Coordinated Decentralized Resource Optimization for Cell-Free ISAC Systems

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Abstract—Integrated Sensing and Communication (ISAC) is emerging as a key enabler for 6G wireless networks, allowing the joint use of spectrum and infrastructure for both communication and sensing. While prior ISAC solutions have addressed resource optimization, including power allocation, beamforming, and waveform design, they often rely on centralized architectures with full network knowledge, limiting their scalability in distributed systems. In this paper, we propose two coordinated decentralized optimization algorithms for beamforming and power allocation tailored to cell-free ISAC networks. The first algorithm employs locally designed fixed beamformers at access points (APs), combined with a centralized power allocation scheme computed at a central server (CS). The second algorithm jointly optimizes beamforming and power control through a fully decentralized consensus ADMM framework. Both approaches rely on local information at APs and limited coordination with the CS. Simulation results obtained using our proposed Python-based simulation framework evaluate their fronthaul overhead and system-level performance, demonstrating their practicality for scalable ISAC deployment in decentralized, cell-free architectures.

Index Terms—Integrated sensing and communication (ISAC), cell-free, resource allocation, distributed optimization, ADMM

I. INTRODUCTION

Integrated Sensing and Communication (ISAC) is envisioned as a cornerstone of 6G wireless systems, enabling joint utilization of spectrum, hardware, and signal processing resources for both high-throughput communication and high-accuracy sensing tasks [1]. Recent research has made significant strides in optimizing various ISAC functionalities, including power allocation (PA) [2], [3], beamforming (BF) [4], waveform design [5], and distributed computation frameworks [6], [7]. Despite this progress, most existing ISAC solutions rely on centralized architectures, where a cloud central server (CS) collects global channel state information (CSI) and performs joint optimization across the network. While such methods may theoretically yield performance gains, in practice, they suffer from critical limitations. Centralized schemes often face bottlenecks due to scalability issues, excessive fronthaul communication, and latency, making them ill-suited for real-time or large-scale deployments. More importantly, these approaches lack adaptability to distributed environments, where local processing and decentralized decision-making are essential due to physical or architectural constraints.

To address these challenges, we develop coordinated decentralized algorithms for BF and PA in cell-free ISAC systems, with the goal of distributing the computation across the network. The first algorithm employs locally designed fixed beamformers at access points (APs) based on predefined methods, while solving a global PA problem centrally at the CS with limited network-level coordination. However, this design constrains the maximum number of simultaneously served users to the number of antennas per AP. To overcome this limitation, we propose a second algorithm that jointly optimizes BF and PA through a fully decentralized consensus ADMM framework. Both algorithms are designed to operate with local CSI and minimal information exchange over the fronthaul network, offering scalable and communication-efficient solutions for decentralized ISAC deployments.

In contrast to existing work that either assumes global CSI or leverages hierarchical and federated learning schemes with limited scalability [6]–[8], our approach emphasizes simple coordination and tractable optimization in a fully decentralized setting. The main contributions are summarized as follows:

- **SplitOpt Algorithm**: We propose an algorithm that employs fixed local BF design at the APs and solves a global PA problem at the CS with limited coordination overhead.
- **JointOpt Algorithm**: We propose a second algorithm that jointly optimizes BF and PA through a fully decentralized consensus ADMM approach, enabling scalability and robustness in locally low-rank scenarios.
- **Open-Source Simulation Framework**: We provide a publicly available Python-based simulation framework that implements the proposed algorithms and supports reproducible and extensible research in decentralized ISAC.
- **Comprehensive Performance Evaluation**: Using our simulation framework, we conduct extensive evaluations of both proposed algorithms, analyzing their sensing and communication performance, scalability, and fronthaul communication overhead. In addition, we implement a centralized solution for the joint BF and PA optimization problem and compare its performance against our ADMM-based decentralized algorithm.

Our results demonstrate the potential of the proposed decentralized optimization methods to serve as scalable solutions for resource allocation in cell-free ISAC systems.

II. ISAC SYSTEM MODEL

We consider an orthogonal frequency division multiplexing (OFDM) system with total operating bandwidth B, number of subcarriers N_{sc}, subcarrier spacing $f_{sc} = B/N_{sc}$, and symbol

period $T_s = T_{sc} + T_{cp}$, where $T_{sc} = 1/f_{sc}$ and T_{cp} is the duration of the cyclic prefix. The network consists of a set $\mathcal{A}$ of APs connected to a CS, with $|\mathcal{A}| = N_{ap}$, and each AP in $\mathcal{A}$ is equipped with an M-element antenna array. A set $\mathcal{U}$ of N_{ue} single-antenna communication users (UEs) is present in the downlink, and one data stream is allocated per user. The set $\mathcal{S}$ indexes the downlink sensing streams with $|\mathcal{S}| = N_s$. The complete set of downlink signal streams sharing the same time-frequency resources is indexed by the set $\mathcal{D} = \mathcal{U} \cup \mathcal{S}$. We consider a resource block (RB) consisting of $Q \leq N_{sc}$ subcarriers and K OFDM symbols. We assume that AP clustering and RB scheduling decisions are announced by the CS to all APs, and that the APs are clustered into a subset of transmit $\mathcal{A}_t$ and receive APs $\mathcal{A}_r$. In this scenario, we focus specifically on PA and BF optimization for the case where a single RB is allocated for joint communication and sensing (JC&S). To this end, we develop decentralized algorithms that reduce fronthaul communication overhead.

