

Department of Atmospheric Sciences
National Central University
Advanced Applied Mathematics
Homework III

2021/10/13/-2021/10/27

(1) 10 pts 考慮 $u'' + u + x = 0$, $u(0) = 0$, $u(1) = 0$ 。如果此微分方程是某變分問題的 Euler-Lagrangian 方程

(a) 用 Rayleigh-Ritz 法求解 u

令可選取的基礎函數為 $x(1-x)$, $x^2(1-x)$, $x^3(1-x)$

為了簡單起見，只用一項來求解。

(b) 請證明其解析解為 $u(x) = \frac{\sin(x)}{\sin(1)} - x$

請繪圖表示兩個解，並比較近似解和解析解在 $x = 0.25, 0.5, 0.75$ 處的值。

Considering $u'' + u + x = 0$, $u(0) = 0$, $u(1) = 0$. If this PDE is the Euler-Lagrangian equation of a variational problem

(a) Use Rayleigh-Ritz method to solve u . You can choose $x(1-x)$, $x^2(1-x)$, $x^3(1-x)$ as your basic functions, but for simplicity, you can use only one term.

(b) Please show that the analytic solution of this equation is

$$u(x) = \frac{\sin(x)}{\sin(1)} - x.$$

Please plot two solutions, and compare the values from these two solutions at $x = 0.25, 0.5, 0.75$.

(2) 20 pts. (Please write the codes and solve the resulting equation with a tri-diagonal matrix all by yourself).

(a) Use cubic spline function to compute the coefficients a_i , b_i , c_i , d_i for each sector, and fill out the parentheses (up to four digits after the decimal point) in the following table.

(b) Plot the distribution of $y = y(x)$.

Data				Results			
i	x_i	y_i	h_i	a_i	b_i	c_i	d_i
0	0	-7	1	()	()	()	()
1	1	-5	1	()	()	()	()
2	2	2	1	()	()	()	()
3	3	18	1	()	()	()	()
4	4	40					

(3) 20 pts. Same as above, but now the values of y are changed, and the distances of each sector are different.

Data				Results			
i	x_i	y_i	h_i	a_i	b_i	c_i	d_i
0	0	20	1	()	()	()	()
1	1	-8	2	()	()	()	()
2	3	10	3	()	()	()	()
3	6	2	4	()	()	()	()
4	10	-7					