Problem Chosen	2021 MCM/ICM Summary Sheet	Team Control Number
A		**2125028**

Decompositions and Interactions of Fungal Species

Summary

Tiny fungi are distributed in almost every corner of the world, having a pivotal influence on the carbon cycle process. This article analyzes the interactions of fungi, which have different growth rates and moisture tolerances by establishing fungi decomposition and fungi population model, and predicts the optimal environmental condition and adverse effects for each species and combinations of fungi, respectively.

First, to model the process of fungi decomposing wood, we divide the decomposition situations into two types: **ideal cylinder** and **plane**. Combining the relevant knowledge of electromagnetic in physics, we find that when the fungi are distributed on the wood surface, Gaussian surface can be used to work out the decomposition rate, and establish **Woody Fibers Decomposition Model Based on Gauss Theorem** and **Ground Litter Decomposition Model Based on Logarithmic-like Function** through fungal activity factor (**ACT**).

Second, to obtain fungal data that can be brought into practical model calculations, we fit two sets of functions from *Figure 1* and *Figure 2* in the problem sheet, and perform **Fungi Selection**. After that, five fictional fungal species are established and denoted as F_A, F_B, F_C, F_D and F_E. Then, in order to analyze the interactions between fungi, we use **Multi-groups Logistic Model** with **Relative Growth Blocking Index** (**RGB**) to predict trends of fungal populations in short term and long term, respectively. For instance, as the interaction between Fungi A and Fungi B, the relative populations maintain exponential growth for $1 \sim 7$ days. After 8 days, the growths of the fungi are restricted. The relative populations of Fungi A and Fungi B eventually stabilize at 796 and 522, respectively.

Third, to predict the optimal environmental condition and adverse effects for each species and combinations of fungi, we conduct a prediction of single fungal species based on **Logistic Model** and another prediction of combinations of fungal species based on **Gray Theory**, respectively. We divide the five fungi into four combinations: F_A & F_B, F_B & F_C, F_C & F_D and F_D & F_E. After the **Gray Prediction**, we get the corresponding optimal environmental conditions and the predicted decomposition rates after 200 days are $\lfloor$semi-arid, $34.7\%\rceil$, $\lfloor$temperate, $32.8\%\rceil$, $\lfloor$temperate/arboreal, $32.7\%\rceil$ and $\lfloor$tropical rain forests, $34.3\%\rceil$.

Finally, from the perspective of the ecosystem, we carry out an ecologically significant analysis of the impact of fungal species biodiversity on the carbon cycle in the nature. Then we conduct sensitivity analysis of **rapid environmental fluctuations** to prove the models are feasible and applicable with high sensitivity, and discuss strengths and weaknesses of our models.

Keywords: Fungal Decomposition; Gauss Theorem; Fungi Selection; Multi-groups Logistic Model; Gray Prediction

Contents

1 Introduction

1.1 Problem Background

The earth's carbon circle is the foundation and important part of life on the planet. The decomposition of mixture is included in the carbon circle on the earth, allowing regeneration and reuse of carbon.

Fungi play a key role in the earth's carbon circle, especially in the decomposition process of ground litter and woody fibers. There are two main and important traits of a fungus. One is **the growth rate of the fungus**, and the other is **the fungal tolerance to moisture**. These two characteristics are prime focuses on fungi of this article.

According to a recent research [1] on fungi, there is a conclusion that the slow growing strains of fungi tend to be better able to survive and grow in the presence of environmental changes with respect to moisture and temperature, while the faster growing strains tend to be less robust to the same changes.

1.2 Restatement of Problem

With basic knowledge of the earth's carbon circle and the vital agents in decomposing wood fibers called fungi, we analyze the relationship of two traits mentioned above and the rate of decomposition. Our team is assigned to solve the problems below.

- ◎ **Problem 1:** Build the **decomposition model** of ground litter and woody fibers by calculating the **fungal activity factor**. Considering the interactions of fungi, fungal growth rate and moisture tolerance, the decomposition model would be optimized.
- ◎ **Problem 2:** Describe the **interactions** between different species of fungi, and characterize the **trends** of short- and long-term interactions.
- ◎ **Problem 3:** Analyze different **influences** on each isolate and fungal combinations **caused by different environments**, including arid, semi-arid, temperate, arboreal, and tropical rain forests.
- ◎ **Problem 4:** Assess the impact of the fungal communities' diversity on the earth' carbon circle efficiency, and explore the importance and role of fungal communities' **biodiversity** in the presence of different degrees of variability in the local environment.

1.3 Analysis of Problem

To work out the four problems, our solutions will be proceeded as follows.

- ◎ **Problem 1:** In the nature, the process of fungi decomposing lignin or cellulose involves complex biochemical reactions. Hence, to simplify the problem, we are going to divide the decomposition situations into two types: ideal cylinder and plane. Based on that, we could conduct a symmetrization operation on decomposition environment of the fungi to facilitate the connection of this process with the **Gauss theorem** and Gauss surface in electromagnetic. Meanwhile, **fungal activity factor** (ACT) will be added to the model, which is determined by the suitable living concentration of specific fungal types, moisture tolerance and the influence between different fungal species.

◎ **Problem 2:** Based on Figure 1 and Figure 2 given in problem sheet, we plan to perform a function fitting derivation, and then carry out the fungi selection operation, which simplifies the overall model building process, and obtains more practical data. When considering two fungal populations living together, due to competition for habitat and food, the interaction between them would hinder each other's growth and reproduction. Therefore, we are going to introduce the **Relative Growth Blocking Index** (RGB) on the basis of Single-group Logistic Model, and establish **Multi-groups Logistic Model** to predict trends of fungal populations in short term and long term.

◎ **Problem 3:** To analyze different influences on each isolate and fungal combinations caused by different environments. We will conduct two predictions for single fungal species and combinations of fungal species, respectively. **Logistic Model** and **Gray Prediction** will be used in the prediction process. Meanwhile, five environmental conditions, including arid, semi-arid, temperate, arboreal, and tropical rain forests, will be candidates for the optimal environmental condition.

◎ **Problem 4:** We will carry out an ecologically significant analysis of the impact of fungal population biodiversity on the **carbon cycle** in authentic nature from the perspective of the ecosystem, and write the important conclusions in the ***article for college level biology textbook***.

Above is a sketch of the analysis process on the four problems.

2 Assumptions

◇ **Assumption 1:** The wood decomposed by fungi is an ideal cylinder, and the fungi are distributed on the surface of the wood.

↪ **Justification:** Symmetrical the decomposition environment of the fungi as much as possible to facilitate the connection of this process with the Gauss theorem and Gauss surface in electromagnetic.

