

ARM: DISCOVERING AGENTIC REASONING MODULES FOR GENERALIZABLE MULTI-AGENT SYSTEMS

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ABSTRACT

Large Language Model (LLM)-powered Multi-agent systems (MAS) have achieved state-of-the-art results on various complex reasoning tasks. Recent works have proposed techniques to automate the design of MASes, eliminating the need for manual engineering. However, these techniques perform poorly, often achieving similar or inferior performance to simple baselines. Furthermore, they require computationally expensive re-discovery of architectures for each new task domain and expensive data annotation on domains without existing labeled validation sets. A critical insight is that simple Chain of Thought (CoT) reasoning often performs competitively with these complex systems, suggesting that the fundamental reasoning unit of MASes, CoT, warrants further investigation. To this end, we present a new paradigm for automatic MAS design that pivots the focus to optimizing CoT reasoning. We introduce the **Agentic Reasoning Module (ARM)**, an agentic generalization of CoT where each granular reasoning step is executed by a specialized reasoning module. This module is discovered through a tree search over the code space, starting from a simple CoT module and evolved using mutations informed by reflection on execution traces. The resulting ARM acts as a versatile reasoning building block which can be utilized as a direct recursive loop or as a subroutine in a learned meta-orchestrator. Our approach significantly outperforms both manually designed MASes and state-of-the-art automatic MAS design methods. Crucially, MASes built with ARM exhibit superb generalization, maintaining high performance across different foundation models and task domains without further optimization.

1 INTRODUCTION

Chain-of-thought (CoT) prompting has emerged as one of the most effective techniques for eliciting complex reasoning from Large Language Models (LLMs) (Wei et al., 2022). By instructing models to generate a series of intermediate steps that lead to a final answer, CoT significantly enhances performance on tasks requiring arithmetic, commonsense, and symbolic reasoning (Nye et al., 2021; Kojima et al., 2022). This simple yet powerful method allows LLMs to break down complex problems into more manageable sub-problems, effectively externalizing the reasoning process over a sequence of generated tokens before arriving at a solution (Wei et al., 2022; Yao et al., 2023a). Recent advancements have also extended CoT with formal verification and multi-agent perspectives, such as MA-LoT (Wang et al., 2025).

Building on the capabilities of individual LLMs, Multi-Agent Systems (MAS) have recently achieved state-of-the-art results on complex reasoning benchmarks (Park et al., 2023; Qian et al., 2023; Hong et al., 2023). These systems typically consist of multiple LLM-powered agents, each assigned a specific role or expertise, orchestrated by a meta-agent or a predefined communication protocol (Wu et al., 2023; Li et al., 2023). While the collaborative nature of MAS enables division

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of labor and synthesis of diverse perspectives (Chen et al., 2023; Dong et al., 2023), recent work has shifted toward the automatic construction of such systems. Emerging automatic MAS generation frameworks demonstrate how agent roles, communication protocols, and workflows can be synthesized directly by LLMs without manual design. For instance, FlowReasoner and AFlow illustrate this trend by automatically generating agent roles and workflows for LLM-based systems, reducing the need for manual design (Zhang et al., 2025c; Kim et al., 2024).

Although MAS approaches have consistently pushed the boundaries of performance, recent studies have revealed a surprising trend: in many cases, a well-prompted single-agent CoT baseline can outperform or perform on par with these complex, multi-agent architectures (Wang et al., 2024; Yao & Yadav, 2025). We also show these observations in our results (Table 1). This finding is significant, as CoT is one of the foundational techniques for LLM reasoning. Its continued competitiveness suggests that the core reasoning unit—the individual thought or step—is of paramount importance. Arguably, the majority of recent research efforts have been dedicated to designing more elaborate MAS frameworks, while the fundamental CoT baseline has remained largely unchanged (Creswell et al., 2022; Chen et al., 2024). Our work pivots from this trend to focus on fundamentally reshaping and enhancing the CoT paradigm for the agentic era by redefining the nature of each reasoning step.

In this work, we introduce the Agentic Reasoning Module (ARM), a novel sequential reasoning approach where each granular step is executed by a specialized, self-contained reasoning agent. The core motivation is to elevate the "thinking" steps of CoT from simple textual continuation to the execution of a sophisticated, agentic block. This block is not manually designed but is instead automatically discovered through an evolutionary process. Starting with a basic CoT procedure, the module is iteratively mutated and refined based on its performance on a generic validation dataset of reasoning problems, resulting in a robust and versatile reasoning procedure that can be applied recursively at each step of solving a challenging multi-step problem.

The prevailing paradigm for MAS design often leads to systems that are highly domain-specific, with individual agents meticulously tuned for particular skills or tasks (Hu et al., 2025; Zhang et al., 2025b). While single-agent systems are generally considered more versatile, they too are often optimized for a narrow set of domains (LaMDAgent, 2025; ScribeAgent, 2024). In contrast, our work focuses on enhancing the universally applicable CoT framework. The agentic block within ARM can be optimized on any generic domain, yielding a general-purpose reasoning technique analogous to the original CoT. We demonstrate that this approach not only achieves superior performance but also exhibits greater generalizability. As we show, MAS built with ARM significantly outperform prominent MAS approaches across diverse agentic datasets without domain-specific tuning.

Our methodology uses the simple yet powerful CoT as a starting seed for the evolutionary discovery of ARM. A meta-agent orchestrates this process, performing a tree search over the code space of possible reasoning modules. Mutations and evolutions are guided by a reflection mechanism that analyzes execution traces from previous attempts, identifying weaknesses and proposing targeted improvements. Furthermore, this meta-agent discovers global strategies to orchestrate collaborations between parallel ARM reasoning traces, effectively creating a high-performance MAS from optimized, homogeneous building blocks. Overall, our work underscores the immense potential of evolving fundamental reasoning methodologies like CoT, presenting a more robust and scalable alternative to the development of increasingly complex and fragile heterogeneous MAS systems. Key contributions of our work are as follows:

- We present the Agentic Reasoning Module (ARM), an evolved and enhanced version of Chain-of-Thought reasoning. We demonstrate that systems built with ARM substantially outperform existing manually designed and automatically discovered multi-agent systems on complex reasoning tasks.
- We show that ARM is a significantly more generalizable reasoning module. MAS constructed with ARM maintain high performance across different underlying foundation models and task domains without requiring re-optimization, highlighting its robustness.
- We provide a rigorous justification and detailed ablations on the validity of our training objective demonstrating the effectiveness of the proposed MAS discovery strategy.

2 RELATED WORKS

Single-Agent and Multi-Agent Reasoning Systems The landscape of LLM-based reasoning is broadly divided into single-agent and multi-agent paradigms. Single-agent systems have demonstrated remarkable capabilities by augmenting the core LLM with sophisticated reasoning and action frameworks. A prominent example is the ReAct framework, which interleaves reasoning steps with actions, enabling the agent to interact with external tools like search engines to gather information and refine its reasoning process (Yao et al., 2023b). Other approaches have focused on enhancing single agents with self-reflection and memory to learn from past mistakes and improve performance iteratively (Shinn et al., 2023; Madaan et al., 2023). While these systems are powerful, their development has often focused on narrower tasks, such as tool-based search, retrieval, and question answering, rather than general-purpose complex reasoning.

In parallel, Multi-Agent Systems (MAS) have emerged as a dominant approach for tackling highly complex problems, often outperforming single-agent counterparts (Park et al., 2023; Qian et al., 2023). Frameworks like AutoGen (Wu et al., 2023), Camel (Li et al., 2023), and MetaGPT (Hong et al., 2023) orchestrate multiple LLM-powered agents, each assigned a specialized role (e.g., programmer, critic, tester). These agents collaborate, debate, and synthesize information to produce solutions for tasks like software development and complex reasoning. A key characteristic of these systems is their heterogeneous nature; each agent is distinct, with a manually engineered role and persona, connected through a predefined and often complex communication topology. In stark contrast, our ARM-based approach constructs a powerful MAS from homogeneous building blocks. The ARM itself is a self-contained, versatile reasoning module that is applied repeatedly, acting as the fundamental unit of thought for all "agents" in the system, thereby simplifying the design while enhancing generalizability.

The Surprising Efficacy of Simple Reasoning Baselines Despite the architectural complexity of many state-of-the-art MAS, a critical and recurring observation is the surprising competitiveness of simple reasoning baselines (Dubey et al., 2023). Foundational techniques like Chain-of-Thought (CoT) (Wei et al., 2022), and simple extensions like Self-Consistency (CoT-SC) which samples multiple reasoning chains and takes a majority vote (Wang et al., 2022), often achieve performance on par with, or even superior to, intricate multi-agent frameworks (Zhang et al., 2025a). This phenomenon is particularly pronounced with the advent of increasingly powerful frontier foundation models (Ke et al., 2025). As these models develop stronger native reasoning abilities, the high-level conceptual guidance provided by a simple CoT prompt is often sufficient to unlock their full potential, rendering the overhead of complex agent orchestration less impactful. This suggests that the primary bottleneck is not necessarily the high-level orchestration strategy but the quality and robustness of the fundamental, step-by-step reasoning process. Our work is directly motivated by this insight, positing that evolving the core reasoning operator—the "thought" in the chain—is a more fruitful direction than designing ever-more-complex superstructures around a static, simple CoT unit.

