NATIONAL INSTITUTE OF TECHNOLOGY, KURUKSHETRA

(An Institution of National Importance under Ministry of Education, Govt. of India)

DEPARTMENT OF MATHEMATICS

END SEMESTER EXAMINATIONS, MAY 2024

B. TECH IV Sem. (ECE -A &B)

MAIC-204: Applied Linear Algebra (Maths-III)

Date: 07-05-2024 **Time:** 09:30 AM to 12:30 PM **Max. Marks:** 50

N.B.: Answer **ALL** questions.

Answers to all parts of each question should be in one place.

Q1	Let $V \equiv$ Set of all polynomials of degree n over real field $\mathbb{R}$, $W \equiv$ Set of all polynomials of degree $\leq n$ over real field $\mathbb{R}$ and $Z \equiv$ Set of all polynomials over real field $\mathbb{R}$. Prove/Disprove that $V(\mathbb{R})$, $W(\mathbb{R})$ and $Z(\mathbb{R})$ are vector space over $\mathbb{R}$. If any structure forms vector space, then discuss their basis and dimension.	[6M]
Q2	Find the range space $R(T)$ and null space $N(T)$ for the given linear operator $T: \mathbb{R}^3(\mathbb{R}) \to \mathbb{R}^3(\mathbb{R})$ such that $T(x,y,z) = (x+y, y+z, x-z)$. Also, find the basis and dimensions of $R(T)$ and $N(T)$.	[6M]
Q3	Find the annihilator space W^0 for the sub space W of $\mathbb{R}^3(\mathbb{R})$ spanned by $(1,2,0)$, $(1,0,5)$ and $(0,-2,5)$. Also, show that $dim\,(W^0) + dim\,(W) = 3$.	[6M]
Q4	Find the eigen values and eigen vectors for the given linear operator $T: \mathbb{R}^2(\mathbb{R}) \to \mathbb{R}^2(\mathbb{R})$ such that $T(x,y) = (y,x)$. Also evaluate i) $\lvert T^{100} + 5T^{20} + 20I \rvert$ and ii) $Trace(T^{100} + 5T^{20} + 20I)$.	[6M]
Q5	Find the characteristic and minimal polynomial for the given linear operator $T: \mathbb{R}^3(\mathbb{R}) \to \mathbb{R}^3(\mathbb{R})$ such that $T(x,y,z) = (5x-6y-6z, -x+4y+2z, 3x-6y-4z)$. Also, discuss the diagonalization of T^{100}.	[6M]
Q6	Define inner product space. Give an example of an inner product space with proper explanation. Define invariant sub-space. Show that for the given linear operator $T: \mathbb{R}^3(\mathbb{R}) \to \mathbb{R}^3(\mathbb{R})$, if T_1 is any polynomial in T then $R(T_1)$ and $N(T_1)$ are invariant under T.	[8M]
Q7	Apply Gram-Schmidt Process on $\{(3,0,4), (-1,0,7), (2,9,11)\}$ to obtain an orthogonal basis for $\mathbb{R}^3(\mathbb{R})$.	[6M]
Q8	Consider the sub-spaces W_1 and W_2 of $\mathbb{R}^3(\mathbb{R})$ given by $W_1 = \{(x,y,z) \mid x+y+z=0\}$, $W_2 = \{(x,y,z) \mid x-y+z=0\}$. If W is a sub-space of $\mathbb{R}^3(\mathbb{R})$ such that a) $W \cap W_2 = Span\{(0,1,1)\}$ b) $W \cap W_1$ is orthogonal to $W \cap W_2$ with respect to usual dot product (inner product) $\mathbb{R}^3(\mathbb{R})$. Then which of the followings are true? i) $W = Span\{(0,1,-1), (0,1,1)\}$ ii) $W = Span\{(1,0,-1), (0,1,-1)\}$ iii) $W = Span\{(1,0,-1), (0,1,1)\}$ iv) $W = Span\{(1,0,-1), (1,0,1)\}$ v) $\dim(W_1) = 1$ vi) $\dim(W_2) = 2$ vii) $\dim(W_1 \cap W_2) = 1$	[6M]

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