National Central University
Department of Atmospheric Sciences
Radar Meteorology Homework III
(2021/10/18 - 2021/10/29)

There are N sets of data (in this case N is an even number), and the observational data X_n (n=0~N-1) can be expressed by Fourier series as:

$$X_n = A_0 + \sum_{m=1}^{\frac{N}{2}-1}\left(A_m \cos\frac{2\pi mn}{N} + B_m \sin\frac{2\pi mn}{N}\right) + A_{\frac{N}{2}} \cos\pi m$$

The coefficients can be obtained by：

$$A_o = \frac{1}{N}\sum_{n=0}^{N-1} X_n \quad , \quad B_0 = 0$$

$$A_m = \frac{2}{N}\sum_{n=0}^{N-1} X_n \cos\frac{2\pi mn}{N} \quad , \quad (m = 1 \sim \frac{N}{2} - 1)$$

$$B_m = \frac{2}{N}\sum_{n=0}^{N-1} X_n \sin\frac{2\pi mn}{N} \quad , \quad (m = 1 \sim \frac{N}{2} - 1)$$

$$A_{\frac{N}{2}} = \frac{1}{N}\sum_{n=0}^{N-1} X_n \cos\pi m \quad , \quad B_{\frac{N}{2}} = 0$$

The variance (S^2_m) of each wave (or harmonic) is:

$$S_m^{\,2} = \frac{A_m^{\,2} + B_m^{\,2}}{2} \quad , \quad (m = 1 \sim \frac{N}{2} - 1)$$

$$S_{\frac{N}{2}}^{\,2} = A_{\frac{N}{2}}^{\,2}$$

, which represents the intensity of this wave (harmonic). If it is divided by the total variance, the result is the contribution from this particular wave.

Note that the total variance is expressed by : $S^2 = \sum_{m=1}^{\frac{N}{2}} S_m^{\,2}$

This table is the temperature (T) distribution with time (n)

n	T
0	100.00
1	129.02
2	127.14
3	102.45
4	89.27
5	101.75
6	117.30
7	110.59
8	89.41
9	82.70
10	98.25
11	110.73
12	97.55
13	72.86
14	70.98
15	82.88

(1) Plot T vs. n.

(2) Conduct Fourier analysis for the temperature data, find the variance of each harmonic, and its contribution to the total variance.

(3) Plot separately the temperature distribution with time for the top 5 harmonics.

(4) Add the top 5 harmonics one by one gradually, and compare with the original data.

(5) Add all harmonics together, and compare with the original data.

(Please write the codes all by yourself).