Automated Design of Multi-Agent Systems Recognizing the significant manual effort required to design effective MAS, recent research has explored automating this process. Approaches like ADAS (Hu et al., 2025), Aflow Zhang et al. (2025b), and Flow-Reasoner (Gao et al., 2025) aim to automatically discover the optimal agent roles and their interaction topology for a given task domain. However, these techniques suffer from two major drawbacks. First, they are computationally expensive, requiring a costly re-discovery process for each new task domain. Second, the discovered systems are often highly specialized and brittle, tuned specifically for the validation data of a single domain. As our results will later show, with the latest generation of foundation models, these automatically discovered systems can be outperformed by simple CoT baselines. Our work diverges from this paradigm. Instead of discovering a complex, domain-specific agent topology, we focus on discovering a single, domain-agnostic reasoning module (ARM). This ARM acts as a universal, high-quality building block that provides superior performance and generalizability without the need for task-specific rediscovery, offering a more scalable and robust path forward for MAS design.

LLM based Prompt Optimizers Recent research has focused on LLMs as prompt optimizers, leveraging their generative and reasoning capabilities to automatically improve prompts within a fixed workflow Zhou et al. (2023); Yang et al. (2024); Khattab et al. (2024); Guo et al. (2024); Novikov et al. (2025); Fernando et al. (2024). Notably, evolutionary approaches coupled with deep reflection over rollouts, such as in GEPA Agrawal et al. (2025), have been shown to offer signifi-

cant advantages in sample efficiency compared to methods that involve updating model weights via Reinforcement Learning.

3 METHODOLOGY: DISCOVERING THE AGENTIC REASONING MODULE

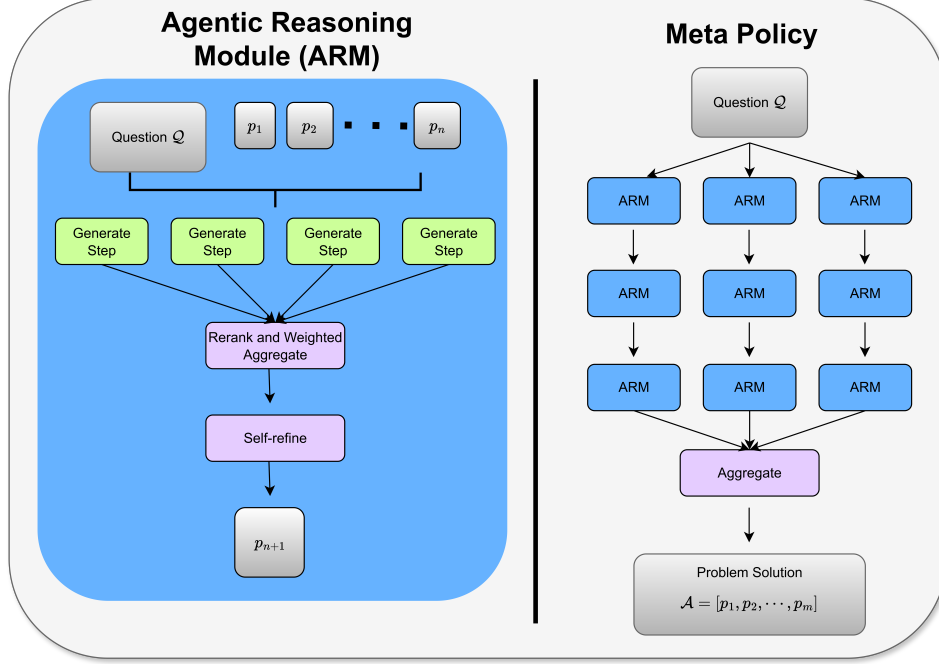


Figure 1: An illustration of the proposed ARM module on the left and the meta policy on the right using "Self refine" as an example MAS. The ARM module takes a question and previous reasoning steps and executes a MAS to get the next step. The meta policy uses ARM as a sub-module and orchestrates the overarching global strategy. Note that this is for illustration only, the actual step generator and the meta policy discovered by Algorithm-1 is more complex (See Appendix).

We introduce the **Agentic Reasoning Module (ARM)**, a self-contained, code-based multi agentic system designed to execute a single, granular step within a complex reasoning process. ARM is conceived as a structured, agentic replacement for a single step in a Chain of Thought (CoT) sequence Wei et al. (2022). While standard CoT prompts an LLM to generate the next reasoning step via naive, monolithic textual generation, an ARM employs an internal multi-agent system (MAS) to produce reasoning steps with greater structure and control.

Following prior work, Hao et al. (2023); Zhang et al. (2024), we define the multi-agentic system as a programming module - a self contained Python function block, while allowing for structured control flow and access to essential APIs such as calling an external LLM, structuring the role and the prompt, and input/output format expectations. Functionally, an ARM accepts the initial problem statement and prior reasoning steps as input, and continues the reasoning until the next logical step in the solution.

3.1 A DECOMPOSABLE FRAMEWORK FOR AGENTIC REASONING

Let the distribution over problem-solution pairs be \mathcal{D} over $(\mathcal{Q}, \mathcal{A})$. A solution \mathcal{A} consists of a sequence of reasoning steps $[p_1, p_2, \dots, p_N]$, where each step p_i belongs to the space of all possible reasoning steps \mathcal{P} . We model the problem-solving process with two key functions:

* **The Step-Generator Module** ($m \in \mathcal{M}$): This is a program that performs a single step of reasoning. It takes the problem question $q \in \mathcal{Q}$ and the history of previous reasoning steps $p_{in} \in \mathcal{P}^*$ as input and returns the next reasoning step $p_{out} \in \mathcal{P}$. Its signature is $m : \mathcal{Q} \times \mathcal{P}^* \rightarrow \mathcal{P}$. An **Agentic Reasoning Module (ARM)** is a structured, code-based implementation of such a module, which can itself be a self-contained MAS.

* **The Meta-Policy** ($\pi \in \Pi$): This is a higher-order program that defines the overarching strategy. It takes a question q and a specific step-generator module m and orchestrates calls to m to generate a complete solution $a \in \mathcal{A}$. Its signature is $\pi : \mathcal{Q} \times \mathcal{M} \rightarrow \mathcal{A}$.

Within this framework, standard Chain of Thought (CoT) can be seen as a simple baseline pairing. It uses a basic step-generator, m_{CoT} , which is a single call to an LLM, and a simple **recursive meta-policy**, π_{Rec} , which applies m_{CoT} repeatedly until a final answer is produced. Our approach independently discovers a more powerful module m^* (the ARM) and a more sophisticated meta-policy π^* .

3.2 DISCOVERING THE OPTIMAL STEP-GENERATOR (m^*)

Our primary goal is to find a step-generator module m^* that is a general-purpose and superior replacement for the simple text generation step in m_{CoT} .

We can formalize a single reasoning step as an update function, $U_{m,q}$, that appends the output of module m to the current reasoning history h :

$$U_{m,q}(h) = h \cdot [m(q, h)]$$

where \cdot denotes list concatenation. A full, n -step reasoning trace generated by the recursive policy π_{Rec} is thus the n -fold composition of this update function: $\pi_{Rec}(q, m) = U_{m,q}^n(\emptyset)$.

Ideally, we would discover the optimal module m^* by maximizing the expected reward \mathcal{R} over the entire problem-solving trace:

$$m^* = \operatorname{argmax}_{m \in \mathcal{M}} \mathbb{E}_{(q,a) \sim \mathcal{D}} [\mathcal{R}(\pi_{Rec}(q, m), a)]$$

However, optimizing this objective directly is intractable due to two main challenges: 1. **Difficult Credit Assignment**: The reward is observed only at the end of a long sequence of steps, making it difficult to determine which specific application of m was responsible for the final outcome. 2. **Unconstrained Search Space**: The space of possible code-based modules \mathcal{M} is vast, making an unguided search highly inefficient.

To address this, we introduce a practical **scaffolded surrogate objective**. Instead of evaluating m on a full rollout generated by itself, we evaluate it within the stable context of a reference trace generated by the baseline m_{CoT} . Specifically, we replace a small, contiguous block of l steps within an n -step CoT trace with our candidate module m . The optimization problem becomes:

$$m^* = \operatorname{argmax}_{m \in \mathcal{M}} \mathbb{E}_{(q,a) \sim \mathcal{D}} [\mathcal{R}(U_{m_{CoT},q}^{n-l-i} \circ U_{m,q}^l \circ U_{m_{CoT},q}^i(\emptyset), a)]$$

where $n = |\pi_{Rec}(q, m_{CoT})|$ is the length of the reference CoT trace, and the starting index i is chosen randomly from $[0, n - 1]$. This formulation isolates the performance contribution of m to a small window, enabling direct credit assignment. Furthermore, the surrounding CoT context provides a powerful inductive bias, constraining the search to modules that behave as effective, incremental reasoning steps. This mirrors the conservative policy-improvement principle Kakade & Langford (2002), where a candidate policy is evaluated under a stable reference distribution to guarantee monotonic improvement. Likewise, the scaffold constrains module updates within a fixed Chain-of-Thought context, ensuring stable, incremental reasoning gains. In our experiments, we find $l = 3$ works well, as it is long enough to expose the module m to critical compositional patterns— $(U_{m_{CoT},q} \circ U_{m,q})$, $(U_{m,q} \circ U_{m,q})$, and $(U_{m,q} \circ U_{m_{CoT},q})$ —while keeping the optimization tractable.

