A Simulation-Based Assessment of Traffic Circle Control

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Summary

The difficulty of evaluating the performance of a control system for a traffic circle lies largely in crucial dependence on the local interactions among individual drivers. Traffic circles are relatively small compared to highways and are therefore susceptible to blockages caused by lane changes, entrances, and exits. A complete model must account for effects of such individual car behavior. Existing models, however, do not track performance at the level of individual cars.

We propose a novel simulator-based approach to evaluating and selecting such control systems. We create a multi-agent discrete-time simulation of behavior under different control systems. The behavior of individual cars in our simulator is determined *autonomously* and *locally*, allowing us to capture the effects of local interactions. In addition, by modeling each car separately, we track the time spent in the traffic circle for each individual car, giving us a more specific measure of performance than the more-commonly-used aggregate rate of car passage.

Measuring the performance of several control strategies using both metrics, we find that the *rate of incoming traffic* and the *number of lanes* in the traffic circle are the major factors for optimal choice of a control system. Based on the simulated performance of traffic circles with varying values of these parameters, we have two different recommendations for traffic control systems based upon the rate of incoming traffic:

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- When the rate of incoming traffic is low, *entering cars should yield to cars already in the circle*.
- When the rate of incoming traffic increases beyond a certain threshold (which should be determined empirically), traffic lights should control entering traffic and the outermost lane of the traffic circle. These lights should be synchronized so that the time between successive lights turning green is the average time needed for a car to travel between them.

For a low rate of incoming traffic, the circle is relatively clear of cars, so entering cars can merge in without blocking the road or slowing the flow. By making entering cars yield to cars in the circle, we maximize the total throughput of cars while maintaining average speed.

When incoming traffic saturates the circle, allowing cars to merge freely into the circle impedes the flow of others. While throughput is still quite high, our simulation predicts that each car will spend an extremely long time in the circle.

Instead, we recommend that traffic lights attenuate the incoming flow of cars. While cars must wait slightly longer to enter, the number of cars in the circle is limited, allowing those cars a reasonable speed. Our simulator predicts that this policy will allow fewer cars to travel through the circle at a much higher speed.

By viewing the performance of the control system at the level of the individual cars, our simulator distinguishes between the performance of these two systems in this case and select the correct system to use.

We therefore recommend as follows: For times with high occupancies and rates of incoming traffic, implement synchronized traffic lights; for other times, require entering cars to yield to cars in the circle. Under this system, the total throughput is maximized while still maintaining an acceptable level of individual performance.

Introduction

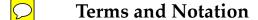


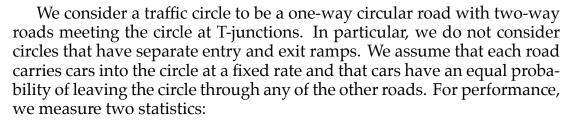
The traffic circle is a type of circular intersection featuring traffic from multiple streets circulating around a central island, usually in one direction. An example is shown in **Figure 1**. Other examples of large traffic circles include Columbus Circle in New York City, while small, one-lane traffic circles often exist in residential neighborhoods.

Traffic circles are often notorious for frequent traffic jams due to their unconventional design, and many methods exist to control traffic in a traffic circle; we investigate their impacts.



Figure 1. An aerial view of Dupont Circle in Washington, DC. Source: U.S. Geological Survey, at http://en.wikipedia.org/wiki/index.html?curid=1017545.





- the average rate at which cars arrive at their desired exit location per time step, the average throughput; and
- the average number of time steps from a car arriving at the back of the queue to enter the circle to when it exits the circle, the average total time.

Problem Background

Modern traffic circles have recently been recognized as safer alternatives to traditional intersections. Research by Zein et al. [1997] and Flannery and Datta [1996] using statistical methods has demonstrated that traffic circles bring added safety to both urban and rural environments. Attempts to understand the specific safety and efficiency benefits of traffic circles have taken four primary approaches: critical-gap estimation, regression studies, continuous models, and discrete models.

