Week 3 Written assignment

1.Reading and Explaining

We want to approximate polynomials $f_p(y) = y^p$ for any $p \in \mathbb{N}$ with tanh neural networks function $\hat{f}_{q,h} := \frac{\delta^q_{hy}[\sigma](0)}{\sigma^{(q)}(0)h^q}$, where $\delta^p_h[f](x) = \Sigma^p_{i=0}(-1)^i\binom{p}{i}f(x + (\frac{p}{2} - i)h)$, and $\sigma(x) = \frac{e^{2x}-1}{e^{2x}+1}$.

In lemma 3.1, the author tell us that if p is odd, then these neural networks are accurate to approximate f_p. And in lemma 3.2, it extends and controls the construction so that all degrees up to s.

For example, if we want to approximate$f(x) = x^2 + x^3$, we first build a shallow tanh-network$\hat{f}_{3,h} := \frac{\delta^3_h[\sigma](0)}{\sigma^{(3)}(0)h^3}$, and there exist the function very close to x^3. As for x^2, we can deal with it by $x^2 = \frac{(x+\alpha)^3-(x-\alpha)^3}{6\alpha}$ according to lemma 3.2.

2.Unanswered Questions

Is there any good neural network to let the approximation of both the Runge function and its derivative are precise?