3.3 DISCOVERING THE OPTIMAL META-POLICY (π^*)

While an optimized step-generator m^* improves the quality of each reasoning step, the high-level strategy π that orchestrates these steps is equally critical. A simple recursive policy, π_{Rec} , may be suboptimal for complex problems that could benefit from strategies like parallel rollouts (for self-consistency) or iterative refinement loops Wang et al. (2023); Madaan et al. (2023).

Searching for an optimal meta-policy π^* by repeatedly evaluating candidates with the full, complex m^* module is computationally prohibitive. Therefore, we adopt a surrogate-based approach here as well. We search for the optimal meta-policy π^* using the fast and computationally cheap baseline step-generator, m_{CoT} , as a stand-in for m^* .

This zero-shot transfer from m_{CoT} to m^* is effective because our step-generator optimization process (Section 3.2) is explicitly designed to produce an m^* that functions as a superior, "drop-in" replacement for m_{CoT} . A meta-policy that effectively orchestrates the simple steps of m_{CoT} is thus highly likely to generalize to orchestrating the more powerful, but functionally analogous, steps of m^* . This allows us to efficiently explore the space of strategies, discovering sophisticated control flows like branching for parallel thought generation or conditional loops for verification, without incurring the high computational cost of using m^* .

3.4 REFLECTION-GUIDED EVOLUTIONARY SEARCH

We discover both the optimal step-generator m^* and meta-policy π^* using a unified **Reflection-Guided Evolutionary Search** algorithm. This algorithm performs a tree search over the programmatic space of valid Python modules, where each node in the tree represents a specific program. The search begins with a root node representing the baseline program (m_{CoT} for the step-generator search and π_{Rec} for the meta-policy search). The search then iteratively performs three steps:

1. **Selection:** A parent node (program) p_{parent} is sampled from the current tree \mathcal{T} using temperature sampling based on its validation performance.
2. **Expansion:** A new child program is generated by a **Reviewer Agent**, an LLM-based agent that reflects on the parent program’s execution traces, correctness, and mutation history to propose a targeted code modification.
3. **Evaluation:** The newly generated program is evaluated to obtain its average reward $\bar{\mathcal{R}}$. For a step-generator module, we use the scaffolded objective from Section 3.2. For a meta-policy, we evaluate its performance on a full problem rollout using m_{CoT} as the step-generator.

This entire process is summarized in Algorithm 1.

3.4.1 THE REVIEWER AGENT

The expansion step is driven by a two-stage **Reviewer Agent** that intelligently mutates existing programs. This agent consists of two LLM-based components:

Critic: The Critic analyzes execution traces from the parent program. It identifies logical errors, inefficiencies, or patterns of failure, providing a concise, natural-language analysis of the program’s strengths and weaknesses.

Designer: The Designer acts as the mutation operator. It takes the original program’s code, its performance history, and the Critic’s analysis as input. Based on this information, it proposes a single, targeted code modification aimed at addressing the identified issues, generating a complete, syntactically valid Python class for the new program.

This reflection-driven process ensures that the search evolves programs purposefully, rather than through random mutations, leading to more efficient discovery of high-performance modules and policies. The prompts used for the Critic and Designer are detailed in the Appendix.

4 ARM SEARCH ALGORITHM

Algorithm 1 provides the full pseudocode of the reflection-guided search algorithm for evolving ARM modules.

5 EXPERIMENTS

5.1 BENCHMARKS

We evaluated our baselines and approach on multiple complex reasoning datasets. To assess complex mathematical reasoning capabilities, we utilized widely studied *American Invitational Mathematics Examination (AIME)*¹ and the *Harvard-MIT Mathematics Tournament (HMMT)*² datasets.

¹https://huggingface.co/datasets/MathArena/aime_2025

²https://huggingface.co/datasets/MathArena/hmmt_feb_2025

Algorithm 1 Reflection-Guided Search

```

1: Input: Initial program  $p_{root}$  (e.g.,  $m_{CoT}$  or  $\pi_{Rec}$ ), evaluation function  $EVALUATE(\cdot)$ , total
   iterations  $K$ , exploration constant  $C$ .
2: Initialize:
3: Tree  $\mathcal{T}$  with a single node for  $p_{root}$ .
4:  $p_{root}.\bar{\mathcal{R}} \leftarrow EVALUATE(p_{root})$   $\triangleright$  Evaluate the baseline program on a validation batch
5:  $p_{root}.N \leftarrow 1$   $\triangleright$  Initialize visit count for the root
6: for  $t = 1$  to  $K$  do
7:  $\triangleright$  1. Select a parent program to mutate
8:  $P(p_i) \leftarrow \frac{\exp(p_i.\bar{\mathcal{R}}/T)}{\sum_{j \in \mathcal{T}} \exp(p_j.\bar{\mathcal{R}}/T)}$ 
9:  $p_{parent} \leftarrow \text{Sample}(\mathcal{T}, P)$ 
10:  $\triangleright$  2. Expand the tree via reflection
11:  $\text{traces} \leftarrow EXECUTE(p_{parent})$   $\triangleright$  Collect execution traces
12:  $\text{history} \leftarrow \text{GETMUTATIONHISTORY}(p_{parent})$ 
13:  $p_{new} \leftarrow \text{REVIEWERAGENT}(p_{parent}, \text{traces}, \text{history})$ 
14:  $\triangleright$  3. Evaluate the new program
15:  $p_{new}.\bar{\mathcal{R}} \leftarrow EVALUATE(p_{new})$ 
16:  $p_{new}.N \leftarrow 1$ 
17:  $\triangleright$  4. Update tree and statistics
18:  $\mathcal{T}.\text{ADDCHILD}(p_{parent}, p_{new})$ 
19:  $p_{parent}.N \leftarrow p_{parent}.N + 1$ 
20: end for
21:
22: return  $\operatorname{argmax}_{p_i \in \mathcal{T}} (p_i.\bar{\mathcal{R}})$   $\triangleright$  Return the program with the highest empirical reward

```

For reasoning evaluations on specialized scientific knowledge, we used *GPQA*, a benchmark containing graduate-level questions in physics, chemistry, and biology designed to be challenging even for human experts (Rein et al., 2023). Finally, to measure practical, up-to-date reasoning and robustness against data contamination, we used *LiveBench Reasoning*³, a dynamic benchmark with continuously evolving questions (Jain et al., 2024).

5.2 BASELINES

We compare our methodology against two distinct groups of multi-agent systems (MAS) baselines: popular handcrafted MAS systems and leading automated MAS generation approaches.

5.2.1 HANDCRAFTED MULTI-AGENT SYSTEMS:

We compare against several strong reasoning baselines. **Chain of Thought (CoT)** Wei et al. (2022) serves as the fundamental baseline, solving tasks through iterative textual reasoning. **CoT-Self Consistency (CoT-SC)** Wang et al. (2023) improves upon CoT by generating $n = 12$ parallel reasoning rollouts and selecting the final answer via a majority vote. **Self-Refine** Madaan et al. (2023) employs a feedback loop where a Large Language Model (LLM) iteratively critiques and refines its own output. Lastly, **LLM-Debate** Du et al. (2023) initializes multiple LLM agents with diverse roles to generate different reasoning paths, fostering a debate to converge on a final solution.

5.2.2 AUTOMATED MULTI-AGENT SYSTEMS:

These baselines include the two leading code based MAS generation approaches: **ADAS** Hu et al. (2025) and **AFlow** Zhang et al. (2025b). These methods employ search algorithms to automatically discover the optimal agent roles and their complex interaction topology for a given task domain

We evaluate the performance of ADAS and AFlow using both the original optimization configuration of using a 20% split of the test dataset as the validation dataset (resulting in a benchmark-optimized MAS for each benchmark) and using the ARM optimization configuration of using the

³<https://huggingface.co/datasets/livebench/reasoning>

1000-sample subset of Open-R1-Mixture-of-Thoughts HuggingFace (2025) as the validation dataset (resulting in a single MAS which we evaluate across all benchmarks without benchmark-specific re-optimization). We denote baselines of the former configuration using “(test set)” and baselines of the latter configuration using “(1000-sample)” in the main results in Table 1.

5.3 MODELS

We use OpenAI’s o4-mini-high OpenAI (2025b) reasoning model as the MAS designer for both the baselines ADAS, AFlow, and our method ARM, as MAS generation requires frontier performance in coding, and instruction following. During validation and inference, we three models as backbone LLMs executing the MAS: two closed source models GPT-4.1-nano OpenAI (2025a), GPT-4o OpenAI et al. (2024) and one open source model Llama-3.3-70B Meta (2024).