• Critical-gap models build from how drivers empirically gauge gaps in traffic before merging or turning into a traffic stream. However, according to Brilon et al. [1997], attempts in the 1980s to model roundabout



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capacity based on gap-acceptance theory were not exceptionally promising; in particular, critical-gap estimation lacked valid procedures as well as general clarity [Brilon et al. 1999]. More recent research applying gap-acceptance models to understanding traffic circles has included Polus et al. [1997] and the modeling of unconventional traffic circles by Bartin et al. [2006].

Nevertheless, regression studies on empirical data



• Regression studies on empirical data made much progress, beginning with Kimber [1980], who studied roundabouts in England and discovered a linear relation relating entry capacity to circulating flow and constants depending on entry width, lane width, the angle of entry, and the traffic circle size. Further regression studies have built extensively on Kimber's work, such as in Polus and Shmueli [1997], which determined the importance of traffic circle diameter in small-to-medium circles.



• Continuous models have included fluid-dynamic models [Helbing 1995; Bellomo et al. 2002; Daganzo 1995; Klar et al. 1996]; but those papers model traffic flow in standard traffic environments, not in traffic circles.



• Discrete models include cellular automata models [Fouladvand et al. 2004; Klar et al. 1996] and discrete stochastic models [Schreckenberg et al. 1995]. Discrete models are suitable for small environments such as traffic circles, where individual car-to-car interactions takes priority over traffic flow as a whole. Discretized approaches have attempted to model multilane traffic flows [Nagel et al. 1998]; but to our knowledge, there has been no research on discrete models of multilane traffic circles of varying sizes.



Our Results



We approach traffic-circle control by first creating a simulator of traffic flow that treats individual cars as autonomous units, allowing us to capture local interactions, such as lane changes and traffic blockages due to cars entering and exiting. We validate this simulator against both a new stylized model of the situation and existing models of traffic-circle flow.



Using the simulator, we implement and test various control systems on different types of traffic circles. Based on the simulated results, we isolate the *rate of incoming traffic* and the *number of lanes* in the traffic circle as the driving factors behind optimal choice of a traffic-control system. We thus recommend two different systems for different circumstances:



- When the rate of incoming traffic is low, we recommend that **entering** cars yield to cars already in the circle.
- When the rate of incoming traffic increases beyond a certain threshold, we recommend traffic lights that control entering traffic and the outermost

lane of the traffic circle. The lights should be synchronized so that the time between successive lights turning green is the average time needed for a car to travel between them.



In subsequent sections, we

- divide the problem into two portions and define our objectives for each;
- introduce the simulator and validate its performance against a mathematical analysis and models from other sources;
- use the simulator to analyze the performance of several types of traffic circles, to produce recommendations for which control systems should be used for each type; and
- provide an overview of the advantages and disadvantages of our approach and give directions for future work.

Simulator



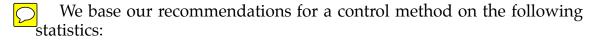
Our goal is a simulator that, given a set of conditions and traffic rules, can produce an accurate prediction of the behavior that will result from following these rules. To achieve this goal, we would like our simulator to fulfill the following requirements:



- The simulator takes into account the local interactions between cars. Because cars enter, exit, and change lanes quite frequently, interactions between cars make a major contribution to the speed and efficacy of a traffic circle.
- The simulator can support variation in the number of cars, size of the circle, and number of lanes.
- The simulator can track properties of both the entire traffic circle and the individual cars passing through it.

The real behavior of cars in a traffic circle may vary widely, but we restrict our simulated cars to idealized behavior: they follow the traffic regulations that we put in place, and no accidents happen.

Control System Evaluation



- 1. Average throughput (the average rate at which cars pass through the traffic circle).
- 2. Average number of cars in the traffic circle.

- 3. Average total time for each car to traverse the traffic circle, including time spent waiting to enter.
- 4. Average time that each car spends driving through the traffic circle.

Statistics 1 and 2 measure global properties of the traffic circle, while statistics 3 and 4 are properties of each individual car. To evaluate a control system, we consider both the global performance and the differences in performance of the system for each individual. In particular, our goals are to:

- 1. Maximize the average throughput of the traffic circle.
- 2. Minimize the total time spent traversing the traffic circle (for individual cars).

We evaluate the performance of a traffic circle by **the rate of cars passing through the circle** (*average throughput*) and **the total time required to traverse the circle** (*average total time*). We choose control methods that perform best with respect to both of these metrics.

