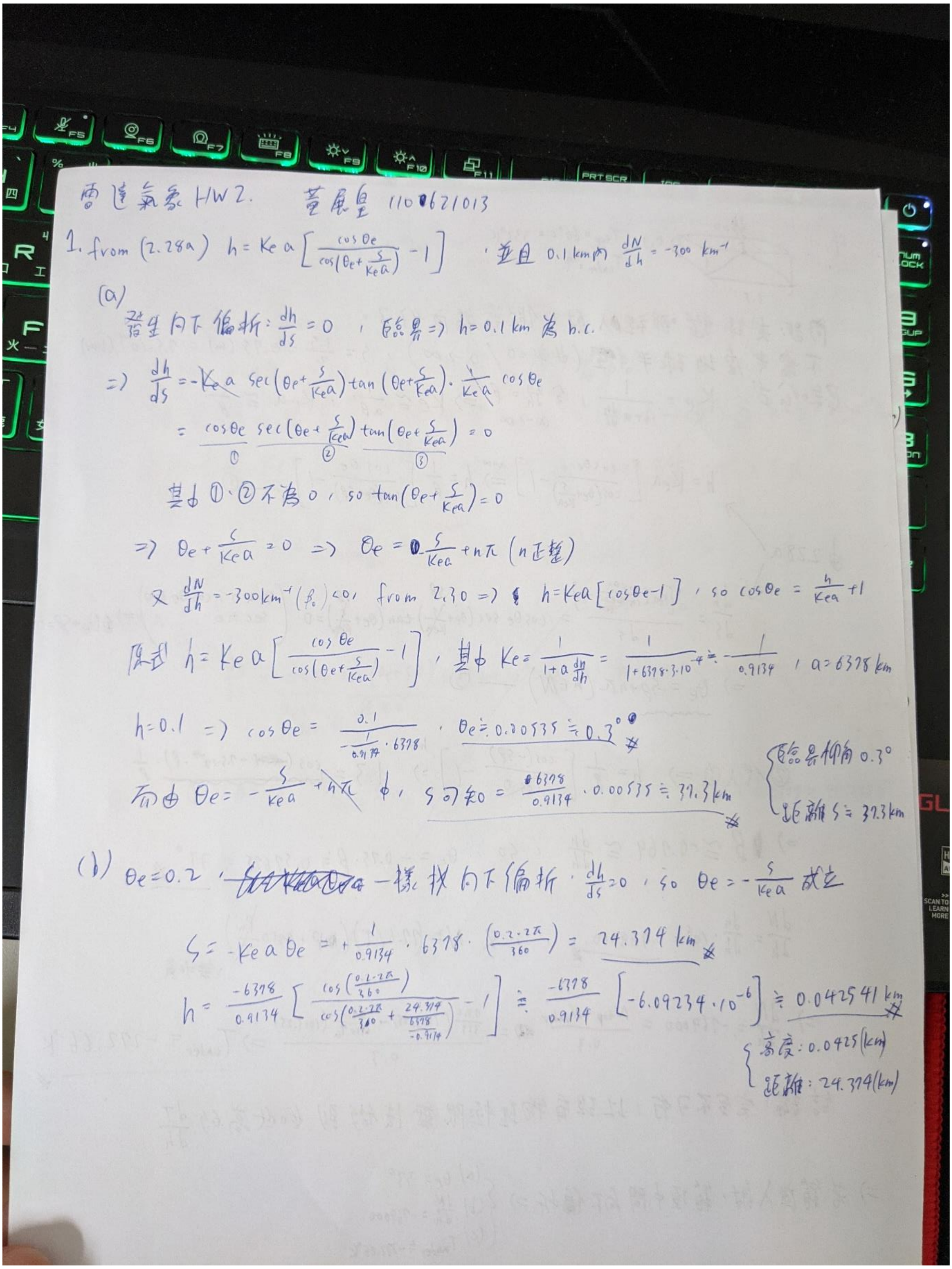
雷達氣象 HW2　黃展皇 110621013

Q1



雷達氣象 HW2.　黃展皇 110621013

1. from (2.28a) $h = K_e a\left[\frac{\cos\theta_e}{\cos(\theta_e+\frac{s}{K_e a})}-1\right]$，並且 0.1 km內 $\frac{dN}{dh} = -300\ km^{-1}$

(a)

發生向下偏折：$\frac{dh}{ds}=0$，臨界 $\Rightarrow h=0.1$ km 為 b.c.