5.4 TRAINING

Our training process is designed to independently discover the two core components of our framework: the optimal step-generator module (m^*) and the optimal meta-policy (π^*). This decoupled approach allows us to first forge a powerful, general-purpose reasoning module and then learn a sophisticated strategy to orchestrate it, all without requiring expensive, domain-specific annotations.

Validation Dataset: For both discovery processes, we utilize the a subset (1000 samples) of the Math and Science splits of the Open-or-Mixture-of-Thoughts HuggingFace (2025) dataset, a general-purpose instruction-following dataset. Our method requires only a one-time, domain-agnostic training phase. The same resulting code artifacts are then deployed across all benchmark domains and foundation models without any task-specific fine-tuning or re-optimization, underscoring the robustness and versatility of our method.

Step-Generator (m^*) Discovery: We discover the ARM module by employing the Reflection-Guided Evolutionary Search detailed in Algorithm 1. The search is initialized with a basic Chain-of-Thought module (m_{CoT}) and iteratively evolves it by maximizing the scaffolded surrogate objective from Section 3.2. This objective evaluates candidate modules within the context of a baseline CoT trace, enabling efficient and stable optimization.

Meta-Policy (π^*) Discovery: The meta-policy is discovered independently using the same evolutionary search algorithm. To ensure computational tractability, this search is performed using the simple and fast baseline module, m_{CoT} , as a surrogate for the more complex m^* (as justified in Section 3.3). This allows us to efficiently explore the space of high-level strategies and discover a sophisticated meta-policy that can be seamlessly paired with the optimized ARM module.

6 RESULTS

We summarize our results in Table 1 and the key findings are as follows:

(1) Naive Operators outperform MAS: Simple basic operators such as CoT, Self-refine, LLM-Debate outperform complex MAS systems like AFlow and ADAS. This highlights an important concern regarding the practicality of recent advancements in MAS. On the other hand, simple reasoning operators such as CoT perform substantially better across tasks, and varied families of LLMs. Our ARM based reasoning approach is step forward to revitalize traditional yet strong reasoning methods like CoT, by advancing their reasoning steps with agentic blocks. Our ARM based approach further improves up the CoT performance and achieves best results all the datasets.

(2) ARM achieving top performance: ARM consistently outperforms all of the operator baselines. Specifically, in complex datasets such as AIME and HMMT, ARM consistently outperforms existing MAS approaches and all the existing baseline operators. This emphasizes the benefits and strong potential of revitalizing proven traditional reasoning methods like CoT.

(3) Effects from stronger foundation LLM: We first note an important observation that with stronger LLMs such as GPT-4o, simple operators such as CoT and CoT-SC outperform complex MASes. Our ARM based reasoning approach further pushes the best performance over the baseline operators with both recent stronger frontier models such as GPT4.1-nano / GPT-4o and older benchmark models such as LLaMa-3.3-70B.

Model	Method	MATH-500	AIME25	HMMT25	GPQA	LiveBench	Average
GPT-4.1-nano	CoT	82.0%	15.1%	9.9%	50.0%	33.1%	38.0%
	CoT-SC	86.2%	<u>21.9%</u>	13.5%	50.6%	36.9%	41.8%
	Self-Refine	84.2%	17.2%	9.4%	50.0%	28.1%	37.8%
	LLM-Debate	84.2%	15.1%	<u>16.7%</u>	52.5%	33.8%	40.5%
	ADAS (test set)	79.8%	12.0%	5.2%	48.1%	31.2%	35.3%
	ADAS (1000-sample)	77.3%	0.0%	6.8%	46.8%	29.4%	32.0%
	AFlow (test set)	74.5%	18.8%	12.0%	39.9%	30.6%	35.2%
	AFlow (1000-sample)	77.0%	16.7%	10.4%	51.3%	30.6%	37.2%
	ARM (Ours)	82.0%	18.2%	14.6%	<u>60.1%</u>	<u>39.4%</u>	<u>42.9%</u>
	ARM + MP (Ours)	<u>86.0%</u>	23.4%	22.4%	61.4%	45.6%	47.8%
GPT-4o	CoT	75.0%	7.3%	0.5%	53.8%	46.2%	36.6%
	CoT-SC	<u>81.8%</u>	12.5%	2.1%	53.2%	42.5%	38.4%
	Self-Refine	77.2	6.8%	2.6%	53.8%	37.5%	35.6%
	LLM-Debate	<u>81.8%</u>	9.9%	3.1%	56.3%	<u>47.5%</u>	39.7%
	ADAS (test set)	65.5%	1.0%	0.0%	46.2%	38.8%	30.3%
	ADAS (1000-sample)	69.0%	0.0%	0.5%	46.8%	41.9%	31.6%
	AFlow (test set)	75.5%	9.9%	3.6%	53.8%	41.9%	36.9%
	AFlow (1000-sample)	48.8%	9.4%	0.0%	50.6%	45.0%	30.8%
	ARM (Ours)	78.3%	<u>13.5%</u>	<u>5.7%</u>	<u>59.5%</u>	<u>47.5%</u>	<u>40.9%</u>
	ARM + MP (Ours)	82.0%	17.2%	9.4%	60.1%	51.9%	44.1%
LLaMA-3.3-70B	CoT	75.0%	6.8%	3.1%	50.0%	38.1%	34.6%
	CoT-SC	78.5%	4.2%	<u>5.7%</u>	53.2%	45.0%	37.3%
	Self-Refine	77.8%	6.8%	4.2%	<u>51.3%</u>	<u>46.9%</u>	37.4%
	LLM-Debate	79.0%	5.7%	4.2%	50.6%	46.2%	37.1%
	ADAS (test set)	67.2%	3.1%	0.0%	47.5%	37.5%	31.0%
	ADAS (1000-sample)	22.2%	3.1%	0.5%	42.4%	46.2%	22.9%
	AFlow (test set)	65.2%	4.7%	0.0%	46.8%	38.1%	31.0%
	AFlow (1000-sample)	63.2%	7.2%	3.1%	46.8%	15.6%	27.2%
	ARM (Ours)	<u>80.0%</u>	8.3%	5.2%	49.6%	46.2%	<u>37.9%</u>
	ARM + MP (Ours)	80.8%	7.8%	6.8%	50.0%	50.0%	39.1%

Table 1: Main results on four complex reasoning benchmarks across three foundation models. We compare against two groups of baselines: (1) foundational reasoning strategies used to build agentic systems (CoT, CoT-SC, Self-Refine, and LLM-Debate), and (2) existing state-of-the-art automatic MAS design methods (ADAS and AFlow). Our approach is presented in two variants: **ARM**, which recursively applies the discovered reasoning module, and our full method, **ARM + MP**, which combines the ARM with a learned Meta-Policy (MP). Best score in each category is **bolded** and second best score is underlined.

7 ANALYSES

To understand the sources of ARM’s effectiveness, we performed two key analyses. First, we provide empirical evidence that our search objective discovers fundamentally more reliable reasoning modules by minimizing their per-step error rate. Secondly, we show the validity of our efficient, decoupled training strategy by demonstrating that the learned meta-policy transfers zero-shot from a simple surrogate to the final ARM, yielding significant performance gains.

7.1 EMPIRICAL VALIDATION OF THE STEP-GENERATOR OBJECTIVE

To empirically validate our theoretical claim (Appendix A) that the scaffolded objective minimizes per-step error, we conducted a targeted ablation study. We executed the top five discovered step-generator modules for a single step, starting from *critical reasoning junctures* identified by an LLM-judge (GPT-OSS-20B) within baseline m_{CoT} traces. The error rate of each single-step output was then evaluated. As shown in Figure 1, a module’s rank, determined by our objective, strongly correlates with a lower per-step error rate at these critical points. This result confirms that our search process successfully discovers modules that are fundamentally more robust at a granular level, validating the core mechanism behind ARM’s performance.

7.2 EMPIRICAL VALIDATION OF META-POLICY TRANSFER

Our methodology relies on a crucial transfer: a meta-policy trained with the simple m_{CoT} module is deployed zero-shot with the powerful, discovered m^* module. The theoretical justification in Appendix B posits this transfer is effective due to two factors: (1) the inherent superiority of the m^* module, and (2) its ability to guide the reasoning process into more productive states. We designed an experiment to empirically disentangle and verify these two sources of gain.

To do this, we measure and compare three distinct performance configurations. First, we establish a **baseline performance** using the meta-policy with the simple m_{CoT} module. Second, to isolate the pure **module improvement gain**, we measure the performance of the powerful m^* module when it takes over from intermediate reasoning states generated by the baseline m_{CoT} . Finally, we measure the **full system performance** of the meta-policy paired with m^* from the start.

Meta Policy Name (abbreviated)	CoT Baseline	CoT→Meta	Meta Policy
VWASCCoT	35.1%	33.7%	42.0%
CWDCWACCCoT	37.2%	39.3%	41.8%
RVDCCWASCCoT	33.7%	40.0%	41.8%
DRWASCCoT	35.5%	34.9%	41.8%
MBECDCCWASCCoT	36.3%	39.2%	41.4%

Figure 2: Validation of the meta-policy transfer for top discovered policies. The table compares performance using the simple surrogate m_{CoT} (**CoT Baseline**) versus the powerful ARM module m^* (**Meta Policy**). The intermediate **CoT→Meta** column isolates the performance gain from the superior m^* module alone by evaluating it on states generated by the baseline.

