Inventing With Polygons

The Movie:

This inventor uses polygons to build amazing expandable structures. Featured: Chuck Hoberman, inventor. *(Movie length: 2:29)*

Background:

The characteristics of polygons and polyhedrons have fascinated humankind for thousands of years. Much of the work of the mathematician Euclid of Alexandria deals with them. The philosopher Plato believed that five very special polyhedrons—which later came to be known as the Platonic solids—formed the basic structure of the universe.

Why this preoccupation? Possibly for two apparently contradictory reasons. On the one hand, polygons and polyhedrons are the most practical of shapes, forming as they do the basic structures of buildings. On the other hand, they are completely abstract constructs which, upon investigation, yield many intriguing patterns and relationships.

Or perhaps it is just that the topic of polygons and polyhedrons is a particularly effective way to introduce students to mathematics itself, because they seem to embody all of the characteristics of the subject: precision, patterns, relationships, beauty, and application.

## Curriculum Connections:

Geometry (volume)

1

This cube has sides of 6 inches each. How is its volume changed if the length of all sides is doubled, to 12 inches?

Geometry (area)

2
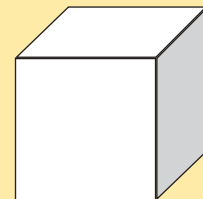
a) Find the surface area of this tetrahedron, assuming all edges are 6 inches long.
   Note: For any equilateral triangle, the height is always $\sqrt{3}/2$ (approximately 0.866) times the length of one of the sides.

b) How is the surface area changed if the length of all edges is doubled, to 12 inches?
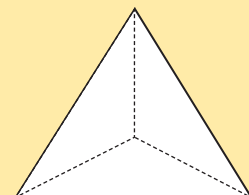
Geometry (Pythagorean theorem)

Suppose you want to construct a symmetric square pyramid, such as the one pictured, in which the height of the pyramid is equal to the length of each side of the square base. If the square base has sides of length 100 millimeters, what should you choose for the length of the triangles' sides?
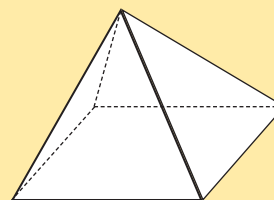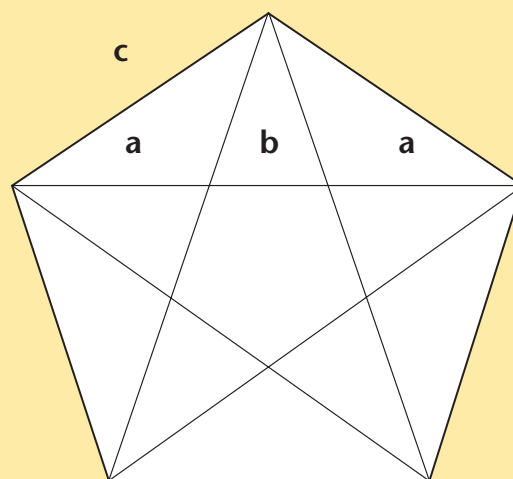
Geometry (polygons), Ratios

This is a regular pentagon with all diagonals drawn in.

a) How many regular pentagons can you find? How can you prove they are regular?

b) How many isosceles triangles can you find? How can you prove they are isosceles?

c) Make measurements to find whether or not these ratios are equivalent:

| First ratio	Second ratio	Equivalent?
$(a + b)/c$	$1/1$	?
$(a + b + a)/(a + b)$	$(a + b)/a$	?
$(a + b)/a$	a/b	?

(Note: The ratio of a/b is called the "Golden Ratio".)

Geometry (polygons)

Make a collapsible polyhedron:

1. Use a copy machine to enlarge the figure at right to a width of approximately 8 inches. Using that as a template, trace and cut out the figure on a sheet of thick cardboard. The cardboard should be composed of three layers: two thin sheets with an inner filling. Cut the outside lines only. Be as accurate as possible.

2. Repeat step 1 with another sheet of cardboard.

3. Score the cardboard along the edges of the inner hexagon, as indicated by dashed lines in the template. Do this for both pieces. When you score, <u>cut through the outer skin only, on only one side of the cardboard.</u>

4. Put the two sheets back to back, with the scored sides facing <u>out.</u> Tape them together only on the 6 outer edges of the figures (indicated by x's on the template).

5. Push in the sides (at points y) as you push up on the bottom (at point z). The two trapezoids at the bottom should fold in so they are slightly inside the polyhedron. The resulting shape should look like that at right.

squares

rhombuses
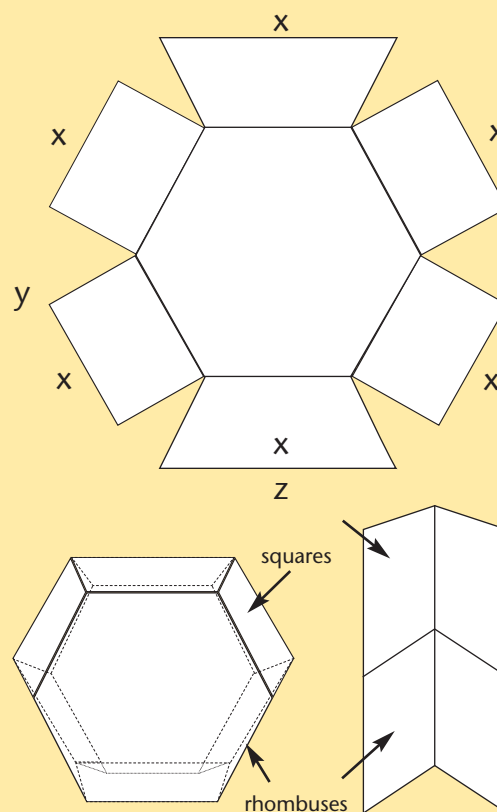
Polyhedral Pillars

From: Chief Structural Engineer
To: Testing Department

For the new museum of architecture, we want to build some new types of structural supports.