◇ **Assumption 2:** The decomposition rate of the fungi is only related to its growth rate, tolerance to moisture and environment temperature.

↪ **Justification:** In the real natural environment, fungi are susceptible to natural disasters. We limit the determination of the factors affecting the decomposition rate of fungi in advance to facilitate subsequent analysis.

◇ **Assumption 3:** After the part of Fungi Selection, we obtain five virtual fungal species, which are typical and representative.

↪ **Justification:** There are thousands of species of fungi, hence, it does not have much practical significance to study a specific species of fungi. The five fungi we set can be analyzed and studied more comprehensively.

◇ **Assumption 4:** The Logistic model is also valid when studying microorganisms, such as fungi.

↪ **Justification:** Logistic model is generally used to model changes in common animals and plants populations. Since fungi are also biological, we will use analogy reasoning to apply it to the study of fungi.

◇ **Assumption 5:** The study on the interactions of the two fungal species combination could show that of more fungal species combination.

↪ **Justification:** In a piece of natural land, although there are more than two types of fungi, the interaction of multiple fungi can be understood as the superimposed effect of multiple combinations.

◇ **Assumption 6:** Fungi play a major role in the decomposition of organic matter in the certain carbon cycle.

↪ **Justification:** In biology, the decomposition process of organic matter is completed by microorganisms, including fungi, bacteria and so on. Since this article only studies fungi, this process is simplified.

3 Notations

Table 1: Important Notations.

Symbol	Description	Unit
$S_{cylinder}$	side surface area of cylinder	m^2
$\boldsymbol{E}$	electric field intensity on the surface of uniformly charged cylinder	N/C
Φ	electric flux	$N \cdot m^2/C$
DR	decomposition rate	Unitless
α	fungi linear density	Unitless
ε_{DR}	environmental decomposition constant	Unitless
ACT	fungal activity factor	Unitless
β	fungi areal density	Unitless
HER	hyphal extension rate	mm/day
RMT	relative moisture tolerance	Unitless
R^2	sample correlation coefficient	Unitless
$N(t)$	current population size	Unitless
RGB	relative growth blocking index	Unitless
X	time series data	Unitless

4 The Models

In the nature, the process of fungi decomposing ground litter and woody fibers involves complex chemical reactions [2]. In order to visualize this objective process mathematically, we simplified the analysis of dead wood and leaves to the study of cylindrical woody fibers and plane irregular ground litter.

4.1 Woody Fibers Decomposition Model

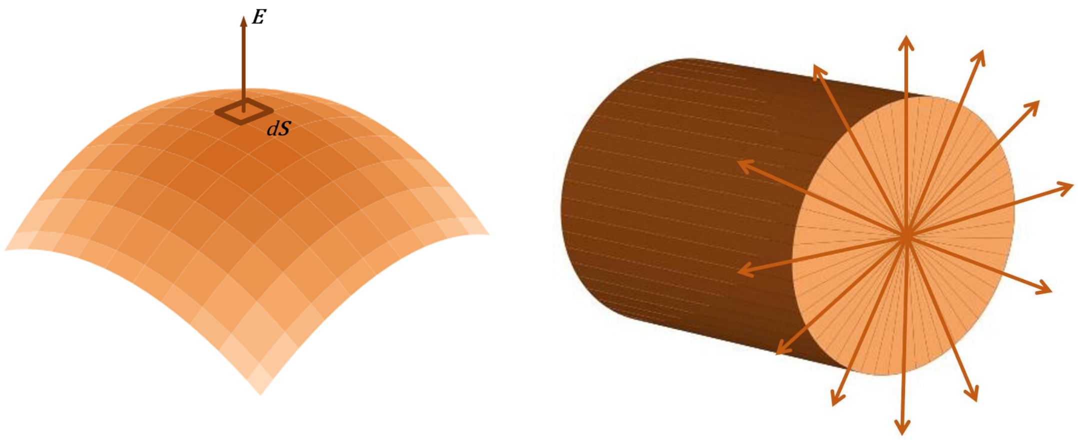
Fungi with the ability to decompose lignin or cellulose [3][4] are distributed on an ideal cylindrical dead wood. Under certain environmental temperature and moisture conditions, the hyphae begin to expand and grow, and fungi secrete biologically active enzymes that can decompose the woody fibers. As time goes by, the outer layer of woody fibers decompose and fall off, and the fungi gradually approach the axis of the dead wood. Ignoring the surface area of cylinder bottoms, use the *cylinder side surface area formula*, and mark it as *Eq. (1)*

$$S_{cylinder} = 2\pi r \cdot h \tag{1}$$

where r represents the radius of dead wood, and h represents the total length of dead wood. As the radius decreases, it comes to a conclusion that the surface area of cylinder also decreases.

Based on the conclusion, it is obvious that the number of fungi per unit area would increase. Meanwhile, fungal hyphae can generally grow to several times their initial size [5]. Therefore, even considering the loss of fungi because of wind and rain, the number of fungi would still increase significantly as their distribution gradually approaches the central axis.

Combining with the relevant knowledge of electromagnetic in physics [6], we find that the distribution of fungi has many similarities with the *Gaussian surface* in the electric field. On the one hand, the distribution of fungi is cylindrical and could be represented by a cylindrical Gaussian surface. On the other hand, as the radius decreases, both the electric field strength and the number of fungi per unit area (also known as fungi decomposition rate) would increase. From *Gauss theorem*, the decomposition cylinder is shown in *Figure 1*, and we can get *Eq. (2)*, *Eq. (3)* and *Eq. (4)* as follows.



(a) Electric field intensity $\boldsymbol{E}$ and area differential $d\boldsymbol{S}$. (b) Ideal cylindrical wood.

Figure 1: Decomposition cylinder.

$$\oint_S \boldsymbol{E} \cdot d\boldsymbol{S} = \frac{1}{\varepsilon} \int_V \rho dV \tag{2}$$

$$\boldsymbol{E} = \frac{\lambda}{2\pi\varepsilon R} \tag{3}$$

$$\Phi = \oint_S \boldsymbol{E} \cdot d\boldsymbol{S} = \oint_S \frac{\lambda}{2\pi\varepsilon R} dS = \frac{\lambda}{2\pi\varepsilon R} 2\pi R l = \frac{\lambda l}{\varepsilon} \tag{4}$$

where $\boldsymbol{E}$ represents electric field intensity on the surface of a uniformly charged cylinder, whose direction is radial, and Φ represents electric flux. Convert the above electromagnetic equations and parameters into formulas for estimating the decomposition rate ($\boldsymbol{DR}$) of fungi in *Eq. (5)*, *Eq. (6)* and *Eq. (7)*

$$\oint_S DR \cdot dS = \frac{1}{\varepsilon_{DR}} \sum N_{inside} = \frac{1}{\varepsilon_{DR}} \int_V \rho dV \tag{5}$$

$$DR = \frac{\alpha}{2\pi\varepsilon_{DR} R} \tag{6}$$

$$ACT = \oint_S DR \cdot dS = \oint_S \frac{\alpha}{2\pi\varepsilon_{DR} R} dS = \frac{\alpha}{2\pi\varepsilon_{DR} R} 2\pi Rh = \frac{\alpha h}{\varepsilon_{DR}} \tag{7}$$

where α represents fungi linear density, DR represents the decomposition rate, ε_{DR} represents environmental decomposition constant determined by surrounding meteorological indicators, and ACT represents the **fungal activity factor**, which is determined by the suitable living concentration of specific fungal types, moisture tolerance and the influence between different fungal species. According to *Eq. (6)*, as fungi gradually approach the central axis, the decomposition rate of fungi increases, which is consistent with the previous analysis.