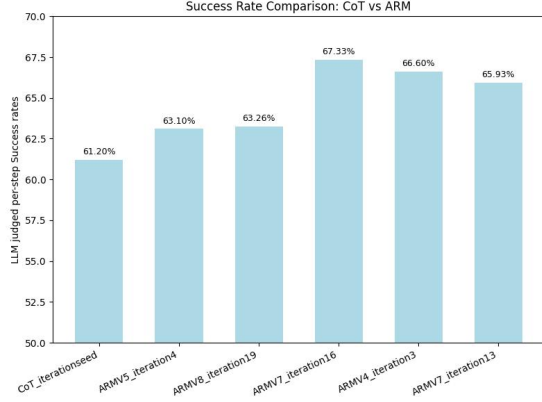


Figure 3: Comparison of LLM judged per-step success rates between the baseline *Chain-of-Thought* (CoT) and multiple *ARM* (CriticChainOfThought) variants. CoT appears first, followed by ARM variants ordered by final performance.

The results, shown in Figure 2, confirm our hypothesis with a clear performance hierarchy. The baseline system performs worst, followed by a significant improvement from simply swapping to the m^* module. The best performance is achieved by the full system, which benefits from both the better module and its ability to find a better reasoning path. This empirically validates the two conditions for successful transfer outlined in AppendixA and confirms the effectiveness of our decoupled discovery strategy.

8 CONCLUSION

We introduced ARM, a modular agentic reasoning framework that revitalizes the traditional Chain-of-Thought (CoT) paradigm by augmenting it with lightweight agentic blocks. Through extensive experiments, we demonstrated that simple operators such as CoT and Self-Refine not only remain highly competitive but, in many cases, outperform complex Multi-Agent Systems (MAS), highlighting the growing gap between empirical performance and the perceived promise of increasingly elaborate MAS designs. Our results show that ARM consistently advances the performance of CoT across diverse reasoning tasks and model families, establishing top-performing results.

Beyond empirical improvements, ARM sheds light on an important perspective: improving the granular step by step reasoning process holds the key to progress in reasoning systems. By preserving the simplicity and generality of CoT steps, while enhancing its reasoning depth and modularity, ARM provides a versatile and powerful foundation that can be applied across tasks and models. ARM represents a step toward a robust and broadly applicable modular reasoning approach with LLMs, paving the way for future research to focus on discovering powerful, reusable reasoning units as a core component of agentic systems.

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A THEORETICAL ANALYSIS

A complete theoretical analysis of the multi-agentic system ARM powered by LLMs is intractable due to the complex, high-dimensional nature of language generation and the non-stationary of the generation process. Therefore, to build a formal intuition for the design choices in our scaffolded search for the step-generator, and the decoupled search for the meta-policy (Algorithm1), we analyze an idealized formulation of the problem as a Markov Decision Process (MDP). This abstracts away the underlying complexities of the text generation and focuses on the dynamics of per-step error and state distribution shift. This analysis provides a formal argument for the soundness of our proposed search objective.

A.1 AN IDEALIZED MDP MODEL OF STEP-WISE REASONING

We model the reasoning process as a Markov decision process (MDP) Sutton & Barto (2018) $\mathcal{M} = (S, A, P, R, \gamma)$:

- **State Space (S):** The state space $S = S_{ok} \cup S_{fail} \cup S_{done}$ is partitioned into three disjoint subsets:
 - S_{ok} : A state $s \in S_{ok}$ represents a partial reasoning trace q, p_1, \dots, p_k that is on a valid path to a solution.
 - S_{fail} : A state $s \in S_{fail}$ has made a critical reasoning error from which recovery is not possible (including terminal states where the reasoning chain ended up on the wrong answer). This is an absorbing region.
 - S_{done} : A state $s \in S_{done}$ represents a reasoning path that has successfully ended on the right answer. This is an absorbing region.
- **Action Space (A):** For a fixed meta-policy π_{Rec} that recursively generates steps until termination (such as the one used by baseline CoT or the ARM-only variant), the meta policy executes a single action at any give state $s \in S_{ok}$: i.e., invokes a step-generator module m to produce the next reasoning step. Thus, the action space is a singleton $\mathcal{A} = \{\text{generate_step}\}$. Hence, the choice of the module m fully defines the transition dynamics of the MDP.
- **Reward Function (R):** A sparse reward function with $R = 1$ given upon transition to a done state.
 - $R(s) = 1$ if $s \in S_{done}$ and 0 otherwise.
- **Transition Dynamics (P):** update function (see 3.2.1) U_m , where $m \in \mathcal{M}$ is characterized by its state-dependent error rate, $\epsilon_m(s)$, which is the probability of making a catastrophic irrecoverable error from state s . Additionally, we introduce For any state $s \in S$, the transition probabilities to the next state s' are defined as:

$$P(s'|s, m) = \begin{cases} p_{done}(s) & \text{if } s' \in S_{done} \\ \epsilon_m(s) & \text{if } s' \in S_{fail} \\ 1 - p_{done}(s) - \epsilon_m(s) & \text{if } s' \in S_{ok} \end{cases}$$

Note that $p_{done}(s)$ is the intrinsic probability of finishing the task by transitioning into a done state from state s . In practice, this is when the model emits `//boxed[Correct Answer]`. This can be assumed to be a property of the task and the progress so far, rather than the generator itself.

- **Value Function:** The value of a state s under module m , denoted by $V_m(s)$ is the probability of eventually reaching a success state in S_{done} (thus receiving a reward) starting from s .

Within in MDP framework, the ideal objective is to discover a module m^* , that maximizes the expected value from the initial state distribution $d_0(s)$

$$m^* = \arg \max_{m \in \mathcal{M}} \mathbb{E}_{s_0 \sim d_0(s)} [V_m(s_0)]$$

This objective poses several major optimization challenges: 1) credit assignment problem over long sequence of steps and 2) unconstrained search space of code modules.

A.2 THEORETICAL GROUNDING FOR THE SCAFFOLDED STEP-GENERATOR SEARCH

The scaffolded objective evaluates a candidate m by *splicing* it into a baseline rollout for a short window $t \in \{i, \dots, i + \ell - 1\}$ while keeping m_{CoT} before and after:

$$\underbrace{U_{m_{\text{CoT}}}^* \circ (U_m^\ell) \circ U_{m_{\text{CoT}}}^i}_{\text{"baseline-candidate-baseline"}}.$$

Let $d_{\text{CoT},t}$ be the state distribution at step t along the (unperturbed) baseline trace. Define the single-step advantage $A_m(s) := V_m(s) - V_{\text{CoT}}(s)$.

Assumption 1 (Finite-horizon conditional stability). *Within the scaffold window and for states along the baseline trace ($s \sim d_{\text{CoT},t}$), the conditional next-state distributions inside \mathcal{S}_{ok} remain close:*

$$\mathbb{E}_{s \sim d_{\text{CoT},t}} \left[D_{\text{TV}}(T_m(\cdot | s, \mathcal{S}_{ok}) \| T_{\text{CoT}}(\cdot | s, \mathcal{S}_{ok})) \right] \leq \beta_{ok} \quad \forall t \in \{i, \dots, i + \ell - 1\}.$$

- **Remark:** This constraint requires that, conditional on the trajectory staying in \mathcal{S}_{ok} , the distribution over possible next states under module m is close on average (in total variation distance) to the distribution under m_{CoT} .

This assumption is grounded in the mechanics of the prompted LLMs. The scaffold provides a strong contextual prior (the preceding CoT steps) and acts as a powerful inductive bias, strongly constraining the module to generate a next step that is stylistically and logically consistent with the atomic reasoning style of CoT. For instance, by defining variables in the same format, using consistent LaTeX for equations, or following an established deductive pattern (e.g., 'Let x be..., then it follows that y ..., therefore z ...' etc.). This incentivizes the LLM to generate a coherent and logically plausible continuation—a necessary condition for remaining in \mathcal{S}_{ok} . Therefore, any "successful" module m must, by necessity, learn to mimic the local *successful* behavior of m_{CoT} while minimizing the transition probability to hazard states \mathcal{S}_{fail} . This is empirically supported by the examples shown in Appendix-B.

Proposition 1 (Scaffolded objective optimizes per-step error rate). *Let w_t be the probability the baseline remains in \mathcal{S}_{ok} up to step t . Then for a universal constant $C > 0$,*

$$V_{\text{scaffold}}(m) - V_{\text{scaffold}}(m_{\text{CoT}}) \geq \sum_{t=i}^{i+\ell-1} \mathbb{E}_{s \sim d_{\text{CoT},t}} [A_m(s)] - C \sum_{t=i}^{i+\ell-1} w_t \beta_{ok}.$$

Moreover, under Assumption 1, $A_m(s)$ is dominated by error-rate reduction:

$$A_m(s) \approx (\varepsilon_{\text{CoT}}(s) - \varepsilon_m(s)) \cdot \mathbb{E}_{s' \sim T_{\text{CoT}}(\cdot | s, \mathcal{S}_{ok})} [V_{\text{CoT}}(s')].$$

Proof of Proposition 1. By Assumption-1, the performance improvement within the ℓ -step scaffold window is lower bounded by the cumulative advantage, minus a small penalty for the distribution shift on successful steps Kakade & Langford (2002); Schulman et al. (2015).

