Graph Based Algorithms for Generating Region Proposals for Object Detection

Lecture Notes on Deep Learning

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Preamble

The problem of object detection and localization that I discussed in the previous lecture becomes more complicated if there is too much competing information in an image.

Just imagine an autonomous trying to identify and localize a stop sign against all the background clutter.

Although we are still ways from creating a 100% foolproof solution for solving a problem like the one mentioned above, we now have techniques that can at least eliminate those portions of an image where we can be sure that the object we are looking for does not exist. Separating out such areas in an image leaves a smaller portion of the image in which we can focus our search for the presence of the object.

The goal of this lecture is to introduce you to some basic computer vision tools that you can bring to bear on making smaller the portion of the image where your search for the object is likely to be more productive.

Preamble (contd.)

Here is making the same point that I made on the previous slide but in a different way: Let's say you want to detect an object(s) in an image, but you neither know where exactly in the image it might be located, nor are you aware of the scale at which the object might have manifested itself.

Under these conditions, in the old days, at each scale of detection, you would run a sliding window through the image, moving it from one pixel to the next, and, at each position of the window, test whether the pixels inside the window speak to the presence of the object.

The main problem with the approach described above is its high computational overhead, especially when an image is mostly empty (with regard to the presence of the object). Detecting objects in a 500×500 image at, say, 4 different scales would involve testing for the presence of the object in a million different windows. Assuming that at most one of these windows contains the object fully, all of the computations in all the other windows would be wasted.

Preamble (contd.)

To get around this computational overhead, more recent research in object detection has taken to first creating region proposals in an image, these being pixel blobs that "look different" from the general background in an image, and then applying the object detection algorithm to just those regions. By looking different I mean that such regions hold clues to the presence of the objects you are looking for.

The flip side of forming region proposals is region discounting, which means eliminating regions of an image that are not likely to contain the objects of interest.

For both the region proposal formation and region discounting, the idea is to first subject an image to some graph-based algorithmic processing to quickly demarcate the regions that are either highly likely to contain the objects of interest or not likely to contain them at all. Subsequently, the regions that survive as highly likely to contain an object are subject to object detection by, say, a CNN.

Preamble (contd.)

I'll start by introducing you to a graph-based algorithm that allows you to aggregate the pixel that are similar to one another in some sense and that, taken together, are dissimilar to the other pixels in the image.

Of the graph-based approaches for forming region proposals, one of the most effective is the Selective Search (SS) algorithm by Uijlings, van de Sande, Gevers, and Smeulders. Perhaps the most important aspect of SS is that it sits on top of the graph-based image segmentation algorithm of Felzenszwalb and Huttenlocher (FH).

I'll show my implementations of both FH and SS through code fragments from my RegionProposalGenerator module that you can download from

https://pypi.org/project/RegionProposalGenerator/

I'll refer to this module by the acronym RPG in this lecture.

Outline

- Graph Based Algorithms for Generating Region Proposals
- 2 The Felzenszwalb and Huttenlocher (FH) Algorithm
- 3 RPG's Implementation of the FH Algorithm
- **4** The Selective Search Algorithm
- 5 SS Algorithm as Implemented in RPG

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What Do the Graph Based Algorithms Do?

- Graph based algorithms represent an image with a graph G = (V, E)in which, at the beginning, the vertices in V are the individual pixels and an edge in E between a pair of vertices is a measure of the similarity between those pixels.
- In the simplest cases, the similarity may depend directly on the difference between the color values at the pixels and also on how far apart the pixels are. In other cases, the value of similarity may depend on a comparison of the neighborhoods around the pixels.
- The goal of such algorithms is to partition an image into regions so that the total similarity weight in each region is maximized, while it is minimized for all pixel pairs for which the pixels are in two different regions.
- In other words, we want to parition a graph G into disjoint collections of vertices so that the vertices in each collection are maximally similar, while, at the same time, the collections are maximally Purdue University
 dissimilar from one another

What Do the Graph Based Algorithms Do? (contd.)

