Design and Analysis of Computer Algorithms - Hw 2

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Problem 1. Consider the binary numbers $x = 10011011$ and $y = 10111010$. Decomposing x using $a = 1001$, $b = 1011$ and y using $c = 1011$, $d = 1010$, compute the product $x * y$ using the ordinary way and the way done in divide-and-conquer method by forming $w1 = a + b$, $w2 = c + d$, $u = w1w2$, $v = ac$, $w = bd$. Recall xy = $2nv + 2\frac{n}{2}(u * v * w) + w$. Each multiplication by 2 amounts to a shift. Count the binary operations in each method.

Problem 2. You are given an infinite array $A[i]$ in which the first n elements are integers in sorted order and the rest are filled with . You are not given n. Describe an algorithm that takes as input an integer x and finds a position in the array containing x, if such a position exists, in $O(\log n)$ time.

Problem 3. Solve each of the recurrence relations and give bound for each. You can use the master theorem if applicable

$$\begin{aligned} T(n) &= 5T(\tfrac{n}{4}) + n \\ T(n) &= 7T(\tfrac{n}{7}) + n \\ T(n) &= 9T(\tfrac{n}{4}) + n^2 \\ T(n) &= 8T(\tfrac{n}{2}) + n^3 \\ T(n) &= 49T(\tfrac{n}{25}) + n^{3.5}\log n \\ T(n) &= T(n^{.5}) + 1 \end{aligned}$$

Problem 4. Find the coefficients of the polynomial $p(x) = a_2x^2 + a_1x + a_0$ such that $p(1) = 2$, $p(2) = 1$, $p(3) = 0$.

Problem 5. Consider the array = $[25, 34, 63, 29, 66, 47, 12, 17]$. Apply the Partition (Split) procedure in Quicksort, as described in class, to this array using the first element as the pivot.