

Test-driven Reinforcement Learning in Continuous Control

Zhao Yu^{*1}, Xiuping Wu^{*2}, Liangjun Ke^{1†},

¹ The State Key Laboratory for Manufacturing Systems Engineering, School of Automation Science and Engineering, Xi'an Jiaotong University, No.28 Xianning West Road, Xi'an, Shaanxi 710049, P.R. China

²Southampton Business School, University of Southampton, Highfield Campus, SO17 1BJ, United Kingdom
yuzhaokz@stu.xjtu.edu.cn, xiuping.wu@soton.ac.uk, keljxjtu@xjtu.edu.cn

Abstract

Reinforcement learning (RL) has been recognized as a powerful tool for robot control tasks. RL typically employs reward functions to define task objectives and guide agent learning. However, since the reward function serves the dual purpose of defining the optimal goal and guiding learning, it is challenging to design the reward function manually, which often results in a suboptimal task representation. To tackle the reward design challenge in RL, inspired by the satisficing theory, we propose a Test-driven Reinforcement Learning (TdRL) framework. In the TdRL framework, multiple test functions are used to represent the task objective rather than a single reward function. Test functions can be categorized as pass-fail tests and indicative tests, each dedicated to defining the optimal objective and guiding the learning process, respectively, thereby making defining tasks easier. Building upon such a task definition, we first prove that if a trajectory return function assigns higher returns to trajectories closer to the optimal trajectory set, maximum entropy policy optimization based on this return function will yield a policy that is closer to the optimal policy set. Then, we introduce a lexicographic heuristic approach to compare the relative distance relationship between trajectories and the optimal trajectory set for learning the trajectory return function. Furthermore, we develop an algorithm implementation of TdRL. Experimental results on the DeepMind Control Suite benchmark demonstrate that TdRL matches or outperforms handcrafted reward methods in policy training, with greater design simplicity and inherent support for multi-objective optimization. We argue that TdRL offers a novel perspective for representing task objectives, which could be helpful in addressing the reward design challenges in RL applications.

Code — <https://github.com/KezhiAdore/TdRL>

1 Introduction

For an intelligent agent, it is crucial to specify the objective that needs to be achieved (Silver et al. 2021; Silver and Sutton 2025). In classical reinforcement learning (RL), the objective is typically specified through the reward functions, which steer the agent toward desired behaviors. (Sutton and Barto 2018) Crucially, reward design accounts for

^{*}These authors contributed equally.

[†]Corresponding author

Copyright © 2026, Association for the Advancement of Artificial Intelligence (www.aaai.org). All rights reserved.

both defining optimal behavior and guiding the learning process, making it inherently challenging (Rajagopal 2023; Knox et al. 2023; Booth et al. 2023). Crafting effective reward functions typically demands domain-specific expertise and reward design experience (Knox et al. 2023; Booth et al. 2023). Nevertheless, the handcrafted reward often represents an imperfect proxy for the true optimization objective, as experts typically view reward as a direct evaluation of the relative goodness of each state-action pair instead of evaluating trajectories.(Booth et al. 2023). This mismatch induces fundamental challenges in reinforcement learning, such as reward hacking (Amodei et al. 2016).

To circumvent the challenges associated with manual reward engineering, several approaches have been developed, mainly including Preference-based RL (PbRL, Christiano et al. 2017), Inverse RL (IRL, Ziebart et al. 2008), and reward functions generated by Large Language Models (LLMs, Kwon et al. 2023).

PbRL learns reward functions or directly optimizes policies based on human preference feedback, which avoids manual reward design and has demonstrated promising performance in robot control and LLM alignment (Christiano et al. 2017; Ouyang et al. 2022). However, PbRL's human-centric nature introduces several limitations (Casper et al. 2023). First, inherent biases in human preference data necessitate dedicated techniques to address them. Second, the reliance on human judgments may constrain the agent's behavior within human cognitive boundaries. IRL infers reward functions from expert demonstrations, thereby avoiding manual reward designing. However, IRL typically demands extensive expert demonstration data, and the inferred reward functions typically exhibit poor generalization when evaluated on out-of-distribution data (Arora and Doshi 2021). Recent studies have demonstrated that LLMs can outperform humans in designing reward functions (Kwon et al. 2023; Yu et al. 2023; Du et al. 2023). However, these approaches depend on a human-specified domain knowledge for reward design, and typically require extensive training feedback to polish the reward function (Cao et al. 2025).

Furthermore, real-world applications typically require agents to simultaneously optimize multiple objectives (Vamplew et al. 2023). This multi-objective optimization presents significant challenges for reward function design, particularly in balancing different objectives.

When solving real-world multi-objective tasks, humans often do not pursue the optimal solution in a certain metric but rather seek a *satisficing solution* (Simon 1947) across multiple objectives. For example, when driving, people do not blindly minimize time consumption, but rather reach the destination within a certain time while ensuring safety, comfort, and compliance with regulations.

Inspired by this, we propose a test-driven reinforcement learning (TdRL) framework. In the TdRL framework, the agent’s objective is passing given tests instead of maximizing the cumulative return, which implies that TdRL seeks to obtain a satisficing solution across multiple objectives rather than an optimal solution in a single metric. Test functions take trajectories as input and output the test results. According to their functionality, test functions can be categorized into two types: pass-fail tests and indicative tests. Pass-fail tests evaluate whether the policy meets the required criteria, while indicative tests provide informative guiding signals for policy learning. Pass-fail tests and indicative tests correspond to defining optimal behavior and guiding the learning process, respectively.

Specifically, the goal of TdRL is to train a policy such that its interaction trajectories with the environment pass all given pass-fail tests. To solve this problem, we first prove that if a trajectory return function assigns higher returns to trajectories closer to the optimal trajectory set (where the trajectory passes all given pass-fail tests), maximum entropy policy optimization based on this return function will yield a policy that is closer to the optimal policy set (where the trajectories generated by the policy pass all given pass-fail tests). Then, we introduce a lexicographic heuristic approach to compare the relative distance relationship between trajectories and the optimal trajectory set. Additionally, we develop an algorithm implementation of TdRL.

The TdRL framework provides several key benefits. First, test functions operate on trajectories instead of state-action pairs, mitigating designer-induced bias (Booth et al. 2023). Second, the task objective is represented by pass-fail tests rather than a scalar reward function, naturally accommodating multi-objective optimization. Finally, test results can evaluate policy performance more accurately than cumulative returns. The main contributions of this paper are summarized as follows:

- To address the reward design challenge, we propose a test-driven reinforcement learning framework, where the task objective is represented by several test functions instead of a single reward function.
- We propose sufficient conditions for trajectory return functions to guarantee policy convergence to the optimal policy set, and present a lexicographic heuristic approach for constructing return functions based on trajectory testing results.
- We develop an algorithm implementation of TdRL and conduct experiments on the DeepMind Control Suite benchmark to show the benefits of TdRL.

2 Related Work

Test-driven reward design has been adapted to RL to ensure robustness and correctness recently (Jaensch et al. 2022; Fischer et al. 2024). Inspired by the test-driven development principle (Beck 2002), Jaensch et al. (2022) proposed an approach that uses test cases to guide reward design, iteratively refining the policy until all tests are passed. Fischer et al. (2024) extended this idea to inverse reinforcement learning, replacing expert demonstrations with scenario-based testing to learn cost functions via Bayesian inference. Pranger et al. (2024) introduced a state-importance-driven testing method for deep RL, prioritizing high-impact states to efficiently detect policy vulnerabilities. These approaches demonstrate the potential of test-driven frameworks to enhance RL in applications.

Preference-based reinforcement learning (PbRL) has emerged to address the challenges of reward design. Instead of relying on handcrafted reward functions, PbRL (Christian et al. 2017) learns reward models from human preferences over agent behaviors, which in turn guides policy optimization, achieving superior performance compared to handcrafted rewards. PEBBLE method (Lee, Smith, and Abbeel 2021) enhances sample efficiency by integrating off-policy learning and unsupervised pre-training. To further reduce human feedback requirements, a semi-supervised method named SURF (Park et al. 2022) combines data augmentation with PbRL to leverage both labeled and unlabeled experience. More recently, Bukharin et al. (2024) proposed a hierarchical reward modeling framework called HERON that structures preference learning based on the importance of feedback signals, improving performance in sparse-reward scenarios.

Test-driven methods have been applied in RL recently, but they often require *manual* design of approaches to process test results. We introduce the first theoretical framework for test-driven reinforcement learning that is learning-based. Algorithmically, we adapt the PbRL paradigm of learning the return function from trajectory comparison.

3 Preliminaries

A Markov Decision Process (MDP) could be described by a tuple $\mathcal{M} = \langle \mathcal{S}, \mathcal{A}, \mathcal{P}, r, \gamma \rangle$ where \mathcal{S} is the state space, \mathcal{A} is the action space, \mathcal{P} is the transition probability function, $r \in \mathcal{R} : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$ is the reward function, and γ is the discount factor. The goal of reinforcement learning (Sutton and Barto 2018) is to find a policy $\pi \in \Pi : \mathcal{S} \times \mathcal{A} \rightarrow [0, 1]$ that maximizes the expected cumulative reward:

$$\pi^* = \arg \max_{\pi} \mathbb{E}_{\pi} \left[\sum_{t=0}^{\infty} r(s_t, a_t) \right]. \quad (1)$$

A trajectory $\tau \in \mathcal{T}$ represents a sequence of state-action pairs generated through policy-environment interactions, beginning at some initial state. The trajectory return function R evaluates a trajectory by summing rewards across all constituent state-action pairs: $R(\tau) = \sum_{(s,a) \in \tau} r(s, a)$. Given these definitions, the reinforcement learning optimization objective is formally expressed as:

$$\pi^* = \arg \max_{\pi} \mathbb{E}_{\tau \sim \pi} [R(\tau)]. \quad (2)$$

3.1 Maximum Entropy Reinforcement Learning

Maximum Entropy Reinforcement Learning (MERL, Haarnoja et al. 2018) extends the standard reinforcement learning framework by incorporating an entropy maximization objective. This approach simultaneously optimizes for both cumulative reward maximization and policy entropy, thereby improving exploration capacity and algorithmic robustness. The objective of MERL could be written as:

$$\max_{\pi} \mathbb{E}_{\tau \sim \pi} \left[\sum_{t=0}^T r(s_t, a_t) + \alpha H(\pi(\cdot | s_t)) \right], \quad (3)$$

where $H(\pi(\cdot | s_t))$ denotes the policy's entropy at state s_t , and α controls the trade-off between the reward and entropy terms. When policy π_1 is optimized via maximum entropy reinforcement learning with respect to the trajectory return function $R(\tau) = \sum_{(s,a) \in \tau} r(s, a)$, the resulting policy π_2 follows:

$$\pi_2(\tau) = \frac{1}{Z} \pi_1(\tau) \exp \left(\frac{1}{\alpha} R(\tau) \right), \quad (4)$$

where Z is the partition function, $\pi(\tau)$ represents the probability of generating trajectory τ using policy π . The proof of Equation (4) is in Appendix E.