 A good tutorial paper to read for such algorithms is "A Tutorial on Spectral Clustering" by Luxburg that is available at:

https://arxiv.org/abs/0711.0189

A central notion in such algorithms is that of graph Laplacian, as you
will see in this lecture. It is the eigendecomposition of the graph
Laplacian that can yield the region proposals. The algorithms of the
sort we will talk about are also known as the graph spectral clustering
algorithms.

Representing an Image with a Graph

- Let's say an image has N pixels, with each pixel represented by the index i whose values go from 0 through N-1.
- Let w_{ij} express the similarity of the pixel i to pixel j. The quantity w_{ij} could depend on, say, (1) the color difference at the two pixels; and (2) inversely on the distance between the two pixels.
- We could also make w_{ij} proportional to some attribute of local grayscale or color variations in the neighborhoods around the pixels i and j, as used in the Census Transform.
- As mentioned earlier, we represent an image by a graph G = (V, E), where the *vertex set* V is the set of pixels, indexed 0 through N-1, and E the set of edges between the vertices. Between the vertices corresponding to the two pixels i and j, $E_{ij} = w_{ij}$.

Representing an Image with a Graph (contd.)

- Although our overall goal is to partition V into subsets $\{V_1, V_2,, V_N\}$ so that the similarity weight within each parition is the highest it can be and across all pairs of partitions the lowest it can be, in the rest of the discussion we will focus specifically on the problem of bipartition, meaning creating two disjoint partitions A and B from the graph subject to some optimality criterion.
- Given a partition (A, B) of V, we can associate a value cut(A, B) with the partition as follows:

$$cut(A, B) = \sum_{i \in A, j \in B} w(i, j)$$

• The edges that go from any vertex in A to any vertex in B are referred to as the **cutset** of the partition. Some folks like to use the word **cutset** only for an optimum partition in which the value of cut(A, B) is the least it can be.

To Gain Insights in What cut(A, B) Stands For

- We consider (A, B) to be an 'optimum' partition if, of all possible choices for (A, B) partitions, it minimizes the value of cut(A, B).
- Using general graph-theoretic terminology, the problem we want to solve is this: Given a general attributed graph in which each edge carries a numerical weight, what is the minimal-weight set of vertices that must be disconnected in order to bipartition the graph?
- Consider a simple graph (that obviously does not represent an image):



• In general, there will exist multiple cutsets for a graph. For example, for the graph shown above, one solution consists of the edges $\{\{b,h\},\{d,g\}\}$, another solution consists of $\{\{b,h\},\{d,g\},\{g,h\}\}$,

Purdund viet silvother of the cutset $\{\{b,h\},\{d,g\},\{b,d\}\}$, and so on.

Gaining Insights in What cut(A, B) Stands For (contd.)

- For the example on the previous slide, a solution that minimizes the cutset weight consists of using edges $\{\{b,h\},\{d,g\}\}$ as the cut set. This optimum solution has a cutset weight of 10.
- Note that the optimum cutset is not unique in the example on the previous slide. Here is another cutset that also has the cutset weight of 10: $\{\{h,i\},\{k,m\}\}$. There exist additional solutions also. For example, $\{\{m,j\},\{i,j\}\}$, with one partition containing only one vertex, etc.
- For another example, consider the following graph:



Gaining Insights in What cut(A, B) Stands For (contd.)

 As with the previous example, we again have a number of cutsets, especially if we include cutsets that result in one-vertex partitions. Of all the solutions that are possible, the following four are particularly interesting:

```
Solution 1: cutset: { {b,h}, {d,g} } cutset weight = 10

Solution 2: cutset: { {b,h}, {g,h} } cutset weight = 13

Solution 3: cutset: { {a,b}, {e,f} } cutset weight = 10

solution 4: cutset: { {g,k} } cutset weight = 5
```

The optimum solution corresponds to Solution 4 with a cutset weight of 5.

Relevance of the Min-Cut Solution to Computer Vision

- It was shown by Greig, Porteous, and Seheult in a paper "Exact
 Maximum A Posteriori Estimation for Binary Images" way back in
 1989 that the problem of finding the best MAP solution to the
 restoration of binary images can be cast as a min-cut problem. The
 solution obtained with min-cut was superior to the one obtained with
 simulated annealing.
- The best way to solve a min-cut problem is with the max-flow algorithms. These algorithms have low-order polynomial complexity.
- As for the name "max-flow" for the algorithms, one can show that in a network of pipes for transporting, say, oil, the maximum flow capacity between any two given points is determined by the pipes that are in the min-cut of the graph that describes the pipe network.