Simulator Details

Assumptions	and	Setup
Table Village		F

- There are two approaches to model the behavior of traffic:
- 1. Make a (usually continuous) abstraction away from the discrete interactions of cars and deal with a more stylized model of the entire system.
- 2. Model the behavior and movement of each car separately.
- Continuous and fluid-like models, as in the first possibility, are suitable under a macroscopic view of traffic, for instance in the study of traffic on long roads or highways. However, for intersections and traffic circles, where car-to-car interactions occur much more frequently, such a model seems inadequate.
 - We follow the second approach to model traffic flow in a traffic circle using a multi-agent discrete time simulation. Our simulation is based around the following two key principles:
 - . It is microscopic.
- 2. Behavior and information are local.
- We do not use an abstract view of traffic as a flow but instead let each car in the traffic circle be its own individual agent. This allows us to account for the effects of car-to-car interaction, particularly in congested situations. From this interaction on the microscopic level, we then examine the macroscopic

consequences of the simulation, instead of beginning with an arbitrary conception of what the macroscopic behavior should be.

Each car is its own independent agent, trying to enter the traffic circle and exit at the desired exit as quickly as possible; no collaboration between cars or higher-level organizational principles exists. Also, only local information, namely the cars in the immediate neighborhood, is available to each individual car.

Γhe Simulator

- The simulation operates using the following model:
- Time is modeled in discrete time steps.
- The traffic circle is a rectangular grid. The width is the number of lanes in the traffic circle, and the height represents the length of the traffic circle. The upper edge wraps around to the lower edge (so that the grid is actually a circle). At any time step, each square of the grid can either be empty or hold one car.
- Certain squares in the outermost lane are *entry squares*. A queue of cars waits at each entry square to enter the circle. (These cars are not located on the grid itself.) The queues start off empty, and for each entry square, there is a fixed probability of a car being added to its queue at each time step.
- Certain squares in the outermost lane are *exit squares*, where cars can exit the circle. When a car is added to the queue for an entry square (and thus to the system), an exit square is chosen at random for the car.
- Each car has a speed indicated by how often it gets the chance to move. For example, faster cars may move at every time step, while slower cars may move less often. This difference simulates differing levels of impatience or aggression among drivers.
- n each time step, the simulation proceeds as follows:
- Determine the subset of all the cars in the system that will move during this time step. Randomly assign the order in which these cars will move.
- 2. Allow each such car to move. Cars move under the following rules: A car that is already in the traffic circle at position (i, j) (i.e., lane i, vertical position j) will, in the following order of preference,
 - (a) Exit if (i, j) is the exit square at which the car wishes to exit.
 - (b) Move forward to (i, j + 1) if (i, j + 1) is unoccupied.
 - (c) Move forward and right to (i + 1, j + 1) if there is a lane to the right and locations (i + 1, j) and (i + 1, j + 1) are both unoccupied.

- (d) Move forward and left to (i-1, j+1) if there is a lane to the left and locations (i-1, j) and (i-1, j+1) are both unoccupied.
- (e) Stay where it is.

An exception occurs for cars that are about to exit—if the vertical distance between the car's current location and its desired exit is less than four times the horizontal distance, then items (b) and (c) above are switched. (This is to ensure that under uncongested situations, cars will be able to exit at their desired exits.)

A car that is the first car in the queue at an entrance location will, in order of preference,

- (a) Move to the entry square if that square is unoccupied.
 - (b) Stay where it is.

All later cars in the queue cannot move for this turn.

In addition to the above rules, certain traffic control systems impose the following additional rules:

- a) Outer-yield: A car at the front of an entrance queue and waiting to enter can enter only when both the entry square and the square directly behind it are empty. That is, if there is a car directly behind the entry square, the entering car must yield to that car.
- (in the rightmost lane) but the queue waiting at that entrance is non-empty, then the car cannot move to that square. If the car has no other possible moves, then it does not move for that turn. This reflects the situation in which cars in the circle need to yield to entering cars.
- At any time step, the light is either green for cars in the circle and red for the waiting queue, or vice versa. If it is green for cars in the circle, then the first car in the waiting queue cannot enter the circle. If it is green for the waiting queue, then no car in the circle can move onto the entry square. In a multilane circle, this traffic light controls traffic only in the outermost lane. This behavior is inspired by the design of metering lights at highway ramps.

