

1. Let $f_l(z) = \sigma(W^{[l]}z + b^{[l]})$ and $W^{[l]}a^{[l-1]} + b^{[l]} = z^{[l]}$ for $l = 1, 2, \dots, L$, then

$$\begin{aligned} a^{[L]'}(x) &= f_L'(a^{[L-1]}(x)) \cdot a^{[L-1]'}(x) \\ &= \sigma'(z^{[L]}) \cdot W^{[L]} \cdot a^{[L-1]'}(x) \end{aligned}$$

note that $\sigma'(z^{[L]}) = (\sigma'(z_1^{[L]}), \sigma'(z_2^{[L]}), \dots)$

Hence $a^{[L]'}(x) = \sigma'(z^{[L]}) \cdot W^{[L]} \cdot \sigma'(z^{[L-1]}) W^{[L-1]} \dots \sigma'(z^{[2]}) W^{[2]}$

$$\nabla a^{[L]}(x) = \left(a^{[L]'}(x) \right)^T$$

$(a^{[L]}(x))$ maps \mathbb{R}^n to \mathbb{R} , so $a^{[L]'}(x)$ is

a row vector $\left(\frac{\partial a^{[L]}}{\partial x_1} \dots \frac{\partial a^{[L]}}{\partial x_n} \right)$