Moving on to Normalized Cuts

- Unfortunately, the min-cut solutions do not always work for solving computer vision problems. They frequently result in highly unbalanced graph bipartitions, unbalanced to the extent that one of the partitions may consist of just a single pixel.
- Of the graph-based algorithms, what has "worked" for image partitioning is the minimization of the Normalized Cut criterion. This criterion, denoted Ncut, seeks a bipartition (A, B) of a graph G = (V, E) that minimizes the following:

$$NCut(A, B) = \frac{cut(A, B)}{assoc(A, V)} + \frac{cut(B, A)}{assoc(B, V)}$$

 $assoc(A, V) = \sum_{u \in A, t \in V} w_{ut}$

The Shi-Malik Algorithm for Minimizing Neut

- Define an indicator vector \vec{x} of size N assuming V is of cardinality N. The i^{th} element x_i of the vector \vec{x} is +1 if vertex i is in A. Otherwise, x_i is -1.
- Define an N element vector $\vec{1}$ as consisting of all 1s.
- We can now express cut(A, B) as

$$cut(A,B) = \sum_{x_i > 0, x_j < 0} -w_i j x_i x_j$$

• We associate with each vertex $v_i \in V$ the degree d_i defined by

$$d_i = \sum_{j=1}^N w_{ij}$$

Obviously, d_i is the sum of all the similarity weights emanating from the vertex i in the graph.

The Shi-Malik Algorithm for Minimizing Neut (contd.)

- We place all the node degrees, d_i 's, on the diagonal of an $N \times N$ matrix D, with its all other elements set to 0. D is called a *degree* matrix. Along the diagonal, the i^{th} element of D is d_i .
- In terms of the indicator vector elements x_i and the degrees d_i , we can now express the formula for the normalized cut as

$$Ncut(A, B) = \frac{\sum_{x_i > 0, x_j < 0} -w_{ij}x_ix_j}{\sum_{x_i > 0} d_i} + \frac{\sum_{x_i < 0, x_j > 0} -w_{ij}x_ix_j}{\sum_{x_i < 0} d_i}$$

The ratios on the right can be expressed more compactly as

$$4Ncut(\vec{x}) = \frac{(\vec{1} + \vec{x})^T (D - W)(\vec{1} + \vec{x})}{k\vec{1}^T D\vec{1}} + \frac{(\vec{1} - \vec{x})^T (D - W)(\vec{1} - \vec{x})}{(1 - k)\vec{1}^T D\vec{1}}$$

where W is the matrix representation of the similarity weights w_{ij} .

The unit vector $\vec{1}$ was defined on the previous slide. The quantity k is given by: $\sum_{i=1}^{n} d_i$

$$k = \frac{\sum_{x_i > 0} d_i}{\sum_{i=1}^{N} d_i}$$

The Shi-Malik Algorithm for Minimizing Neut (contd.)

- Note that k is a normalized sum of all the similarity weights in just the partition A. Therefore, 1-k would be a normalized sum of all the similarity weights in just the partition B.
- The expression for $Ncut(\vec{x})$ shown on the previous slide can be further simplified to

$$Ncut(\vec{x}) = \frac{\vec{y}^T (D - W) \vec{y}}{\vec{y}^T D \vec{y}}$$

$$\vec{y} = (\vec{1} + \vec{x}) - b(\vec{1} - \vec{x})$$

$$b = \frac{k}{1 - k}$$

• The form D-W is famous unto itself. Recall that D is a diagonal matrix whose i^{th} element is sum of all the similarity weights emanating from the vertex i in the graph.

