The derivation of equations (2.20a) and (2.20b)

Starting from Snell's law for spherically stratified media:

$$R \cdot n(R) \cdot \cos\theta = \text{constant} = C \qquad (1)$$

when $R = a$, $\theta = \theta_e$ (at surface) ➔ $C = an(0)\cos\theta_e$

From Fig A.1, R is the earth's radius plus the height reached by the EM waves

$$d\ell = \left[(R \cdot d\psi)^2 + (dR)^2\right]^{1/2}$$

$$\cos^2\theta = \frac{(R \cdot d\psi)^2}{d\ell^2} = \frac{(R \cdot d\psi)^2}{(R \cdot d\psi)^2 + (dR)^2} = \frac{R^2}{R^2 + (\frac{dR}{d\psi})^2}$$

$$\cos^2\theta \cdot R^2 + \cos^2\theta \cdot (\frac{dR}{d\psi})^2 = R^2$$

$$\cos^2\theta \cdot (\frac{dR}{d\psi})^2 = R^2 \sin^2\theta, \qquad (\frac{dR}{d\psi})^2 = R^2 \tan^2\theta$$

therefore：the change of angle ψ w.r.t. R $\frac{d\psi}{dR} = \frac{1}{R}\cot\theta$

$$\psi(R') = \int_{R_0}^{R'} \frac{\cot\theta}{R} dR + \psi(R_0)$$

a known number related to the radar's position

ψ times a is the distance of the great arch along the earth surface

$$S = a \cdot \psi(R_2) - a \cdot \psi(R_1) = \int_{R_1}^{R_2} \frac{a\cot\theta}{R} dR$$

From equation (1) $\cos\theta = \frac{C}{R \cdot n(R)}$ → $\cot\theta = \frac{C}{(R^2 n^2 - C^2)^{1/2}}$

The above equation becomes:

$$\int_{R_1}^{R_2} \frac{aC}{R(R^2 n^2 - C^2)^{1/2}} dR$$

$R = a + h'$ $\qquad dR = dh'$

R: a to *a+h*

h: 0 to *h*

thus,

$$S(h) = \int_0^h \frac{aC\,dh'}{(a+h')[(a+h')^2 n^2(h') - C^2]^{1/2}} \qquad (2.20a)$$

The decrease of P (or T) will cause N to decrease (or increase). In the atmosphere, both P and T decrease with height. However, since the fractional decrease in P is larger than in T in the troposphere, the N normally decreases with height

When radar elevation angle $\cong 0$ and $\frac{dN}{dh} \le -157 km^{-1}$ (derived later), h is the height, EM beams are bent toward the ground.

The bending phenomena can easily take place when inversion occurs (dT/dZ > 0)

Spherically stratified Atmosphere

Assume $T = T(Z)$, $q_v = q_v(Z)$

$$S(h) = \int_0^h \frac{aCdh'}{R\left[R^2 n^2(h') - C^2\right]^{1/2}} \qquad (2.20a)$$

$$C = an(0)\cos\theta_e \qquad (2.20b)$$

$S(h)$: great circle distance (along earth sfc)

a: earth radius

R: $a + h'$

h: height above the sfc

$n(0)$: refractive index at the radar location

θ_e : elevation angle of the radar

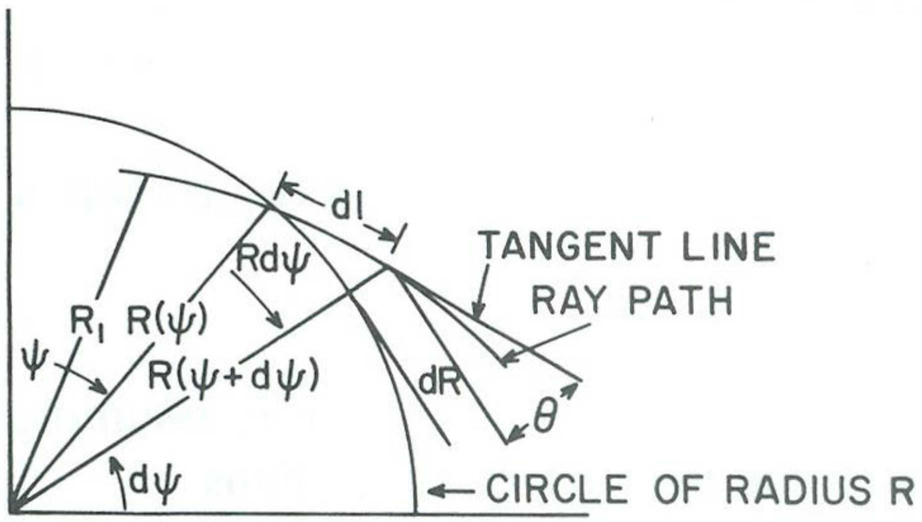

Fig. A.1 Ray path in a spherically stratified medium.