4.2 Ground Litter Decomposition Model

Due to the irregular shape of ground litter, it is hard to represent ground litter as a special geometric. Therefore, we simplified it as the process of fungi decomposition of a plane, shown as follows.

Figure 2: Decomposition plane.

Similar to the *Woody Fibers Decomposition Model*, fungi that could decompose cellulose or lignin are distributed on a flat surface, gradually decomposing ground litter from outside to inside. By consulting the literature [7], the decomposition rate function of fungi decomposing ground litter on the plane is similar to the **logarithmic function** in *Eq. (8)* as follows.

$$DR \propto \ln(1 + \frac{1}{4d}) \tag{8}$$

Combined with the analysis of environmental decomposition constant (ε_{DR}) and fungi activity factor (ACT) in *Model 4.1*, we deduce the **decomposition rate of fungi in the plane distribution** as *Eq. (9)*

$$DR = \frac{\beta}{\varepsilon_{DR}} \ln(1 + \frac{1}{4d}),\ d \in (0.05,\ +\infty) \tag{9}$$

where β represents fungi areal density, and d represents the depth of fungi's current decomposition.

Considering that fungal decomposing needs to be carried out at a certain distance below ground level, we set a lower limit of 0.05 meters for the current depth of fungal decomposition, which avoids invalid information when $d \longrightarrow 0^+$, $DR \longrightarrow +\infty$.

4.3 Fungi Selection

There are many types of fungi in the nature, and the selection of specific fungal species for research would be a complicated problem. Therefore, with *Figure 1* and *Figure 2* in the problem sheet, we fit the three functions corresponding with three curves of *hyphal extension rate* (mm/day) *- decomposition rate (% mass loss over 122 days)* under conditions of $22°C$, $16°C$ and $10°C$. We also fit the linear function corresponding with the *relative moisture tolerance - decomposition rate curve* under relative scaling. The fitting functions are listed in *Eq. (10)* and *Eq. (11)* as follows.

$$\begin{cases} DR_{22°C} = 5.46\ln(HER) + 16.95 + \varepsilon_1 \\ DR_{16°C} = 4.94\ln(HER) + 8.31 + \varepsilon_2 \\ DR_{10°C} = 2.22\ln(HER) + 6.03 + \varepsilon_3 \end{cases} \tag{10}$$

where $\varepsilon_i(i = 1,\ 2,\ 3) \sim N(0, \sigma^2)$, HER represents the **hyphal extension rate** and DR represents the **decomposition rate**.

$$\log(DR) = 0.995RMT + 1.882 \tag{11}$$

where RMT represents the relative moisture tolerance. In the regression fitting process, the sample correlation coefficient is R^2 [8], the results of which are listed as follows.

Table 2: Sample correlation coefficient results.

Temperature ($°C$)	R^2
22	0.1662
16	0.3921
10	0.2504

Since R^2 does not significantly tend to zero, three random disturbance terms are added to improve the accuracy of the fitting.

In order to obtain representative fungal species, we take temperature, moisture and hyphal extension rate into account in the **fungi selection**. Finally, select five typical fungal species as follows.

Table 3: Five selected typical fungal species.

Fungi	Optimal Temperature ($°C$)	Relative Moisture	HER (mm/day)
Fungi A (F_A)	22	-1.0	5
Fungi B (F_B)	16	-0.5	5
Fungi C (F_C)	10	0	5
Fungi D (F_D)	16	0.5	5
Fungi E (F_E)	22	1.0	5

Substitute the data in the above table into *Eq. (10)* and *Eq. (11)* and perform logarithmic process. We obtain the hyphal extension rate of five fungal species ($F_A \sim F_E$) as $5\ mm/day$.

The corresponding decomposition rates in different environmental conditions are listed as follows.

Table 4: Decomposition rates in different environmental conditions.

Fungi	Decomposition Rate $(\%)$
F_A	13.32
F_B	8.83
F_C	5.74
F_D	9.32
F_E	14.31

Table 3 and *Table 4* provides data support for research on the interactions of different fungal species, which makes it convenient for the modeling and analysis later.

4.4 Fungi Interaction Model

The *Logistic model* [9] can precisely analyze the growth phenomenon of animals, human and plants who naturally confront obstacles of survival attribute to the excess of the total population and the lack of food resources, so does microorganism, fungi. Therefore, it is suitable to use the **Logistic model** to study the interactions of different kinds of fungi.

When a single fungal population lives, its number changes obey the **Single-group Logistic model**.

$$\frac{1}{N(t)}\frac{dN(t)}{dt} = r(1-\frac{N(t)}{N_{max}}) \tag{12}$$

where $N(t)$ represents current population size, and r represents the growth rate of population. When considering two fungal populations living together, due to competition for habitat and food, the interaction between them would hinder each other's growth and reproduction. Therefore, we introduce the **Relative Growth Blocking Index** (RGB) on the basis of Single-group Logistic model, and establish **Multi-groups Logistic Model** [10] as follows.

$$\begin{cases} \frac{1}{N_x(t)}\frac{dN_x(t)}{dt} = r_x(1-\frac{N_x(t)}{N_{xmax}} - RGB_x\frac{N_y(t)}{N_{y_{max}}}) \\ \\ \frac{1}{N_y(t)}\frac{dN_y(t)}{dt} = r_y(1-\frac{N_y(t)}{N_{y_{max}}} - RGB_y\frac{N_x(t)}{N_{xmax}}) \end{cases} \tag{13}$$