A. Communication Model

The channel between UE u and AP a on a given subcarrier and OFDM symbol (resource element) in the selected RB is denoted by $\mathbf{h}_{au}$. We adopt a block-fading channel model where the given random channel realization is constant over the duration of the JC&S RB. Moreover, each AP is assumed to have perfect CSI for all UEs on the JC&S RB. The subsequent analysis is conducted on a per-subcarrier basis; hence, for notational simplicity, we omit the subcarrier index when it is clear from the context.

The downlink signal at AP a for the k-th symbol is

$$\mathbf{x}_a[k] = \sum_{i \in \mathcal{D}} \mathbf{w}_{ai} \mathrm{s}_i[k] = \sum_{u \in \mathcal{U}} \mathbf{w}_{au} \mathrm{s}_u[k] + \sum_{j \in \mathcal{S}} \mathbf{w}_{aj} \mathrm{s}_j[k]$$
$$= \mathbf{W}_a \mathbf{s}[k] = \mathbf{W}_a^{(c)} \mathbf{s}^{(c)}[k] + \mathbf{W}_a^{(s)} \mathbf{s}^{(s)}[k], \qquad (1)$$

where $\mathbf{w}_{ai} \in \mathbb{C}^M$ is the BF vector for the i-th stream and $\mathrm{s}_i[k]$ is the complex message on the k-th OFDM symbol. The vectors $\mathbf{w}_{ai}$ are collected as the columns of $\mathbf{W}_a = \left[\mathbf{W}_a^{(c)} \mid \mathbf{W}_a^{(s)} \right] \in \mathbb{C}^{M \times (N_{ue}+N_s)}$, and $\mathbf{s}[k] = \{\mathrm{s}_i[k]\}, \forall i \in \mathcal{D}$, where $\{\cdot\}$ represents the collection of elements in vector (or matrix) form. The total available power at each AP is $\mathbb{E}[\|\mathbf{x}_a[k]\|^2] = P_{\max}$, which can be allocated to both communication and sensing through the design of $\mathbf{W}_a^{(c)}$ and $\mathbf{W}_a^{(s)}$. Therefore, the received signal at UE u due to the k-th symbol is given as

$$\mathrm{y}_u[k] = \underbrace{\sum_{a \in \mathcal{A}_t} \mathbf{h}_{au}^H \mathbf{w}_{au} \mathrm{s}_u[k]}_{\text{Comm Desired Signal (CDS)}} + \underbrace{\sum_{a \in \mathcal{A}_t} \sum_{u' \in \mathcal{U} \setminus u} \mathbf{h}_{au}^H \mathbf{w}_{au'} \mathrm{s}_{u'}[k]}_{\text{Multi-User Interference (MUI)}}$$
$$+ \underbrace{\sum_{a \in \mathcal{A}_t} \sum_{j \in \mathcal{S}} \mathbf{h}_{au}^H \mathbf{w}_{aj} \mathrm{s}_j[k]}_{\text{Sensing-to-Comm Interference (S2CI)}} + \underbrace{\mathrm{z}_u[k]}_{\text{Noise}}, \qquad (2)$$

where the noise term is assumed to be zero-mean with variance σ_z^2. We define $\mathbf{s}_i = [\mathrm{s}_i[1], \cdots, \mathrm{s}_i[K]]^\top \in \mathbb{C}^K, \forall i \in \mathcal{D}$, as the vector of all K samples from signal stream i, and we assume the radar and communication signals are statistically

independent [9], i.e., $\mathbb{E}[\mathbf{s}_i \mathbf{s}_i^H] = \mathbf{I}_K$ and $\mathbb{E}[\mathbf{s}_i \mathbf{s}_j^H] = \mathbf{0}_K$ for $i, j \in \mathcal{D}$ and $i \neq j$. Using these assumptions, the communication SINR for UE u is given in (3), presented at the top of the next page.

We further define the downlink channel from AP a to all users as $\mathbf{H}_a = [\mathbf{h}_{a1}, \cdots, \mathbf{h}_{aN_{ue}}]^\top \in \mathbb{C}^{N_{ue} \times M}$. We collect the noise-free signals received by all users in the vector

$$\mathbf{y}_{\text{DL}}[k] = \sum_{a \in \mathcal{A}_t} \mathbf{H}_a \mathbf{W}_a \mathbf{s}[k] \triangleq \mathbf{C}\, \mathbf{s}[k] \in \mathbb{C}^{N_{ue}}, \qquad (4)$$

for $\mathbf{C} \triangleq \sum_{a \in \mathcal{A}_t} \mathbf{H}_a \mathbf{W}_a \in \mathbb{C}^{N_{ue} \times (N_{ue}+N_s)}$. With an ideal BF matrix $\mathbf{W}_a$ we have $\mathbf{C} = \left[\, \mathbf{I}_{N_{ue}} \mid \mathbf{0} \,\right]$, i.e., the interference is effectively canceled and MUI = S2CI = 0.

