ay$. (1.5)

(b) Find the work done in moving a particle in the force field $\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along a straight line from (0, 0, 0) to (2, 1, 3). (1.5)

(c) Find the value of λ, for the differential equation $(xy^2 + \lambda x^2 y)dx + (x + y)x^2\, dy = 0$ is exact. (1.5)

(d) Solve $x^2 = 1 + p^2$. (1.5)

(e) Solve $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 0$. (1.5)

(f) Show that $P_n(1) = 1$ for all n. (1.5)

(g) Show that the function $u = e^{-2xy} \sin (x^2 - y^2)$ is harmonic. (1.5)

(h) Write C-R equations in polar form. (1.5)

(i) Evaluate $\int_0^{1+i} (x^2 - iy)dz$ along the path $y = x$. (1.5)

(j) Find the residue at each pole of $f(z) = \frac{\sin z}{z \cos z}$ inside the circle $|z| = 2$. (1.5)

PART-B

2. (a) Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. (8)

(b) Find by double integration, the centre of gravity of the area of the cardiod $r = a(1 + \cos\theta)$. (7)

3. (a) Solve $(y^3 - 2x^2y)dx + (2xy^2 - x^3)dy = 0$. (8)

(b) Solve Bernoulli equation $x^2dy + y(x + y)dx = 0$. (7)

4. (a) Solve the differential equation in power series

$$2x(1-x)\frac{d^2y}{dx^2}+(1-x)\frac{dy}{dx}+3y=0.$$ (8)

(b) Using the Method of Variation of parameters, solve $y'' - 2y' + y = e^x \log x$. (7)

5. (a) Determine the analytic function whose real part is $e^{2x}(x\cos 2y - y\sin 2y)$. (8)

(b) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, o, -i$. Hence find the image of $|z| < 1$. (7)

6. (a) Evaluate $\oint_C \frac{3z^2+7z+1}{z+1}dz$, where C is the circle

(i) $|z| = 1.5$.

(ii) $|z + i| = 1$. (8)

(b) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region

(i) $|z| < 1$.

(ii) $1 < |z| < 2$.

(iii) $|z| > 2$. (7)

7. (a) Verify Stoke's Theorem for the vector field $\vec{F} = (2x - y)\hat{i} + yz^2\hat{j} + y^2z\hat{k}$ over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection in xy-plane.

(b) Show that $\frac{d}{dx}[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x)$.

Sr. No.051202

August/September 2022

B.Tech (ME(Hindi Medium)/(ME/RAI)) - II SEMESTER

Mathematics-II (Calculus, Ordinary Differential Equations and Complex Variables)

(BSCH-106A/BSC-106A)

Time: 3 Hours

Max. Marks:75

Instructions:

1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*
4. *The candidate is required to attempt the question paper in the language as per his/her medium of instruction.*

PART -A

Q1 (a) Evaluate (1.5)

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} x^2\, y\, dx\, dy$$

मूल्यांकन करो।

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} x^2\, y\, dx\, dy$$

(b) Find the area bounded between $r = 2\, sin\theta$ and $r = 4\, sin\theta$ (1.5)

$r = 2\, sin\theta$ और $r = 4\, sin\theta$ के बीच घिरा हुआ क्षेत्रफल ज्ञात कीजिए।

(c) Evaluate (1.5)

$$\int_0^1 \int_0^2 \int_1^2 x^2 y z\, dx\, dy\, dz$$

मूल्यांकन करो।

$$\int_0^1 \int_0^2 \int_1^2 x^2 y\, z\, dx\, dy\, dz$$

(d) Find the integrating factor of the differential equation (1.5)

$$(x^2 - 3xy)dx + (x^2 - xy)dy = 0$$

विभेदक समीकरण का एकीकृत गुणांक ज्ञात कीजिये ।

$$(x^2 - 3xy)dx + (x^2 - xy)dy = 0$$

(e) Solve the following differential equation (1.5)

$$p^2 - 7p + 12 = 0$$

निम्नलिखित विभेदक समीकरण को हल कीजिये

$$p^2 - 7p + 12 = 0$$

(f) Solve the following differential equation (1.5)

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{2x}$$

दोहरे अभिन्न का उपयोग करके $y = 2 - x$ और $x^2 + y^2 = 4$ से घिरे क्षेत्रों में से छोटे को ज्ञात कीजिए।