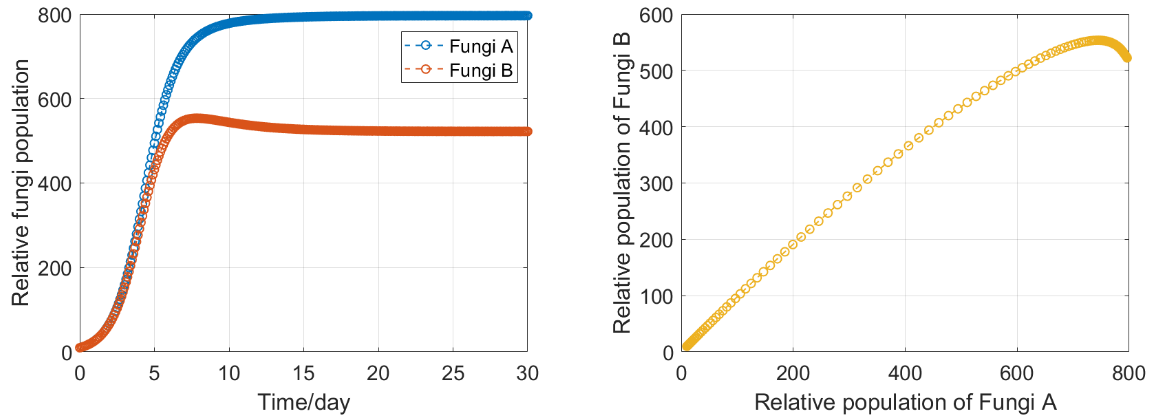
where $N_x(t)$ and $N_y(t)$ represents the numbers of the two populations x and y, r_x and r_y represents the inherent growth rate of populations, N_{xmax} and $N_{y_{max}}$ are maximum capacities of populations, RGB_x means for resources that support x, the consumption of y is RGB_x times the consumption of x per unit quantity, and RGB_y has the similar definition to RGB_x.

Taking fungal species F_A and F_B proposed in the previous article as examples, analyze and predict the interaction between them. Calculate the values of parameters and list them as follows.

Table 5: Values of parameters.

Parameter	RGB_A	RGB_B	r_A	r_B
Value	0.39	0.60	1	1

Draw the change curves of $N_A(t)$ and $N_B(t)$ as well as the phase trajectory of these two function variables as follows.

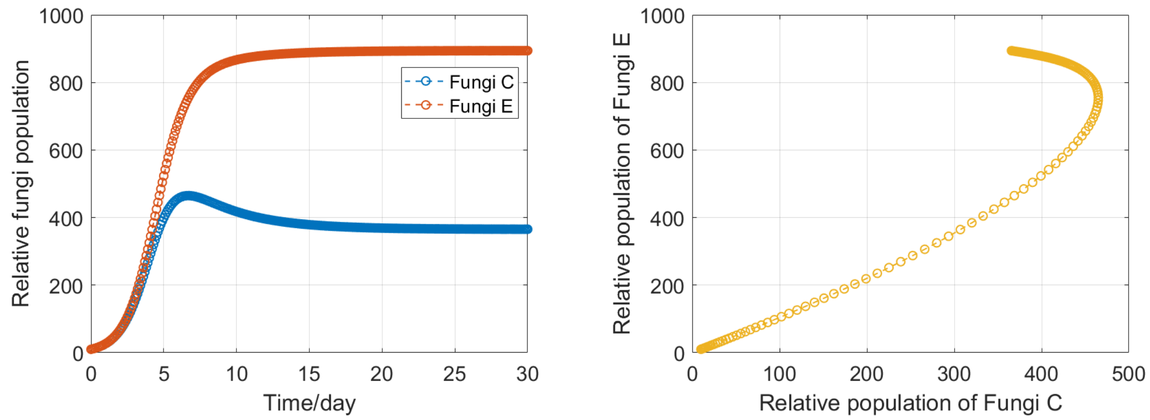


(a) Curves of $N_A(t)$ and $N_B(t)$. (b) Phase trajectory of $N_A(t)$ and $N_B(t)$.

Figure 3: The interaction of F_A & F_B.

The dynamics of the interaction between Fungi A and Fungi B are characterized and described in the above figures. **In the short-term period** (about $1 \sim 7$ days), F_A and F_B are in a period of rapid growth and reproduction, the curves of which maintain exponential growth. **In the long term** (after 8 days), the growths of F_A and F_B would be both restricted by environmental resources. Meanwhile, because F_A has a stronger ability to decompose, it has a stronger blocking effect on F_B. The final relative population numbers tend to 796 and 522 respectively and remain stable.

In the same way, we calculate and analyze the interaction of Fungi C and Fungi E, and obtain the $N_C(t)$ and $N_E(t)$ function curves as well as the phase trajectory of the two function variables as follows.



(a) Curves of $N_C(t)$ and $N_E(t)$. (b) Phase trajectory of $N_C(t)$ and $N_E(t)$.

Figure 4: The interaction between F_C & F_E.

In a short period of time (about $1 \sim 5$ days), the numbers of F_C and F_E grow exponentially. **From a long-term perspective** (after 6 days), since the discrepancy of decomposition ability between F_E and F_C is greater than that between F_A and F_B, the blocking effect of F_E on F_C is extremely obvious. The final relative populations numbers of F_C and F_E tend to 365 and 894 respectively and remain stable.

4.5 Fungal Species Development Predictions

4.5.1 Single Fungal Species

It is simple to analyze and predict a single species of fungus, because in this ideal environment, the growth and reproduction process of a single fungus can be accurately represented by the **Logistic Model**. Substitute data of $F_A \sim F_E$ into the previous differential equation *Eq. (12)* and get *Eq. (14)* as follows.

$$\begin{cases} \frac{1}{N_A(t)}\frac{dN_A(t)}{dt} = r_A(1 - \frac{N_A(t)}{N_{Amax}}) \\ \frac{1}{N_B(t)}\frac{dN_B(t)}{dt} = r_B(1 - \frac{N_B(t)}{N_{Bmax}}) \\ \frac{1}{N_C(t)}\frac{dN_C(t)}{dt} = r_C(1 - \frac{N_C(t)}{N_{Cmax}}) \\ \frac{1}{N_D(t)}\frac{dN_D(t)}{dt} = r_D(1 - \frac{N_D(t)}{N_{Dmax}}) \\ \frac{1}{N_E(t)}\frac{dN_E(t)}{dt} = r_E(1 - \frac{N_E(t)}{N_{Emax}}) \end{cases} \tag{14}$$

The function curves of $N_A(t)$, $N_B(t)$, $N_C(t)$, $N_D(t)$, and $N_E(t)$ are as follows.