$\Rightarrow \frac{dh}{ds} = -\cancel{K_e a}\sec(\theta_e+\frac{s}{K_e a})\tan(\theta_e+\frac{s}{K_e a})\cdot\frac{1}{\cancel{K_e a}}\cos\theta_e$

$= \underbrace{\cos\theta_e}_{①}\ \underbrace{\sec(\theta_e+\frac{s}{K_e a})}_{②}\ \underbrace{\tan(\theta_e+\frac{s}{K_e a})}_{③} = 0$

其中 ①、② 不為 0，so $\tan(\theta_e+\frac{s}{K_e a}) = 0$

$\Rightarrow \theta_e + \frac{s}{K_e a} = 0 \Rightarrow \theta_e = -\frac{s}{K_e a} + n\pi$ (n正整)

又 $\frac{dN}{dh} = -300 km^{-1}$ ($\beta_0$) < 0，from 2.30 $\Rightarrow$ $h = K_e a[\cos\theta_e - 1]$，so $\cos\theta_e = \frac{h}{K_e a}+1$

原式 $h = K_e a\left[\frac{\cos\theta_e}{\cos(\theta_e+\frac{s}{K_e a})}-1\right]$，其中 $K_e = \frac{1}{1+a\frac{dn}{dh}} = \frac{1}{1+6378\cdot 3\cdot 10^{-4}} \doteq -\frac{1}{0.9134}$，$a = 6378$ km

$h=0.1 \Rightarrow \cos\theta_e = \frac{0.1}{-\frac{1}{0.9134}\cdot 6378}$，$\theta_e \doteq 0.00535 \doteq 0.3°$ #

而由 $\theta_e = -\frac{s}{K_e a} + \cancel{n\pi}$ 中，s 可知 $= \frac{6378}{0.9134}\cdot 0.00535 \doteq 37.3$ km #

$\begin{cases} \text{臨界仰角 } 0.3° \\ \text{距離 } s \doteq 37.3\text{ km} \end{cases}$

(b) $\theta_e = 0.2$，~~[illegible]~~ 一樣找向下偏折，$\frac{dh}{ds}=0$，so $\theta_e = -\frac{s}{K_e a}$ 成立

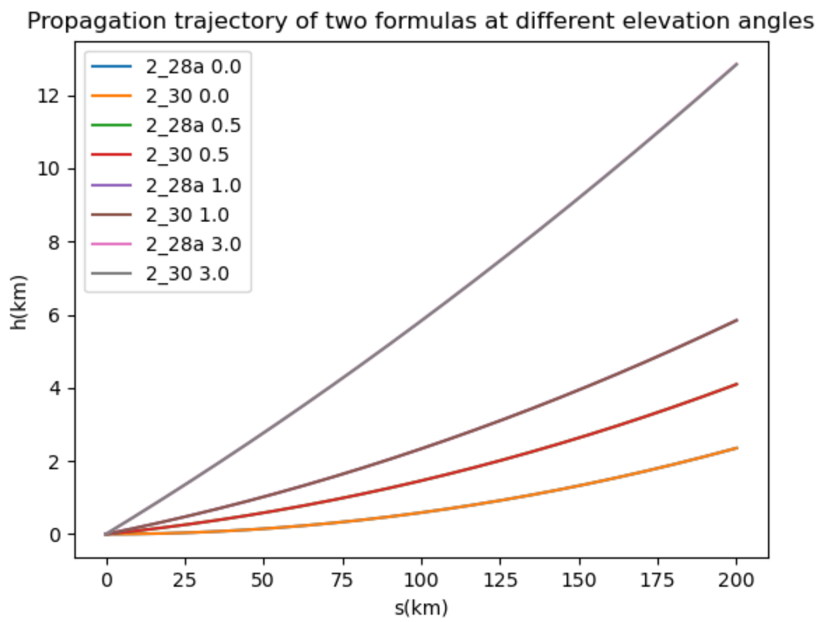
$s = -K_e a\,\theta_e = +\frac{1}{0.9134}\cdot 6378\cdot\left(\frac{0.2\cdot 2\pi}{360}\right) = 24.374$ km #

$h = \frac{-6378}{0.9134}\left[\frac{\cos(\frac{0.2\cdot 2\pi}{360})}{\cos(\frac{0.2\cdot 2\pi}{360}+\frac{24.374}{\frac{6378}{-0.9134}})}-1\right] \doteq \frac{-6378}{0.9134}\left[-6.09234\cdot 10^{-6}\right] \doteq 0.042541$ km #