B. Sensing Model

We assume that the CS dedicates a subset $\mathcal{A}_r$ of the APs for receiving the sensing echo signals on the selected JC&S RB. The sensing task considered in this work is single target detection through a multi-static setup. We assume that the target location has a dominant line-of-sight (LoS) path to the APs in $\mathcal{A}_t$ and $\mathcal{A}_r$. For the sensing channel, we consider a single-point reflector with beam-space channel model [10]

$$\mathbf{H}_{a_t a_r}^{(s)}[q, k] = \beta_{a_t a_r} \Omega_{a_t a_r}[q, k] \mathbf{a}(\phi_{a_r}, \theta_{a_r}) \mathbf{a}^H(\phi_{a_t}, \theta_{a_t}), \quad (5)$$

in which, $\beta_{a_t a_r} \sim \mathcal{CN}(0, \zeta_{a_t a_r}^2)$ is the channel gain that combines the path-loss and radar cross section (RCS) effect, $\Omega_{a_t a_r}[q, k] = \exp(-j2\pi\tau_{a_t a_r} q f_{sc}) \exp(j2\pi f_{D,a_t a_r} k T_s)$ is the combined phase shift due to the path delay $\tau_{a_t a_r}$ and the Doppler frequency $f_{D,a_t a_r}$, ϕ_{a_r} and θ_{a_r} are the 3D angles of arrival (AoA) at AP a_r, and ϕ_{a_t} and θ_{a_t} are the AoDs from AP a_t. The vector $\mathbf{a}(\phi, \theta)$ represents the array steering vector for an azimuth angle ϕ and elevation angle θ. We use the Swerling-I model [11] for the RCS, allowing us to assume constant channel gain $\beta_{a_t a_r}$ across the entire JC&S RB. The analysis is performed for each subcarrier; thus, the subcarrier index q is dropped. In the following, we use $\mathbf{a}_{a_r}$ and $\mathbf{a}_{a_t}$ to represent the array steering vectors at the receiver and transmitter, respectively, for notational brevity.

We also assume that the direct channels between transmit and receive APs are known to the receive AP, e.g., through pilot-based estimation prior to transmission on the JC&S RB. Hence, the influence of the direct channels can be effectively removed. The received signal for sensing at AP $a_r \in \mathcal{A}_r$ at the k-th symbol time is

$$\mathbf{y}_{a_r}^{(s)}[k] = \underbrace{\sum_{a_t \in \mathcal{A}_t} \sum_{j \in \mathcal{S}} \mathbf{H}_{a_t a_r}^{(s)}[k] \mathbf{w}_{a_t j} \mathrm{s}_j[k]}_{\text{Sensing Only (SO)}}$$
$$+ \underbrace{\sum_{a_t \in \mathcal{A}_t} \sum_{u \in \mathcal{U}} \mathbf{H}_{a_t a_r}^{(s)}[k] \mathbf{w}_{a_t u} \mathrm{s}_u[k]}_{\text{Comm-to-Sensing Contribution (C2SC)}} + \mathbf{z}_{a_r}[k], \qquad (6)$$

where both the SO and C2SC contribute to increasing the sensing parameter estimation accuracy. By applying the receiver

$$\text{SINR}_u^{(c)} = \frac{\mathbb{E}[|\text{CDS}|^2]}{\mathbb{E}[|\text{MUI}|^2] + \mathbb{E}[|\text{S2CI}|^2] + \mathbb{E}[|\text{Noise}|^2]} = \frac{|\sum_{a \in \mathcal{A}_t} \mathbf{h}_{au}^H \mathbf{w}_{au}|^2}{\sum_{u' \in \mathcal{U} \setminus u} |\sum_{a \in \mathcal{A}_t} \mathbf{h}_{au}^H \mathbf{w}_{au'}|^2 + \sum_{j \in \mathcal{S}} |\sum_{a \in \mathcal{A}_t} \mathbf{h}_{au}^H \mathbf{w}_{aj}|^2 + \sigma_z^2} \quad (3)$$

array steering vector $\mathbf{a}_{a_r}$ as a matched filter to the received signal as $\mathrm{r}_{a_r}[k] = \mathbf{a}_{a_r}^H \mathbf{y}_{a_r}^{(s)}[k]$, we obtain

$$\mathrm{r}_{a_r}[k] = M \sum_{a_t \in \mathcal{A}_t} \beta_{a_t a_r} \Omega_{a_t a_r}[k] \mathbf{a}_{a_t}^H \mathbf{W}_{a_t} \mathbf{s}[k] + \widetilde{z}_{a_r}[k], \quad (7)$$

where $\widetilde{z}_{a_r}[k] = \mathbf{a}_{a_r}^H \mathbf{z}_{a_r}[k]$. Collecting the filtered received signals over all K symbols in the JC&S RB, we form the RB-level received signal at each AP a_r as

$$\mathrm{r}_{a_r}^{\text{RB}} = \underbrace{M \sum_{a_t \in \mathcal{A}_t} \beta_{a_t a_r} \mathbf{a}_{a_t}^H \mathbf{W}_{a_t} \sum_{k=1}^{K} \Omega_{a_t a_r}[k] \mathbf{s}[k]}_{\text{Sensing Desired Signal (SDS)}} + \underbrace{\sum_{k=1}^{K} \widetilde{z}_{a_r}[k]}_{\text{Sensing Noise}}. \quad (8)$$

Using (8) we can express the sensing SNR based on the filtered signal $\mathrm{r}_{a_r}^{\text{RB}}$ at receive AP a_r as the ratio of the SDS and sensing noise (SN) power:

$$\begin{aligned} \text{SNR}_{a_r}^{(s)} &= \frac{\mathbb{E}[|\text{SDS}|^2]}{\mathbb{E}[|\text{SN}|^2]} = \frac{M}{\sigma_{z,a_r}^2} \sum_{a_t \in \mathcal{A}_t} \zeta_{a_t a_r}^2 \|\mathbf{a}_{a_t}^H \mathbf{W}_{a_t}\|_2^2, \\ &= \frac{M}{\sigma_{z,a_r}^2} \sum_{a_t} \zeta_{a_t a_r}^2 \Big(\sum_{j \in \mathcal{S}} |\mathbf{a}_{a_t}^H \mathbf{w}_{a_t j}|^2 + \sum_{u \in \mathcal{U}} |\mathbf{a}_{a_t}^H \mathbf{w}_{a_t u}|^2 \Big), \end{aligned} \quad (9)$$

where σ_{z,a_r}^2 is the noise variance at AP a_r. In the following section, we present optimization strategies aimed at jointly maximizing communication SINR and sensing SNR.