We consider two methods of synchronizing the traffic lights around the circle:

- i. All lights turn green and red simultaneously.
- ii. The difference in time between each traffic light turning green and the next light turning green (and also the difference between their reds) is directly proportional to the distance between the two lights. The proportionality constant is chosen so that a car waiting at a traffic light that begins to move when that light turns green will reach the next light just as it turns green.

- 3. For each entry queue, add a car to the end of that queue with the fixed probability for that entry location.
- 1. Have the traffic lights change if it is the correct time step to do so.

Validation Against Existing Empirical Models

The two criteria on which we evaluate the various traffic control systems, average throughput and average total time, are not unrelated. In fact, our simulations indicate that increasing one comes at the cost of increasing the other. For the outer-yield system, we show in **Figure 2** average total time against reserve throughput (maximum throughput minus average throughput).

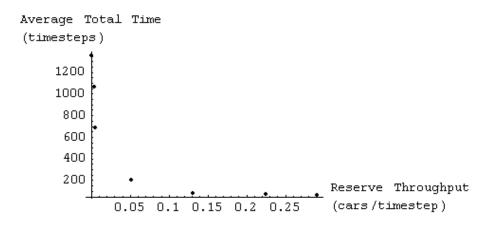


Figure 2. Average total time vs. reserve throughput.

This inverse relationship is intuitive, since greater throughput indicates a greater volume of traffic on the road and hence both slower driving speeds and longer wait times to enter the circle. This result also matches the relationship between average total time and reserve capacity given by the Kimber-Hollis delay equation in Brilon and Vandehey [1998]. This agreement indicates that the results of our simulator are reasonable.

Validation Against a Simple Model

To provide further verification of the accuracy of our simulator, we compare large-scale features of its output to a mathematical model for a simple case. In particular, we consider a single-lane traffic circle in which cars entering the circle yield to cars in the circle. For simplicity, we assume that all cars move at the same speed, one square per time step. We assume that roads at traffic circle are all two-way, so that each entry point is also an exit point. The model is given as follows.

Suppose that there are n entry/exit roads to the traffic circle, and that all cars have an equal probability of leaving through each of the n roads. For i=1 to n, let r_i be the probability that a new car appears at road i at any time step. Let x_i be the volume density of traffic in the segment of the circle between roads i and i+1. The expected change in the number of cars between roads i and i+1 is given by a sum of four terms:

- The probability that a car will leave this segment through exit i+1 is $\frac{1}{n} \cdot x_i$, since x_i is the probability that there is a car in the exit square and the probability that this car wishes to exit is $\frac{1}{n}$.
- The probability that a car will move from this segment to the next one is $x_i \cdot \frac{n-1}{n} \cdot (1-x_{i+1})$, since $\frac{n-1}{n}$ is the probability that the car in the exit square will not exit and $1-x_{i+1}$ is the probability that the square after the exit square, which is the first square of the next segment, is unoccupied.
- The probability that a car will move from the previous segment to this segment is, similarly, $x_{i-1} \cdot \frac{n-1}{n} \cdot (1-x_i)$.
- The probability that a car will enter through entrance i is the probability p of a sufficiently large space at entrance i for a car to enter, times the probability that there is a car there waiting to enter. This latter probability can be calculated as

$$r_i + r_i(1 - r_i)(1 - p) + r_i(1 - r_i)^2(1 - p)^2 + \dots = \frac{r_i}{r_i + p - r_i p},$$

- since there is an r_i probability of a car arriving at entrance i this time step, an $r_i(1-r_i)(1-p)$ probability of a car arriving at entrance i at the last time step (but not this time step) and remaining until this time step, etc. In our simulation, $p=(1-x_{i-1})(1-x_i)$, since a car can enter the circle if the entry square and the previous square are unoccupied.
- So the expected change in the number of cars in this segment in one time step is

$$\Delta c_i = -x_i \cdot \frac{1}{n} - x_i \cdot \frac{n-1}{n} \cdot (1 - x_{i+1}) + x_{i-1} \cdot \frac{n-1}{n} \cdot (1 - x_i) + \frac{r_i(1 - x_{i-1})(1 - x_i)}{r_i + (1 - x_{i-1})(1 - x_i) + r_i(1 - x_{i-1})(1 - x_i)}.$$