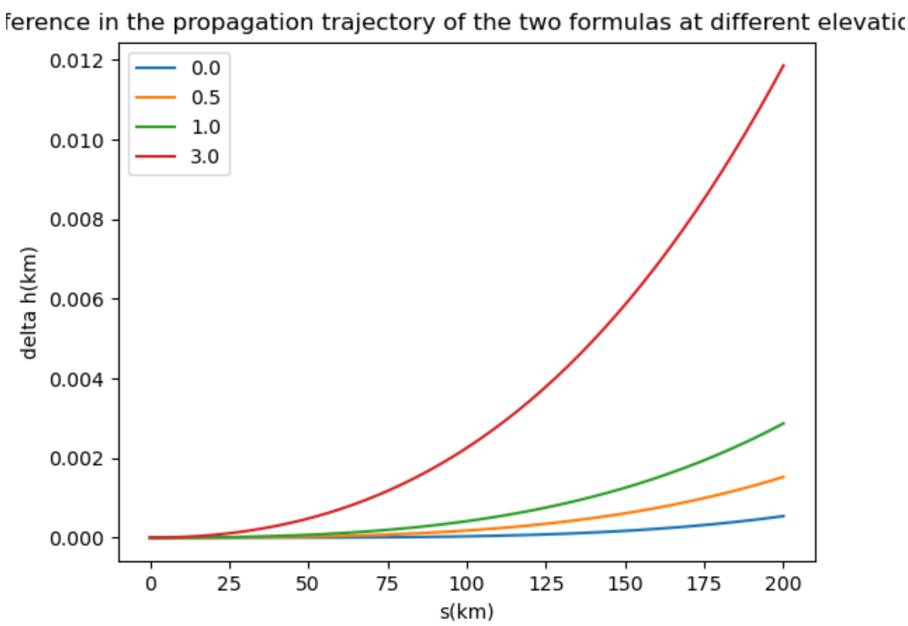
$\begin{cases} \text{高度：} 0.0425\text{(km)} \\ \text{距離：} 24.374\text{(km)} \end{cases}$

Q2 如下圖，基本上 2.28a 與 2.30 算出的傳播軌跡很相近，基本上沒有差別

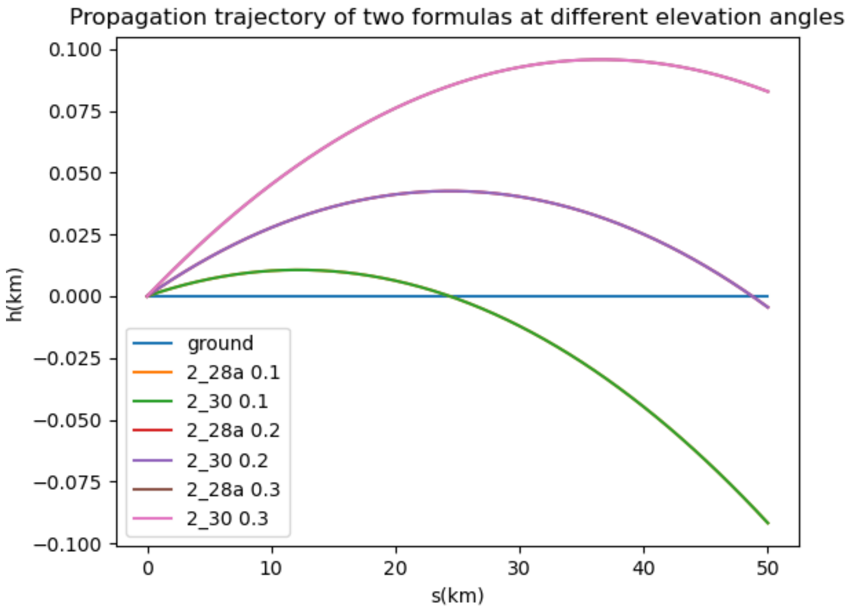
並且傳播路徑有上凹的彎曲(距離越遠，軌跡隨高度增加越明顯)




