自动驾驶控制算法第三讲

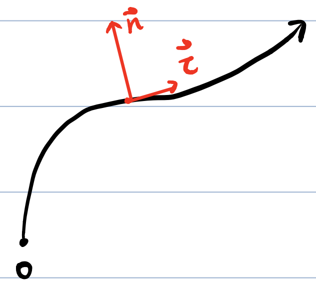
运动学方程 $\dot{x} = v\cos\varphi$

$\dot{y} = v\sin\varphi$

$\dot{\varphi} = \frac{v\tan\delta}{L}$ $\quad \dot{\varphi} = \frac{v}{R} \Rightarrow \frac{1}{R} = \frac{\tan\delta}{L} \quad \tan\delta = \frac{L}{R}$

动力学方程：考虑轮胎特性

当选取 Frenet 坐标系时，可以将纵向控制与横向控制解耦



$\vec{v} = \frac{ds}{dt}$ $\quad \vec{a_\tau} = \frac{d^2s}{dt^2}$ $\quad a_n = \frac{v^2}{R} = \frac{v^2\tan\delta}{L}$

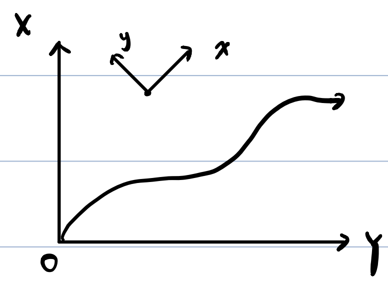
s 与 a_τ 直接相关 $\quad a_n$ 与 v, δ 有关，当纵向控制稳定以后，v 变化不大

a_n 与 δ 有关

运动学 $\begin{cases} \dot{x} = v\cos\varphi \\ \dot{y} = v\sin\varphi \\ \dot{\varphi} = v\frac{\tan\delta}{L} \end{cases}$ $\quad \left.\begin{matrix} v \\ \delta \end{matrix}\right\} \Rightarrow \dot{\varphi} \Rightarrow \left.\begin{matrix} \varphi \\ v \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \dot{x} \\ \dot{y} \end{matrix}\right\} \Rightarrow \begin{matrix} x \\ y \end{matrix}$

动力学 + Frenet 坐标系 $\begin{matrix} a_\tau \Rightarrow s \\ \delta \Rightarrow d \end{matrix}$ 解耦

$a_\tau = \frac{d^2s}{dt^2}$ $\quad d = f(v, \delta)$

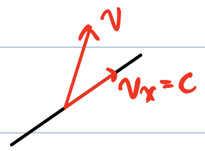


$\left.\begin{matrix} \text{首先建立 } \underline{y = f(v, \delta)} \\ \text{动力学方程} \\ \text{再建立 } \underline{y = g(d)} \\ \text{坐标变换} \end{matrix}\right\} \Rightarrow d = h(v, \delta)$

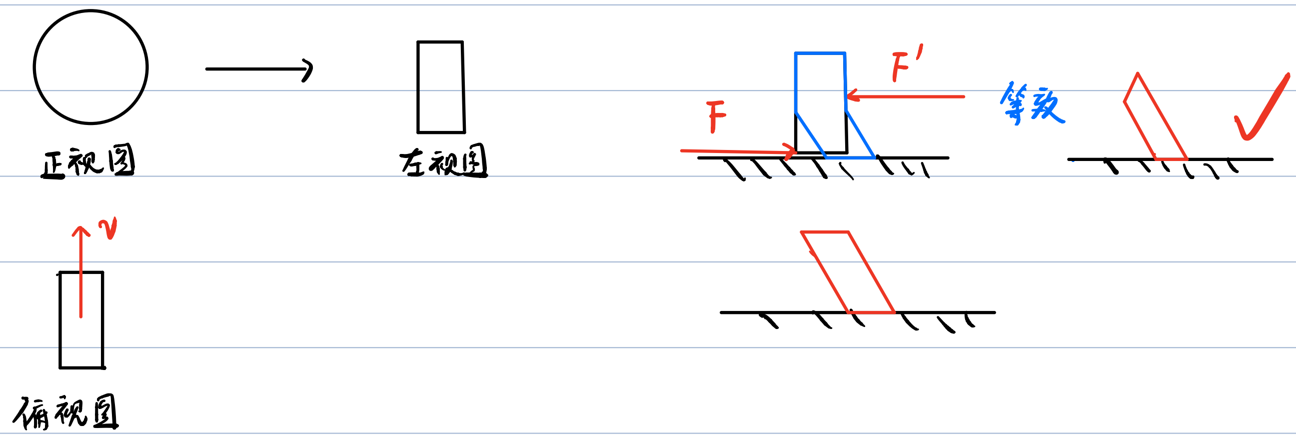
x	s
y	d
$\dot{x}$	$\dot{s}$
$\dot{y}$	$\dot{d}$