3.2 Preference-based Reinforcement Learning

Preference-based reinforcement learning (Christiano et al. 2017) is a class of reinforcement learning algorithms that learns a policy by comparing the rewards of different trajectories. A preference relation \succ is defined on \mathcal{T} as follows:

$$\tau_1 \succ \tau_2 \Leftrightarrow R(\tau_1) \geq R(\tau_2). \quad (5)$$

Following the Bradley-Terry model (Bradley and Terry 1952), we could use a reward function estimate \hat{r} as the preference predictor, as \hat{r} could be viewed as a latent factor to explain the preference. We assume the preference probability of a trajectory depends exponentially on its return:

$$\hat{P}[\tau_1 \succ \tau_2] = \frac{\exp(\hat{R}(\tau_1))}{\exp(\hat{R}(\tau_1)) + \exp(\hat{R}(\tau_2))}, \quad (6)$$

For a given dataset \mathcal{D} of triples (τ_1, τ_2, μ) , where τ_1 and τ_2 are different trajectories and μ is a distribution over $\{1, 2\}$ indicating which trajectory is preferred, we could learn the reward function by minimize the following loss function :

$$\begin{aligned} \mathcal{L}(\hat{r}) = & - \sum_{(\tau_1, \tau_2, \mu) \in \mathcal{D}} \mu(1) \log \hat{P}[\tau_1 \succ \tau_2] \\ & + \mu(2) \log(1 - \hat{P}[\tau_1 \succ \tau_2]). \end{aligned} \quad (7)$$

4 Method

In this section, we introduce Test-driven reinforcement learning. We first illustrate the notation and goal in TdRL, then we establish sufficient conditions for trajectory return functions to guarantee policy convergence to the optimal policy set. Additionally, we introduce a lexicographic heuristic approach to compare trajectories for learning trajectory return function. Finally, we detail the implementation of the TdRL algorithm. Figure 1 demonstrates the main procedure of the TdRL algorithm. It follows an iterative procedure mainly consisting of four stages:

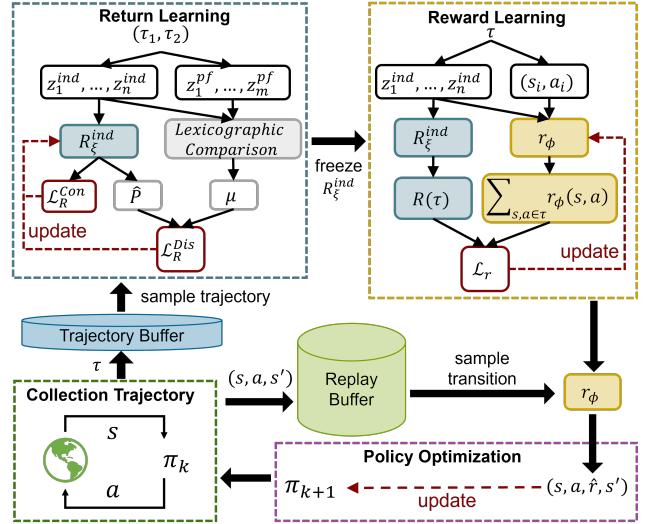


Figure 1: The main procedure of the TdRL algorithm.

- *Collection Trajectory*: The agent interacts with the environment to collect trajectories.
- *Return Learning*: The return function is updated by the comparison results of trajectories.
- *Reward Learning*: The reward function is updated and rewards are recalculated for transitions in the replay buffer.
- *Policy Optimization*: The policy network is optimized using transitions in the replay buffer.

4.1 Notation and Goal

TdRL aims to find a policy whose interaction trajectories with the environment pass all given pass-fail tests. Test functions map a trajectory to a test result. Specifically, test functions can be categorized into two types based on their functionality: pass-fail tests and indicative tests. A pass-fail test outputs a binary value judging whether a trajectory meets the required criteria. In contrast, an indicative test outputs a real number, quantifying the performance of a trajectory in a specific metric. Formally, we use $z^{pf} \in \mathcal{Z}^{pf} : \mathcal{T} \rightarrow \{0, 1\}$ to denote pass-fail test function, and $z^{ind} \in \mathcal{Z}^{ind} : \mathcal{T} \rightarrow \mathbb{R}$ to denote indicative test function.

The task objective in TdRL can be formulated as $\langle S, \mathcal{A}, \mathcal{P}, Z^{pf}, Z^{ind}, \gamma \rangle$, where $Z^{pf} = \{z_1^{pf}, \dots, z_m^{pf}\}$ represents a set of m pass-fail test functions and $Z^{ind} = \{z_1^{ind}, \dots, z_n^{ind}\}$ represents a set of n indicative test functions. The goal of TdRL is to find a policy π^* such that the resulting trajectory τ from the interaction between π^* and the environment satisfies:

$$\mathbb{E}_{\tau \sim \pi^*} \left[\sum_{i=1}^m z_i^{pf}(\tau) \right] = m. \quad (8)$$

Following the above definition, we define the optimal trajectory set as the collection of all trajectories that pass all pass-fail tests, denoted as:

$$\tilde{\mathcal{T}} = \{\tau | \tau \in \mathcal{T}, \sum_{i=1}^m z_i^{pf}(\tau) = m\}. \quad (9)$$

Let $\tilde{\mathcal{T}}_i = \{\tau | \tau \in \mathcal{T}, z_i^{pf}(\tau) = 1\}$ denote the set of trajectories that pass the pass-fail test z_i^{pf} . Then, we have $\tilde{\mathcal{T}} = \cap_{i=1}^m \tilde{\mathcal{T}}_i$.

Similarly, we define the optimal policy set as the collection of all policies whose interaction with the environment generates trajectories that pass all pass-fail tests, denoted as:

$$\tilde{\Pi} = \{\pi | \pi \in \Pi, \mathbb{E}_{\tau \sim \pi} [\sum_{i=1}^m z_i^{pf}(\tau)] = m\}. \quad (10)$$

Let $\tilde{\Pi}_i = \{\pi | \pi \in \Pi, \mathbb{E}_{\tau \sim \pi} [z_i^{pf}(\tau)] = 1\}$ denote the set of policies whose interaction with the environment generates trajectories that pass the pass-fail test z_i^{pf} . Then, we have $\tilde{\Pi} = \cap_{i=1}^m \tilde{\Pi}_i$.

4.2 Test Functions to Return Function

The goal of TdRL is to find a policy π^* that generates trajectories passing all pass-fail tests. In other words, we need to find a policy in $\tilde{\Pi}$. To achieve this, we could construct a trajectory return function R such that policy optimization based on this function could converge to a policy belonging to $\tilde{\Pi}$. Let $d(\tau_1, \tau_2)$ denote the distance between trajectories τ_1 and τ_2 in the trajectory space, and the distance between a trajectory τ and a trajectory set $\tilde{\mathcal{T}}$ is defined as:

$$d(\tau, \tilde{\mathcal{T}}) = \min_{\tau' \in \tilde{\mathcal{T}}} d(\tau, \tau'). \quad (11)$$

For a policy π , let $P_\pi : \mathcal{T} \rightarrow [0, 1]$ denote the distribution of trajectories generated through interaction with the environment, satisfying $\int_{\mathcal{T}} P_\pi(\tau) d\tau = 1$. The distance between policies π_1 and π_2 is defined by the Wasserstein- p distance (Villani et al. 2008) between their corresponding trajectory distributions P_{π_1} and P_{π_2} :

$$d(\pi_1, \pi_2) = W_p(P_{\pi_1}, P_{\pi_2}) \\ = \left(\inf_{\gamma \in \Gamma(P_{\pi_1}, P_{\pi_2})} \int_{\mathcal{T} \times \mathcal{T}} d(\tau_1, \tau_2)^p d\gamma(\tau_1, \tau_2) \right)^{\frac{1}{p}}, \quad (12)$$

where $\Gamma(P_{\pi_1}, P_{\pi_2})$ denotes the set of all joint probability distributions whose marginal distributions are P_{π_1} and P_{π_2} , respectively. The distance between policy π and policy set $\tilde{\Pi}$ is defined as:

$$d(\pi, \tilde{\Pi}) = \min_{\pi' \in \tilde{\Pi}} d(\pi, \pi'). \quad (13)$$

Theorem 1. If there exists a trajectory return function $R(\tau)$ that is monotonically non-increasing with respect to the distance between a trajectory τ and the optimal trajectory set $\tilde{\mathcal{T}}$, such that:

$$d(\tau_1, \tilde{\mathcal{T}}) \leq d(\tau_2, \tilde{\mathcal{T}}) \implies R(\tau_1) \geq R(\tau_2).$$

Suppose policy π_2 is obtained by optimizing policy π_1 using a maximum entropy algorithm with respect to R , Then, policy π_2 is closer to the optimal policy set $\tilde{\Pi}$ than π_1 :

$$d(\pi_1, \tilde{\Pi}) \geq d(\pi_2, \tilde{\Pi}).$$

The proof of Theorem 1 is shown in Section F. Theorem 1 demonstrates that if there exists a trajectory return function R that assigns higher values to trajectories closer to the optimal trajectory set $\tilde{\mathcal{T}}$, then performing maximum entropy policy optimization on this return function will yield a policy that is closer to the policy set $\tilde{\Pi}$ than the original policy. To obtain a trajectory reward function R that satisfies the above conditions, we consider two key issues: how to construct the trajectory return function and how to design a loss function for learning such a return function.

There typically exist numerous natural indicative signals in environments. The key challenge lies in effectively combining these indicative signals into a reward signal that can properly guide the learning process (Juechems and Summerfield 2019; Silver and Sutton 2025). Motivated by this insight, our approach does not directly construct a function mapping trajectories to returns. Instead, we regard indicative test results as indicative signals, and we utilize indicative test functions to construct a trajectory return function. Appendix G provides a detailed interruption about the indicative test function combination.