Graph Laplacian and Its Properties

- The matrix L = D W is known as the graph Laplacian of a similarity matrix W. It has the following interesting properties:
 - 1 L is symmetric and positive semidefinite;
 - 2 Its smallest eigenvalue is always 0 and the corresponding eigenvector is $\vec{1}$, meaning a vector of all 1's.; and
 - 3 All of its eigenvalues are non-negative.
- On the next slide you will see a variant of the graph Laplacian known as the symmetric normalized graph Laplacian and given by

$$L_{sym} = D^{-\frac{1}{2}}(D-W)D^{-\frac{1}{2}}$$

• The form for $Ncut(\vec{x})$ shown on the previous slide is an example of what is known as the Raleigh Quotient that looks like

$$R(A, \vec{x}) = \frac{\vec{x}^T A \vec{x}}{\vec{x}^T \vec{x}}$$

Using the Raleigh Quotient Exactly

• We get the Raleigh Quotient form exactly for Ncut if set

$$\vec{y} = D^{-\frac{1}{2}}\vec{z}$$

• Substituting the above in the formula for Ncut, we get

$$\min_{\vec{x}} Ncut(\vec{x}) = \min_{\vec{z}} \frac{\vec{z}^T L_{sym} \vec{z}}{\vec{z}^T \vec{z}}$$

where L_{sym} was defined on the previous slide.

- It follows from the properties of the graph Laplacian as stated on the previous slide that L_{sym} is also symmetric positive semidefinite, that its smallest eigenvalue is 0 and that the corresponding eigenvector is $\vec{z} = D^{-\frac{1}{2}}\vec{1}$.
- We therefore use the next to the smallest eigenvalue and its corresponding eigenvector as the solution for \vec{x} for bipartitioning a graph.

Some Results Obtained with Ncut Minimization and Clustering

Here is a result from our own paper:

https://engineering.purdue.edu/RVL/Publications/Martinez04OnCombining.pdf

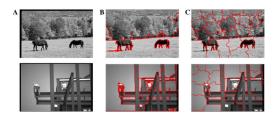


Fig. 9. For the two images shown, it is necessary to extract several segments corresponding to highly localized detail. (A) Original images. (B) Segmentations obtained using Koontz–Fukunaga clustering with $e_2 = 100$. (C) Segmentations obtained using k-means clustering with $e_2 = 100$.

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The Felzenszwalb and Huttenlocher (FH) Algorithm

 The graph-based algorithm by Felzenszwalb and Huttenlocher has emerged as a strong competitor to the graph spectral clustering based method described in the previous section. You can download the FH paper from:

http://people.cs.uchicago.edu/~pff/papers/seg-ijcv.pdf

- FH algorithm is a recursive merging algorithm in the graph G=(V,E) representation of an image in which initially the vertices in V represent the individual pixels and the edges in E represent the pairwise similarity between the pixels. Subsequently, each vertex in V represents a blob obtained by previous merging steps, and each edge a measure of similarity between a pair of blobs.
- FH bases its blob merging decisions on two quantities: (1) the largest inter-pixel color difference at adjacent pixels within each blob; and (2) the smallest inter-pixel difference for a pair of pixels that are in

Purduo different blobs.

The FH Algorithm (contd.)

- The largest value of the inter-pixel color difference at a pair of adjacent pixels within a blob is represented by Int(u) for a vertex u in the graph.
- In order to account for the fact that, at the beginning, each vertex consists of only one pixel [which would not allow for the calculation of Int(u)], the unary property of the pixels at a vertex is extended from Int(u) to MInt(u) with the addition of a vertex-size dependent number equal to k/|C| where k is a user-specified parameter and |C| the cardinality of the set of pixels represented by the vertex u in the graph.
- As mentioned above, initially the edges in the graph representation of an image are set to the color difference between the two 8-adjacent pixels that correspond to two different vertices, meaning to two different blobs.

The FH Algorithm (contd.)