Q3 (a) Solve the following differential equation: (8)

$$(x^2 y^2 + xy + 1)\, y\, dx + (x^2 y^2 - xy + 1)\, x\, dy = 0$$

निम्नलिखित विभेदक समीकरण को हल कीजिये

$$(x^2 y^2 + xy + 1)\, y\, dx + (x^2 y^2 - xy + 1)\, x\, dy = 0$$

(b) Solve the following differential equation: (7)

$$x^2 \left(\frac{dy}{dx}\right)^2 - 2\, xy \frac{dy}{dx} + 2\, y^2 - x^2 = 0$$

निम्नलिखित विभेदक समीकरण को हल कीजिये

$$x^2 \left(\frac{dy}{dx}\right)^2 - 2\, xy \frac{dy}{dx} + 2\, y^2 - x^2 = 0$$

Q4 (a) Solve the following differential equation: (8)

$$2 \frac{d^2y}{dx^2} + 5 \frac{dy}{dx} + 2y = 5 + 2x$$

निम्नलिखित विभेदक समीकरण को हल कीजिये

$$2 \frac{d^2y}{dx^2} + 5 \frac{dy}{dx} + 2y = 5 + 2x$$

(b) Apply the method of variation of parameters to solve the equation: (7)

$$(1-x) \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = (1-x)^2$$

समीकरण को हल करने के लिए मापदंडों की भिन्नता की विधि लागू करें:

$$(1-x) \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = (1-x)^2$$

Q5 (a) Using the Cauchy – Riemann equations, show that: (8)

(i). $f(z) = |z|^2$ is not analytic at any point.

(ii). $f(z) = \bar{z}$ is not analytic at any point.

कौची - रीमैन समीकरणों का उपयोग करके, दिखाएं कि:

(i). $f(z) = |z|^2$ किसी भी बिंदु पर विश्लेषणात्मक नहीं है।

(ii). $f(z) = \bar{z}$ किसी भी बिंदु पर विश्लेषणात्मक नहीं है।

(b) Evaluate the integral: (7)

$$\oint_c \frac{z^2 + 1}{z\,(2z - 1)}\, dz \quad ; \quad c : |z| = 1$$

अभिन्न का मूल्यांकन करें:

$$\oint_c \frac{z^2 + 1}{z\,(2z - 1)}\, dz \quad ; \quad c : |z| = 1$$

Q6 (a) Find all possible Taylor's and Laurent series expansions for the function (8)

$$f(z) = \frac{1}{(1-z)} \qquad about\ z = 0$$

फ़ंक्शन के लिए सभी संभव टेलर और लॉरेंट श्रृंखला विस्तार खोजें

$$f(z) = \frac{1}{(1-z)} \qquad about\ z = 0$$

(b) Show that the function (7)

(i). $cosec\, z$ has a simple pole at $z = 0$.

(ii). $\frac{1}{z^2-1}$ has simple pole at $z = 1 \ \ and \ \ z = -1$.

दिखाएँ कि फ़ंक्शन

(i). $cosec\, z$ में $z = 0$ पर एक सरल ध्रुव है।

(ii). $\frac{1}{z^2-1}$ में $z = 1$ और $z = -1$ पर सरल ध्रुव है।

Q7 (a) Solve the following differential equation: (8)

$$x^2 \frac{d^2y}{dx^2} + 8x\frac{dy}{dx} + 13y = \log x$$

निम्नलिखित विभेदक समीकरण को हल कीजिए:

$$x^2 \frac{d^2y}{dx^2} + 8x\frac{dy}{dx} + 13y = \log x$$

(b) Compute the residues at all the singular points of the following functions: (7)

(i). $f(z) = z \sin\left(\frac{1}{z}\right)$

(ii). $f(z) = z \cos\left(\frac{1}{z}\right)$

निम्नलिखित फलन के सभी एकवचन बिंदुओं पर अवशेषों की गणना करें:

(i). $f(z) = z \sin\left(\frac{1}{z}\right)$

(ii). $f(z) = z \cos\left(\frac{1}{z}\right)$

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Total Pages : 4

Roll No. 22001013001

013202

July, 2023

B.Tech. (ME) - IInd SEMESTER

Calculus, ODE and Complex Variables

(BSC-106A/BSCH-106A)

Time: 3 Hours] [Max. Marks. : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) State Liouville's Theorem. (1.5)

(b) State Cauchy Residue theorem. (1.5)

(c) Show that the function $\sin z$ is analytic in the finite z-plane. (1.5)

(d) Show that the transformation $w = e^z$ is always conformal. (1.5)

(e) Find the solution of the differential equation

$$\frac{dy}{dx} - y = 5 \sin x. \qquad (1.5)$$

(f) What is the general form of the Bessel's differential equation? (1.5)

(g) Find the integrating factor of the following differential equation : (1.5)

$$(x^3 + xy^4)\,dx + 2y^3 dy = 0.$$

(h) State Green's Thorem. (1.5)