Specifically, we employ a fully connected network that takes an input vector of dimension n and produces a scalar output parameterized by ξ to construct a return mapping function R_ξ^{ind} . Then, the trajectory return function R could be constructed as a composite function of R_ξ^{ind} and the n indicative test functions:

$$R(\tau) = R_\xi^{ind}(z_1^{ind}(\tau), z_2^{ind}(\tau), \dots, z_n^{ind}(\tau)). \quad (14)$$

Following the Bradley-Terry model (Bradley and Terry 1952), if we consider the distance to the optimal trajectory set as a measure of trajectory quality, we can estimate the probability of distance relationships between trajectories by the return function. For convenience, we define $\tilde{d}(\tau)$ as the distance between trajectory τ and the optimal trajectory set $\tilde{\mathcal{T}}$. Then, the probability of τ_1 being closer to $\tilde{\mathcal{T}}$ than τ_2 can be estimated as:

$$\hat{P}[\tilde{d}(\tau_1) < \tilde{d}(\tau_2)] = \frac{\exp(R(\tau_1))}{\exp(R(\tau_1)) + \exp(R(\tau_2))}. \quad (15)$$

Suppose $\mu \in \{0, 0.5, 1\}$ represents the true probability that τ_1 is closer to $\tilde{\mathcal{T}}$ than τ_2 , where $\mu = 0$ means $\tilde{d}(\tau_1) > \tilde{d}(\tau_2)$, $\mu = 1$ means $\tilde{d}(\tau_1) < \tilde{d}(\tau_2)$, and $\mu = 0.5$ means $\tilde{d}(\tau_1) = \tilde{d}(\tau_2)$. Then, we could construct the following distance-based loss function:

$$\mathcal{L}_R^{Dis} = - \sum_{(\tau_1, \tau_2, \mu) \in \mathcal{D}} \left[\mu \cdot \log \hat{P}[\tilde{d}(\tau_1) < \tilde{d}(\tau_2)] \right. \\ \left. + (1 - \mu) \cdot \log \left(1 - \hat{P}[\tilde{d}(\tau_1) < \tilde{d}(\tau_2)] \right) \right], \quad (16)$$

where \mathcal{D} is a dataset of trajectory pairs (τ_1, τ_2) and their corresponding probabilities μ . Furthermore, to ensure numerical stability during return function learning, we introduce a penalty term:

$$\mathcal{L}_R^{Penalty} = \sum_{(\tau_1, \tau_2, \mu) \in \mathcal{D}} \sum_{i \in \{1, 2\}} \left(R(\tau_i) - \tilde{R}(\tau_i) \right)^2, \quad (17)$$

where \tilde{R} denotes the trajectory return values computed before network updating.

Then, we decompose the learned trajectory return function into a state-action reward function for policy learning. We parameterize the reward function as $r_\phi(s, a)$ and construct the reward learning loss:

$$\mathcal{L}_r = \sum_{\tau \in \mathcal{D}} \left[R(\tau) - \sum_{(s, a) \in \tau} r_\phi(s, a) \right]^2. \quad (18)$$

By minimizing the loss \mathcal{L}_R^{Dis} and $\mathcal{L}_R^{Penalty}$, we can learn the return mapping function R_ξ^{ind} . Then we could decompose the trajectory return to state-action reward by learning a reward function through minimizing the loss \mathcal{L}_r .

Computing \mathcal{L}_R^{Dis} requires the relative distance relationships between trajectories. However, during the learning process, the trajectories contained in the optimal trajectory set $\tilde{\mathcal{T}}$ remain unknown, making it intractable to directly compute the distances $\tilde{d}(\tau)$. Notably, we do not require exact values of $\tilde{d}(\tau)$ when computing the loss \mathcal{L}_R^{Dis} , but only need to determine the relative distance relationships between trajectories, specifically, whether $\tilde{d}(\tau_1) > \tilde{d}(\tau_2)$, $\tilde{d}(\tau_1) < \tilde{d}(\tau_2)$ or $\tilde{d}(\tau_1) = \tilde{d}(\tau_2)$. In the next section, we will introduce a lexicographic heuristic approach to construct the relative distance relationships between trajectories.

4.3 Lexicographic Trajectory Comparison

For trajectories τ_1 and τ_2 , we need to compare their distances to the optimal trajectory set $\tilde{\mathcal{T}}$, i.e., $\tilde{d}(\tau_1)$ and $\tilde{d}(\tau_2)$. However, directly computing $\tilde{d}(\tau)$ is infeasible as $\tilde{\mathcal{T}}$ is unknown. Under such limited information conditions, we employ a lexicographic heuristic (Özgür Şimşek 2020) approach to compare the distances. The lexicographic heuristic is efficient, practical, and adaptable, which is a fast and frugal strategy originally from human decision-making (Gigerenzer and Goldstein 1996; Katsikopoulos 2013; Katsikopoulos et al. 2021). Specifically, the lexicographic method sequentially evaluates information in priority order and makes decisions based on the first criterion that meets predefined thresholds (e.g., exceeding certain values).

Building upon the definitions of pass-fail tests and indicative tests, along with the relationship between $\tilde{\mathcal{T}}_i$ and $\tilde{\mathcal{T}}$, we could derive the following priors (The interpretation of the comparison priors is detailed in Appendix H):

- Pass-fail tests take precedence over indicative tests
- A trajectory satisfying more pass-fail tests is closer to $\tilde{\mathcal{T}}$
- A trajectory passing more challenging tests (corresponding to smaller $\tilde{\mathcal{T}}_i$) is closer to $\tilde{\mathcal{T}}$
- All trajectories within $\tilde{\mathcal{T}}$ have zero distance to $\tilde{\mathcal{T}}$
- Under-optimized indicators should be prioritized

Building upon the established priors, we could compute the probability μ that τ_1 is closer to $\tilde{\mathcal{T}}$ than τ_2 through the following lexicographical procedure (more details are shown in Appendix H):

Algorithm 1: TdRL

Require: frequency of return network update K
Require: pass-fail tests $\{z_1^{pf}, z_2^{pf}, \dots, z_m^{pf}\}$, indicative tests $\{z_1^{ind}, z_2^{ind}, \dots, z_n^{ind}\}$

- 1: Initial π_θ , R_ξ^{ind} , r_ϕ , and $\tau = \emptyset$
- 2: Initial trajectory buffer $\mathcal{D} = \emptyset$, replay buffer $\mathcal{B} = \emptyset$.
- 3: **for** each iteration **do**
- 4: Sample action a from $\pi_\theta(\cdot | s)$
- 5: Exec action a in environment and get $(s', done)$
- 6: Update Trajectory $\tau \leftarrow \tau \cup \{(s, a, s')\}$
- 7: Store transition $\mathcal{B} \leftarrow \mathcal{B} \cup \{(s, a, s', r_\phi(s, a), done)\}$
- 8: **if** $done$ **then**
- 9: Store trajectory $\mathcal{D} \leftarrow \mathcal{D} \cup \tau$, reset $\tau = \emptyset$
- 10: **end if**
- 11: **if** iteration $\%K == 0$ **then**
- 12: **for** each gradient step **do**
- 13: Sample minibatch $\{(\tau_1, \tau_2)_j\}_{j=1}^D \sim \mathcal{D}$
- 14: Compute μ for each pair (τ_1, τ_2) by the lexicographic trajectory comparison approach
- 15: Optimize \mathcal{L}_R^{Dis} in (16) and $\mathcal{L}_R^{Penalty}$ in (17) with respect to ξ
- 16: Optimize \mathcal{L}_r in (18) with respect to ϕ
- 17: **end for**
- 18: Relabel entire replay buffer \mathcal{B} using r_ϕ
- 19: **end if**
- 20: **for** each gradient step **do**
- 21: Sample random minibatch from \mathcal{B}
- 22: Update policy network θ
- 23: **end for**
- 24: **end for**

1. If $\sum_{i=1}^m z_i^{pf}(\tau_j) = m, \forall \tau \in \{\tau_1, \tau_2\}$, return $\mu = 0.5$.
2. Compare pass-fail test passing count:
 - If $\sum_{i=1}^m z_i^{pf}(\tau_1) > \sum_{i=1}^m z_i^{pf}(\tau_2)$, return $\mu = 1$
 - If $\sum_{i=1}^m z_i^{pf}(\tau_1) < \sum_{i=1}^m z_i^{pf}(\tau_2)$, return $\mu = 0$
3. Sort pass-fail tests in ascending order of historical pass rates (i.e., descending difficulty) as $\{z_{k_1}^{pf}, \dots, z_{k_m}^{pf}\}$. Sequentially compare $z_{k_i}^{pf}(\tau_1)$ and $z_{k_i}^{pf}(\tau_2)$.
 - If $z_{k_i}^{pf}(\tau_1) > z_{k_i}^{pf}(\tau_2)$, return $\mu = 1$
 - If $z_{k_i}^{pf}(\tau_1) < z_{k_i}^{pf}(\tau_2)$, return $\mu = 0$
4. Sort indicative tests in descending order of the skewness of historical test results (least-optimized first) as $\{z_{l_1}^{ind}, \dots, z_{l_n}^{ind}\}$. Sequentially compare $z_{l_i}^{ind}(\tau_1)$ and $z_{l_i}^{ind}(\tau_2)$.
 - If $z_{l_i}^{ind}(\tau_1) > z_{l_i}^{ind}(\tau_2)$, return $\mu = 1$
 - If $z_{l_i}^{ind}(\tau_1) < z_{l_i}^{ind}(\tau_2)$, return $\mu = 0$
5. Return $\mu = 0.5$, i.e., trajectories are indistinguishable

4.4 TdRL Algorithm

Algorithm 1 presents the detailed procedure of the TdRL algorithm. TdRL employs \mathcal{L}_R^{Dis} to learn the return function

and constrains the update step via $\mathcal{L}_R^{Penalty}$. However, since the gradient magnitudes of the two losses can not be directly comparable, selecting appropriate weights for balancing them is challenging. Specifically, \mathcal{L}_R^{Dis} is a cross-entropy loss, whose gradient scales with the probability difference, whereas $\mathcal{L}_R^{Penalty}$ is an MSE loss, whose gradient depends on the variation in the return output. To address this issue, we propose two methods: *gradient norm* (GN) and *early stop* (ES). Both methods compute the gradients of \mathcal{L}_R^{Dis} ($\nabla_{\xi} \mathcal{L}_R^{Dis}$) and $\mathcal{L}_R^{Penalty}$ ($\nabla_{\xi} \mathcal{L}_R^{Penalty}$) before each network update and use their sum to update the network parameters. The key distinction lies in the fact that GN rescales the MSE gradient to match the cross-entropy gradient L2-norm when the former exceeds the latter, and ES stops training if the MSE gradient L2-norm surpasses a predefined multiple (K^{ES}) of the cross-entropy gradient L2-norm.

5 Experiments

We design our experiments to investigate the benefits of using test functions for representing task objectives, as well as the performance of the TdRL algorithm.

5.1 Setups

We evaluate TdRL on several continuous robot control tasks from DeepMind Control Suite (DM-Control, Tunyasuvunakool et al. 2020). We employ the Soft Actor-Critic (SAC, Haarnoja et al. 2018) algorithm as the backbone for the implementation of TdRL. To enhance experience diversity in early training, we use unsupervised RL for warm-up. Hyperparameters for the algorithm implementation are detailed in Appendix C. In Section 4.4, we introduce two approaches—*gradient norm* and *early stop*—to balance the two loss terms \mathcal{L}_R^{Dis} and $\mathcal{L}_R^{Penalty}$ in return function learning. For notational clarity, we refer to the TdRL variants employing these methods as TdRL-GN and TdRL-ES.