One idea we had was to use irregular polyhedrons to support the load of the ceiling and roof.

Can you do a quick rough test of this to see what polyhedrons might be the best for this purpose?

Here's what I 'd like you to do:

1. Build three polyhedrons using the basic shapes of triangles, rhombuses, squares, trapezoids and hexagons. Use thin cardboard and tape.

2. Work out a way to numerically compare their ability to support a load.

3. Give me a report explaining what you did and what your results were. Tell me also if you think that your results are reliable, and why or why not. (Be sure to include instructions for building the polyhedral you used.)

Below are templates for the polygons you are to use.
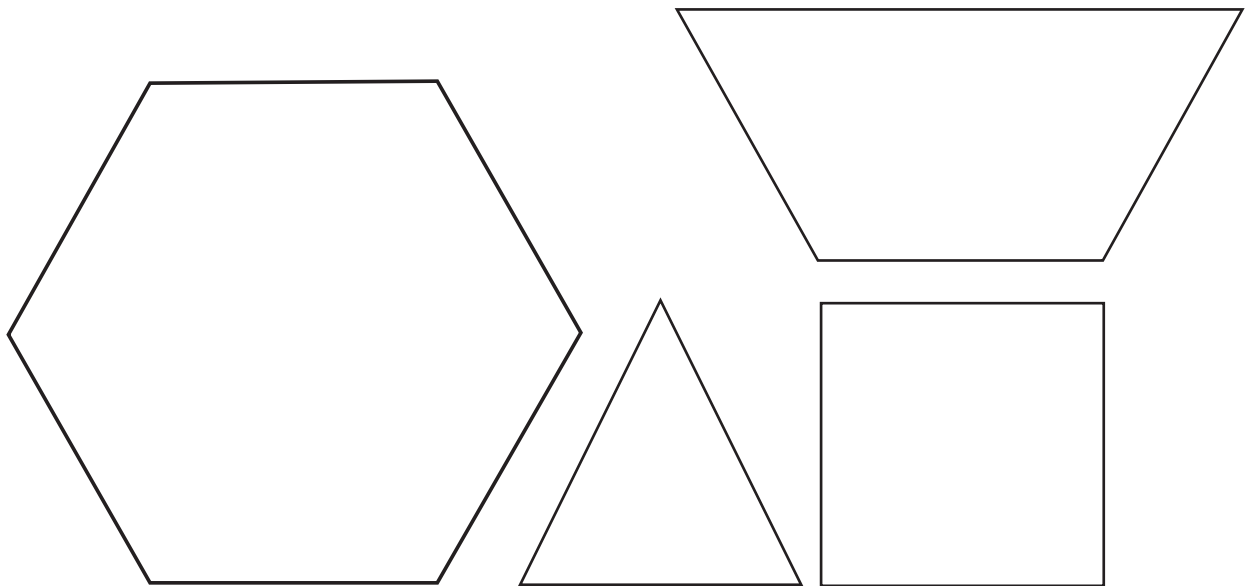
Teaching Guidelines: Polyhedral Pillars
Math Topics: Geometry (polygons, solids)

Materials:

Thin poster board or shoebox cardboard, transparent tape, kitchen scale

Procedure:

This project should be done by students in teams of two, three or four.

Prepare for the assignment by cutting out an adequate supply of polygons, so that each student or team has at least 12 triangles, 8 rhombuses, 8 squares, 8 trapezoids, and 4 hexagons. (You may wish to have students cut out the polygons as the first step of the activity.) Use thin cardboard, such as that used to make shoe boxes.

You will also need one roll of transparent tape for each team.

A kitchen scale should be available so that students can measure the amount of load they are putting on the polyhedral.

Distribute the handout and discuss it. Ensure that students understand the assignment. You may wish to discuss some ideas about possible ways to measure the ability of the polyhedral pillars to support a certain amount of weight.
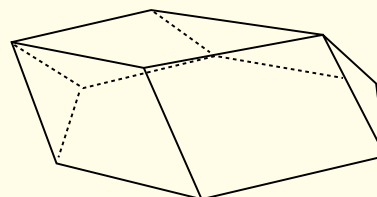
Give students a schedule for working on the assignment and a due date.

This assignment can be somewhat simplified if you provide students with models of polyhedrons to copy. Four are shown below.
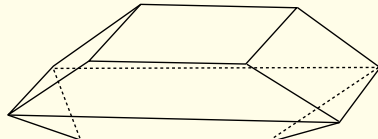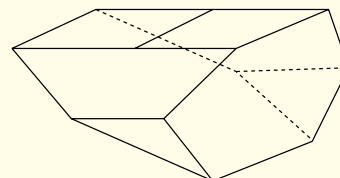
4 trapezoids
4 triangles
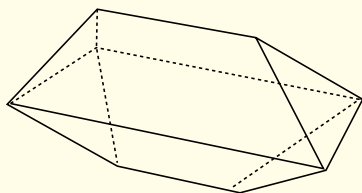
1 hexagon
3 rhombuses
2 triangles
2 squares

4 trapezoids
2 triangles
3 squares

2 hexagons
2 trapezoids
2 triangles
2 squares

If you enjoyed this Futures Channel Movie, you will probably also like these:

Geometry and Structural Engineering, #1009	Structural engineers use shapes to design huge buildings and bridges.
The ABC's of Architecture, #4010	When Penn Station needed a new front entrance, they called upon architect Frances Halsband, and she called upon her knowledge of geometry.
Designing Sunglasses, #4012	Watch as a new model of sunglasses goes from design sketch to finished product.