- That is, the edge E(u, v) between two vertices u and v in the graph is set to the smallest value of the inter-pixel color difference for two adjacent pixels that belong to the two vertices.
- At each iteration of the algorithm, two vertices u and v are merged provided E(u,v) is less than the smaller of the MInt(u) or MInt(v) attributes at the two vertices. My experience is that for most images the algorithm terminates of its own accord after a small number of iterations while the vertex merging condition can be satisfied.
- Since the algorithm is driven by the color differences between 8-adjacent pixels, the FH algorithm is likely to create too fine a segmentation of an image. The segments produced by FH can be made larger by using the logic of SS that allows blobs of pixels to merge into larger blobs provided doing so makes sense based on the inter-blob values for mean color levels, color variances, texture values,

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RPG's Implementation of FH

- In FH, the fundamental condition for merging the pixel blobs corresponding to two different vertices u and v is: The value of the E[u,v], which keeps track of the smallest inter-pixel color difference for two pixels that are each other's 8-neighbors, with one pixel in blob u and the other in blob v, must be less than the smaller of the Int values for either of the two blobs.
- In order to carry out this comparison efficiently, the code you see in Slide 31 also associates another attribute with each edge, Mint[u,v], that stores the minimum of the Int values at u and v.
- When two blobs u and v are merged, they must both be dropped from the list of blobs maintained by the system and we must also drop the edge corresponding to them.
- It stands to reason that the edges that do not satisfy the condition for merging of the blobs represented by the vertices at their two ends should be eiliminated from further consideration. This is what is Purdue University
 accomplished by the statements labeled (B) (C) and (D) in Slide 31

RPG's Implementation of FH (contd.)

- After the step at the end of the last slide, all other edges are good candidates for the merging of the blobs represented by their vertex ends.
- From the remaining edges, we now take up one edge at a time in order of inreasing value of the Mint value associated with the edges.
 This is on account of the call to sorted() in line (A).
- Therefore the edge whose vertices we want in line (E) is the one with the smallest value of Mint value, that is, with the smallest of either of the two values for Int attribute at the two vertices that define the edge.
- One thing that requires care in coding FH is that we want to go through all the edges in the sorted_edges while we are deleting the vertices that are merged and the edges that are no longer relevant because of vertex deletion.

RPG's Implementation of FH (contd.)

- You have to be careful when debugging the code in the the main for loop. The problem is that the sorted edge list is made from the ogiginal edge list which is modified by the code in the for loop.
- Let's say that the edge (u,v) is a good candidate for the merging of the pixel blobs corresponding to u and v. After the main for loop shown on the next slide has merged these two blobs corresponding to these two vertices, the u and v vertices in the graph do not exist and must be deleted. Deleting these two vertices requires that we must also delete from E all the other edges that connect with either u and v.
- Shown on the next slide is RPG's main loop that implements the FH algorithm.

RPG's Implementation of FH (contd.)

```
sorted_vals_and_edges = list( sorted( (v,k) for k,v in E.items() ) )
sorted_edges = [x[1] for x in sorted_vals_and_edges]
edge counter = 0
for edge in sorted_edges:
                                                                                                     mm (4)
   if edge not in E: continue
   edge counter += 1
   if E[edge] > MInt[edge]
                                                                                                     ## (B)
       del E[edge]
                                                                                                     ## (C)
        del MInt[edge]
                                                                                                     ** (D)
        continue
   ### Let us now find the identities of the vertices of the edge whose two vertices
   ### are the best candidates for the merging of the two pixel blobs.
   vert1.vert2 = int(edge[:edge.find('.')]), int(edge[edge.find('.')+1 :])
                                                                                                     ## (E)
   if (vert1 not in V) or (vert2 not in V): continue
   affected edges = |
   for edg in E:
        end1,end2 = int(edg[:edg.find(',')]), int(edg[edg.find(',')+1 :])
        if (vert1 == end1) or (vert1 == end2) or (vert2 == end1) or (vert2 == end2):
            affected edges.append(edg)
       print("\n\naffected edges to be deleted: %s" % str(affected edges))
   for edg in affected edges:
        del E[edg]
        del MInt[edg]
   merged blob = V[vert1] + V[vert2]
   V[index for new vertex] = merged blob
   if self.debug:
       print("\n\n\filter Index: %d] index for new vertex: %d and the merged blob: %s" % (master iteration index, index for new vertex, str(merwed blob)))
        We will now calculate the Int (Internal Difference) and MInt property to be
   ### to be associated with the newly created vertex in the graph:
   within blob edge weights = []
   for u1 in merged blob;
       i = u1[0] * arr width + u1[1]
       for u2 in merged blob:
            1 = u2[0] * arr width + u2[1]
            if i > 1:
                ij_key = "%d,%d" % (i,j)
                if ij_key in initial_graph_edges:
                    within blob edge weights.append( initial graph edges[ ii kev ] )
   Int_prop[index_for_new_vertex] = max(within_blob_edge_weights)
   IRT_prop[index_ior_new_vertex; = max_vertex_jor_new_vertex; + kay / float(len(merged_blob))
### Now we must calculate the new graph edges formed by the connections between the newly
   ### formed node and all other nodes. However, we first must delete the two nodes that
        we just merged:
   del V[verti]
   del V[vert2]
   del Int_prop[verti
   del Int_prop[vert2]
   del MInt_prop[verti
   del MInt_prop[vert2]
       if v == index_for_new_vertex: continue
            we need to store the edge weights for the pixel-to-pixel edges
            in the initial graph with one pixel in the newly constructed
        ### blob and other in a target blob
        pixels_in_v = V[v]
        for u_pixel in merged_blob:
            i = u_pixel[0] * arr_width + u_pixel[1]
            inter_blob_edge_weights = []
            for v_pixel in pixels_in_v:
                j = v_pixel[0] * arr_width + v_pixel[1]
                    ij_key = "%d,%d" % (i,j)
                    ij_key = "%d,%d" % (j,i)
                if ij_key in initial_graph_edges:
                    inter_blob_edge_weights.append( initial_graph_edges[ij_key ] )
            if len(inter_blob_edge_weights) > 0:
                uv_key = str("%d,%d" % (index_for_new_vertex,v))
                E[uv_key] = min(inter_blob_edge_weights)
                MInt[uv_key] = min( MInt_prop[index_for_new_vertex], MInt_prop[v] )
```

