assignment 1

soham chougule 111903107

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1 geometric meaning of y' = f(x,y)

A first-order ODE

$$\frac{d(y)}{dx} = f(x, y) \tag{1}$$

has a simple geometric interpretation. From calculus you know that the derivative of y'(x) of y(x) is the slope of . Hence a solution curve of (1) that passes through a point must have, at that point, the slope equal to the value of f at that point; that is,

$$\mathrm{y}(x_0)=\mathrm{f}(x_0,y_0)$$

Using this fact, we can develop graphic or numeric methods for obtaining approximate solutions of ODEs (1). This will lead to a better conceptual understanding of an ODE (1). Moreover, such methods are of practical importance since many ODEs have complicated solution formulas or no solution formulas at all, whereby numeric methods are needed.
Graphic Method of Direction Fields. Practical Example Illustrated . We can show directions of solution curves of a given ODE (1) by drawing short straight-line segments (lineal elements) in the xy-plane. This gives a direction field (or slope field) into which you can then fit (approximate) solution curves. This may reveal typical properties of the whole family of solutions.

$$y' = y + x \tag{2}$$

obtained by a CAS (Computer Algebra System) and some approximate solution curves fitted in.

1.1 Numeric Method by Euler

$$y_1=y_0 + \mathrm{hf}(x_0,y_0)$$
$$y_2=y_1 + \mathrm{hf}(x_1,y_1)$$

in general,

$$y_n=y_{n-1} + \mathrm{hf}(x_{n-1},y_{n-1})$$

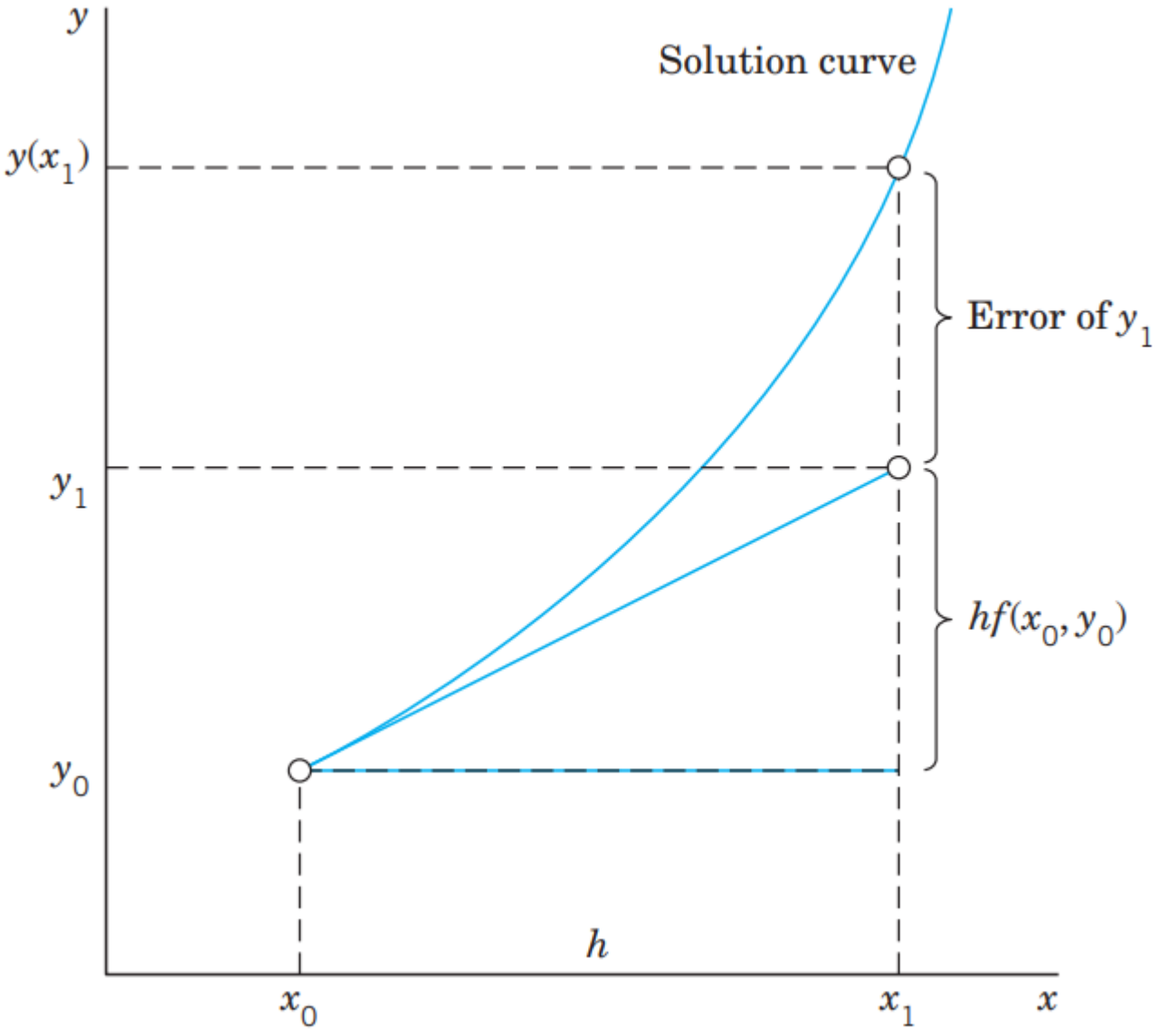
where the step h equals, e.g., 0.1 or 0.2 (as in Table 1.1) or a smaller value for greater accuracy

Table 1.1 shows the computation of steps with step for the ODE (2) and initial condition corresponding to the middle curve in the direction field. We shall solve the ODE exactly in Sec. 1.5. For the time being, verify that the initial value problem has the solution y = e^x - x - 1 . The solution curve and the values in Table 1.1 are shown in Fig. 9. These values are rather inaccurate. The errors are shown in Table 1.1 as well as in Fig. 9. Decreasing h would improve the values, but would soon require an impractical amount of computation.

Table 1: demo table

Table Head	**output points**		
	output	***x0***	***x1***
input points	output curves through points	y0	y1

[a]solution points


y
Solution curve
$y(x_1)$
Error of y_1
y_1
$hf(x_0, y_0)$
y_0
h
x_0
x_1
x