將 2.28a-2.30 可以得到兩者差距僅有數公尺(2.28a>2.30)並隨仰角與距離提昇

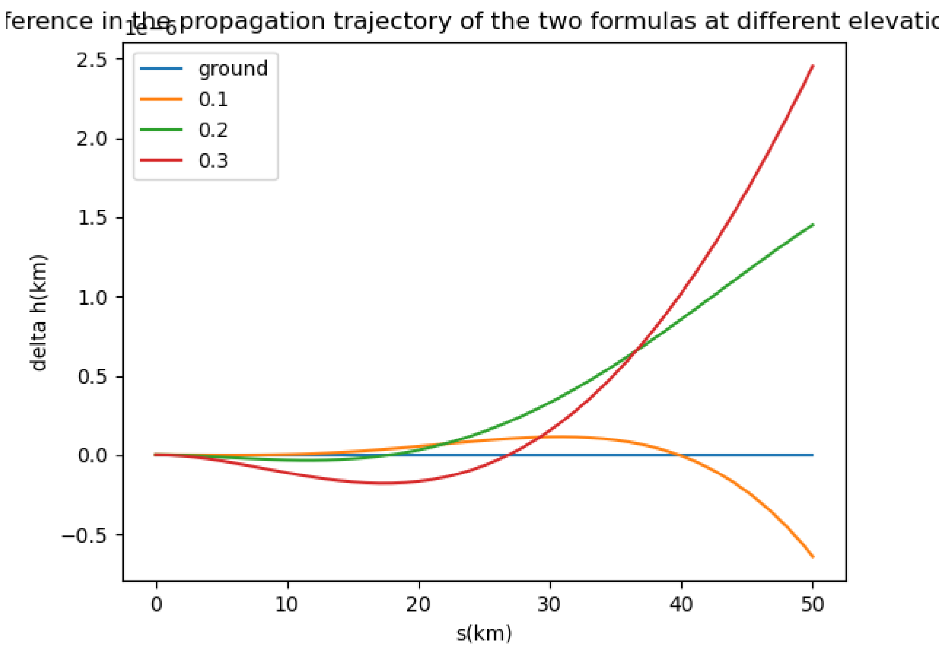


Q3 如下圖，在低仰角高 dn/dh 時電磁波傳播可能有 traped 的現象

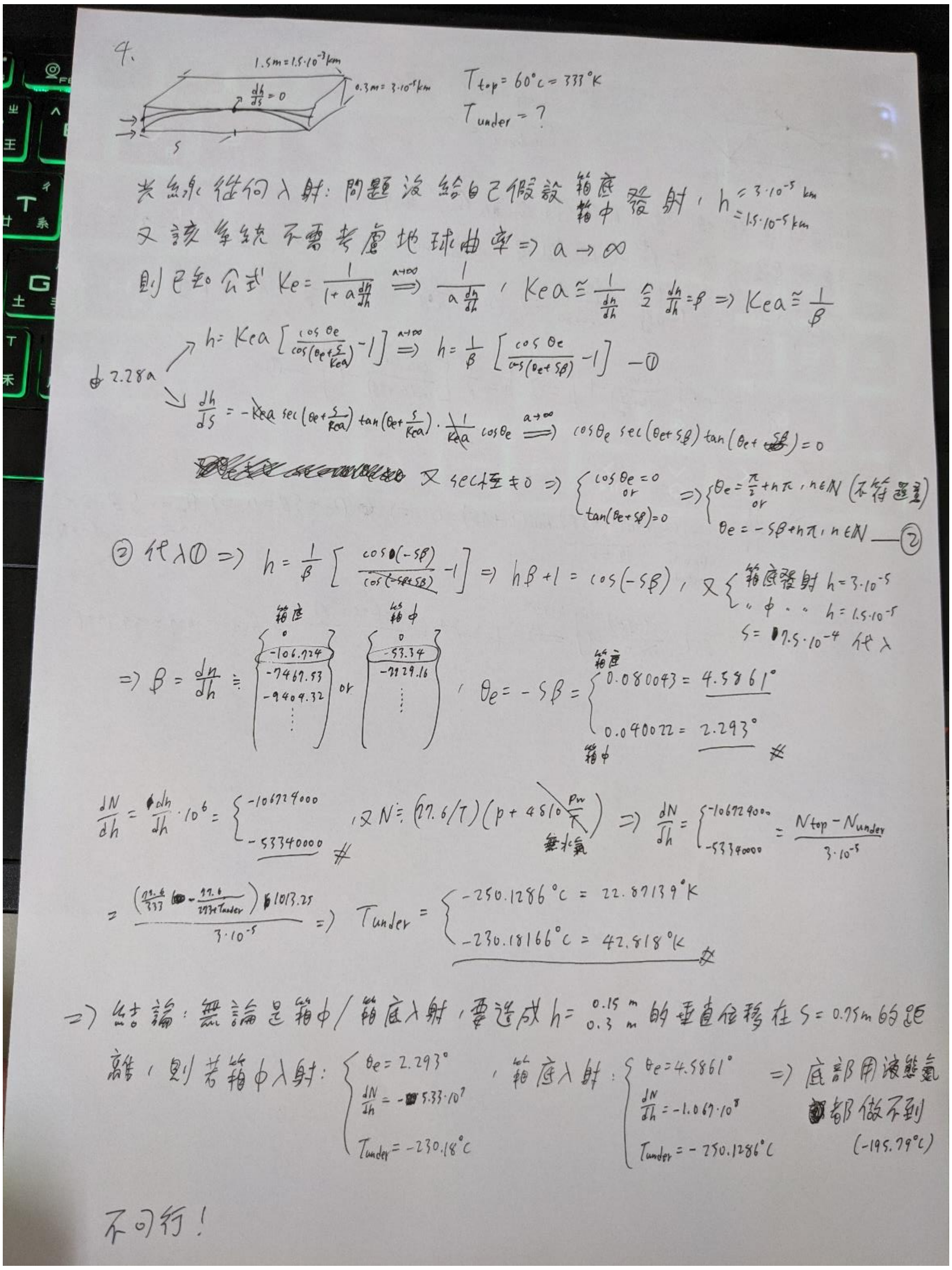



同樣將 2.28a-2.30，可發現在不同仰角時，2.28 與 2.30 算出的 h 會不同，並

且沒有明顯的大小關係



Q4 推導過程如下圖，其中 dn/dh 以數值方法求解如下下圖



4.

1.5m = 1.5·10⁻³km　0.3m = 3·10⁻⁴km　$\frac{dh}{ds}=0$　s

$T_{top} = 60°c = 333°K$

$T_{under} = ?$

光線從何入射：問題沒給自己假設箱底/箱中發射，$h = 3\cdot10^{-5}$ km / $= 1.5\cdot10^{-5}$ km

又該系統不需考慮地球曲率 $\Rightarrow a \to \infty$

則已知公式 $K_e = \frac{1}{1+a\frac{dn}{dh}} \xRightarrow{a\to\infty} \frac{1}{a\frac{dn}{dh}}$，$K_e a \cong \frac{1}{\frac{dn}{dh}}$ 令 $\frac{dn}{dh} = \beta \Rightarrow K_e a \cong \frac{1}{\beta}$

由 2.28a

$h = K_e a\left[\frac{\cos\theta_e}{\cos(\theta_e + \frac{s}{K_e a})} - 1\right] \xRightarrow{a\to\infty} h = \frac{1}{\beta}\left[\frac{\cos\theta_e}{\cos(\theta_e + s\beta)} - 1\right]$ —①

$\frac{dh}{ds} = -\cancel{K_e a}\sec(\theta_e + \frac{s}{K_e a})\tan(\theta_e + \frac{s}{K_e a})\cdot\frac{1}{\cancel{K_e a}}\cos\theta_e \xRightarrow{a\to\infty} \cos\theta_e\sec(\theta_e + s\beta)\tan(\theta_e + s\beta) = 0$