5.2 Main Results

We treat the environment rewards in DM-Control as the oracle rewards. To focus on the efficiency of TdRL, we only use the components of the oracle rewards in environments to construct the test functions. For instance, in the *Walker-Walk* task, the oracle reward is derived from three components: torso height, torso upright, and move speed. Our test functions for this task also only use these components. Detailed test functions for each task are provided in Appendix I. More experiment results are shown in Section D

TdRL eliminates manual objective weighting while achieving performance comparable to or better than carefully handcrafted reward functions. Figure 3 demonstrates that TdRL achieves comparable or superior performance to SAC with oracle rewards in continuous robot control tasks. Since TdRL requires learning the reward function during policy optimization, its performance improvement during early training stages is slower compared to using oracle rewards. Furthermore, both TdRL-GN and TdRL-ES exhibit strong performance, indicating the effectiveness of combining cross-entropy and MSE loss in both approaches.

TdRL can also be applied to on-policy RL algorithms. Although TdRL is theoretically grounded in maximum entropy reinforcement learning, the TdRL algorithm can also be applied to on-policy RL methods. We maintain the reward learning component of TdRL while replacing its policy update algorithm with Proximal Policy Optimization (PPO, Schulman et al. 2017). Figure 3 demonstrates that TdRL with PPO achieves comparable performance to PPO with oracle rewards on several tasks, while exhibiting significant performance gaps on others. These results suggest TdRL’s potential for integration with on-policy reinforcement learning algorithms.

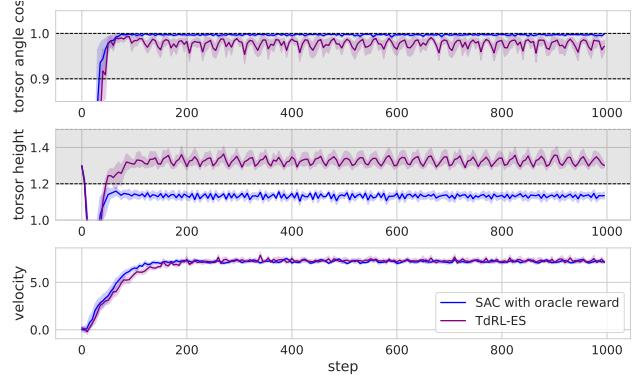


Figure 2: Performance comparison in multi-objectives between SAC with oracle reward and TdRL in the *Walker-Run* task. The gray shaded area represents the predefined performance threshold for each metric.

TdRL achieves a satisfying solution across multiple objectives rather than an optimal solution in a single metric. Figure 2 shows that the performance comparison in multi-objectives between SAC with oracle reward and TdRL in the *Walker-Run* task. In the *Walker-Run* task, there are three primary objectives. The cosine of the torso angle should be within [0.9, 1], ensuring the robot’s upper body remains upright. The torso height should exceed 1.2, ensuring the robot maintains a standing posture. The velocity in the x-axis should reach 8. The reward function for this task in DM-Control is defined as $((3 * stand + upright)/4) * (5 * move + 1)/6$, where *upright*, *stand*, and *move* respectively represent the rewards of uprightness, standing height, and move speed of the robot. Figure 2 shows that despite *stand* having a higher weight than *upright* in the reward function, the trained policy achieves the desired uprightness but fails to maintain sufficient standing height.

In TdRL, however, for each of the three metrics - uprightness, stand height, and move speed - we constructed both pass-fail tests and indicative tests (the test functions are detailed in the Appendix I). Notably, we do not need to preset the weights to combine them; instead, the pass-fail tests define the passing conditions for each metric. Figure 2 demonstrates that the TdRL policy fulfills all task metrics more effectively. While TdRL does not always achieve the optimum in individual metrics (e.g., upper-body uprightness), it consistently attains a satisfying solution across multiple ob-

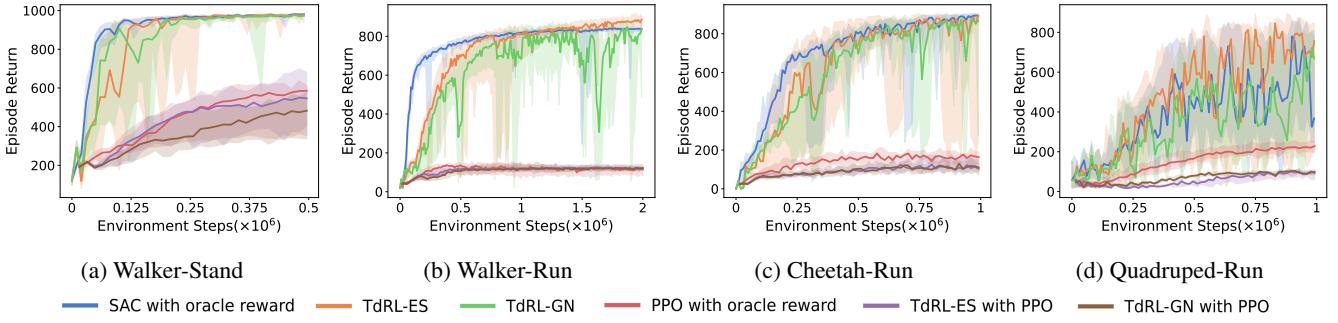


Figure 3: Performance comparison of algorithms on DM-Control tasks. Each algorithm runs with 10 different random seeds. Following Agarwal et al. (2021), the solid lines represent the interquartile mean (IQM) of episode returns while the shaded areas indicate 95% confidence intervals.

jectives. In the *Walker-Run* task, TdRL matches the speed of the SAC using oracle reward while satisfying both uprightness and stand height requirements.

TdRL focuses on the performance of trajectories rather than the quality of state-action pairs. Figure 2 shows that the TdRL-trained policy underperforms the policy trained by SAC with oracle rewards across all metrics in stability. This discrepancy likely arises because TdRL operates at the trajectory level rather than evaluating state-action pairs. Such trajectory-level evaluation is more intuitive for designers, avoiding task objectives that overly emphasize state quality (Booth et al. 2023), which may lead to reward hacking (Amodei et al. 2016). Furthermore, TdRL can also enhance stability for specific metrics through introducing additional test functions (e.g., the variance of the robot’s standing height).

5.3 Ablation Study

Impact of reward learning approaches Figure 4(a) illustrates the impact of different reward learning approaches in the TdRL algorithm. TdRL with no penalty denotes the variant without applying $\mathcal{L}_R^{Penalty}$ to constrain the trajectory return function learning, while TdRL with direct reward learning refers to directly learning the reward function from trajectory comparisons rather than first learning the return function followed by decomposition. Experimental results demonstrate that both omitting the penalty term and directly learning the reward function lead to training instability and degraded policy performance. Specifically, the absence of a penalty causes uncontrolled growth in return values during learning, potentially inducing numerical instability. When directly learning the reward function, *tanh* activation is typically applied in the reward network’s output layer to bound its outputs, which requires continuous rescaling during later training stages to accommodate reward learning, ultimately resulting in training instability and performance deterioration.

Hyperparameters choice TdRL-ES stops training when the L2-norm of $\mathcal{L}_R^{Penalty}$ ’s gradient exceeds a predefined multiple (K^{ES}) of \mathcal{L}_R^{Dis} ’s gradient L2-norm. The scaling factor K^{ES} is a tunable parameter requiring configuration. We investigated how different multiple K^{ES} values affect

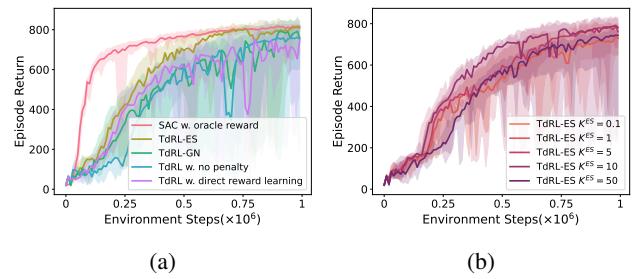


Figure 4: The left figure shows the performance of TdRL with different reward learning methods, while the right figure shows the performance of TdRL-ES with varying values of multiple K^{ES} .

algorithm performance. Figure 4(b) indicates that both excessively large and small multiple K^{ES} values degrade performance. Based on experimental results, we recommend setting multiple K^{ES} to 10 as a general guideline.

6 Conclusion

To tackle the challenges in reward design for reinforcement learning, this paper introduces a test-driven reinforcement learning (TdRL) framework. Instead of relying on a scalar reward function, TdRL represents task objectives using multiple test functions. The test functions are defined on trajectories rather than state-action pairs, which is more intuitive for designers. Besides, each test function only needs to represent a single objective, and designers do not need to consider their weights, greatly reducing the complexity of the reinforcement learning task design.

We propose sufficient conditions for trajectory return functions to guarantee policy convergence to the optimal policy set. Then, we introduce a lexicographic trajectory comparison approach for return learning. Furthermore, we present an implementation of TdRL, and the experimental results show that TdRL matches or outperforms reward-based methods in task training, with greater design simplicity and inherent support for multi-objective optimization. TdRL offers a viable approach to tackling reward design challenges in reinforcement learning.

7 Acknowledgments

We thank all anonymous reviewers for their constructive feedback, which helped improve our paper. We also thank Mr. Yuxin Cheng from the University of Hong Kong for his helpful discussions.