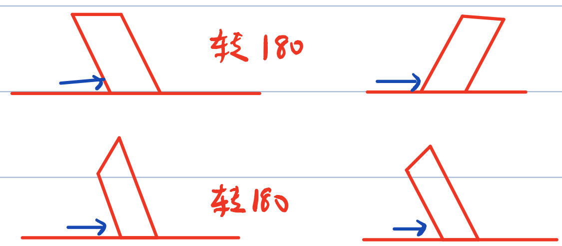
二自由度车辆动力学方程

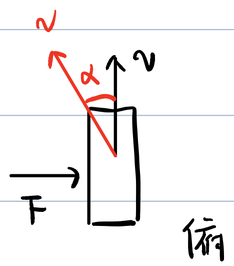
假设前轮转角 δ 较小，假设 $v_x = C$



轮胎的侧偏特性



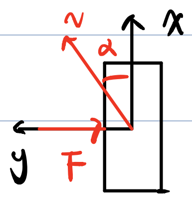




α 侧偏角　定义 $F=C\alpha$　C 侧偏刚度

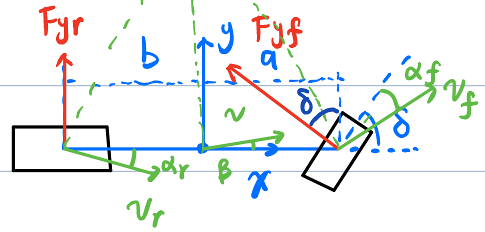
侧偏刚度 × 侧偏角 = 侧向力

侧偏刚度一定是负数



负的侧偏力 ⇒ 正的侧偏角

自行车模型



f: front   r: rear

α_r α_f 都是负的

$\Sigma F_y = m a_y \Rightarrow m a_y = F_{yf}\cos\delta + F_{yr}$

$\Sigma M = I\ddot{\varphi} \Rightarrow F_{yf}\cos\delta \cdot a - F_{yr}\cdot b = I\ddot{\varphi}$

假设 δ 较小 $\cos\delta \approx 1$

$m a_y = F_{yf} + F_{yr} = C_{\alpha f}\alpha_f + C_{\alpha r}\alpha_r$

$I\ddot{\varphi} = F_{yf}\cdot a - F_{yr}\cdot b = aC_{\alpha f}\alpha_f - bC_{\alpha r}\alpha_r$

a_y 与 y 的关系，以及 α_f, α_r 的具体表达式

$v_y = \dot{y}$ $a_y = \ddot{y} + v_x\dot{\varphi}$

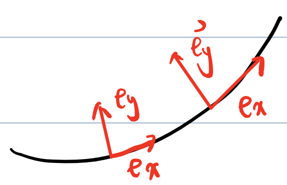
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d(v_x\vec{e_x} + v_y\vec{e_y})}{dt}$$

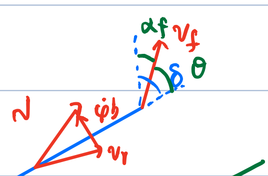
在直角坐标系中 $\vec{e_x}$, $\vec{e_y}$ 为常矢量 $\dot{\vec{e_x}}$, $\dot{\vec{e_y}} = 0$

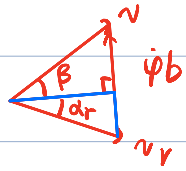
$$\Rightarrow \vec{a} = \frac{dv_x}{dt}\vec{e_x} + v_x\cdot\frac{d\vec{e_x}}{dt} + \frac{dv_y}{dt}\vec{e_y} + v_y\cdot\frac{d\vec{e_y}}{dt} \neq 0$$

在直角坐标 $\vec{a} = \dot{v}_x\vec{e_x} + \dot{v}_y\vec{e_y}$ $\therefore a_y = \dot{v}_y$

在车身坐标 $\vec{e_x}$, $\vec{e_y}$ 不是常矢量 (Frenet 公式) (非惯性系)



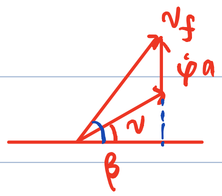




$v\cos\beta = v_x$

$\dot{\varphi} b - v\sin\beta = \dot{\varphi} b - v_y$

$\tan\alpha_r = \dfrac{\dot{\varphi} b - v_y}{v_x} \approx \alpha_r$ $\because$ α_r 是负的 $\therefore$ $\alpha_r = \dfrac{v_y - \dot{\varphi} b}{v_x}$



$\tan\theta = \dfrac{\dot{\varphi} a + v_y}{v_x}$ $\alpha_f = \theta - \delta = \dfrac{\dot{\varphi} a + v_y}{v_x} - \delta$

$$m a_y = C_{\alpha f}\alpha_f + C_{\alpha r}\alpha_r \Rightarrow m(\dot{v}_y + v_x\dot{\varphi}) = C_{\alpha f}\left(\frac{\dot{\varphi} a + v_y}{v_x} - \delta\right) + C_{\alpha r}\left(\frac{v_y - \dot{\varphi} b}{v_x}\right)$$

$$I\ddot{\varphi} = aC_{\alpha f}\alpha_f - bC_{\alpha r}\alpha_r \Rightarrow I\ddot{\varphi} = aC_{\alpha f}\left(\frac{\dot{\varphi} a + v_y}{v_x} - \delta\right) - bC_{\alpha r}\left(\frac{v_y - \dot{\varphi} b}{v_x}\right)$$

$v_y \Leftrightarrow \dot{y}$

$$\begin{pmatrix} \ddot{y} \\ \ddot{\varphi} \end{pmatrix} = \begin{pmatrix} \dfrac{C_{\alpha f} + C_{\alpha r}}{m v_x} & \dfrac{aC_{\alpha f} - bC_{\alpha r}}{m v_x} - v_x \\ \dfrac{aC_{\alpha f} - bC_{\alpha r}}{I v_x} & \dfrac{a^2 C_{\alpha f} + b^2 C_{\alpha r}}{I v_x} \end{pmatrix} \begin{pmatrix} \dot{y} \\ \dot{\varphi} \end{pmatrix} + \begin{pmatrix} -\dfrac{C_{\alpha f}}{m} \\ -\dfrac{aC_{\alpha f}}{I} \end{pmatrix} \delta$$

设 $x = \begin{pmatrix} \dot{y} \\ \dot{\varphi} \end{pmatrix}$ $u = \delta$ $\dot{x} = Ax + Bu$ 通过控制 δ，实现对 y, φ 的控制