(i) Evaluate the integral

$$\int_0^1 \int_1^2 x(x+y)\,dy\,dx. \qquad (1.5)$$

(j) Find the general solution of the following differential equation :

$$y = (x-a)p - p^2, \text{ where } p = \frac{dy}{dx}. \qquad (1.5)$$

$p^2 = p(x-a) - y = 0$

$p = c$

$c^2 = c(x-a) - y = 0$

$a = 1,\ b = (x-a),\ c = -y$

PART–B

2. (a) Evaluate the integral $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\,dx$ by changing the order of integration. (7)

(b) Evaluate the integral $\iint_S \vec{F}\cdot\vec{n}\,ds$ if $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ and S is part of the surface $x^2 + y^2 + z^2 = 1$, which lies in the first octant. (8)

$e^{\int p.dx}$ $e^{\int \frac{1}{2}dx}$

3. (a) Find the General solution of the differential equation $y'' + y = \operatorname{cosec} x \cot x$ using the method of variation of parameters. (7).

(b) Find the series solution in series of power of x of the following differential equation :

$$4x\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0.$$ (8)

4. (a) Solve the following differential equation :

$$p^2 + 2xp - 3x^2 = 0$$

where $p = \dfrac{dy}{dx}$. $e^{-\int p.dx}$ (7)

(b) Solve the differential equation :

$(2x - y)dy + (2y + x)\, dx.$ (8)

5. (a) Show that the function $u(x, y) = 2x + y^3 - 3x^2y$ is harmonic. Find its conjugate harmonic function $v(x, y)$ and the corresponding analytic function $f(z)$. (7)

(b) Find the image of the closed half disk

$$|z| \le 1,\ Im\ z \ge 0$$

under the bilinear transformation (8)

$$w = \frac{z}{z+1}.$$

6. (a) Evaluate the integral $I = \int_0^{2\pi} \frac{d\theta}{2+\sin\theta}$. (8)

(b) Find the residue at all the singular points of the following functions : (7)

$$f(z) = \frac{1}{z^3 + z^5}.$$

7. (a) Expand the function $f(z) = 1/z$ about $z = 2$ in Taylor's series. Obtain its radius of convergence. (7)

(b) State and prove Stoke's Theorem. (8)

August/September-2022 **Sr. No** 015201

B.Tech.(ECE/ENC/EEIOT)- II SEMESTER

(Calculus, Ordinary Differential Equation and Complex Variable)(BSC-106D)

Time: 3 Hours **Max. Marks:75**

Instructions:
1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART-A

Que.1(a)Evaluate $\int_0^1 \int_y^{y^2+1} x^2 y\,dx\,dy$ (1.5)

(b) Find the area lying between the parabola $y = 4x - x^2$ and the line $y = x$. (1.5)

(c) Solve $(xy^3+y)dx+(2x^2y^2+x+y^4)dy = 0$. (1.5)

(d) Solve the differential equation:

$y = 2px+p^4x^2$ (solvable for y). (1.5)

(e) Solve $(D^4+6D^2+9)\,y = 0$, where $D= d/dx$. (1.5)

(f) Write the Bessel's differential equation of order n. (1.5)

(g) State C-R Equations. (1.5)

(h) Define conformal mapping. (1.5)

(i) State Cauchy's integral theorem and Cauchy's integral formula. (1.5)

(j) State Cauchy's Residue Theorem. (1.5)

PART-B

Que.2 (a)Change the order of integration in the given integral and then evaluate $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dy\,dx$ (7)

(b)Verify the Green's theorem in the plane for $\oint_C (3x^2-8y^2)dx+(4y-6xy)dy$, where C is the boundary of the region defined by $x = 0, y = 0, x + y = 1$. (8)

Que.3 (a)Solve the differential equation $(2y\sin x+3y^4\sin x\cos x)dx - (4y^3\cos^2 x+\cos x)dy = 0$. (7)

(b)Solve the differential equation: $y = 2px+y^2p^3$(Solvable for x). (8)

Que.4 (a)Using variation of parameter ,solve $(D^2- 6D + 9)y = \frac{e^{3x}}{x^2}$, where $D = d/dx$. (7)

(b)Express $4x^3- 2x^2- 3x + 8$ in terms of Legendre's polynomial. (8)

Que.5 (a)Show that the function $u = e^{-2xy}\sin(x^2-y^2)$ is harmonic. Find the conjugate function 'v' and express u+iv as an analytic function of z. (7)

(b)Under the transformation $w = 1/z$, find the image of the given curve: $|z - 2i| = 2$ (8)

Que.6 (a)Expand $\frac{e^{2z}}{(z-1)^3}$ about the singularity $z = 1$ in Laurent's series. (7)

(b)Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$ using Residue theorem. (8)

Que.7 (a) Find the volume bounded by the cylinder $x^2+y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. (7)

(b)Find the sum of the residues of the function $f(z) = \frac{\sin z}{z\cos z}$ at its poles inside the circle $|z| = 2$ (8)