

1. Let $a_i^{[0]} = \sigma(z_i^{[0]})$ for $i=1 \dots n_L$

$$\text{since } \frac{\partial a_i^{[0]}}{\partial z_j^{[0]}} = \frac{\partial \sigma(z_i^{[0]})}{\partial z_j^{[0]}} = \begin{cases} \sigma'(z_i^{[0]}), & \text{if } i=j \\ 0, & \text{if } i \neq j \end{cases}$$

$$\Rightarrow \frac{\partial a^{[0]}}{\partial z^{[0]}} = \text{diag}(\sigma'(z^{[0]}))$$

$$\text{Therefore, } \frac{\partial a^{[0]}}{\partial a^{[L-1]}} = \frac{\partial a^{[0]}}{\partial z^{[0]}} \cdot \frac{\partial z^{[0]}}{\partial a^{[L-1]}} = \text{diag}(\sigma'(z^{[0]})) W^{[0]}$$

$$\nabla_a^{[0]}(x) = \frac{\partial a^{[0]}}{\partial a^{[L-1]}} \cdot \frac{\partial a^{[L-1]}}{\partial a^{[L-2]}} \dots \frac{\partial a^{[2]}}{\partial a^{[1]}} = (\text{diag}(\sigma'(z^{[0]})) W^{[0]}) \cdot (\text{diag}(\sigma'(z^{[1]})) W^{[1]}) \cdot \dots \cdot (\text{diag}(\sigma'(z^{[L-1]}) W^{[L-1]})$$

2. Is there any specific example about approximation theory?