In equilibrium, this change should be 0 for all segments, giving a system of equations in the x_i . If we consider the case where the roads have equal incoming traffic, i.e. r_i is the same for all i, then by symmetry the x_i are the same for all i, and we may solve the equation

$$\Delta c = -x \cdot \frac{1}{n} - x \cdot \frac{n-1}{n} \cdot (1-x) + x \cdot \frac{n-1}{n} \cdot (1-x) + \frac{r(1-x)^2}{r + (1-x)^2 + r(1-x)^2} = 0$$

numerically for x in terms of r. Here, x is the traffic volume density for the circle as a function of r, the rate at which cars enter the circle through each

road. The result of numerically solving for x is shown in **Figure 3** together with the corresponding plot generated by our simulator. The black data points were generated by our simulator and the curve was produced by the rudimentary model.

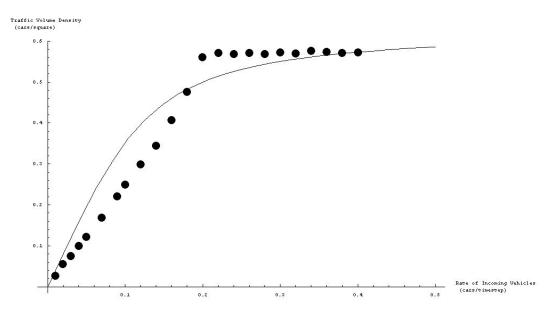


Figure 3. Traffic volume density vs. rate of incoming vehicles.

The volume density of both seems to grow in a somewhat linear fashion for low rates of incoming vehicles. When the number of incoming vehicles increases, the traffic circle appears to become saturated at a fixed density. The simulator and our mathematical model agree on these large-scale features. Disagreement about the critical rate of incoming vehicles might be explained by the fact that our mathematical model essentially considers the cars in each segment as equivalent, hence ignores the small-scale interactions that occur near gridlock.

Bredictions and Analysis

We apply the simulator to analyze different types of traffic circles.

Criteria

We characterize traffic circles by the following variables:

1. **Rate of incoming vehicles:** This is a result of the amount of traffic present on the roads entering the traffic circle and will influence the total number of vehicles trying to enter the circle and hence the traffic in the circle.

- 2. **Length:** This affects the number of cars that can be contained in the circle at a single time, which has many implications for how the entry mechanism of the circle should be determined.
- 3. **Number of lanes:** This affects both the number of cars that can be in the circle at a time and their maneuverability around each other. Because cars can more easily pass one another with more lanes, increasing the number of lanes may reduce the effects of traffic blockages.
- 4. **Number of incoming roads:** This affects the rate at which cars need to enter and exit the traffic circle, which may influence the magnitude of traffic blockages.

We wish to consider systems that are relatively close to conventional systems, since it would be impractical and hazardous to introduce radically different systems unfamiliar to drivers who do not encounter traffic circles frequently. Therefore, we will evaluate the performance of the following traffic control systems when we vary the parameters for our traffic circle:

- 1. **Outer yield:** Cars attempting to enter the circle yield to cars already in the circle at all times.
- 2. **Inner yield:** Cars already in the circle yield to cars attempting to enter the circle.
- 3. **Simultaneous lights:** The intersections between the circle and other roads are controlled by traffic lights that all turn green/red at the same time. The traffic lights apply only to the outermost lane of the traffic circle.
- 4. **Synchronized lights:** The intersections between the circle and other roads are controlled by traffic lights for which the time interval between a light turning green and the next light turning green is proportional to the distance between the two lights. The traffic lights apply only to the outermost lane of the traffic circle.

With the exception of the traffic lights, these control systems are all similar to existing control systems. However, there is a crucial difference between our traffic-light system and standard traffic lights: Stopping only the outer lane of the traffic circle allows traffic in the inner lane to proceed undisturbed, improving throughput. This approach is a hybrid of normal traffic lights and metering lights for congested highways.

Analysis

To analyze the effects of control systems, we run each on circles with varying parameters and create plots of average throughput and average total time per car for each strategy (**Figures 4–8**).