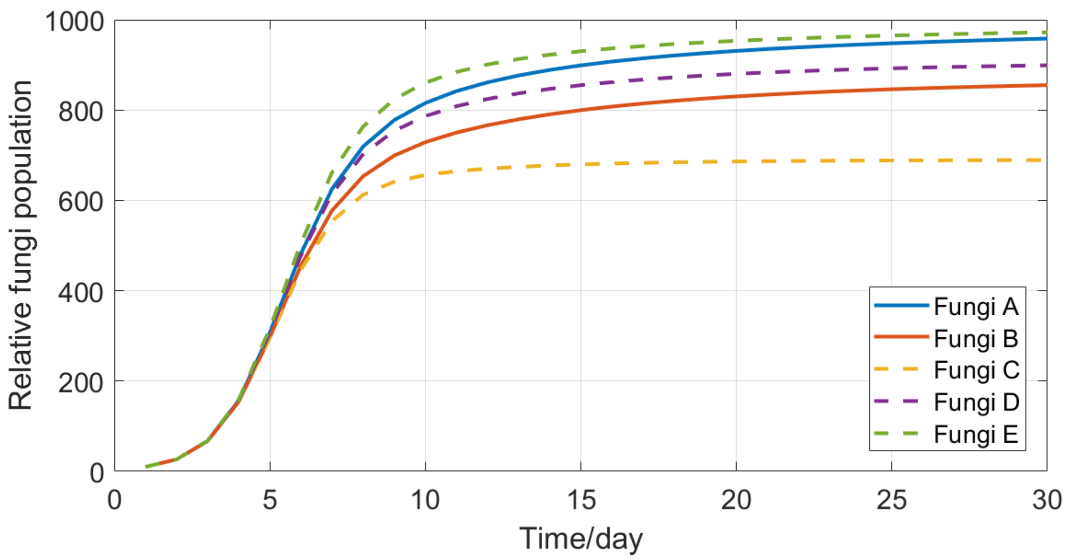


Figure 5: Function curves of each single fungus.

The fungal data in the above figure are all based on the research and analysis of the environmental climate in *Table 3*. The temperature, moisture and so on indicated in the table are the most suitable environmental conditions for the five fungi. Looking up the climatic data, it is found that the optimal growth environment of the five fungi corresponds exactly to the five typical climatic characteristics, including arid, semi-arid, temperate, arboreal and tropical rain forests. Therefore, it is obvious that when five species of fungi $F_A \sim F_E$, grow and reproduce under the optimal conditions, they can exert their greatest advantages in decomposing lignin or cellulose. On the contrary, these fungi will be at disadvantages due to excessive environmental blocking.

Table 6: Corresponding optimal environment for the five fungi.

Fungi	Corresponding Environment
F_A	Arid
F_B	Semi-arid
F_C	Temperate
F_D	Arboreal
F_E	Tropical rain forests

The above table illustrates the corresponding optimal environment when the five fungi live alone, but it is quite different from the real situation in the nature. The following will further analyze the combinations between fungal species.

4.5.2 Combinations of Fungal Species

The distribution of fungi in the nature is diverse and wide, and there are often hundreds of microorganisms in one square meter of natural land [11]. In order to simplify the study on the topic that **combinations of fungal species likely to persist in different environment conditions**, we only analyze and predict the growth and decomposition ability of two fungi in combination.

Since the known information is not sufficient and the internal relationship of fungal species combinations in different environments has a degree of uncertainty, we use **Gray Prediction Model** [12] and time series data based on the **Decomposition Models** in *Model 4.1* and *Model 4.2* to analyze and predict the living situation of the fungal species combination and the change trend of decomposition ability.

Mark the known group of time series data as follows.

$$X^0 = (x^0(1),\ x^0(2),\ x^0(3),\ \ldots,\ x^0(n))$$

To ensure the stability of the data, we perform **Accumulated Generating Operation** (AGO) [12] on the data. The AGO process would be executed multiple times until the time series data is stable enough, and we get the following time series data.

$$X^1 = (\sum_{i=1}^{1} x^0(i),\ \sum_{i=1}^{2} x^0(i),\ \ldots,\ \sum_{i=1}^{n} x^0(i))$$

The time series X^1 is then fitted by a first-order differential equation given by *Eq. (15)*

$$\frac{dx^1}{dt} = ax^1 = b \tag{15}$$

where the parameters a and b are the development coefficient and control variable.

According to the **whitening gray derivatives** of discrete data points, the following equation is obtained.

$$\frac{dX^1(t)}{dt} = x^1(k) - x^1(k-1) = x^0(k) \tag{16}$$

A new variable $z^1(k)$, which is known as the whitening value of $x^1(k)$, is defined by the following equation.

$$z^1(k) = 0.5x^1(k) + 0.5x^1(k-1) \tag{17}$$

Then we use **the least square method** to calculate the parameters.

$$\hat{\delta} = (B^\top B)^{-1} B^\top Y \tag{18}$$

where

$$\hat{\delta} = [\hat{a},\ \hat{b}]^\top,\ B = \begin{bmatrix} -z^1(2) & 1 \\ -z^1(3) & 1 \\ \vdots & \vdots \\ -z^1(n) & 1 \end{bmatrix},\ Y = \begin{bmatrix} x^0(2) \\ x^0(3) \\ \vdots \\ x^0(n) \end{bmatrix}$$

Finally, use **inverse AGO** and get the predicted value $x^2(k+1)$ as follows.

$$\hat{x}^2(k+1) = \hat{x}^1(k+1) - \hat{x}^1(k) = (1-e^a)(x^1(1) - \frac{b}{a})e^{-at},\ k = 1,\ 2,\ \ldots,\ n$$

When new data are added, the parameters a and b in the Gray Prediction Model would be **updated** to ensure the accuracy of prediction.

Due to the large differences in the suitable environment among the five fungi, we divide the five fungi into four combinations, including F_A & F_B, F_B & F_C, F_C & F_D, and F_D & F_E, based on the conclusion of *Table 6* for analysis and prediction.

Take combination F_A & F_B as an example. Calculate the time series data of the decomposition rate for the previous 122 days using the data in *Eq. (6)*, *Eq. (9)*, and *Table 4*, and mark it as follows.

$$X^0_{A\&B} = (x^0_{A\&B}(1),\ x^0_{A\&B}(2),\ \ldots,\ x^0_{a\&B}(122))$$

Then the time series data is substituted into Gray Prediction Model, and the prediction data is calculated as follows.

$$X^2_{A\&B} = (x^2_{A\&B}(123),\ x^2_{A\&B}(124),\ \ldots,\ x^2_{A\&B}(200))$$

Draw the decomposition rate of fungal species combination F_A & F_B in five climates, including arid, semi-arid, temperate, arboreal, and tropical rain forests, as shown in the figure below.

