1、 Consider $dx_t = f(x_t, t)\,dt + g(x_t, t)\,dW_t$

with Fokker Planck equation

$$\frac{\partial}{\partial t} p = -\frac{\partial}{\partial x}(fp) + \frac{1}{2}\frac{\partial^2}{\partial x^2}(g^2 p) \quad — ①$$

We want to find $v(x_t, t)$ such that $d(x_t) = v(x_t, t)\,dt$

Since $\frac{\partial p}{\partial t} = -\frac{\partial}{\partial x}(vp)$

By ①, $\frac{\partial}{\partial x}(vp) = \frac{\partial}{\partial x}(fp) - \frac{1}{2}\frac{\partial^2}{\partial x^2}(g^2 p)$

$$\Rightarrow vp = fp - \frac{1}{2}\frac{\partial}{\partial x}(g^2 p)$$

$$v = f - \frac{1}{2}\frac{1}{p}\frac{\partial}{\partial x}(g^2 p)$$

$$v = f - \frac{1}{2}(\partial_x g^2) - \frac{1}{2} g^2 \frac{\partial_x p}{p}$$

By $\frac{\partial_x p}{p} = \partial_x \log p$, $v(x, t) = f(x_t, t) - \frac{1}{2}\frac{\partial}{\partial x} g^2(x_t, t) - \frac{g^2(x_t, t)}{2}\frac{\partial}{\partial x}\log p(x_t, t)$

$$dx_t = \left(f(x_t, t) - \frac{1}{2}\frac{\partial}{\partial x} g^2(x_t, t) - \frac{g^2(x_t, t)}{2}\frac{\partial}{\partial x}\log p(x_t, t)\right) dt$$

2、详见 problem 2.pdf

3、Unanswered Question:

Can Backward SDE actually back to initial data distribution in practice?