III. PROBLEM FORMULATION AND DISTRIBUTED ALGORITHMS

Our goal is to develop distributed optimization solutions for resource allocation that enable decentralized processing while minimizing information exchange over the fronthaul network. In this section, we provide two optimization strategies for BF and PA. The first approach splits the BF and PA optimization by allowing each AP to locally design its beamformers, while delegating PA to centralized optimization at the CS. This approach limits the maximum number of UEs simultaneously served by local beamformers to the number of antennas per AP. Our second approach overcomes this limitation by formulating a multi-objective weighted sum maximization problem that jointly optimizes BF and PA. To solve this problem in a fully decentralized manner, we develop an algorithm based on consensus ADMM. These two approaches are detailed in the following subsections.

A. SplitOpt (Distributed BF + Centralized PA)

In the first approach, we adopt a fixed local BF method and optimize PA centrally at the CS. To decouple BF and PA, we define a power splitting ratio (PSR) ρ_a for each AP a, such that the communication power is given by $P_a^{(c)} = \rho_a P_{\max}$ and the sensing power by $P_a^{(s)} = (1 - \rho_a) P_{\max}$.

1) Distributed BF at APs: For the local design of the beamformers, we will approximately solve the equations $\sum_{a \in \mathcal{A}_t} \mathbf{H}_a \mathbf{W}_a = \mathbf{C}$ for the ideal $\mathbf{C} = \begin{bmatrix} \mathbf{I}_{N_{ue}} & | & \mathbf{0} \end{bmatrix}$ using regularized zero-forcing (RZF). To decouple this problem for all APs $a \in \mathcal{A}_t$, we define the matrix $\mathbf{C}_a = \alpha_a \mathbf{C}$ and approximately solve the equation $\mathbf{H}_a \mathbf{W}_a = \mathbf{C}_a$ for each $a \in \mathcal{A}_t$. The weights are defined as $\alpha_a = \mathcal{M}(a)/\sum_{a' \in \mathcal{A}_t} \mathcal{M}(a')$, where $\mathcal{M}(a)$ is a metric designed to capture the quality of the channel $\mathbf{H}_a$. In this paper, we will set $\mathcal{M}(a)$ as the inverse of the condition number of $\mathbf{H}_a$. This choice implies that APs with poorly conditioned channels (i.e., higher condition numbers) will have a reduced contribution to the communication BF matrix. To determine the weights α_a, each AP sends its $\mathcal{M}(a)$ value to the CS, and the CS broadcasts the sum $\sum_{a' \in \mathcal{A}_t} \mathcal{M}(a')$ to all transmit APs. Then, each AP approximately solves the local system of equations $\mathbf{H}_a \mathbf{W}_a = \mathbf{C}_a$ for $\mathbf{W}_a$ using RZF, which leads to the solution

$$\widetilde{\mathbf{W}}_a^{(c)} = \left(\mathbf{H}_a^H \mathbf{H}_a + \varsigma \mathbf{I} \right)^{-1} \mathbf{H}_a^H \mathbf{C}_a. \quad (10)$$

After applying the power constraint, the BF matrix is

$$\mathbf{W}_a^{(c)} = \sqrt{\rho_a P_{\max}} \; \widetilde{\mathbf{W}}_a^{(c)} / \|\widetilde{\mathbf{W}}_a^{(c)}\|_F, \quad (11)$$

which is designed at each AP for the communication users. We refer to this BF approach as local modified RZF (LM-RZF).

For sensing, the transmit APs employ a null-space conjugate (NS-C) BF design [12] by projecting the beams pointed toward the target location onto the null space of the communication channel matrix. The projection matrix at transmit AP a is

$$\mathbf{P}_a^\perp = \mathbf{I} - \mathbf{H}_a^H \left(\mathbf{H}_a \mathbf{H}_a^H + \epsilon \mathbf{I} \right)^{-1} \mathbf{H}_a, \quad (12)$$

and defining each sensing stream as $\widetilde{\mathbf{w}}_{aj} = \mathbf{P}_a^\perp \mathbf{a}_a$, we obtain the sensing BF matrix $\widetilde{\mathbf{W}}_a^{(s)} = \{\widetilde{\mathbf{w}}_{aj}\}$ for all $j \in \mathcal{S}$. With the required power constraint, the NS-C beamforming becomes

$$\mathbf{W}_a^{(s)} = \sqrt{(1 - \rho_a) P_{\max}} \; \widetilde{\mathbf{W}}_a^{(s)} / \|\widetilde{\mathbf{W}}_a^{(s)}\|_F, \quad (13)$$

and the overall BF matrix $\mathbf{W}_a$ is obtained by concatenating the communication and sensing beamformers.