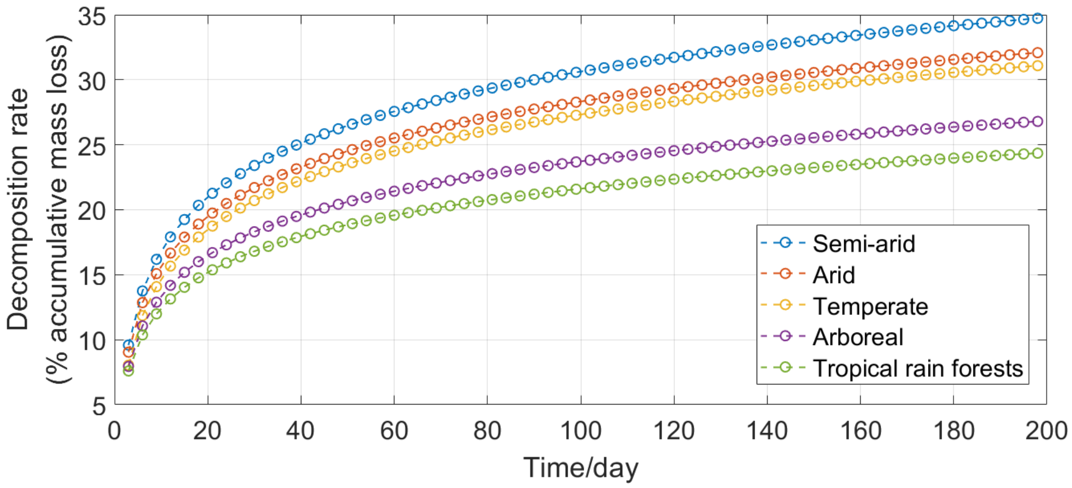


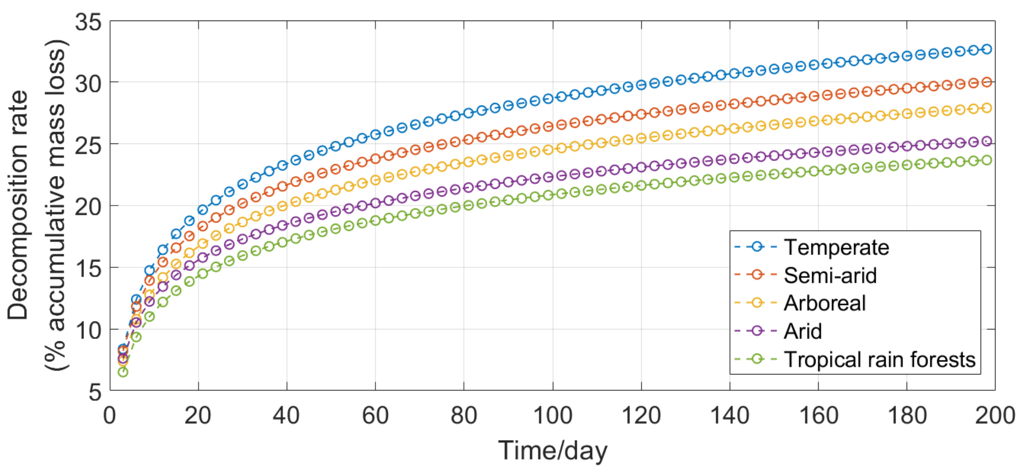
Figure 6: Decomposition rate of fungal species combination F_A & F_B in five climates.

Combination F_A & F_B is most suitable for the decomposition of lignin or cellulose in semi-arid climate. From the prediction result, the cumulative decomposition rate after 200 days can reach 34.7%.

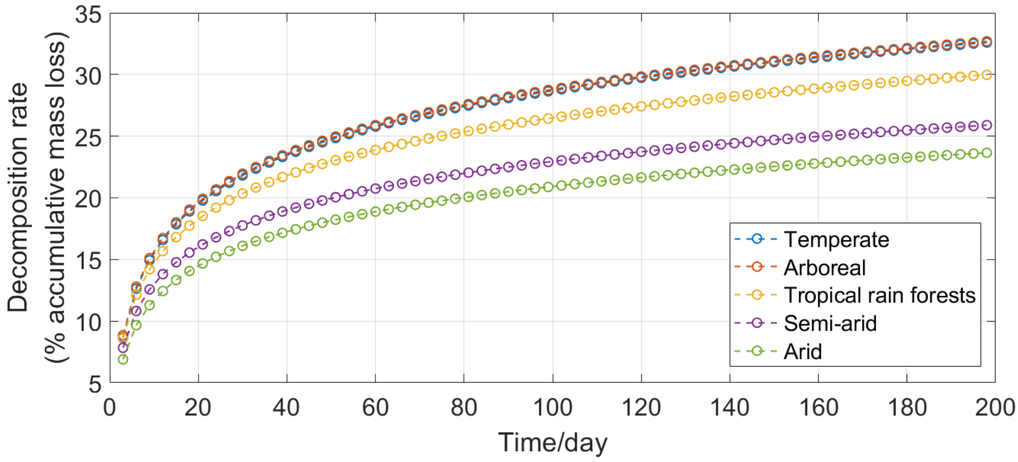
In the same way, the time series data of the other three fungal species combinations, F_B & F_C, F_C& F_D and F_D& F_E, are calculated as follows.

$$\begin{cases} X^0_{B\&C} = (x^0_{B\&C}(1),\ x^0_{B\&C}(2),\ \ldots,\ x^0_{B\&C}(122)) \\ X^0_{C\&D} = (x^0_{C\&D}(1),\ x^0_{C\&D}(2),\ \ldots,\ x^0_{C\&D}(122)) \\ X^0_{D\&E} = (x^0_{D\&E}(1),\ x^0_{D\&E}(2),\ \ldots,\ x^0_{D\&E}(122)) \end{cases}$$

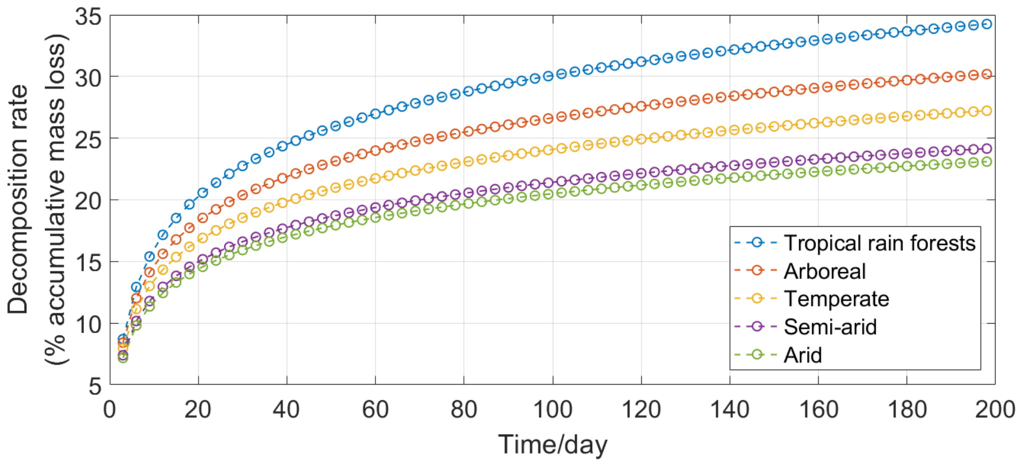
Substituting Gray Prediction Model to obtain the following decomposition rate prediction curves in different climates in turn.