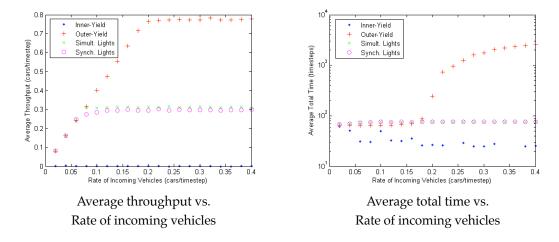


Figure 4. Performance for 1 lane, length 100, 4 roads, rate variable.

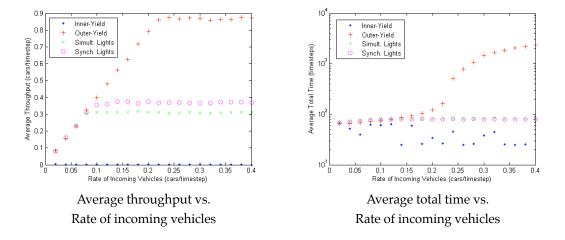


Figure 5. Performance for 3 lanes, length 100, 4 roads, rate variable.

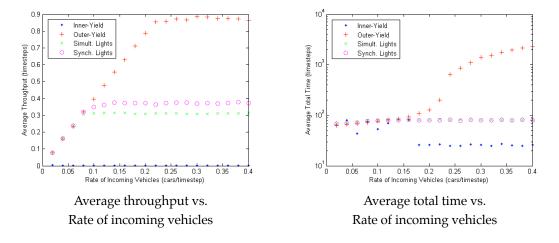


Figure 6. Performance for 5 lanes, length 100, 4 roads, rate variable.

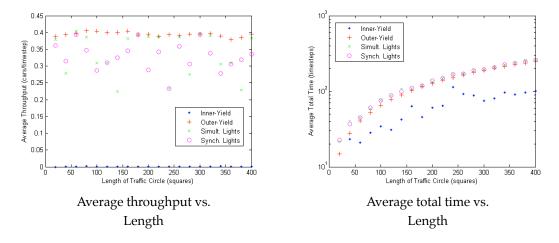


Figure 7. Performance for 3 lanes, rate 0.1, 4 roads, length variable.

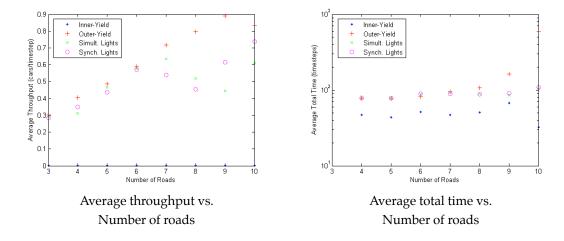


Figure 8. Performance for 3 lanes, rate 0.1, length 100, roads variable.

Our goal is to determine which parameters have the greatest effect on performance of the control systems. From the plots, we make the following observations:

- In almost all the plots, the inner-yield system has almost no throughput, since the cars in the road become gridlocked because they too often yield to incoming cars and therefore cannot exit. The low value of the average total time for this system results from the fact that the only cars that can exit do so before the road becomes entirely gridlocked. As a result, we reject the inner-yield system.
- As the rate is varied in **Figures 4–6**, the throughputs of the outer-yield system and the traffic-light systems correspond for small values of the rate. However, for each system, the throughput reaches a plateau beyond a certain value of the rate. At this point, the circle has been saturated, meaning that it can no longer accept more cars from the incoming roads.

- The throughput value at which saturation occurs is much higher for the outer-yield system. However, the amount of time required for each individual car to pass through the traffic circle under the outer-yield system is extremely high, almost an order of magnitude higher than needed under either traffic-light system.
- When there are either 3 or 5 lanes, the synchronized traffic-light system allows slightly greater throughput than the simultaneous traffic-light system. This might be explained by the fact that, with more lanes, cars can move in a more uniform manner, allowing them to use the synchronized lights and move through the circle more quickly.
- The number of lanes and the number of roads do not have a significant effect on either the throughput or the total time in the outer-yield system or in either of the traffic-light systems. However, traffic lights may perform worse for some values of the distance between roads, perhaps due to synchronization issues.