References

- Agarwal, R.; Schwarzer, M.; Castro, P. S.; Courville, A.; and Bellemare, M. G. 2021. Deep reinforcement learning at the edge of the statistical precipice. In *Proceedings of the 35th International Conference on Neural Information Processing Systems, NIPS ’21*. Red Hook, NY, USA: Curran Associates Inc. ISBN 9781713845393.
- Amodei, D.; Olah, C.; Steinhhardt, J.; Christiano, P.; Schulman, J.; and Mané, D. 2016. Concrete Problems in AI Safety. arXiv:1606.06565.
- Arora, S.; and Doshi, P. 2021. A survey of inverse reinforcement learning: Challenges, methods and progress. *Artificial Intelligence*, 297: 103500.
- Beck. 2002. *Test Driven Development: By Example*. USA: Addison-Wesley Longman Publishing Co., Inc. ISBN 0321146530.
- Booth, S.; Knox, W. B.; Shah, J.; Niekum, S.; Stone, P.; and Allievi, A. 2023. The Perils of Trial-and-Error Reward Design: Misdesign through Overfitting and Invalid Task Specifications. *Proceedings of the AAAI Conference on Artificial Intelligence*, 37(5): 5920–5929.
- Bradley, R. A.; and Terry, M. E. 1952. Rank analysis of incomplete block designs: I. the method of paired comparisons. *Biometrika*, 39(3/4): 324–345.
- Bukharin, A.; Li, Y.; He, P.; and Zhao, T. 2024. Deep Reinforcement Learning from Hierarchical Preference Design. arXiv:2309.02632.
- Cao, Y.; Zhao, H.; Cheng, Y.; Shu, T.; Chen, Y.; Liu, G.; Liang, G.; Zhao, J.; Yan, J.; and Li, Y. 2025. Survey on Large Language Model-Enhanced Reinforcement Learning: Concept, Taxonomy, and Methods. *IEEE Transactions on Neural Networks and Learning Systems*, 36(6): 9737–9757.
- Casper, S.; Davies, X.; Shi, C.; Gilbert, T. K.; Scheurer, J.; Rando, J.; Freedman, R.; Korbak, T.; Lindner, D.; Freire, P.; Wang, T. T.; Marks, S.; Ségerie, C.-R.; Carroll, M.; Peng, A.; Christoffersen, P. J. K.; Damani, M.; Slocum, S.; Anwar, U.; Siththanjanan, A.; Nadeau, M.; Michaud, E. J.; Pfau, J.; Krasheninnikov, D.; Chen, X.; Langosco, L.; Hase, P.; Biyik, E.; Dragan, A. D.; Krueger, D.; Sadigh, D.; and Hadfield-Menell, D. 2023. Open Problems and Fundamental Limitations of Reinforcement Learning from Human Feedback. *Trans. Mach. Learn. Res.*, 2023.
- Christiano, P.; Leike, J.; Brown, T. B.; Martic, M.; Legg, S.; and Amodei, D. 2017. Deep reinforcement learning from human preferences. arXiv:1706.03741.
- Du, Y.; Watkins, O.; Wang, Z.; Colas, C.; Darrell, T.; Abbeel, P.; Gupta, A.; and Andreas, J. 2023. Guiding Pre-training in Reinforcement Learning with Large Language Models. In Krause, A.; Brunskill, E.; Cho, K.; Engelhardt, B.; Sabato, S.; and Scarlett, J., eds., *Proceedings of the 40th International Conference on Machine Learning*, volume 202 of *Proceedings of Machine Learning Research*, 8657–8677. PMLR.
- Fischer, J.; Werling, M.; Lauer, M.; and Stiller, C. 2024. Test-Driven Inverse Reinforcement Learning Using Scenario-Based Testing. In *2024 IEEE Intelligent Vehicles Symposium (IV)*, 827–834.
- Gigerenzer, G.; and Goldstein, D. G. 1996. Reasoning the fast and frugal way: models of bounded rationality. *Psychological review*, 103(4): 650.
- Haarnoja, T.; Zhou, A.; Hartikainen, K.; Tucker, G.; Ha, S.; Tan, J.; Kumar, V.; Zhu, H.; Gupta, A.; Abbeel, P.; and Levine, S. 2018. Soft Actor-Critic Algorithms and Applications. *CoRR*, abs/1812.05905.
- Jaensch, F.; Kübler, K.; Schwarz, E.; and Verl, A. 2022. Test-Driven Reward Function for Reinforcement Learning: A Contribution towards Applicable Machine Learning Algorithms for Production Systems. *Procedia CIRP*, 112: 103–108. 15th CIRP Conference on Intelligent Computation in ManufacturingEngineering, 14–16 July 2021.
- Juechems, K.; and Summerfield, C. 2019. Where Does Value Come From? *Trends in Cognitive Sciences*, 23(10): 836–850.
- Katsikopoulos, K. V. 2013. Why Do Simple Heuristics Perform Well in Choices with Binary Attributes? *Decision Analysis*, 10(4): 327–340.
- Katsikopoulos, K. V.; Şimşek, Ö.; Buckmann, M.; and Gigerenzer, G. 2021. *Classification in the Wild: The Science and Art of Transparent Decision Making*. The MIT Press. ISBN 9780262363228.
- Knox, W. B.; Allievi, A.; Banzhaf, H.; Schmitt, F.; and Stone, P. 2023. Reward (Mis)design for autonomous driving. *Artificial Intelligence*, 316: 103829.
- Kwon, M.; Xie, S. M.; Bullard, K.; and Sadigh, D. 2023. Reward Design with Language Models. In *The Eleventh International Conference on Learning Representations*.
- Lee, K.; Smith, L. M.; and Abbeel, P. 2021. PEBBLE: Feedback-Efficient Interactive Reinforcement Learning via Relabeling Experience and Unsupervised Pre-training. In Meila, M.; and Zhang, T., eds., *Proceedings of the 38th International Conference on Machine Learning*, volume 139 of *Proceedings of Machine Learning Research*, 6152–6163. PMLR.
- Ouyang, L.; Wu, J.; Jiang, X.; Almeida, D.; Wainwright, C. L.; Mishkin, P.; Zhang, C.; Agarwal, S.; Slama, K.; Ray, A.; Schulman, J.; Hilton, J.; Kelton, F.; Miller, L.; Simens, M.; Askell, A.; Welinder, P.; Christiano, P.; Leike, J.; and Lowe, R. 2022. Training language models to follow instructions with human feedback. In *Proceedings of the 36th International Conference on Neural Information Processing Systems, NIPS ’22*. Red Hook, NY, USA: Curran Associates Inc. ISBN 9781713871088.
- Park, J.; Seo, Y.; Shin, J.; Lee, H.; Abbeel, P.; and Lee, K. 2022. SURF: Semi-supervised Reward Learning with Data Augmentation for Feedback-efficient Preference-based Reinforcement Learning. In *International Conference on Learning Representations*.

- Pranger, S.; Chockler, H.; Tappler, M.; and Könighofer, B. 2024. Test Where Decisions Matter: Importance-driven Testing for Deep Reinforcement Learning. In *The Thirty-eighth Annual Conference on Neural Information Processing Systems*.
- Rajagopal, S. 2023. What’s Next if Reward is Enough? Insights for AGI from Animal Reinforcement Learning. *Journal of Artificial General Intelligence*, 14(1): 15–40.
- Schulman, J.; Wolski, F.; Dhariwal, P.; Radford, A.; and Klimov, O. 2017. Proximal Policy Optimization Algorithms.
- Silver, D.; Singh, S.; Precup, D.; and Sutton, R. S. 2021. Reward is enough. *Artificial Intelligence*, 299: 103535.
- Silver, D.; and Sutton, R. S. 2025. Welcome to the era of experience. *Google AI*, 1.
- Simon, H. A. 1947. Administrative behavior; a study of decision-making processes in administrative organization. *Administrative behavior; a study of decision-making processes in administrative organization.*, xvi, 259–xvi, 259. Place: Oxford, England Publisher: Macmillan.
- Sutton, R. S.; and Barto, A. G. 2018. *Reinforcement Learning: An Introduction*. Cambridge, MA, USA: A Bradford Book. ISBN 978-0-262-03924-6.
- Tunyasuvunakool, S.; Muldal, A.; Doron, Y.; Liu, S.; Bohez, S.; Merel, J.; Erez, T.; Lillicrap, T.; Heess, N.; and Tassa, Y. 2020. dm_control: Software and tasks for continuous control. *Software Impacts*, 6: 100022.
- Vamplew, P.; Smith, B. J.; Källström, J.; Ramos, G.; Rădulescu, R.; Roijers, D. M.; Hayes, C. F.; Hentz, F.; Mannion, P.; Libin, P. J.; Dazeley, R.; and Foale, C. 2023. Scalar Reward is Not Enough. In *Proceedings of the 2023 International Conference on Autonomous Agents and Multiagent Systems*, AAMAS ’23, 839–841. Richland, SC: International Foundation for Autonomous Agents and Multiagent Systems. ISBN 9781450394321.
- Villani, C.; et al. 2008. *Optimal transport: old and new*, volume 338. Springer.
- Yu, W.; Gileadi, N.; Fu, C.; Kirmani, S.; Lee, K.-H.; Arenas, M. G.; Chiang, H.-T. L.; Erez, T.; Hasenclever, L.; Humprik, J.; brian ichter; Xiao, T.; Xu, P.; Zeng, A.; Zhang, T.; Heess, N.; Sadigh, D.; Tan, J.; Tassa, Y.; and Xia, F. 2023. Language to Rewards for Robotic Skill Synthesis. In *7th Annual Conference on Robot Learning*.
- Ziebart, B. D.; Maas, A.; Bagnell, J. A.; and Dey, A. K. 2008. Maximum entropy inverse reinforcement learning. In *Proceedings of the 23rd National Conference on Artificial Intelligence - Volume 3*, AAAI’08, 1433–1438. AAAI Press. ISBN 9781577353683.
- Özgür Şimşek. 2020. Lexicographic Decision Rule. In *Oxford Research Encyclopedia of Politics*. Oxford University Press.

A Limitation

Our work has several limitations. First, our theoretical analysis assumes the maximum entropy policy optimization framework. While empirical results show that TdRL works

well with other RL algorithms (e.g., PPO), it lacks theoretical guarantees beyond maximum entropy optimization. Second, our trajectory comparison relies on a lexicographic heuristic. Future studies should investigate methods with stronger theoretical foundations. Moreover, the test functions employed in this study are manually designed. Future work could explore integrating large language models to assist test function design.

B TdRL Algorithm Analysis

Computational Efficiency Analysis. For each trajectory, its test results are deterministic and fixed during training, so caching these results can reduce computational overhead. However, in trajectory comparison, the order of test functions may change as training progresses, necessitating re-comparing the trajectories whenever the reward network is updated. The primary computational overhead of TdRL arises from these comparisons. Given M reward network updates, each involving N trajectory pairs, the total computational overhead is $\mathcal{O}(MN)$. Moreover, since trajectory comparisons are mutually independent, they can be executed in parallel to improve computational efficiency.

Does TdRL eliminate the burden of reward design?

Firstly, the transformation of problems often leads to more diverse solutions. PbRL turns reward engineering into the collection of human preference labels, while IRL turns it into the collection of demonstration data. Difficulties in transformation can be addressed from a new perspective. Secondly, recent literature (Rajagopal 2023; Knox et al. 2023; Booth et al. 2023) point out that in the practice of reinforcement learning reward design, the problems with reward design are usually caused by designers focusing on the state and neglecting its impact on the trajectory. TdRL directly focuses on the quality of the trajectory during design tests, which is more intuitive. Additionally, reward design often requires considering trade-offs between multiple optimization objectives, which is challenging and usually requires a significant amount of time and resources for tuning. TdRL only needs to set the required satisfaction thresholds for each optimization objective, without designing weights between them; the trade-off between multiple optimization objectives is automatically completed during the reward learning in the TdRL algorithm. In summary, we believe that TdRL, compared to reward engineering, reduces the difficulty of reward design and offers better support for multi-objective optimization. Moreover, this shift also provides a new perspective for addressing future challenges in reward design.

Compare with other reward design/learning methods. Table 1 compares TdRL (Test-driven Reinforcement Learning), PbRL (Preference-based Reinforcement Learning), IRL (Inverse Reinforcement Learning), and Reward Engineering. TdRL is independent of human preference labels or expert demonstrations, and its core learning process relies on trajectory-level evaluation. In contrast, PbRL requires human-provided preference labels and also employs trajectory evaluation. IRL relies heavily on expert demonstrations to infer reward functions. Finally, Reward Engineering, which involves the manual design of reward functions, requires neither preference labels nor demonstrations;

Table 1: Comparison of TdRL and other reward design/learning methods.