(a) F_B & F_C



(b) F_C & F_D



(c) F_D & F_E

Figure 7: Decomposition rate of fungal species combinations in five climates.

The optimal environment and the predicted decomposition rate after 200 days under different combinations of fungal species are summarized in the following table.

Table 7: Summary of optimal environment and predicted decomposition rate.

Combinations	Optimal Environment	Decomposition Rate Predicted Value
F_A&F_B	Semi-arid	34.7%
F_B&F_C	Temperate	32.8%
F_C&F_D	Temperate / Arboreal	32.7%
F_D&F_E	Tropical rain forests	34.3%

According to the above table, we find that the combination of F_A & F_B has the highest decomposition rate in semi-arid region, which is 34.7%, and the decomposition rates of other species combinations in their optimal environment are also relatively high, which are all above 32%.

5 Analysis of the Impacts of Fungal Diversity

In the previous analysis and prediction, we obtained the predicted value of decomposition rate under different combinations of fungi. Compared with decomposition rates of the five fungi living alone, it is found that the decomposition rates of the fungal combinations is significantly higher than that of the single species of fungi living alone.

In the combination F_A & F_B, although both are drought-tolerant fungal species suitable for arid or semi-arid climates, they still maintain a certain degree of decomposition ability in the arboreal or tropical rain forests environments, the predicted decomposition rate of which after 200 days is 24.4%. In the combination F_D & F_E, although both are water-requiring fungal species suitable for arboreal and tropical rain forests climates, they can still maintain the capacity to decompose lignin or cellulose in the arid and semi-arid regions, the predicted decomposition rate of which after 200 days is 22.1%.

It could be inffered from the analysis above that when the diversity of fungal population [13] is further improved, the total decomposition rate in the area where the fungi are located would increase significantly. The diversity of fungal communities could greatly improve the overall efficiency of a certain system with respect to the breakdown of ground litter.

Since drought-tolerant fungi can survive in hot flashes, and water-requiring fungi can also live in arid areas, it is possible to introduce fungal species in a certain area as much as possible as long as ensuring that there is no harm such as biological invasion. By doing so to respond to varying degrees of changes in the local natural environment.

For instance, if the five fungi distribute in the tropical rain forests, as the destruction of the rainforest by humans intensifies, the environment of the rainforest deteriorates with moisture decreasing and temperature rising. At this time, drought-tolerant species F_A and F_B would play their superior role to maintain the stability of the decomposition process in the rainforest ecosystem as much as possible, and avoid that the carbon cycle [14] is out of balence.

6 Sensitivity Analysis

6.1 Analysis of Rapid Environmental Fluctuations

In this article, to analyze the interaction of the mixed growth of multiple fungi, we establish Multi-groups Logistic Model. However, we do not make a detailed analysis of the environmental changes. Hence, we are going to examine the sensitivity of our model to rapid fluctuations in the environment.

In the real natural environment, disasters that are not conducive to the survival and reproduction of living things happen from time to time. When a disaster occurs, the natural environment fluctuates rapidly, and the survival of organisms in the area is threatened and restricted. Combining our model and the characteristics of fungi, we find that when the environment fluctuates rapidly, the inherent growth rate of fungi will change rapidly in a short time. Meanwhile, the natural maximum capacity of fungi will be reduced accordingly.

6.2 Sensitivity Test

Based on the analysis above, we modify the Multi-groups Logistic Model.

$$\begin{cases} \frac{1}{N_x(t)}\frac{dN_x(t)}{dt} = r_x(t)(1 - \frac{N_x(t)}{N'_{xmax}} - RGB_x\frac{N_y(t)}{N'_{ymax}}) \\ \\ \frac{1}{N_y(t)}\frac{dN_y(t)}{dt} = r_y(t)(1 - \frac{N_y(t)}{N'_{ymax}} - RGB_y\frac{N_x(t)}{N'_{xmax}}) \end{cases} \tag{19}$$

We take F_A and F_B as examples to analyze the sensitivity of our model when environment fluctuates rapidly. Because they are all drought-tolerant fungi, when rapid fluctuations in the environment have an adverse effect, they will appear as heavy precipitation in a short period of time. We artificially set rapid fluctuations in the environment on the fifth day and draw curve figure of $N'_A(t)$ and $N'_B(t)$ as follows.

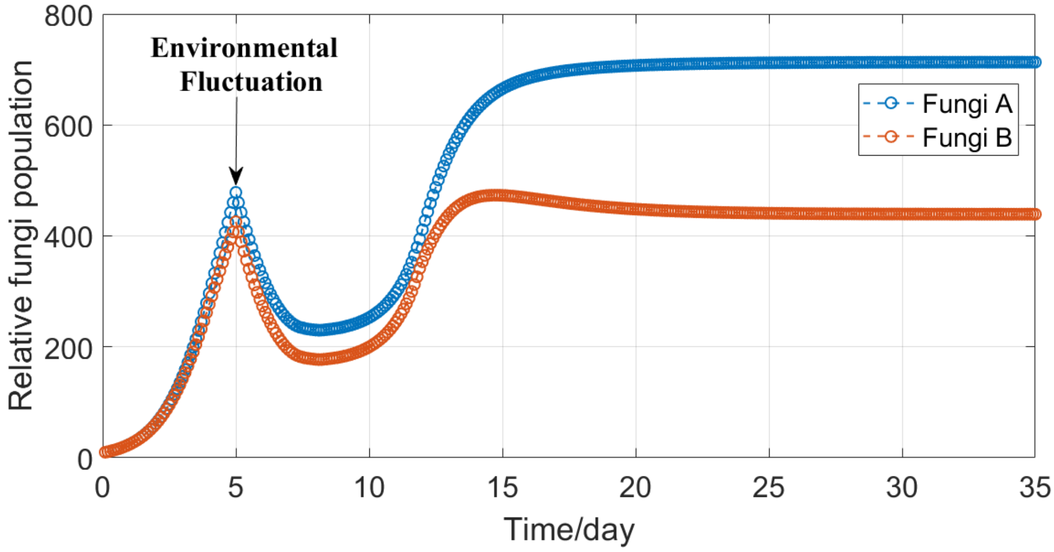


Figure 8: Curves of $N'_A(t)$ and $N'_B(t)$.