2) Centralized PA at CS: To optimize the PA between communication and sensing, we define SINR-based and SNR-based utility functions for communication and sensing, respectively, and formulate a problem that maximizes sensing utility under a communication utility constraint. This formulation leads to the computation of optimal PSRs ρ_a^* for each AP $a \in \mathcal{A}_t$. Assuming LM-RZF for communication BF design, which essentially removes MUI and S2CI, the SINR (3) reduces to approximately $(P_{\max}/\sigma_z^2) U_{\text{comm}}$ for the communication utility function U_{comm} defined as

$$U_{\text{comm}} \triangleq \left| \sum_{a \in \mathcal{A}_t} \frac{\alpha_a}{\|\widetilde{\mathbf{W}}_a^{(c)}\|_F} \sqrt{\rho_a} \right|^2 = \left(\widetilde{\boldsymbol{\alpha}}^\top \boldsymbol{\rho}^{1/2} \right)^2, \quad (14)$$

Algorithm 1 SplitOpt (DistBF-CentPA)

1: **Initialization:**
2:   APs receive transmit signal information from the CS.
3: **Distributed BF at each AP $a \in \mathcal{A}_t$:**
4:   Send $\mathcal{M}(a)$ to the CS and receive $\sum_{a'} \mathcal{M}(a')$ in return.

5:   Employ LM-RZF using (11) for communication UEs.
6:   Employ NS-C using (13) for target(s).
7:   Measure $\widetilde{\alpha}_a$ and $\widetilde{\Delta}_a$ and share with the CS.
8: **Centralized PA at the CS:**
9:   Receive vectors $\widetilde{\boldsymbol{\alpha}}$ and $\widetilde{\boldsymbol{\Delta}}$ from APs $a \in \mathcal{A}_t$.
10:   Solve problem (16) using projected gradient ascent.
11:   Send optimal PSRs $\{\rho_a^*\}$ to APs $a \in \mathcal{A}_t$.

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in which $\widetilde{\alpha}_a = \alpha_a / \|\widetilde{\mathbf{W}}_a^{(c)}\|_F$, $\widetilde{\boldsymbol{\alpha}} = \{\widetilde{\alpha}_a\}$, and $\boldsymbol{\rho} = \{\rho_a\}$, for $a \in \mathcal{A}_t$. Similarly, assuming NS-C for sensing BF design, the SNR-based sensing utility function is defined as

$$U_{\text{sens}} \triangleq \sum_{a \in \mathcal{A}_t} \zeta_a^2 \left(\rho_a \|\mathbf{a}_a^H \widehat{\mathbf{W}}_a^{(c)}\|_2^2 - \rho_a \|\mathbf{a}_a^H \widehat{\mathbf{W}}_a^{(s)}\|_2^2 \right)$$
$$= \sum_{a \in \mathcal{A}_t} \zeta_a^2 \rho_a \Delta_a \triangleq \widetilde{\boldsymbol{\Delta}}^\top \boldsymbol{\rho}, \quad (15)$$

where $\widehat{\mathbf{W}}_a = \widetilde{\mathbf{W}}_a / \|\widetilde{\mathbf{W}}_a\|_F$ for both communication and sensing matrices, $\Delta_a = \|\mathbf{a}_a^H \widehat{\mathbf{W}}_a^{(c)}\|_2^2 - \|\mathbf{a}_a^H \widehat{\mathbf{W}}_a^{(s)}\|_2^2$, $\widetilde{\Delta}_a = \zeta_a^2 \Delta_a$, and $\widetilde{\boldsymbol{\Delta}} = \{\widetilde{\Delta}_a\}$ for $a \in \mathcal{A}_t$. Here, the subscript a_r is removed for brevity and should be clear from the context.

The global PA optimization problem, which aims to maximize the sensing utility subject to a constraint on the communication utility, is formulated as follows:

$$\max_{\{\rho_a\}, \varepsilon \geq 0} \quad \widetilde{\boldsymbol{\Delta}}^\top \boldsymbol{\rho} - \xi \varepsilon \quad (16a)$$
$$\text{s.t.} \quad \widetilde{\boldsymbol{\alpha}}^\top \boldsymbol{\rho}^{1/2} + \varepsilon \geq \sqrt{\gamma}, \quad (16b)$$
$$0 \leq \rho_a \leq 1, \ \forall a \in \mathcal{A}_t, \quad (16c)$$

where γ denotes the minimum communication utility requirement, ε is a slack variable used to ensure the feasibility of constraint (16b) for large values of γ, and ξ is a parameter used to penalize the objective for slack usage. The optimization problem in (16) is a constrained concave maximization problem with convex constraints, which can be efficiently solved using the projected gradient ascent algorithm. To solve this problem centrally at the CS, two real-valued vectors $\widetilde{\boldsymbol{\alpha}}$ and $\widetilde{\boldsymbol{\Delta}}$ must be shared with the CS. Accordingly, each AP $a \in \mathcal{A}_t$ transmits its local values $\widetilde{\alpha}_a$ and $\widetilde{\Delta}_a$ to the CS and receives the optimal PSR ρ_a^* in return. The procedure for implementing the *SplitOpt* algorithm is summarized in Algorithm 1.