Side Note: For the purpose of a conservative lower bound, we treat this term as a penalty. However, in practice such conditional distributional shift may be beneficial: an advanced module m , potentially using mechanisms like self-consistency or debate, could guide the trajectory toward higher-value states even within \mathcal{S}_{ok} . This would add a positive contribution, but our main result does not rely on that stronger assumption.

For a step t and a baseline state $s \in \mathcal{S}_{ok}$, the Bellman equation with our transitions gives

$$V_m(s) = p_{\text{done}}(s) \cdot 1 + (1 - p_{\text{done}}(s) - \varepsilon_m(s)) \cdot \mathbb{E}_{s' \sim T_m(\cdot | s, \mathcal{S}_{ok})} [V_m(s')].$$

Subtract the corresponding identity for $V_{\text{CoT}}(s)$ and rearrange:

$$\begin{aligned} A_m(s) &= V_m(s) - V_{\text{CoT}}(s) \\ &= (\varepsilon_{\text{CoT}}(s) - \varepsilon_m(s)) \underbrace{\mathbb{E}_{s' \sim T_{\text{CoT}}(\cdot | s, \mathcal{S}_{ok})} [V_{\text{CoT}}(s')]}_{\text{error-rate reduction term}} \\ &\quad + \underbrace{(1 - p_{\text{done}}(s) - \varepsilon_m(s)) (\mathbb{E}_{s' \sim T_m} [V_m(s')] - \mathbb{E}_{s' \sim T_{\text{CoT}}} [V_{\text{CoT}}(s')])}_{\Delta(s)}. \end{aligned}$$

By Assumption 1, the conditional distributions inside \mathcal{S}_{ok} are close in total variation, hence (by standard TV–expectation inequalities) for some constant $C' > 0$,

$$|\mathbb{E}_{s' \sim T_m}[V_m(s')] - \mathbb{E}_{s' \sim T_{CoT}}[V_{CoT}(s')]| \leq C' \beta_{ok}.$$

Since $0 \leq 1 - p_{\text{done}}(s) - \varepsilon_m(s) \leq 1$, we have $|\Delta(s)| \leq C' \beta_{ok}$. Taking expectations over $s \sim d_{CoT,t}$ and summing from $t = i$ to $i + \ell - 1$, the additive “shift” terms accumulate only along trajectories that have not yet absorbed, which contributes the factor w_t ; let $C \geq C'$ absorb constants and the bound on $w_t \leq 1$. This yields the stated lower bound. The approximation claim follows by dropping $\Delta(s)$, which is precisely the small conditional-shift term bounded via β_{ok} .

This leaves an approximation where the advantage is primarily driven by the reduction in the probability of making a catastrophic error. This proposition is directly supported by the empirical results in Figure-3, which shows a strong correlation between a module’s rank and its per-step error rate. \square

A.3 THEORETICAL GROUNDING FOR THE DECOUPLED META-POLICY SEARCH

We now justify the zero-shot transfer of a meta-policy π^* discovered using the surrogate m_{CoT} to the final module m^* . The goal is to show that the expected value of the full system improves, i.e.,

$$V(\pi^*(m^*)) \geq V(\pi^*(m_{CoT})).$$

Proposition 2 (Module improvement on baseline states). *The scaffolded meta policy objective optimizes for a module m^* that has a non-negative expected advantage over the states induced by the baseline policy:*

$$\mathbb{E}_{s \sim d_{m_{CoT}}}[V_{m^*}(s)] \geq \mathbb{E}_{s \sim d_{m_{CoT}}}[V(s)].$$

Proposition 3 (Beneficial distribution shift). *A superior module m^* , which has a lower error rate on-expectation compared to the baseline (i.e., $\mathbb{E}_{s \sim d_{m_{CoT}}}[\epsilon_{m^*}(s)] \leq \mathbb{E}_{s \sim d_{m_{CoT}}}[\epsilon_{m_{CoT}}(s)]$), induces a stationary state distribution d_{m^*} that is weighted towards higher-value states.*

$$\mathbb{E}_{s \sim d_{m^*}}[V_{m^*}(s)] \geq \mathbb{E}_{s \sim d_{CoT}}[V_{m^*}(s)]$$

- **Remark:** Reducing per-step hazard (\mathcal{S}_{fail} states) increases expected survival time in \mathcal{S}_{ok} , shifting probability away from \mathcal{S}_{fail} . Since $V_{m^*}(s)$ is larger on \mathcal{S}_{ok} than on \mathcal{S}_{fail} , the expected value under d_{m^*} is weakly greater.

We now justify the zero-shot transfer of a meta-policy π^* discovered using the surrogate m_{CoT} to the final module m^* . We aim to show that the expected value of the system improves: $\bar{V}_{m^*} \geq \bar{V}_{m_{CoT}}$. This transfer relies on two premises.

Theorem 1 (Monotonic Improvement of Meta-Policy Transfer). *Let $\pi(m)$ denote the system using meta-policy π and step-generator m , and let $V(\pi(m))$ be its expected reward from the initial state distribution. If Proposition 2 and Proposition 3 hold, then the transfer of a meta-policy π^* from m_{CoT} to m^* is guaranteed to not degrade performance:*

$$V(\pi^*(m^*)) \geq V(\pi^*(m_{CoT})).$$

Proof Sketch: We can decompose the difference in expected values as follows:

$$V(\pi^*(m^*)) - V(\pi^*(m_{CoT})) = \mathbb{E}_{s \sim d_{m^*}}[V_{m^*}(s)] - \mathbb{E}_{s \sim d_{m_{CoT}}}[V(s)].$$

We can add and subtract the term $\mathbb{E}_{s \sim d_{m_{CoT}}}[V_{m^*}(s)]$ to obtain:

$$\begin{aligned} V(\pi^*(m^*)) - V(\pi^*(m_{CoT})) &= \underbrace{\left(\mathbb{E}_{s \sim d_{m^*}}[V_{m^*}(s)] - \mathbb{E}_{s \sim d_{m_{CoT}}}[V_{m^*}(s)] \right)}_{\text{Gain from Distribution Shift}} \\ &\quad + \underbrace{\left(\mathbb{E}_{s \sim d_{m_{CoT}}}[V_{m^*}(s)] - \mathbb{E}_{s \sim d_{m_{CoT}}}[V(s)] \right)}_{\text{Gain from Module Improvement}}. \end{aligned}$$

The first term (*Gain from Distribution Shift*) is non-negative by Proposition-3. The second term (*Gain from Module Improvement*) is non-negative by Proposition-2. Since both terms are non-negative, their sum is non-negative, proving the theorem.

This theoretical decomposition is empirically validated by the experiment in Figure 3. The performance of *CoT*→*Meta* relative to *CoT Baseline* measures the Gain from Module Improvement, while the additional gain seen in the full *Meta Policy* system reflects the Gain from Distribution Shift. The positive results for both components provide strong empirical support for this theoretical justification.

B STEP GENERATOR ROLL OUT EXAMPLES

B.1 EXAMPLE-1

ARM Rollout

```

### Step 2
Express the sides in terms of the common ratio  $(2R)$ :
\[
a = 2R \sin 60^\circ = 2R \times \frac{\sqrt{3}}{2} = R \sqrt{3},
\]
\[
b = 2R \sin 45^\circ = 2R \times \frac{\sqrt{2}}{2} = R \sqrt{2},
\]
\[
c = 2R \sin 75^\circ.
\]
Recall that  $(\sin 75^\circ = \sin(45^\circ + 30^\circ))$ :
\[
\sin 75^\circ = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ
\]

$$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}.$$

Hence,
\[
c = 2R \times \frac{\sqrt{6} + \sqrt{2}}{4} = R \times \frac{\sqrt{6} + \sqrt{2}}{2}.
\]

```

CoT Rollout

```

### Step 2
Expressing the sides in terms of the circumradius  $(R)$ :
\[
a = 2R \sin A = 2R \sin 60^\circ = 2R \times \frac{\sqrt{3}}{2} = R \sqrt{3},
\]
\[
b = 2R \sin B = 2R \times \frac{\sqrt{2}}{2} = R \sqrt{2},
\]
\[
c = 2R \sin C = 2R \times \sin 75^\circ.
\]
Recall that:
\[
\sin 75^\circ = \sin (45^\circ + 30^\circ) = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ = \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}.
\]
Therefore:
\[
c = 2R \times \frac{\sqrt{6} + \sqrt{2}}{4} = R \times \frac{\sqrt{6} + \sqrt{2}}{2}.
\]

```

ARM Rollout

```

### Step 3
Now, determine the new temperature after doubling the kinetic energy:
\[
T_2 = 526.3 \text{ K}
\]
Convert this back to Celsius:
\[
T_2 (^\circ\text{C}) = 526.3 - 273.15 = 253.15^\circ\text{C}
\]
This is a significant increase from the initial  $(-10^\circ\text{C})$ , indicating the molecules now have enough energy corresponding to about  $(253^\circ\text{C})$ .

