Lab Assignment-2

MTH 308 and & MTH 308B: Numerical Analysis and Scientific Computing-I

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1. This example illustrates the effects of truncation error and rounding error. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function. The derivative of f at $x \in \mathbb{R}$ is given by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}, \quad \text{if the limit exists.}$$

For given $h \in \mathbb{R}$, $h \neq 0$, an approximation of $f'(x)$ is given by

$$f'_h(x) = \frac{f(x+h) - f(x)}{h}.$$

with a given h. The absolute error for this approximation is

$$AE_{x,f}(h) = |f'(x) - f'_h(x)|.$$

For the function $f(x) = \sin x$, $x \in \mathbb{R}$, the derivative of f at $x = 1$ is $f'(1) = \cos(1)$. For $h \in \mathbb{R}$, $h \neq 0$, we can calculate the approximation $f'_h(1)$ and the absolute error as $|f'(1) - f'_h(1)|$. Write a program that prints the output in a tabular format as shown below where $h = 10^{-k}$ $(k = 1, 2, \ldots, 18)$. Output of your program should appear in places marked (- - -). Print the real variables in exponetial format using 6 decimal places. For evaluation of $\cos(1)$, $\sin(1 + h) - \sin(1)$ use inbuilt trigonometric math fuctions in C/Matlab.

k	h	f'_h	$f'(1)$	Abs. Error
1	− − −	− − −	− − −	− − −
.	− − −	− − −	− − −	− − −
.	− − −	− − −	− − −	− − −
18	− − −	− − −	− − −	− − −.

If possible plot the graph of the absolute error h-vs-$AE_{1,\sin}(h)$ (in suitable scale).

End.