B. JointOpt (Decentralized ADMM)

In the second approach, we formulate a joint BF and PA optimization problem and modify it to enable a decentralized solution using the ADMM algorithm [13]. For the optimization objective, we consider maximizing a weighted sum of utility functions for communication and sensing. However, in this case, the communication utility is defined via a max-min

formulation, aiming to maximize the minimum SINR across all UEs. Hence, the communication utility is defined as

$$U_{\text{comm}} \triangleq \min_{u \in \mathcal{U}} \ \text{SINR}_u^{(c)}, \quad (17)$$

for $\text{SINR}_u^{(c)}$ defined in (3). We define the sensing utility function U_{sens} using the SNR expression in (9) such that $\text{SNR}^{(s)} = (M/\sigma_z^2) U_{\text{sens}}$, which results in

$$U_{\text{sens}} \triangleq \sum_{a \in \mathcal{A}_t} \zeta_a^2 \|\mathbf{a}_a^H \mathbf{W}_a\|_2^2. \quad (18)$$

The global optimization problem based on these utility functions will result in a non-smooth (due to the min operator) and non-convex (due to the SINR expression) objective function, making it challenging to apply standard distributed optimization methods without further reformulation.

To address these challenges, we employ an epigraph reformulation by introducing an SINR floor γ that is maximized subject to the constraint that all UE SINRs are greater than this threshold. Accordingly, the modified problem is given as

$$\max_{\gamma, \mathbf{W}_a} \quad \lambda \gamma + (1 - \lambda) \sum_{a \in \mathcal{A}_t} \zeta_a^2 \|\mathbf{a}_a^H \mathbf{W}_a\|_2^2 \quad (19a)$$
$$\text{s.t.} \quad \text{SINR}_u^{(c)} \geq \gamma, \ \forall u \in \mathcal{U}, \quad (19b)$$
$$\|\mathbf{W}_a\|_F^2 \leq P_{\text{max}}, \ \forall a \in \mathcal{A}_t, \quad (19c)$$

where $\lambda \in [0, 1]$ is a trade-off parameter. Larger values of λ prioritize SINR for the UEs, whereas smaller values place greater emphasis on delivering power to the sensing direction.

Although problem (19) features a linear objective in γ, the constraint (19b) is still non-convex and coupled. To decouple the problem, we adopt a local approximation of the SINR that can be independently evaluated at each AP $a \in \mathcal{A}_t$. Following the similar SINR approximation in [14], we use Minkowski's inequality to derive the local SINR contribution constraint as

$$|\mathbf{h}_{au}^H \mathbf{w}_{au}|^2 \geq \eta_{au} \gamma_a (\psi_u + \sigma_z)^2, \ \forall u \in \mathcal{U}, \quad (20)$$

that forces each AP to contribute a fair share toward achieving the SINR target. The parameter η_{au} is a fractional weight that quantifies the responsibility of AP a in contributing to the SINR of user u. A straightforward choice is to assign equal responsibility to all APs by setting $\eta_{au} = 1/N_{ap}$. However, alternative weighting strategies may also be considered based on factors such as channel gain or user-AP association.

The interference produced by AP a at user u is proportional to $\psi_{au} = (\sum_{i \in \mathcal{D} \setminus u} |\mathbf{h}_{au}^H \mathbf{w}_{ai}|^2)^{1/2}$, and we define $\psi_u = \sum_{a \in \mathcal{A}_t} \psi_{au}$ for each user u. We further define the real-valued vector $\boldsymbol{\psi} = \{\psi_u\} \in \mathbb{R}^{N_{ue}}$ for all $u \in \mathcal{U}$. This interference vector must be known to all APs $a \in \mathcal{A}_t$. The local constraint (20) is still not compatible with standard CVX solvers since it is in the form of convex $\geq$ affine. We adopt a linear approximation of $g(\mathbf{w}_{au}) = |\mathbf{h}_{au}^H \mathbf{w}_{au}|^2$ based on the first-order Taylor approximation around a given point $\mathbf{w}_{au}^{[t]}$ as

$$\bar{g}(\mathbf{w}_{au}; \mathbf{w}_{au}^{[t]}) = 2\Re\{(\mathbf{h}_{au}^H \mathbf{w}_{au}^{[t]})^* \mathbf{h}_{au}^H \mathbf{w}_{au}\} - |\mathbf{h}_{au}^H \mathbf{w}_{au}^{[t]}|^2, \quad (21)$$

and we use this approximation instead to make the problem solvable with standard CVX solvers.

Algorithm 2 JointOpt (DecentADMM)

1: **Initialization:**
2:   Initialize $\mathbf{W}_a^{[0]}$, $\gamma_a^{[0]}$, and $\nu_a^{[0]}$ for all $a \in \mathcal{A}_t$.
3:   CS broadcasts initial $\boldsymbol{\psi}^{[0]}$ to all APs $a \in \mathcal{A}_t$.
4: **Local Update at each AP** $a \in \mathcal{A}_t$**:**
5:   Obtain $\gamma^{[t-1]}$, $\boldsymbol{\psi}^{[t-1]}$, $\nu_a^{[t-1]}$, and $\xi^{[t-1]}$.
6:   Solve the QCQP problem in (24) via CVX.
7:   Measure $\psi_{au}^{[t]}$ for all $u \in \mathcal{U}$ and share with the CS.
8: **Global Update at the CS:**
9:   SINR floor update $\gamma^{[t]} = \frac{1}{N_{ap}} \sum_{a \in \mathcal{A}_t} (\gamma_a^{[t]} + \nu_a^{[t-1]})$
10:   Interference update $\psi_u^{[t]} = \sum_{a \in \mathcal{A}_t} \psi_{au}^{[t]}, \forall u \in \mathcal{U}$.
11:   Broadcast $\gamma^{[t]}$ and $\boldsymbol{\psi}^{[t]}$ to all APs $a \in \mathcal{A}_t$.
12: **Local Dual and Slack Updates:**
13:   Update dual variable $\nu_a^{[t]} = \nu_a^{[t-1]} + (\gamma_a^{[t]} - \gamma^{[t]})$
14:   Slack penalty update $\xi^{[t]} = \min(\xi_{\max}, \xi^{[t-1]} + \Psi t)$
15: **Check Convergence:**
16:   If $\max_a |\gamma_a - \gamma| < \epsilon_{\text{conv}}$, exit.
17:   Else, set $t \leftarrow t + 1$ and return to step 4.