	TdRL	PbRL	IRL	Reward Engineering
No preference label required	✓	✗	✓	✓
No demonstration required	✓	✓	✗	✓
Trajectory-level evaluation	✓	✓	/	✗

however, it focuses on state evaluation rather than trajectory evaluation, often resulting in suboptimal reward functions during the design process.

C Hyperparameter

Table 2 uses some standard reinforcement learning abbreviations for conciseness: Learning rates are abbreviated as "lr" (e.g., actor lr), "rew" and "ret" shorten reward and return-related terms (e.g., rew lr is reward learning rate). These conventions align with common RL literature to ensure clarity while saving space.

D More Experimental Results

Table 3 shows the performance of policies trained by SAC with oracle rewards and TdRL algorithms in multiple DM-Control tasks. The table details the metrics of the policy with the highest episode reward during training, where each policy is tested ten times, and the average performance and standard deviation of these ten tests are recorded. Black values in the table indicate that the performance meets the target, blue values indicate that the performance does not meet the target, and bold values represent the highest value among all algorithms for that metric.

Performance Comparison. The experimental results in Table 3 show that in multiple tasks, TdRL can achieve or exceed the performance of SAC with oracle rewards, and it can demonstrate better performance when SAC with oracle rewards fails to guarantee certain metrics meet the target. Additionally, for some metrics in certain tasks (e.g., speed in Walker-Run), neither the policies trained by SAC with oracle rewards nor TdRL can achieve the task objectives, but TdRL generally exhibits better performance on these metrics. This could be because TdRL optimizes the unmet metrics as much as possible after ensuring other metrics meet the target, whereas SAC with oracle rewards may continue improving other metrics to further improve cumulative rewards, which may lead to reward hacking.

Novel Task. Figure 5 presents frames from the *Walker-Run* task, comparing policies trained by SAC with oracle reward and TdRL. The TdRL-trained policy consistently

Table 2: Hyperparameter Settings

	SAC	TdRL	PPO	PPO TdRL
discount factor(γ)		0.99		
critic hidden dim		1024		
actor hidden dim		1024		
critic depth		2		
actor depth		2		
optimizer		Adam		
adam β_1		0.9		
adam β_2		0.999		
actor lr	0.0005		0.0003	
critic lr	0.0005		0.0003	
alpha lr	0.0001		/	
batch size	1024		64	
critic τ	0.005		/	
unsupervised steps	/ 9000		/ 9000	
trajectory max num	/ 100		/ 100	
segment size	/ 50		/ 50	
rew lr	/ 0.0003		/ 0.0003	
ret lr	/ 0.0003		/ 0.0003	
rew ensemble	/ 3		/ 3	
ret ensemble	/ 3		/ 3	
rew batch size	/ 128		/ 128	
ret batch size	/ 128		/ 128	
rew update num	/ 50		/ 50	
ret update num	/ 50		/ 50	
ret change penalty	/ 0.1		/ 0.1	
rew update interval	/ 5000		/ 5000	
ret update interval	/ 5000		/ 5000	
es multiple	/ 10		/ 10	

raises the legs higher during each running stride than the policy trained by oracle reward, thereby maintaining the desired torso height throughout the trajectory. Furthermore, we evaluated TdRL on a new task, *Walker-JumpRun*, which extends the *Walker-Run* task by introducing an additional test for maximum torso height. As shown in Figure 5, to satisfy the maximum height requirement, the torso must perform repeated jumps during locomotion. Simultaneously, to maximize forward speed, the torso retracts its legs mid-jump, enabling greater forward displacement per jump. These results demonstrate that TdRL can more readily accomplish complex tasks compared to manual reward function design by incorporating additional test functions.

Sensitivity Analysis. Table 4 shows the performance metrics tested on the *walker-run* environment by systematically varying the target speed and the torso angle threshold. The three metrics recorded in each cell of the results table are: torso angle / average height / average speed. It shows that a clear performance trade-off exists. Specifically, higher target speeds (e.g., 8 m/s) become increasingly difficult to achieve as the torso angle constraint is tightened (from 0.9 to 0.99 rad), evidenced by the significant drop in the achieved speed metric (from 7.34 to 5.55). Conversely, at lower target speeds (e.g., 2 m/s), the agent can maintain a very close torso angle and consistent height across all thresholds. This indi-

Table 3: Performance Comparison of Different Methods

env	metric	target	SAC (Oracle Reward)	TdRL-ES	TdRL-GN
Walker-Stand	upright stand height	[0.9, 1] [1.2, $+\infty$]	0.94 \pm 0.03 1.23 \pm 0.02	0.95 \pm 0.02 1.26 \pm 0.02	0.94 \pm 0.03 1.25 \pm 0.03
	upright stand height speed	[0.9, 1] [1.2, $+\infty$] [8, $+\infty$]	0.97 \pm 0.02 1.12 \pm 0.01 6.80 \pm 0.08	0.94 \pm 0.02 1.30 \pm 0.01 6.78 \pm 0.07	0.93 \pm 0.02 1.26 \pm 0.02 6.45 \pm 0.14
Cheetah-Run	speed	[10, $+\infty$]	9.44 \pm 0.19	9.56 \pm 0.37	9.88 \pm 0.08
Quadruped-Run	upright speed	[0.9, 1] [5, $+\infty$]	0.94 \pm 0.03 3.95 \pm 0.09	0.93 \pm 0.04 4.00 \pm 0.13	0.91 \pm 0.13 4.56 \pm 0.49

Table 4: Performance metrics for different speed thresholds and torso angle threshold

speed \ torso angle	0.9	0.95	0.98	0.99
2	0.94/1.31/2.05	0.97/1.30/2.02	0.98/1.30/3.87	0.99/1.26/1.98
4	0.95/1.26/4.01	0.97/1.24/3.98	0.98/1.27/3.96	0.99/1.24/3.93
6	0.95/1.30/5.86	0.96/1.32/5.81	0.98/1.29/5.64	0.99/1.26/5.18
8	0.93/1.28/7.34	0.94/1.32/7.27	0.98/1.29/6.31	0.99/1.25/5.55

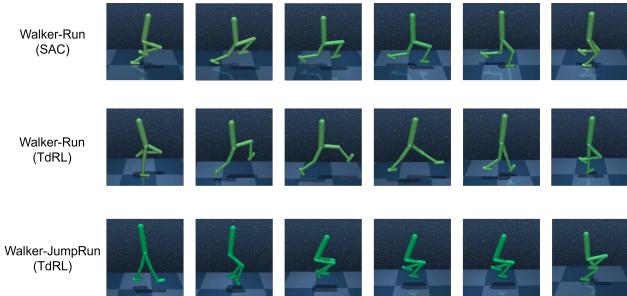


Figure 5: The frames of policies trained by SAC with oracle reward and TdRL in the Walker-Run task, as well as the performance of the policy trained by TdRL on a new task, *Walker-JumpRun*. *Walker-JumpRun* adds a test of the maximum torso height in trajectory based on the Walker-Run task, as detailed in Appendix I.

cates that the policy’s ability to precisely control its torso angle while maintaining stability is compromised when pushed to its maximum velocity limits. Moreover, the overall experimental results demonstrate that TdRL tends to prioritize achieving the more attainable threshold metrics when thresholds vary, while making efforts to satisfy other challenging performance metrics.

E Proof of Equation (4)

When policy π_1 is optimized via maximum entropy reinforcement learning with respect to the trajectory return function $R(\tau) = \sum_{(s,a) \in \tau} r(s,a)$, the resulting policy π_2 follows:

$$\pi_2(\tau) = \frac{1}{Z} \pi_1(\tau) \exp\left(\frac{1}{\alpha} R(\tau)\right).$$

Proof. The optimization objective of Maximum Entropy

Reinforcement Learning (MERL) is:

$$\max_{\pi} \mathbb{E}_{\tau \sim \pi} \left[\sum_{t=0}^T r(s_t, a_t) + \alpha H(\pi(\cdot | s_t)) \right],$$

where $\alpha > 0$ is a temperature parameter that balances reward maximization and entropy maximization, and the entropy of the policy at state s_t is defined as

$$H(\pi(\cdot | s_t)) = -\mathbb{E}_{a_t \sim \pi(\cdot | s_t)} [\log \pi(a_t | s_t)].$$

Since the trajectory distribution depends on the policy’s action distribution, and the environment dynamics are fixed, the total entropy contribution over the trajectory can be expressed as

$$\begin{aligned} \sum_{t=0}^T H(\pi(\cdot | s_t)) &= -\mathbb{E}_{\tau \sim \pi} \left[\sum_{t=0}^T \log \pi(a_t | s_t) \right] \\ &= -\mathbb{E}_{\tau \sim \pi} [\log \pi(\tau)] + \text{const}, \end{aligned}$$

where the constant comes from the environment dynamics and initial state distribution, which do not depend on π .

Therefore, the maximum entropy objective can be equivalently written as

$$\max_{\pi} \mathbb{E}_{\tau \sim \pi} [R(\tau) - \alpha \log \pi(\tau)].$$

The objective above can be viewed as maximizing the expected return regularized by the negative entropy of the trajectory distribution. Equivalently, this corresponds to minimizing the Kullback-Leibler (KL) divergence between the policy-induced trajectory distribution $\pi(\tau)$ and a reward-weighted distribution. Formally, define a base trajectory distribution $\pi_1(\tau)$ induced by some prior policy or uniform policy:

$$\pi_1(\tau) = p(s_0) \prod_{t=0}^T \pi_1(a_t | s_t) p(s_{t+1} | s_t, a_t).$$

The maximum entropy objective implies the optimal trajectory distribution $\pi_2(\tau)$ satisfies:

$$\pi_2 = \arg \min_{\pi} D_{\text{KL}} \left(\pi(\tau) \middle\| \frac{1}{Z} \pi_1(\tau) \exp \left(\frac{1}{\alpha} R(\tau) \right) \right),$$

where Z is the partition function ensuring normalization:

$$Z = \int \pi_1(\tau) \exp \left(\frac{1}{\alpha} R(\tau) \right) d\tau.$$

Solving this minimization yields the optimal distribution:

$$\pi_2(\tau) = \frac{1}{Z} \pi_1(\tau) \exp \left(\frac{1}{\alpha} R(\tau) \right).$$

□

F Proof of Theorem 1

Lemma 1. Let the trajectory return function $R(\tau)$ be monotonically non-increasing with respect to the distance between a trajectory τ and the optimal trajectory set $\tilde{\mathcal{T}}$:

$$d(\tau_1, \tilde{\mathcal{T}}) \leq d(\tau_2, \tilde{\mathcal{T}}) \implies R(\tau_1) \geq R(\tau_2).$$