Some Results Obtained with FH as Reported by the Authors



Figure 3: A baseball scene (432×294 grey image), and the segmentation results produced by our algorithm ($\sigma = 0.8, k = 300$).



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The Selective Search (SS) Algorithm for Region Proposals

 If for whatever reason an algorithm for generating region proposals is creating too fine a division of the image, you can use the Selective Search (SS) algorithm proposed by Uijlings, van de Sande, Gevers, and Smeulders for merging them into larger proposals. You can access their paper at:

http://www.huppelen.nl/publications/selectiveSearchDraft.pdf

 In the publication mentioned above, the logic of SS sits on top of the image partitions produced by the FH algorithm and that's what I have also implemented in my previously mentioned Python module RegionProposalGenerator. I'll refer to this module by RPG in the rest of this section.

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The Selective Search (SS) Algorithm for Region Proposals

- In RPG, the recursive blob-merging logic of SS is based on comparing the adjacent pixel blobs on the basis of the following four properties:
 - pairwise adjacency
 - 2 pairwise comparison of color homogeneity
 - opairwise comparison of grayscale variance
 - pairwise comparison of LBP textures

To order to appreciate what is meant by color homogeneity and LBP (Local Binary Patterns) texture values, I'll refer to reader to the following report by me:

https://engineering.purdue.edu/kak/Tutorials/TextureAndColor.pdf

 The RPG module maintains an ever-increasing integer index as the ID for each blob and associates a pair_id with every pair of blobs discovered in an image as shown in the code fragment shown on the next slide.

SS Algorithm (contd.)

 The comparative properties for each pair are calculated as shown below and subsequently stored in the dictionary all_pairwise_similarities:

```
for blob_id_1 in pixel_blobs:

for blob_id_2 in pixel_blobs:

pair_id = str('Al_A'', 'Al_A'', '
```

The unary properties of the blobs are calculated as follows. Note that
we consider the blobs in the reverse order of their sizes — just in case
we want to ignore the very smallest of the blobs. With the option
reverse=True,sorted returns a list in the decreasing order of the
sorting criterion:

```
sorted_blobs = sorted(pixel_blobs, key=lambda x: len(pixel_blobs[x]), reverse=True)
for blob_id in sorted_blobs
    pixel_blob = pixel_blobs[blob_id]
    pixel_vals_color = [im_array_color[pixel[0],pixel[1],:].tolist() for pixel in pixel_blob]
    pixel_vals_color = [im_array_color[pixel[0],pixel[1] for pixel in pixel_blob])
    pixel_vals_gray = np.array([im_array_gray[pixel] for pixel in pixel_blob])
    color_seen_vals[blob_id] = finet(sem([pix[]] for pixel in pixel_vals_color])) / float(len(pixel_vals_color)) for j in range(3) ]
    gray_seen_vals[blob_id] = np.seen(pixel_vals_gray)
    texture vals[blob_id] = estimate blo texture(pixel_blob_in array gray)
```

SS Algorithm (contd.)