From the above figure, it is found that the fungal population maintained an exponential growth in the first 5 days. After rapid environmental fluctuations occurred on the $5th$ day, the populations of F_A and F_B both declined significantly. Around the $11th$ day, the population began to increase. After 20 days, the populations of F_A and F_B both stabilized, at 713 and 439, respectively. Compared with the stable fungal population of F_A and F_B without the influence of rapid environmental fluctuations, the values have dropped by 10.4% and 15.9%, respectively.

Therefore, when rapid environmental fluctuations occur, the **advanced Multi-groups Logistic Model** could well reflect the influence of harsh environments on the growth and reproduction of fungi, which shows that the model has **good sensitivity**.

7 Strengths and Weaknesses

7.1 Strengths

- Using Gaussian surface and Gauss theorem in physical electromagnetism, we deduce the process of fungi wood decomposition in accordance with objective laws and mathematical logic.

- Based on *Figure 1* and *Figure 2* given in problem sheet, after performing the function fitting derivation, we carry out the fungi selection operation, which simplifies the overall model building process, and obtains more practical data.
- Starting from Single-group Logistic Model, we use Multi-groups Logistic Model to accurately and comprehensively describe the interactions between different species of fungi.
- In the case of insufficient time series data, we use the Gray Prediction Model to predict and analyze the survival of fungal species combination and the change trend of decomposition in different environmental conditions.
- From the perspective of the ecosystem, we carry out an ecologically significant analysis of the impact of fungal population biodiversity on the carbon cycle in authentic nature.

7.2 Weaknesses

- F_A, F_B, F_C, F_D, and F_E are all virtual fungal species based on the analysis of problem sheet. Whether the corresponding species actually exists in the nature, remains to be studied.
- When modeling and analyzing the interactions within the fungal species combination, we only analyzed the combinations of two fungal species.
- Logistic model is generally used to model changes in common animals and plants populations. When it is applied to the study of microorganisms (fungi), we are uncertain about the validity of the results.

8 Article for College Level Biology Textbook

Please turn to the next page.

1 Decomposer plays a vital role in the carbon cycle

Key Points

1. The stronger the ecosystem in nature, the more diverse the types of its decomposers.
2. Abundant species of fungi will increase the efficiency of the carbon cycle.
3. The protection of microorganisms urgently needs people's efforts.

▲**Fig. 1** Decomposers in the nature.

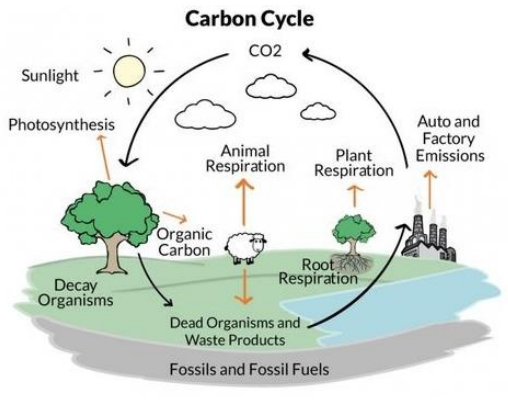
Ecosystem refers to the collective name of all the interacting organisms and the environment in a specific environment. As one of the most important roles in the ecosystem, the decomposer undertakes most of the decomposition tasks. While helping to make full use of the energy in the ecosystem, it also acts as a bridge from organic carbon to inorganic carbon. It is the existence of decomposers that the carbon in the biological and inorganic worlds can circulate and flow, and energy can also rely on carbon as a carrier to deduct the vibrant earth in the process of flow.

Any ecosystem has a food chain, and the material and energy flowing into the ecosystem can flow through the food chain. Although there are many types of organisms in the ecosystem and they also play different roles in the ecosystem, they can be classified into three categories: producers, consumers, and decomposers according to the effects they cause in energy and matter. The bottom layer is the "producer", which uses sunlight to perform photosynthesis and uses water and carbon dioxide and other inorganic substances to synthesize organic matter green plants; the upper layer is the "consumer" at all levels, which rely on the producer to supply material and energy; After death, the "decomposers" will feed on their corpses. Because the decomposer has the ability to convert macromolecular substances into small molecular substances, decomposer and the existence of producers together construct the carbon cycle structure of the ecosystem.

However, there are a large number of organisms corresponding to the types of decomposers in nature. Will the biological competition between them affect the decomposition rate of the environment? What is the impact of the growth of the decomposer population on producers and consumers? What are the factors that contribute to the efficiency of the decomposition? With these questions in mind, we will use the modeling analysis results to answer this question.

We chose fungi as an example, and conducted a study on the degradation of plant ground litter by fungi.

Under different climatic conditions, different types of fungi will also show different activities, and their decomposing properties as decomposers are also very different. Through separate grouping experiments, we found that as the main two decomposers of plant corpses, white fungus and brown fungus exhibit different characteristics in terms of degradation rate and environmental adaptability. When two decomposers with significant competitive relationship are placed in the same environment at the same time, the two fungi show stronger vitality and more adequate decomposition rate than a single species. After further modeling and prediction of experimental data, we found that when the biodiversity of fungi is very high, its role in improving the efficiency of the carbon cycle will become more important.



▲**Fig. 2** Carbon circle of the ecosystem.

Therefore, this conclusion can also explain why the stronger the ecosystem in nature, the more diverse the types of its decomposers. The larger the ecosystem, the higher the efficiency of material circulation and energy flow, because when the total material is limited, only accelerated flow can meet the energy needs of each part of the ecosystem. It is with a sufficient variety of decomposers that can ensure the high efficiency of its material circulation, thereby further stabilizing the overall structure of the biosphere. This is why the larger the ecosystem, the more stable it is.

▲**Fig. 3** Pollution leads to devastating blow of microorganism.

Most of the decomposers in the environment are microorganisms (such as bacteria, fungi, etc.). They are often large in number, but they are very sensitive to environmental changes. At the same time, they are extremely weak in mobility and do not have the ability to migrate when the environment deteriorates. Therefore, when the environment is affected by human activities, it is precisely these vital, ubiquitous, but difficult to detect microorganisms that are destroyed first. At the same time, because the damage to the microbial population is difficult to detect in a short time, the protection of microorganisms has been lacking.

So far, various countries have basically promulgated various animal and plant protection laws, but laws and regulations for the protection of microorganisms have been absent for a long time. This shows that people are still far from enough about the role and understanding of microorganisms in the ecosystem.

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