Let $P_{\pi_1}(\tau)$ denote the trajectory distribution of the baseline policy, and the maximum entropy optimization yields the trajectory distribution of the improved policy as

$$P_{\pi_2}(\tau) = \frac{1}{Z} P_{\pi_1}(\tau) \exp \left(\frac{1}{\alpha} R(\tau) \right),$$

where Z is the partition function. By mapping trajectories to their distances $\rho = d(\tau, \tilde{\mathcal{T}})$, we define the corresponding probability density functions $P_{\pi_i}(\rho)$. Then, the likelihood ratio function

$$r(\rho) := P_{\pi_2}(\rho) / P_{\pi_1}(\rho)$$

is a monotonically non-increasing function of the distance ρ

Proof. Let us define the mapping:

$$\Phi : \tau \mapsto \rho = d(\tau, \tilde{\mathcal{T}}),$$

which projects the trajectory space onto the non-negative real axis. Assume the trajectory space is equipped with a measure μ , and the baseline trajectory distribution $P_{\pi_1}(\tau)$ represents a probability density function with respect to μ . From probability theory, the marginal probability density is defined as:

$$P_{\pi_1}(\rho) = \int_{\tau \in \Phi^{-1}(\rho)} P_{\pi_1}(\tau) d\mu(\tau).$$

We define the conditional probability density (the distribution of trajectories τ at fixed distance ρ) as:

$$P_{\pi_1}(\tau | \rho) = \frac{P_{\pi_1}(\tau)}{P_{\pi_1}(\rho)}, \quad \forall \tau \in \Phi^{-1}(\rho).$$

By the definition of the maximum entropy policy distribution:

$$P_{\pi_2}(\tau) = \frac{1}{Z} P_{\pi_1}(\tau) \exp \left(\frac{1}{\alpha} R(\tau) \right),$$

the corresponding marginal probability density becomes:

$$\begin{aligned} P_{\pi_2}(\rho) &= \int_{\Phi^{-1}(\rho)} P_{\pi_2}(\tau) d\mu(\tau) \\ &= \frac{1}{Z} \int_{\Phi^{-1}(\rho)} P_{\pi_1}(\tau) \exp \left(\frac{1}{\alpha} R(\tau) \right) d\mu(\tau). \end{aligned}$$

Expressed in conditional probability form:

$$\begin{aligned} P_{\pi_2}(\rho) &= \frac{1}{Z} P_{\pi_1}(\rho) \int_{\Phi^{-1}(\rho)} P_{\pi_1}(\tau | \rho) \exp \left(\frac{1}{\alpha} R(\tau) \right) d\mu(\tau). \end{aligned}$$

We define:

$$r(\rho) := \frac{P_{\pi_2}(\rho)}{P_{\pi_1}(\rho)} = \frac{1}{Z} \mathbb{E}_{\tau \sim P_{\pi_1}(\cdot | \rho)} \left[\exp \left(\frac{1}{\alpha} R(\tau) \right) \right].$$

Given the monotonic relationship:

$$d(\tau_1, \tilde{\mathcal{T}}) \leq d(\tau_2, \tilde{\mathcal{T}}) \implies R(\tau_1) \geq R(\tau_2),$$

the reward function $R(\tau)$ is a monotonically non-increasing function of distance ρ . Consequently, for any $\rho \geq 0$, the range of $R(\tau)$ over the trajectory set $\Phi^{-1}(\rho)$ forms a bounded interval:

$$R(\Phi^{-1}(\rho)) := \{R(\tau) \mid \tau \in \Phi^{-1}(\rho)\} \subseteq [m(\rho), M(\rho)],$$

where:

$$m(\rho) := \inf_{\tau \in \Phi^{-1}(\rho)} R(\tau), \quad M(\rho) := \sup_{\tau \in \Phi^{-1}(\rho)} R(\tau).$$

Moreover, $M(\rho)$ is a monotonically non-increasing function of ρ :

$$p_1 \leq p_2 \implies M(p_1) \geq M(p_2).$$

Thus, all trajectory rewards within $\Phi^{-1}(\rho)$ lie within a bounded interval, and as ρ increases, the upper bound of $R(\tau)$ decreases. We now employ Jensen's inequality to analyze the monotonicity of $r(\rho)$:

Since the exponential function is convex:

$$\begin{aligned} r(\rho) &= \frac{1}{Z} \mathbb{E}_{\tau | \rho} \left[\exp \left(\frac{1}{\alpha} R(\tau) \right) \right] \\ &\geq \frac{1}{Z} \exp \left(\frac{1}{\alpha} \mathbb{E}_{\tau | \rho}[R(\tau)] \right). \end{aligned}$$

Defining the conditional expected reward:

$$\bar{R}(\rho) := \mathbb{E}_{\tau \sim P_{\pi_1}(\cdot | \rho)}[R(\tau)],$$

which inherits the monotonically non-increasing property from $R(\tau)$. This yields the lower bound:

$$r(\rho) \geq \frac{1}{Z} \exp \left(\frac{1}{\alpha} \bar{R}(\rho) \right),$$

where the right-hand side is monotonically non-increasing. Furthermore, the upper bound is given by:

$$\begin{aligned} r(\rho) &\leq \frac{1}{Z} \exp \left(\frac{1}{\alpha} \max_{\tau \in \Phi^{-1}(\rho)} R(\tau) \right) \\ &= \frac{1}{Z} \exp \left(\frac{1}{\alpha} M(\rho) \right), \end{aligned}$$

which is also monotonically non-increasing. Therefore, $r(\rho)$ is sandwiched between monotonically non-increasing functions, and under typical conditions where the conditional distribution becomes increasingly concentrated, $r(\rho)$ itself converges to being strictly monotonically non-increasing. \square

Lemma 1 shows that if a return function $R(\tau)$ is monotonically non-increasing with the distance of a trajectory τ from the optimal trajectory set, then the policy optimized under maximum entropy reinforcement learning will assign higher probabilities to trajectories nearer to the optimal trajectory set.

Lemma 2. Consider a trajectory space (\mathcal{T}, d) equipped with a distance metric that maps a trajectory τ to its distance from the optimal trajectory set $\tilde{\mathcal{T}}$:

$$\rho := d(\tau, \tilde{\mathcal{T}}) \in [0, +\infty).$$

The trajectory distributions induced by policies π_1 and π_2 are represented by probability density functions $P_{\pi_1}(\rho)$ and $P_{\pi_2}(\rho)$, respectively, where $\rho \in \mathbb{R}_{\geq 0}$. Suppose there exists a non-increasing function $r(\rho)$ such that for all $\rho \geq 0$,

$$\frac{P_{\pi_2}(\rho)}{P_{\pi_1}(\rho)} = r(\rho),$$

meaning that π_2 assigns higher relative probability density to trajectories with smaller ρ (i.e., closer to $\tilde{\mathcal{T}}$). The optimal policy set $\tilde{\Pi}$ consists of policies that generate trajectory distributions concentrated at $\rho = 0$ (i.e., a Dirac delta distribution δ_0). Then, the following inequality holds:

$$d(\pi_1, \tilde{\Pi}) \geq d(\pi_2, \tilde{\Pi}).$$

Proof. Since all policies in $\tilde{\Pi}$ induce the Dirac delta trajectory distribution δ_0 , the Wasserstein- p distance between any policy π and $\tilde{\Pi}$ is given by:

$$d(\pi, \tilde{\Pi}) = W_p(P_\pi, \delta_0).$$

When the target distribution is a Dirac measure, the Wasserstein distance simplifies to:

$$W_p(P_\pi, \delta_0)^p = \int_0^{+\infty} \rho^p dP_\pi(\rho) = \mathbb{E}_{\tau \sim \pi} [\rho(\tau)^p].$$

Thus, we need to prove:

$$\mathbb{E}_{\tau \sim \pi_1} [\rho(\tau)^p] \geq \mathbb{E}_{\tau \sim \pi_2} [\rho(\tau)^p].$$

From our assumptions, $P_{\pi_2}(\rho) = r(\rho)P_{\pi_1}(\rho)$, where $r(\rho)$ is a non-increasing function. This implies that for any $\rho_1 \leq \rho_2$:

$$\frac{P_{\pi_2}(\rho_1)}{P_{\pi_1}(\rho_1)} = r(\rho_1) \geq r(\rho_2) = \frac{P_{\pi_2}(\rho_2)}{P_{\pi_1}(\rho_2)}.$$

In other words, π_2 redistributes probability mass from smaller to larger ρ values less aggressively than π_1 . Rearrangement Inequality states that if $f(\rho)$ is non-decreasing and $g(\rho)$ is non-increasing, then:

$$\int_0^{+\infty} f(\rho)g(\rho)d\rho \leq \int_0^{+\infty} f(\rho)\tilde{g}(\rho)d\rho,$$

where $\tilde{g}(\rho)$ is the non-increasing rearrangement of $g(\rho)$. Application:

Let $f(\rho) = \rho^p$ (clearly non-decreasing). Let $g(\rho) = P_{\pi_1}(\rho)$, and since $P_{\pi_2}(\rho) = r(\rho)P_{\pi_1}(\rho)$ with $r(\rho)$ non-increasing, $P_{\pi_2}(\rho)$ represents a non-increasing rearrangement of $P_{\pi_1}(\rho)$.

By the rearrangement inequality, we have:

$$\begin{aligned} \int_0^{+\infty} \rho^p P_{\pi_2}(\rho)d\rho &= \int_0^{+\infty} \rho^p r(\rho)P_{\pi_1}(\rho)d\rho \\ &\leq \int_0^{+\infty} \rho^p P_{\pi_1}(\rho)d\rho. \end{aligned}$$

This establishes:

$$\mathbb{E}_{\tau \sim \pi_2} [\rho(\tau)^p] \leq \mathbb{E}_{\tau \sim \pi_1} [\rho(\tau)^p].$$

From the expression of Wasserstein- p distance, we directly obtain:

$$W_p(P_{\pi_1}, \delta_0)^p \geq W_p(P_{\pi_2}, \delta_0)^p \implies d(\pi_1, \tilde{\Pi}) \geq d(\pi_2, \tilde{\Pi}).$$

\square

Based on Lemma 1 and Lemma 2, we can readily derive that:

If exist a trajectory return function $R(\tau)$ that is monotonically non-increasing with respect to the distance between a trajectory τ and the desired trajectory set $\tilde{\mathcal{T}}$, such that:

$$d(\tau_1, \tilde{\mathcal{T}}) \leq d(\tau_2, \tilde{\mathcal{T}}) \implies R(\tau_1) \geq R(\tau_2).$$

Suppose policy π_2 is obtained by optimizing policy π_1 using a maximum entropy algorithm with respect to R . Then, policy π_2 is closer to the optimal policy set $\tilde{\Pi}$ than π_1 :

$$d(\pi_1, \tilde{\Pi}) \geq d(\pi_2, \tilde{\Pi}).$$

G Indicative Test Combination

In this section, we explore the combination of multiple indicative tests into a return function.