In general, the outer-yield and traffic-light methods have an advantage over the inner-yield method, and the correct choice of control system is largely determined by the rate of incoming vehicles on each of the entry roads.

Recommendations

Since the number of lanes and the number of incident roads do not significantly affect average throughput or average traversal time, we can restrict our attention to the rate of incoming cars and the number of lanes in the circle.

The rate of incoming cars accounts for a large part of the variation in performance, as can be seen in **Figures 4–6**.

- For values of this rate between 0 and 0.1 cars per time step, the performance of the outer-yield and traffic-light systems is identical, since in this range traffic is light and there is very little interaction between cars.
- As the rate increases to between 0.1 and 0.2 cars per time step, the traffic circle reaches its maximum throughput under the traffic-light system, while the average total time stays fixed. However, under the outer-yield system, the throughput continues to increase but at the cost of a rather dramatic increase in average total time. In this range, choosing between the outer-yield system and the traffic-light systems involves a tradeoff between throughput, the *quantity* of cars passing through, and total time, the *speed* at which cars pass through.
- Finally, as the the rate increases above 0.2 cars per time step , the circle becomes saturated with cars, meaning that the average total time for the outer-yield system increases dramatically and there is gridlock, meaning

that cars move extremely slowly and must wait a very long time to pass through. Under the traffic-light systems, however, a smaller number of cars can pass through, but the average total time required for them to pass remains similar to that with a much lower rate. Since the inner-yield system requires an extremely large total time in this range, the traffic-light systems are clearly superior for a rate of above 0.2 cars per time step.

In each of these cases, the synchronized traffic lights allow for higher throughput than simultaneous traffic lights.

We can now make the following recommendations:

- For a low rate of entering cars, no traffic lights should be used. Instead, cars already in the circle should be given the right of way, and cars entering the circle should yield.
- As the rate of entering cars increases, synchronized traffic lights should be considered for the outermost lane (only), to ensure a reasonable traversal time for most cars.
- For large rates of entering cars, as may occur during rush hour, synchronized traffic lights should be used, to ensure that the traffic circle does not become congested. By preserving a reasonable flow of cars within the circle, synchronized traffic lights allow a slightly smaller number of cars to pass through the circle much more quickly, which is preferable to deadlock for all cars.

For low and high rates, our recommendations agree with practice. An intersection with little traffic may have no traffic signals (alternatively, a traffic circle is installed explicitly in place of traffic lights). For highways, it is common to use metering lights during peak hours to regulate entry of vehicles, to ensure that cars already on the road can move at a reasonable speed. Our recommendations seem to be a mix of these two ideas applied to traffic circles.

Conclusions

Strengths

Our simulator takes into account the behavior and outcomes of individual cars traveling through a traffic circle. By doing so, we are able to detect interactions at a microscopic level and to track the performance of a traffic control system for each individual rather than only in aggregate. Doing this allows our model to evaluate the effects of cars changing lanes and entering and from specific lanes. We validated the simulator against both an existing empirical model and the results of a simple model for the steady-state limit.

We can simulate the performance of a widely varied spectrum of traffic control systems on a range of different traffic circles. Our results allow us to isolate the rate of incoming cars and the number of lanes in the traffic circle as the two parameters key to determining a good control system.

We recommend the either an outer-yield system or synchronized traffic lights to control the traffic circle, depending on the rate at which cars enter the circle.

Weaknesses

While our simulator attempts to model the behavior of drivers fairly accurately, it cannot completely capture the dynamics of lane-changing and braking. Further, while using a discrete-time, discrete-space model for the simulator allows us to capture the local multiagent nature of individual drivers, it forces us to make simplifications about the continuity of car movement and about simultaneous actions.

In addition, our simulation does not take into account the fact that in an actual traffic circle, the inner lanes have shorter length than the outer lanes.

We consider only traffic lights with simultaneous or synchronized light changes, and it is infeasible computationally for us to consider a wider variety of switching approaches.

Alternative Approaches and Future Work

We could evaluate the safety of a control system by counting the number of conflicting desired movements at the local level. We could then compare systems by safety as well as by performance and hence evaluate the claim that certain types of traffic circles are safer than intersections [Flannery and Datta 1996].

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