又 sec 恆 ≠ 0 $\Rightarrow \begin{cases}\cos\theta_e = 0 \\ \text{or} \\ \tan(\theta_e + s\beta) = 0\end{cases} \Rightarrow \begin{cases}\theta_e = \frac{\pi}{2} + n\pi,\ n\in\mathbb{N}\ (\text{不符題意}) \\ \text{or} \\ \theta_e = -s\beta + n\pi,\ n\in\mathbb{N}\end{cases}$ —②

② 代入 ① $\Rightarrow h = \frac{1}{\beta}\left[\frac{\cos(-s\beta)}{\cancel{\cos(-s\beta+s\beta)}} - 1\right] \Rightarrow h\beta + 1 = \cos(-s\beta)$，又 $\begin{cases}\text{箱底發射 } h = 3\cdot10^{-5} \\ \text{〃 中 〃 〃 } h = 1.5\cdot10^{-5}\end{cases}$ $s = 7.5\cdot10^{-4}$ 代入

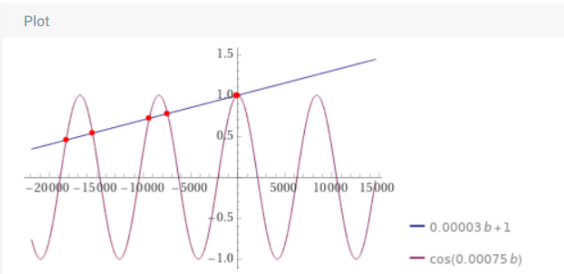
$\Rightarrow \beta = \frac{dn}{dh} \doteq$ 箱底 $\begin{Bmatrix}0 \\ -106.724 \\ -7467.53 \\ -9409.32 \\ \vdots\end{Bmatrix}$ or 箱中 $\begin{Bmatrix}0 \\ -53.34 \\ -7729.16 \\ \vdots\end{Bmatrix}$，$\theta_e = -s\beta = \begin{cases}\text{箱底 } 0.080043 = 4.5861° \\ \text{箱中 } 0.040022 = 2.293°\end{cases}$ #