```

CoT Rollout

```

### Step 3
Determine the new temperature after doubling the kinetic energy:
Since kinetic energy  $\propto T$ ,
\[
T_2 = 2 \times T_1 = 526.3 \text{ K}
\]
Convert this back to Celsius:
\[
T_2 (^\circ\text{C}) = T_2 (\text{K}) - 273.15 = 526.3 - 273.15 \approx 253.15^\circ\text{C}
\]
This temperature is well above the boiling point of water at 1 atm (100C). Therefore, the molecules' kinetic energy would be sufficient to overcome intermolecular forces and convert the water from the solid phase (ice) into the gas phase (vapor).

```

B.3 EXAMPLE-3

ARM Rollout

Step 1

To determine the correct substitution from equation 12.9 to equation 12.10, we need to understand the context of these equations and the variables involved. Equation 12.9 likely involves a momentum-like variable (P^+) or a gamma matrix that acts on spinors, and the goal is to simplify or rewrite this expression into a form involving gamma matrices or energy-momentum components.

Typically, in relativistic quantum mechanics or quantum field theory, the transformation from the light-cone momentum (P^+) to gamma matrices involves choosing a specific gamma matrix to project or decompose spinor components. The options provided suggest that the substitution involves gamma matrices and possibly the momentum operator or gamma matrices related to time.

Given that, the known structures are:

- (P^+) : light-cone momentum component, usually $(P^+ = P^0 + P^3)$.
- (γ^u) : a gamma matrix with a specific index (u) , possibly a spatial index.
- (γ^0) : the gamma matrix corresponding to time.
- $(\gamma^0 P)$: a product of (γ^0) with some momentum operator (P) .

Since the transformation is from equation 12.9 to 12.10, and the options involve gamma matrices and momenta, the typical approach in such derivations involves rewriting the projection operator or the momentum in terms of gamma matrices.

CoT

Step 1

To determine the correct substitution, first understand the context: equation 12.9 involves the momentum operator (P^μ) , and equation 12.10 is likely a transformed version involving gamma matrices. In relativistic quantum mechanics, transformations often involve gamma matrices (γ^μ) and the gamma matrices' role in formulating Dirac equations.

Given the options:

- A: (P^+)
- B: (γ^u)
- C: (γ^0)
- D: $(\gamma^0 P)$

and based on the typical transformations, the substitution from (P^μ) to a form involving gamma matrices usually involves (γ^0) because it relates to the time component of the gamma matrices and appears frequently in the Dirac formalism.

In particular, if the original equation involves (P^μ) , and the transformed equation involves a gamma matrix, then the key is recognizing the common substitution that converts momentum operators into gamma matrix expressions, especially the time component (P^0) .

Therefore, the step is to analyze which gamma matrix substitution correctly transforms the operator form in equation 12.9 into the one in 12.10.

C BEST ARM DISCOVERED: CRITICCHAINOFTHOUGHTV7

The following is the Python implementation of the best ARM discovered by our algorithm.

```

1 import asyncio
2
3 class CriticChainOfThoughtV7:
4     def __init__(self, llm):
5         self.llm = llm
6
7     async def forward(self, problem, partial_progress):
8         # 1. Generate four candidate next steps in parallel
9         candidate_tasks = [
10             self.llm.generate_step(problem, partial_progress)
11             for _ in range(4)
12         ]
13         candidates = await asyncio.gather(*candidate_tasks)
14
15         # 2. Critique candidates in two groups of two, in parallel
16         critique_tasks = []
17         groups = [
18             (0, 2, ("rating_1", "rating_2"), ("critique_1",
19 ↪ "critique_2")),
19             (2, 4, ("rating_3", "rating_4"), ("critique_3",
20 ↪ "critique_4"))
21         ]
22         for start, end, rating_names, critique_names in groups:
23             context = [
24                 {
25                     "name": "Problem",
26                     "data": problem,
27                     "description": "The problem to solve."
28                 },
29                 {
30                     "name": "Partial Progress",
31                     "data": partial_progress,
32                     "description": "The partial solution so far."
33                 },
34                 {
35                     "name": "Candidate Next Steps",
36                     "data": "\n\n".join(
37                         f"### Candidate Next Step
38 ↪ {i+1}\n{candidates[i]}"
39                         for i in range(start, end)
40                     ),
41                     "description": "Two candidate next steps
42 ↪ formatted with markdown subheaders."
43                 }
44             ]
45             instructions = (
46                 "You are given a problem, the current partial
47 ↪ solution, and two candidate next reasoning steps.\n"
48                 "For each candidate, provide:\n"
49                 f"- {rating_names[0]} and {rating_names[1]}: a single
50 ↪ integer rating from 1 to 10 indicating its fit as the next
51 ↪ reasoning step (10 is best).\n"
52                 f"- {critique_names[0]} and {critique_names[1]}: a
53 ↪ one-sentence critique highlighting each candidate's strengths
54 ↪ and weaknesses.\n"
55                 f"Name the fields exactly {rating_names[0]},
56 ↪ {critique_names[0]}, {rating_names[1]}, {critique_names[1]}."
57             )
58             response_format = [
59                 {

```

```

51         "name": rating_names[0],
52         "description": f"Integer rating (1-10) for
↳ Candidate Next Step {start+1}."
53     },
54     {
55         "name": critique_names[0],
56         "description": f"One-sentence critique of
↳ Candidate Next Step {start+1}."
57     },
58     {
59         "name": rating_names[1],
60         "description": f"Integer rating (1-10) for
↳ Candidate Next Step {start+2}."
61     },
62     {
63         "name": critique_names[1],
64         "description": f"One-sentence critique of
↳ Candidate Next Step {start+2}."
65     }
66 ]
67 critique_tasks.append(
68     self.llm.chat_completion(context, instructions,
↳ response_format)
69 )
70
71 critiques = await asyncio.gather(*critique_tasks)
72
73 # 3. Parse ratings and identify the two highest-rated
↳ candidates
74 ratings = [
75     int(critiques[0]["rating_1"]),
76     int(critiques[0]["rating_2"]),
77     int(critiques[1]["rating_3"]),
78     int(critiques[1]["rating_4"])
79 ]
80 sorted_indices = sorted(range(4), key=lambda i: ratings[i],
↳ reverse=True)
81 top1_idx, top2_idx = sorted_indices[0], sorted_indices[1]
82 top1_candidate = candidates[top1_idx]
83 top2_candidate = candidates[top2_idx]
84
85 # 4. Final head-to-head comparison between the top two
↳ candidates
86 context_final = [
87     {
88         "name": "Problem",
89         "data": problem,
90         "description": "The problem to solve."
91     },
92     {
93         "name": "Partial Progress",
94         "data": partial_progress,
95         "description": "The partial solution so far."
96     },
97     {
98         "name": "Candidate Next Steps",
99         "data": (
100             f"### Candidate A\n{top1_candidate}\n\n"
101             f"### Candidate B\n{top2_candidate}"
102         ),
103         "description": "Two top candidate next steps
↳ formatted with markdown subheaders."
104     }
105 ]
106 instructions_final = (

```



```

107         "Compare Candidate A and Candidate B as the next
    ↪ reasoning step for the given problem and partial progress.\n"
108         "Provide:\n"
109         "- winner: choose either 'Candidate A' or 'Candidate B'
    ↪ indicating which step is better.\n"
110         "- justification: one-sentence justification for your
    ↪ choice."
111     )
112     response_format_final = [
113         {
114             "name": "winner",
115             "description": "Either 'Candidate A' or 'Candidate B'
    ↪ indicating the better next step."
116         },
117         {
118             "name": "justification",
119             "description": "One-sentence justification for the
    ↪ choice."
120         }
121     ]
122     final_decision = await self.llm.chat_completion(
123         context_final, instructions_final, response_format_final
124     )
125
126     if final_decision["winner"].strip() == "Candidate A":
127         selected_candidate = top1_candidate
128         runnerup_candidate = top2_candidate
129     else:
130         selected_candidate = top2_candidate
131         runnerup_candidate = top1_candidate
132
133     # 5. Post-selection adversarial critique with severity rating
134     context_flaw = [
135         {
136             "name": "Problem",
137             "data": problem,
138             "description": "The problem to solve."
139         },
140         {
141             "name": "Partial Progress",
142             "data": partial_progress,
143             "description": "The partial solution so far."
144         },
145         {
146             "name": "Selected Candidate Next Step",
147             "data": f"### Selected Candidate Next
    ↪ Step\n(selected_candidate)",
148             "description": "The final chosen candidate next
    ↪ reasoning step formatted with a markdown subheader."
149         }
150     ]
151     instructions_flaw = (
152         "You are given a problem, the current partial solution,
    ↪ and a selected next reasoning step.\n"
153         "Identify any major flaw or missing piece of reasoning in
    ↪ the selected step.\n"
154         "Provide:\n"
155         "- flaw: either the single word 'None' if there is no
    ↪ flaw, or a brief description of the flaw.\n"
156         "- severity: a single integer rating from 1 to 10
    ↪ indicating how severe the flaw is (10 is critical).\"
157     )
158     response_format_flaw = [
159         {
160             "name": "flaw",