To enable decentralized ADMM, we introduce a local consensus variable γ_a for each AP $a \in \mathcal{A}_t$, and enforce global consensus through a corresponding update step. By employing the local SINR approximation and the consensus variable, the global problem in (19) can be decomposed into parallel subproblems solvable independently at each AP $a \in \mathcal{A}_t$. The augmented Lagrangian for local updates is obtained as

$$\mathcal{L}_\varrho^{(a)} = \lambda \gamma_a + (1-\lambda)\|\mathbf{a}_a^H \mathbf{W}_a\|_2^2 - \frac{\varrho}{2}(\gamma_a - \gamma + \nu_a)^2, \quad (22)$$

where ϱ is the penalty parameter, γ_a and γ are local and global consensus variables, respectively, and ν_a is the dual variable. Since the term $f(\mathbf{W}_a) = \|\mathbf{a}_a^H \mathbf{W}_a\|_2^2$ is not concave, the augmented Lagrangian cannot be maximized directly using CVX solvers. Thus, we use a concave surrogate for f based on the first-order Taylor series expansion as

$$\bar{f}(\mathbf{W}_a; \mathbf{W}_a^{[t]}) = 2\Re\{(\mathbf{a}_a^H \mathbf{W}_a^{[t]})\mathbf{W}_a^H \mathbf{a}_a\} - \|\mathbf{a}_a^H \mathbf{W}_a^{[t]}\|_2^2, \quad (23)$$

which is concave and is used as a lower bound approximation. The augmented Lagrangian defined by replacing $f(\mathbf{W}_a)$ with the concave surrogate $\bar{f}(\mathbf{W}_a; \mathbf{W}_a^{[t]})$ is denoted by $\bar{\mathcal{L}}_\varrho^{(a)}$.

To ensure that the local subproblems remain feasible, we introduce non-negative slack variables $\varepsilon_{au} \geq 0$ for each user $u \in \mathcal{U}$, and penalize them in the objective using a large weight $\xi \gg 1$. Finally, the local optimization problem at each transmit AP $a \in \mathcal{A}_t$ has the following form

$$\max_{\gamma_a, \mathbf{W}_a, \varepsilon_{au}} \bar{\mathcal{L}}_\varrho^{(a)} - \xi \sum_{u \in \mathcal{U}} \varepsilon_{au} \quad (24a)$$

$$\text{s.t.} \quad \bar{g}(\mathbf{w}_{au}; \mathbf{w}_{au}^{[t]}) \geq \eta_{au} \gamma_a (\psi_u + \sigma_z)^2 - \varepsilon_{au}, \quad (24b)$$

$$\|\mathbf{W}_a\|_F^2 \leq P_{\max}, \; \forall u \in \mathcal{U}. \quad (24c)$$

The local problem in (24) is a quadratically-constrained quadratic program (QCQP), which can be efficiently solved using standard CVX solvers. The complete procedure of the *JointOpt* algorithm is summarized in Algorithm 2.
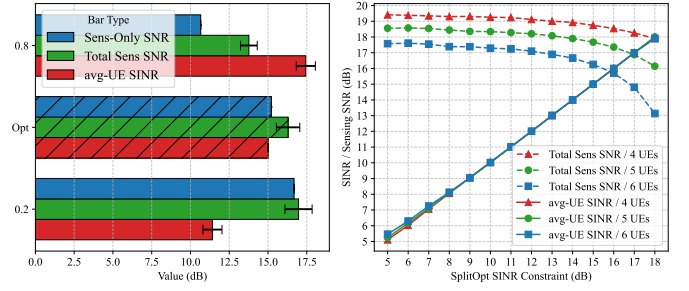

Fig. 1. Performance of Algorithm 1. (Left) SINR and sensing SNR comparison for fixed PSRs (0.2, 0.8) and optimal PSRs (Opt). (Right) SINR and sensing SNR achieved under different SINR constraints γ with $N_{ue} \in \{4, 5, 6\}$.

IV. SIMULATION RESULTS

In addition to the theoretical analysis, we developed a Python-based simulation framework, which is publicly available on GitHub at [15]. The simulator implements the proposed algorithms and provides a flexible platform for further research and development on decentralized ISAC systems. All datasets and seed numbers used for the figures in this paper are also provided in the repository. We also provide a centralized solution for problem (19), with the full results made available in our repository. Due to space limitations, centralized results are included in the paper for only one simulation scenario. We simulate an environment in which the APs are uniformly distributed on a circle of radius 650m, and each is equipped with a uniform circular array (UCA) of size M. The UEs and a single target are randomly placed within a concentric circle of radius 1km, and the closest AP to the target location is selected as the sensing receive AP. The ratio of $P_{\max}$ to the noise power is $P_{\max}/\sigma_z^2 = 20$ dB, the target RCS variance is $\zeta_a^2 = 0.5$ (-3 dB), and the communication channels are generated based on the 3GPP Rician model for urban micro cells [10], [16].