$\frac{dN}{dh} = \frac{dn}{dh}\cdot10^6 = \begin{cases}-106724000 \\ -53340000\end{cases}$ #，又 $N \doteq (77.6/T)(p + \cancel{4810\frac{p_w}{T}})$ 無水氣 $\Rightarrow \frac{dN}{dh} = \begin{cases}-106724000 \\ -53340000\end{cases} = \frac{N_{top} - N_{under}}{3\cdot10^{-5}}$

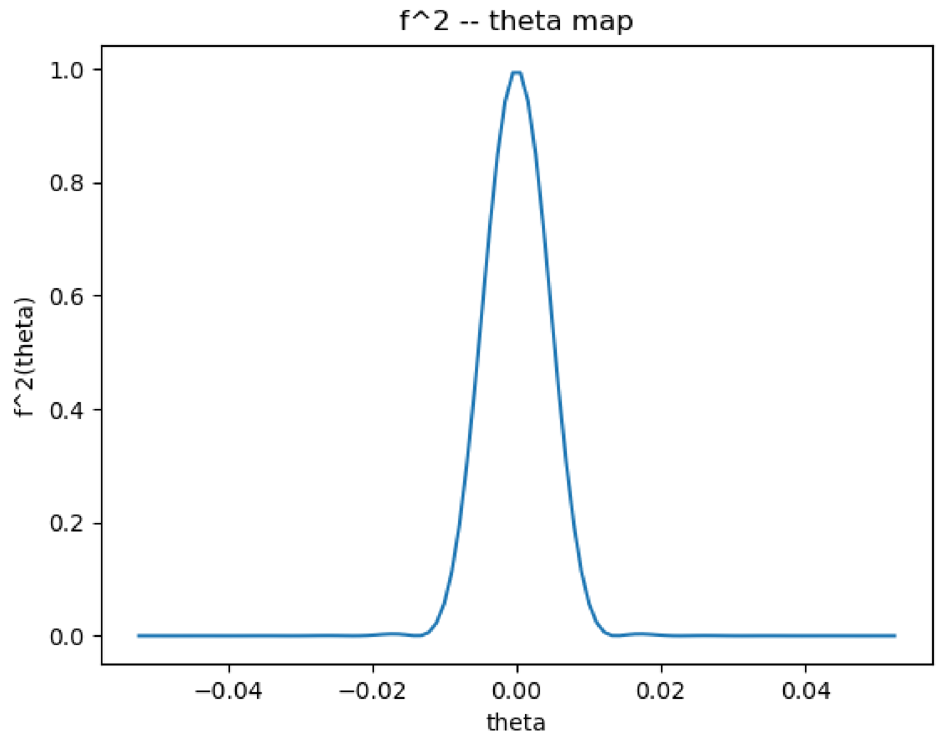
$= \frac{\left(\frac{77.6}{333} - \frac{77.6}{273+T_{under}}\right)1013.25}{3\cdot10^{-5}} \Rightarrow T_{under} = \begin{cases}-250.1286°c = 22.87139°K \\ -230.18166°c = 42.818°K\end{cases}$ #

$\Rightarrow$ 結論：無論是箱中/箱底入射，要造成 $h = \begin{matrix}0.15\text{ m} \\ 0.3\text{ m}\end{matrix}$ 的垂直位移在 $s = 0.75$m 的距離，則若箱中入射：$\begin{cases}\theta_e = 2.293° \\ \frac{dN}{dh} = -5.33\cdot10^7 \\ T_{under} = -230.18°c\end{cases}$，箱底入射：$\begin{cases}\theta_e = 4.5861° \\ \frac{dN}{dh} = -1.067\cdot10^8 \\ T_{under} = -250.1286°c\end{cases}$ $\Rightarrow$ 底部用液態氮都做不到 (−195.79°c)

不可行！



Q5 如圖，明顯有個主峰值在 theta=0 處，並且有微弱的 side log




放大 0.01~0.02 區間可看到確實有 side log 的情況，但相當小