Shown below is RPG's logic for the merging of the blobs recursively:

```
while ss_iterations < 1:
   sorted_up_blobs = sorted(merged_blobs, key=lambda x: len(merged_blobs[x]))
    sorted_down_blobs = sorted(merged_blobs, key=lambda x: len(merged_blobs[x]), reverse=True)
   for blob_id_1 in sorted_up_blobs:
        if blob id_1 not in merged_blobs: continue
                                                        # the largest blob is typically background
        for blob_id_2 in sorted_down_blobs[:-1]:
            if blob_id_2 not in merged_blobs: continue
            if blob id_1 not in merged_blobs: break
            if blob_id_1 > blob_id_2:
                pair_id = "%d,%d" % (blob_id_1,blob_id_2)
                if (pairwise_color_homogeneity_val[pair_id][0] < self.color_homogeneity_thresh[0])\
                  (pairwise_color_homogeneity_val[pair_id][i] < self.color_homogeneity_thresh[i])\
                   (pairwise_color_homogeneity_val[pair_id][2] < self.color_homogeneity_thresh[2])\
                   (pairwise_gray_var_comp[pair_id] < self.gray_var_thresh)
                  (pairwise texture comp[pair id] < self.texture homogeneity thresh):
                   if self.debug
                       print("\n\n\nmerging blobs of id %d and %d" % (blob id 1. blob id 2))
                   new merged blob = merged blobs[blob id 1] + merged blobs[blob id 2]
                   merged blobs next blob id = new merged blob
                   del merged blobs[blob id 1]
                   del merged blobs blob id 2
                   ### We need to estimate the unary properties of the newly created
                   eee blobs
                   pixel_vals_color = [im_array_color[pixel[0],pixel[1],:].tolist() for pixel in
                                                                            new merged blob]
                   pixel_vals_gray = np.array([im_array_gray[pixel] for pixel in new_merged_blob])
                   color mean vals[next blob id] = [float(sum([pix[i] for pix in \
                       pixel vals color)) / float(len(pixel vals color)) for i in range(3)]
                   gray_mean_vals[next_blob_id] = np.mean(pixel_vals_gray)
                   gray_vars[next_blob_id] = np.var(pixel_vals_gray)
                   texture vals[next blob id] = estimate lbp texture(new merged blob, im array gray)
                   ### Now that we have merged two blobs, we need to create entries
                   ### in pairwise dictionaries for entries related to this new blob
                   for blb id in sorted up blobs:
                        if blb id not in merged blobs: continue
                       if next_blob_id > blb_id:
                            pair id = "%d.%d" % (next blob id. blb id)
                            pairwise adjacency[pair id] = \
         True if are_two_blobs_adjacent(new_merged_blob, pixel_blobs[blb_id]) else False
                            pairwise_color_homogeneity_val[pair_id] =
         [abs(color mean vals[next blob id][i] - color mean vals[blb id][i]) for i in range(3)]
                           pairwise_gray_homogeneity_val[pair_id] = \
                                 abs(gray_mean_vals[next_blob_id] - gray_mean_vals[blb_id])
                            pairwise gray var comp[pair id] = \
                                abs(gray_vars[next_blob_id] - gray_vars[blb_id])
                            pairwise texture comp[pair id] =
                        np.linalg.norm(texture vals[next blob id] - texture vals[blb id])
           next blob id += 1
```

Some Results Obtained with SS as Reported by the Authors

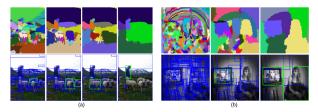


Figure 2: Two examples of our selective search showing the necessity of different scales. On the left we find many objects at different scales. On the right we necessarily find the objects at different scales as the girl is contained by the tv.