To evaluate the performance of Algorithm 1, we simulate a scenario with 4 APs each with a UCA of size 10, one sensing target, and 6 UEs. We compare the performance of local BF with fixed PSRs $\rho_a \in \{0.2, 0.8\}$ against the optimal PSRs ρ_a^* obtained using the *SplitOpt* algorithm. The results are presented in Fig. 1 (left), illustrating that a PSR of 0.2 prioritizes sensing performance, whereas a PSR of 0.8 favors higher communication SINR. In contrast, the *SplitOpt* algorithm satisfies an SINR constraint of 15 dB for all users while simultaneously achieving a high sensing SNR. In Fig. 1 (right), the SINR and sensing SNR results achieved by the *SplitOpt* algorithm are presented for different utility constraints γ and varying numbers of users. The results show that the SINR constraint is satisfied across all cases, while the sensing SNR begins to decline once the SINR constraint exceeds approximately 13 dB. Moreover, the degradation in sensing SNR is more pronounced as the number of users increases since satisfying the SINR constraint requires allocating a larger portion of the available power to communication. This analysis demonstrates that Algorithm 1 provides an effective decentralized solution, requiring minimal coordination with the CS.
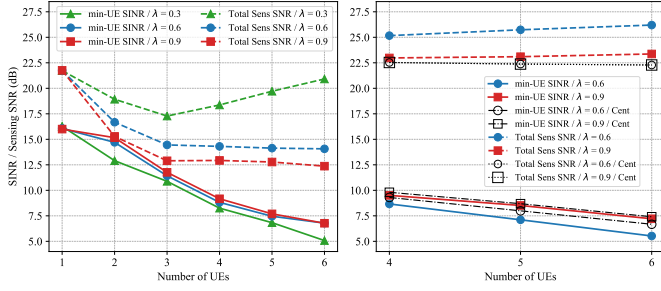
Fig. 2. Performance of Algorithm 2. (Left) SINR and sensing SNR vs different number of users for trade-off parameter $\lambda \in \{0.3, 0.6, 0.9\}$. (Right) SINR and sensing SNR with $N_{ap} = 11$ APs and $M = 3$ antennas for both the proposed ADMM-baed decentralized solution and the centralized solution.

TABLE I
COMPARING FRONTHAUL COMMUNICATION OVERHEAD FOR EACH AP

	Centralized	**SplitOpt**	**JointOpt**
Data to share	$\mathbf{H}_a$, $\mathbf{W}_a$	$\mathcal{M}(a)$, $\widetilde{\alpha}_a$, $\widetilde{\Delta}_a$, ρ_a^*	$\gamma^{[t]}$, $\boldsymbol{\psi}^{[t]}$
Size of data	$2 \times \mathbb{C}^{M \times |\mathcal{D}|}$	$4 \times \mathbb{R}^1$	$\mathbb{R}^1$, $\mathbb{R}^{|\mathcal{U}|}$
# of scalars	280	4	$7 \times T_{\text{ADMM}}$
Scalable	NO	YES	YES

Using the same simulation settings except for varying the number of users from 1 to 6, we next used Algorithm 2 for joint BF and PA design. The results are presented in Fig. 2 (left) for the trade-off parameter values $\lambda \in \{0.3, 0.6, 0.9\}$. The results show that as the number of users increases from 1 to 6, the minimum user SINR guaranteed by the *JointOpt* algorithm decreases from approximately 16 dB to 6 dB. Additionally, the advantage of reducing the trade-off parameter λ to prioritize sensing performance is clearly illustrated: $\lambda = 0.3$ achieves the highest sensing SNR while maintaining only a slight reduction in SINR compared to higher λ values.

Algorithm 2 is particularly advantageous for scenarios with fewer antennas per AP than the number of users, where locally low-rank channels render Algorithm 1 inapplicable. To illustrate this, we simulate a setup with 11 APs (10 Tx APs) each with 3 antennas, serving $\{4, 5, 6\}$ users and sensing a single target location. The total number of transmit antennas (30) is the same as in the previous scenarios. Fig. 2 (right) demonstrates that high sensing SNR is maintained with a minimum user SINR between 5–10 dB, and that the centralized solution to problem (19) performs comparably to our decentralized algorithm. These results confirm that both algorithms enable efficient decentralized BF and PA in cell-free ISAC systems, supporting flexible trade-offs across diverse scenarios. Table I compares the two algorithms against centralized solutions in terms of coordination overhead, highlighting a significant reduction in information exchange. The number of real scalars to share over the fronthaul is provided for one of the scenarios with $M = 10$ antennas per AP, serving $N_{ue} = 6$ UEs and a single target. T_{ADMM} denotes the number of ADMM iterations, which was on the order of 5 to 10 for our simulations.

V. CONCLUSION

In this paper, we proposed two coordinated decentralized algorithms for BF and PA in cell-free ISAC systems. The first algorithm splits BF and PA by employing fixed local beamformers and centralized power optimization with limited coordination. The second algorithm jointly optimizes BF and PA through a fully decentralized consensus ADMM approach, enabling scalable operation even in the presence of locally low-rank channels. To support further research and reproducibility, we developed and released a Python-based simulation framework implementing both algorithms. Simulation results showed that, compared to centralized solutions, both algorithms achieve effective sensing and communication performance while significantly reducing fronthaul overhead.

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