Figure 6 shows the combination of pass-fail tests, where the optimal trajectory set $\tilde{\mathcal{T}}$ is the intersection of $\tilde{\mathcal{T}}_i, \forall i \in \{1, 2, \dots, m\}$. The policy optimization objective seeks to find a policy whose interaction with the environment generates trajectories that all fall within the optimal trajectory set $\tilde{\mathcal{T}}$. Therefore, we can construct a trajectory reward function such that trajectories closer to the optimal trajectory set $\tilde{\mathcal{T}}$ receive higher rewards.

The lower plot in Figure 6 exhibits this property, but since its derivative vanishes almost everywhere, it cannot provide meaningful optimization directions. Thus, we design a trajectory reward function that offers such directional guidance. Figure 7 illustrates this function: it not only assigns

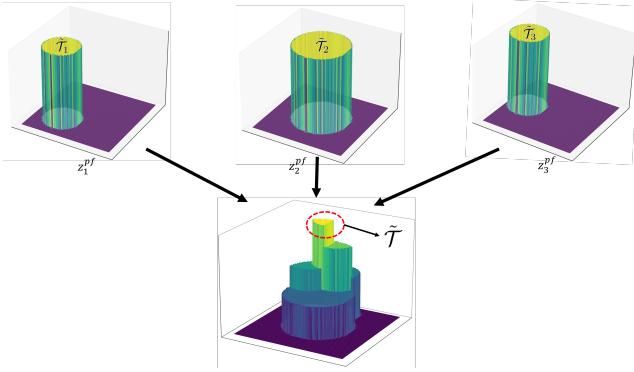


Figure 6: A combination of pass-fail tests. The top row displays individual pass-fail test functions, with yellow regions denoting trajectories satisfying $z_i^{pf}(\tau) = 1$ (passing test). The bottom row presents the summation function of these pass-fail tests, where the output value equals the count of passed tests. The maximal-value region (where $\sum_{i=1}^m z_i^{pf}(\tau) = m$) identifies trajectories belonging to $\tilde{\mathcal{T}}$ that satisfy all test conditions.

higher rewards to trajectories nearer to $\tilde{\mathcal{T}}$ but also provides gradient information usable for policy optimization.

A trajectory return function with the desired properties can be constructed by combining indicative test functions. Figure 7 illustrates this composition process. However, constructing such a return function via linear weighted summation is challenging: trajectories satisfying all test criteria rarely excel in any individual metric, and linear combinations often induce metric conflicts or excessive emphasis on one metric. To address this, we use a nonlinear neural network to integrate the indicative test functions into the trajectory reward function.

H Interpretation of Trajectory Comparison

This section provides a detailed explanation of the following trajectory comparison priors:

- Pass-fail tests take precedence over indicative tests
- A trajectory satisfying more pass-fail tests is closer to $\tilde{\mathcal{T}}$
- A trajectory passing more challenging tests (corresponding to smaller $\tilde{\mathcal{T}}_i$) is closer to $\tilde{\mathcal{T}}$
- All trajectories within $\tilde{\mathcal{T}}$ have zero distance to $\tilde{\mathcal{T}}$
- Under-optimized indicators should be prioritized

The optimization objective in TdRL requires trajectories to satisfy all pass-fail tests, while indicative tests provide quantitative measurements of specific aspects. In other words, pass-fail tests directly reflect the policy optimization goals, while indicative tests represent potential optimization pathways. This fundamental distinction establishes the priority hierarchy: **pass-fail tests take precedence over indicative tests during trajectory evaluation**.

In pass-fail tests, for the collection of sets $\{\tilde{\mathcal{T}}_1, \tilde{\mathcal{T}}_2, \dots, \tilde{\mathcal{T}}_m\}$, consider their intersection family (i.e., all sets of the form $\bigcap_{i \in I} \tilde{\mathcal{T}}_i$ where $I \subseteq \{1, 2, \dots, m\}$).

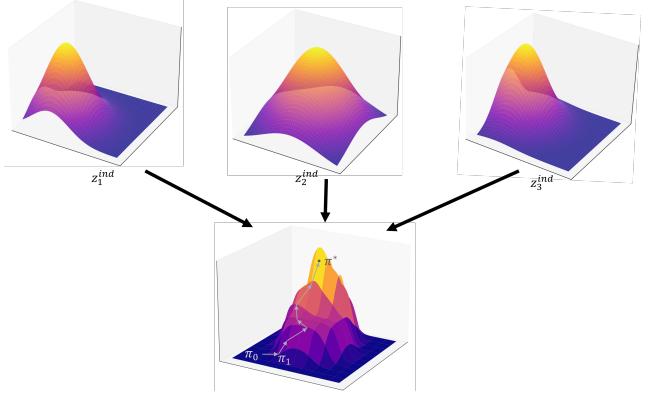


Figure 7: A combination of indicative tests. The top row displays individual indicative test functions, where the vertical axis represents the test results of trajectories. The bottom row illustrates the trajectory return function obtained by combining these test functions. Policy optimization performed on this return function enables the agent to generate trajectories that converge to the optimal trajectory set (highlighted in Figure 6) through environment interactions.

There exists a minimal set $\tilde{\mathcal{T}}_\tau$ in this family that contains trajectory τ , with $\tilde{\mathcal{T}} \subseteq \tilde{\mathcal{T}}_\tau$. The smaller $\tilde{\mathcal{T}}_\tau$, the tighter the upper bound on $\tilde{d}(\tau)$. The equality $\tilde{d}(\tau) = 0$ holds iff $\tilde{\mathcal{T}}_\tau = \tilde{\mathcal{T}}$. In this intersection family, the intersection typically becomes smaller as the number of intersecting sets increases. Therefore, we could derive that **a trajectory satisfying more pass-fail tests is closer to $\tilde{\mathcal{T}}$** .

Similarly, if a pass-fail test is more difficult, the set of trajectories that can pass this test becomes smaller, thus we have that **a trajectory passing more challenging tests (corresponding to smaller $\tilde{\mathcal{T}}_i$) is closer to $\tilde{\mathcal{T}}$** . However, directly comparing the size of a difficult test's passing trajectory set with the intersection of multiple simple tests' passing sets is challenging. Thus, we cannot definitively determine their relative priorities. Given that our ultimate objective is to pass all pass-fail tests, this work prioritizes the quantity of passed tests over their difficulty. This prioritization can, of course, be adapted based on specific task requirements.

When pass-fail test functions cannot distinguish between trajectories (either all tests fail or trajectories pass identical test sets), we lose relative performance information between trajectories. In such cases, metric-based tests provide finer-grained differentiation. However, since multiple metric tests typically exist, their prioritization becomes necessary. Empirically, trajectories passing all pass-fail tests tend to exhibit consistently high metric test scores. Therefore, our training process emphasizes that **under-optimized indicators should be prioritized**, quantified using the skewness of historical metric test data distributions.

Algorithm 2 presents the detailed pseudocode for the lexicographic trajectory comparison method.

Algorithm 2: TdRL

Input: τ_1, τ_2 **Output:** μ

- 1: **if** $\sum_{i=1}^m z_i^{pf}(\tau_1) = \sum_{i=1}^m z_i^{pf}(\tau_2) = m$ **then**
- 2: return $\mu = 0.5$
- 3: **end if**
- 4: **if** $\sum_{i=1}^m z_i^{pf}(\tau_1) > \sum_{i=1}^m z_i^{pf}(\tau_2)$ **then**
- 5: return $\mu = 1$
- 6: **end if**
- 7: **if** $\sum_{i=1}^m z_i^{pf}(\tau_1) < \sum_{i=1}^m z_i^{pf}(\tau_2)$ **then**
- 8: return $\mu = 0$
- 9: **end if**
- 10: **for** $i = 1$ to m **do**
- 11: **if** $z_{k_i}^{pf}(\tau_1) > z_{k_i}^{pf}(\tau_2)$ **then**
- 12: return $\mu = 1$
- 13: **end if**
- 14: **end for**
- 15: **for** $i = 1$ to n **do**
- 16: **if** $z_i^{ind}(\tau_1) > z_i^{ind}(\tau_2)$ **then**
- 17: return $\mu = 1$
- 18: **end if**
- 19: **end for**
- 20: **if** μ is not assigned yet **then**
- 21: return $\mu = 0.5$
- 22: **end if**

I Test Function Settings

This section provides a detailed description of the test function settings for each task in the experimental part of the paper. To standardize notation, we adopt the prefix "pf-" for pass-fail tests and "ind-" for indicative tests.

- **Walker-Stand**

- **pf-upright**: verifies if the cosine of torso angle remains within $[0.9, 1]$ at all timesteps.
- **pf-height**: verifies if the torso's height stays within $[1.2, +\infty]$ at all timesteps.
- **ind-upright**: counts the number of timesteps where the cosine of torso angle remains within $[0.9, 1]$.
- **ind-height**: counts the number of timesteps where the torso's height stays within $[1.2, +\infty]$.

- **Walker-Run**

- **pf-upright**: verifies if the cosine of torso angle remains within $[0.9, 1]$ at all timesteps.
- **pf-height**: verifies if the torso's height stays within $[1.2, +\infty]$ at all timesteps.
- **pf-speed**: verifies if the mean speed in x-axis of torso stays within $[8, +\infty]$ across the trajectory.
- **ind-upright**: counts the number of timesteps where the cosine of torso angle remains within $[0.9, 1]$.
- **ind-height**: counts the number of timesteps where the torso's height stays within $[1.2, +\infty]$.
- **ind-speed**: mean speed in x-axis across the trajectory.

- **Walker-JumpRun**

- **pf-upright**: verifies if the cosine of torso angle remains within $[0.9, 1]$ at all timesteps.

- **pf-height**: verifies if the torso's height stays within $[1.2, +\infty]$ at all timesteps.
- **pf-jump**: verifies if the torso's max height stays within $[1.5, +\infty]$ in the trajectory.
- **pf-speed**: verifies if the mean speed in x-axis of torso stays within $[8, +\infty]$ across the trajectory.
- **ind-upright**: counts the number of timesteps where the cosine of torso angle remains within $[0.9, 1]$.
- **ind-height**: counts the number of timesteps where the torso's height stays within $[1.2, +\infty]$.
- **ind-jump**: the torso's max height in the trajectory.
- **ind-speed**: mean speed in x-axis across the trajectory.
- **Cheetah-Run**

 - **pf-speed**: verifies if the mean speed in x-axis of body stays within $[5, +\infty]$ across the trajectory.
 - **ind-speed**: mean speed in x-axis across the trajectory.
 - **Quadruped-Run**

 - **pf-upright**: verifies if the cosine of torso angle remains within $[0.9, 1]$ at all timesteps.
 - **pf-speed**: verifies if the mean speed in x-axis of torso stays within $[10, +\infty]$ across the trajectory.
 - **ind-upright**: counts the number of timesteps where the cosine of torso angle remains within $[0.9, 1]$.
 - **ind-speed**: mean speed in x-axis across the trajectory.