

# 1. Score matching

goal: estimate probability density function

$$\text{learn } S(x) = \nabla \log p(x)$$

Loss:

$$L_{SM}(\theta) = E_{x \sim p(x)} \|S(x; \theta) - \nabla_x \log p(x)\|^2$$

→ hard to know

$$L_{2SM}(\theta) = E_{x \sim p(x)} [\|S(x; \theta)\|^2 + 2 \nabla_x \cdot S(x; \theta)]$$

They are equivalent

Choosing score matching (DSM)

$x_0$ : original data

$p_0(x_0)$ : original pdf

$x$ : perturbed data

$p(x)$ : pdf of perturbed data

$$\text{Find } S_0(x; \theta) = \nabla_x \log p_0(x)$$

$$L_{DSM} = E_{x_0 \sim p_0(x_0)} E_{(x|x_0) \sim p(x|x_0)} [S_0(x; \theta) - \nabla_x \log p(x|x_0)]$$

Diffusion probabilistic models (DDPM):

training

$$x_0 \sim q(x_0)$$

$$t \sim \text{uniform}([1, 2, \dots, T])$$

$$\epsilon \sim N(0, I)$$

Gradient descent on

$$\nabla_{\theta} \| \epsilon - \epsilon_{\theta}(\sqrt{\bar{\alpha}_t} x_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon, t) \|^2$$

2. When was the concept of score matching first proposed?