```

```

161         "description": "Either the single word 'None' if
    ↪ there is no flaw, or a brief description of a major flaw in
    ↪ the selected step."
162     },
163     {
164         "name": "severity",
165         "description": "Integer rating (1-10) indicating
    ↪ severity of the flaw (10 is most severe).",
166     }
167 ]
168 flaw_response = await self.llm.chat_completion(
169     context_flaw, instructions_flaw, response_format_flaw
170 )
171 flaw = flaw_response["flaw"].strip()
172 severity = int(flaw_response["severity"])
173
174 # 6. Compute dynamic severity threshold based on rating gap
175 gap = ratings[top1_idx] - ratings[top2_idx]
176 if gap <= 1:
177     threshold = 5
178 elif gap == 2:
179     threshold = 6
180 else:
181     threshold = 7
182
183 # 7. If a severe flaw is detected above the dynamic
    ↪ threshold, fall back
184 if flaw.lower() != "none" and severity >= threshold:
185     return runnerup_candidate
186 return selected_candidate

```

Listing 1: Code for CriticChainOfThoughtV7, performance: 38.0

D BEST META-POLICY DISCOVERED: VERIFIEDWEIGHTEDADAPTIVELFCONSISTENTCHAINOFTHOUGHT

The following is the Python implementation of the best meta-policy discovered by our algorithm.

```

1 import asyncio
2 from agent.solution import Solution, Step
3 from judge_utils import judge_equality
4
5 class VerifiedWeightedAdaptiveSelfConsistentChainOfThought:
6     def __init__(self, llm, block):
7         self.llm = llm
8         self.block = block
9
10    async def forward(self, problem):
11        # Helper: generate one chain up to 8 steps, then complete via
    ↪ LLM if needed
12        async def generate_chain():
13            solution = Solution()
14            for _ in range(8):
15                next_step = await self.block.forward(problem,
    ↪ str(solution))
16                solution.add_step(Step(str(next_step)))
17                if solution.is_completed():
18                    return solution
19                completion = await self.llm.complete_solution(problem,
    ↪ str(solution))
20                solution.add_step(Step(str(completion)))
21            return solution

```

```

22
23     # Helper: confidence scoring (1-5)
24     async def score_chain(chain):
25         context = [
26             {"name": "Problem", "data": problem, "description":
↪ "The original problem statement."},
27             {"name": "Chain", "data": str(chain),
↪ "description": "Full chain-of-thought reasoning plus final
↪ answer."}
28         ]
29         instructions = (
30             "You are evaluating the chain-of-thought solution for
↪ the given problem. "
31             "On a scale from 1 (very uncertain) to 5 (very
↪ confident), rate your confidence "
32             "that the final answer is correct. Output ONLY the
↪ integer confidence (1-5)."
33         )
34         response_format = [{"name": "Confidence", "description":
↪ "Integer from 1 to 5"}]
35         resp = await self.llm.chat_completion(context,
↪ instructions, response_format)
36         # parse safely
37         try:
38             conf = int(resp["Confidence"].strip())
39         except Exception:
40             conf = 1
41         return max(1, min(conf, 5))
42
43     # Helper: verify logical consistency (Yes/No)
44     async def verify_chain(chain):
45         context = [
46             {"name": "Problem", "data": problem, "description":
↪ "The original problem statement."},
47             {"name": "Chain", "data": str(chain),
↪ "description": "Full chain-of-thought reasoning plus final
↪ answer."}
48         ]
49         instructions = (
50             "Review the chain-of-thought reasoning for the given
↪ problem. "
51             "Is the reasoning free of logical errors or
↪ contradictions? "
52             "Output ONLY 'Yes' if it is fully logical, otherwise
↪ output 'No'."
53         )
54         response_format = [{"name": "Valid", "description": "Yes
↪ or No"}]
55         resp = await self.llm.chat_completion(context,
↪ instructions, response_format)
56         valid = resp.get("Valid",
↪ "").strip().lower().startswith("y")
57         return valid
58
59     # Weighted vote helper
60     def find_best_weighted(chains_list, conf_list):
61         weight_sums = {}
62         total = sum(conf_list)
63         for chain, cf in zip(chains_list, conf_list):
64             ans = chain.answer()
65             weight_sums[ans] = weight_sums.get(ans, 0) + cf
66         best_ans, best_w = None, -1
67         for ans, w in weight_sums.items():
68             if w > best_w:
69                 best_ans, best_w = ans, w

```

```

70         return best_ans, best_w, total
71
72     # 1) Generate initial 3 chains in parallel
73     initial = [generate_chain() for _ in range(3)]
74     chains = await asyncio.gather(*initial)
75
76     # 2) Score and verify each chain
77     score_tasks = [score_chain(ch) for ch in chains]
78     verify_tasks = [verify_chain(ch) for ch in chains]
79     confidences = await asyncio.gather(*score_tasks)
80     valids = await asyncio.gather(*verify_tasks)
81
82     max_chains = 7
83
84     # 3) Adaptive sampling with verification gating
85     while True:
86         # Determine which chains to consider: only verified if
87         ↪ any, else all
88         if any(valids):
89             considered_chains = [ch for ch, v in zip(chains,
90             ↪ valids) if v]
91             considered_confs = [cf for cf, v in
92             ↪ zip(confidences, valids) if v]
93         else:
94             considered_chains = chains
95             considered_confs = confidences
96
97         best_ans, best_weight, total_weight =
98         ↪ find_best_weighted(considered_chains, considered_confs)
99         # stop if weighted majority reached or chain cap
100         if best_weight > total_weight / 2 or len(chains) >=
101         ↪ max_chains:
102             break
103
104         # else generate one more chain, score & verify, then loop
105         new_chain = await generate_chain()
106         chains.append(new_chain)
107         new_conf = await score_chain(new_chain)
108         confidences.append(new_conf)
109         new_valid = await verify_chain(new_chain)
110         valids.append(new_valid)
111
112     # 4) Select final chain: consensus & highest confidence among
113     ↪ considered
114     if any(valids):
115         final_pool = [ (ch, cf) for ch, cf, v in zip(chains,
116         ↪ confidences, valids) if v and judge_equality(ch.answer(),
117         ↪ best_ans) ]
118     else:
119         final_pool = [ (ch, cf) for ch, cf in zip(chains,
120         ↪ confidences) if judge_equality(ch.answer(), best_ans) ]
121
122     selected_chain = None
123     top_conf = -1
124     for ch, cf in final_pool:
125         if cf > top_conf:
126             selected_chain, top_conf = ch, cf
127
128     # Fallback if nothing selected
129     if selected_chain is None:
130         selected_chain = chains[-1]
131
132     return selected_chain

```

Listing 2: Code for VerifiedWeightedAdaptiveSelfConsistentChainOfThought, performance: 38.0

E REPRODUCIBILITY STATEMENT

Upon publication, we commit to releasing the open-source code for our framework, including all discovered Agentic Reasoning Modules, meta-policies, and the specific prompts used for the Reviewer Agent. Our experiments were conducted using a mix of closed and open-source models. The MAS designer utilized OpenAI’s `o4-mini-high`. The reasoning modules were executed on GPT-4.1-nano, GPT-4o, and the open-source Llama-3.3-70B. All evaluation benchmarks, including MATH500, AIME, and HMMT, are publicly available.

E.1 ARM IMPLEMENTATION DETAILS

The 1000-sample subset of Open-R1-Mixture-of-Thoughts was created by taking the math and science splits of the original dataset, filtering to samples which the provided Deepseek-R1 reasoning trace had length between 8k to 10k tokens (to filter to samples of appropriate difficulty), and randomly sampling 1000 problems from the filtered problems.

We run both the step-generator module optimization and the meta-policy optimization for 20 iterations. Both optimizations are performed using GPT-4.1-nano as the MAS executor model.

Whenever sampling from the MAS executor model, we use a temperature of 1.0 with a top_p of 0.95.

E.2 BASELINE IMPLEMENTATION DETAILS

As in the ARM implementation, whenever sampling from the MAS executor model, we use a temperature of 1.0 with a top_p of 0.95.

- CoT: We use a simple CoT prompt that instructs the model to reason step-by-step and follow the final answer format.
- CoT-SC: We use $n = 12$ parallel reasoning traces.
- Self-Refine: We limit to a maximum of 5 self refining iterations.
- LLM-Debate: We use 4 LLM agents debating for a maximum of 3 rounds.
- ADAS: We use the provided codebase, following the recommended run configuration. For a fair comparison to other baselines, we make a one line addition to the optimizer prompt to disallow arbitrary Python code execution within the discovered MASes, since other baselines do not utilize code execution. For the 1000-sample optimization, we use GPT-4.1-nano as the MAS executor model during optimization, following ARM’s implementation.
- AFlow: We use the provided codebase, following the recommended run configuration. We allow the optimizer to utilize the Custom, AnswerGenerate, and ScEnsemble operators. For the 1000-sample optimization, we use GPT-4.1-nano as the MAS executor model during optimization, following ARM’s implementation.