Team Control Number

201906014

Problem Chosen

A

2019

ShuWei Cup

Summary

In this paper, the principal component analysis method was selected to comprehensively evaluate the status quo of common diseases of people, and the following five problems were reasonably solved by combining relevant knowledge such as queuing theory.

In response to question one, considering the limitations of short-term model in predicting population aging and the uncertainties of many factors, we found out the proportion of aging population in China in recent years from the National Bureau of Statistics and established a grey dynamic model for predicting aging.At the same time, through the multiple linear regression equation, combined with the curve fitting method to derive the medical demand prediction model formula, the hospital and the government medical policy formulation has certain significance.

In response to question two, based on the comprehensive consideration of the five-year prevalence of five common diseases, this paper replaced the five principal components in the main componen t-ts comprehensive evaluation model, and obtained the ranking and comprehensive evaluation resu -lts of the five diseases in liaoning province from 2012 to 2016 (see table 1.1).Adjust the future m-edical demand distribution and focus according to the ranking in the table.

In response to question three, this paper establishes the queuing model of multi-service desk in hos -pital.According to queuing theory, hospital queuing system can be roughly divided into multi-que -ue model with infinite source and single-queue model with infinite source. Through analysis and comparison, it can be concluded that single-queue queuing service model is more efficient than multi-queue model and patients' waiting time is shorter

In response to question four, this paper establishes the hospital competition model.According to the inherent growth rate of the hospital, the number of two kinds of hospitals in the future is predic -ted.By using the analytic hierarchy process, this paper analyzes two kinds of competitive market situations and finds out the competitive strategies of future hospitals.

In response to question five,the prediction results of the model are combined.I wrote a proposal for the "14thFive-Year plan" to the relevant medical departments.

Keyword: *Grey prediction;Principal component analysis;Queuing theory;The population competition*

Content

1. Introduction

1.1 Background

With the rapid development of China's economy and society, the aging trend and the acceleration of industrialization and urbanization, residents' demand for hospital health services continues to increase, and medical and health work has met a series of new challenges. With the increasing number of private hospitals, private hospitals and public hospitals have formed different levels of competition and cooperation. How to coordinate the development of private hospitals and public hospitals, jointly meet the growing general medical needs of residents, and comprehensively improve the quality of medical services has become the main direction of work.

1.2 Work

Task 1 : According to the per capita income of residents in recent years, the age structure population and the level of economic development can reasonably predict the aging trend and medical needs of China in the next few years.

Task 2 : Taking Liaoning province as an example, this paper analyzes the most common diseases in Liaoning province in the future and provides Suggestions for the overall development of the province.

Task 3 : Different types of patients need different tests in the hospital, different test items in different locations, and the number of people waiting in line also varies greatly. Please solve the queuing problem according to the queuing theory and get the relevant optimal queuing method.

Task 4 : Considering the complex cooperation and competition between private hospitals and public hospitals, please propose the best cooperation and comp etition strategies among several hospitals.

Task 5 : Write 1-2 pages of Suggestions on medical management and medical services for relevant medical institutions to provide reference for the preparation of the "145th five-year plan".

2. Problem analysis

2.1Analysis of question one

Question one allows us to make reasonable predictions about aging trends and medical needs in terms of demographics, per capita income, etc. This is an analytical prediction model. We used grey prediction model $GM(1,1)$ to predict the aging trend, and then screened out reasonable influence indicators, namely different parameters, through multiple linear regression, and obtained the multiple regression equation for reasonable prediction and analysis, and drew the conclusion..

2.2 Analysis of question two

Taking Liaoning province as an example, through collecting the prevalence of major diseases in Liaoning in recent years, we adopted the principal component analysis method to put forward Suggestions on the future development direction of hospitals in Liaoning province, which is the decision-making evaluation model. Principal component analysis was used to get the ranking of typical diseases and to make a reasonable prediction of the future focus of the hospital.

2.3 Analysis of question three

Hospital has a lot of diagnosis program, each check project corresponding to multiple doctors, so we call the doctor treat service, called every doctor service desk, so that patients to leave the process of input, services, the output, because the hospital was full of many service desk, so can each service desk and row of multiple service desk team.

2.4 Analysis of question four

Question 4 requires us to put forward the optimal cooperation and competition strategies among several hospitals by combining the cooperation and competition between private hospitals and private hospitals. The relationship between public and private hospitals is guided by the growth of medical demand and the cost of public hospitals. Next, we will collect the number growth of public hospitals and private hospitals in recent years and use the method of nested population competition model to predict the future change trend of the two competitive factors of public hospitals and private hospitals in this environment in China. The competition between public hospitals and private hospitals is analyzed by means of hierarchical analysis, so as to realize the final diversification of running hospitals.

3. Symbol and Assumptions

3.1 Symbol Description

Symbol	Instructions
λ	Average arrival rate
μ	Average service rate
S	Service stations
P_0	Probability that the service desk is idle
L_q	Average queue length
$\sigma(k)$	scale comparison.
$\rho(k)$	smooth ratio
μ_j	The sample mean of the j th indicator
s_j	The sample standard deviation of the j th indicator
ε_i	The I principal component
α_p	The cumulative contribution rate of principal component $\varepsilon_1, \varepsilon_2, \ldots \varepsilon_p$.
b_j	The information contribution rate of principal component ε_i
$x(t)$	respectively the numberof public hospitals

r_1, r_2	inherent growth rates.
n_1, n_2	maximum capacity
s_1	public hospitals
s_2	medical resources of private hospital s

3.2 Fundamental assumptions

①Assuming there are no major disasters and policy changes in liaoning province.
②The hypothesis is that interactions between diseases can be eliminated.
③Assuming that patients arrive randomly and independently of each other.
④The queuing system has no capacity limits and does not consider priority comparisons.
⑤Assuming that the patient's waiting time is infinite, each service desk does not affect each other.
⑥Suppose that the medical resources of one public hospital can supply about two private hospitals.

4. Model

4.1Model establishment and solution of problem 1

Prediction model of population aging trend
Model building

Through the aging rate data in recent years, we use grey prediction to make a reasonable prediction of the aging rate in the future.

Tip 4.1.1 aging rate from 2011 to 2018

year	Aging rate /%
2011	8.26
2012	8.45
2013	8.68
2014	8.97
2015	9.33
2016	9.8

2017	10.35
2018	10.92

Grey series prediction model is one of the grey prediction methods, which mainly refers to using the $GM(1,1)$ model to predict the quantity of time series data. The $GM(1,1)$ model has only one exponential component, so the change is monotonous. The $GM(1,1)$ model is established as follows:

Step 1: we constructed the original data sequence according to different aging rates in different years: $x^{(0)} = \left\{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\right\}$

Step 2: make a cumulative generation of the original data：

$$x^{(1)} = \left\{x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)\right\}$$

Among them， $x^{(1)}(k) = \sum_{j=1}^{1} x^{(0)}(j)$, $k = 1, 2, \cdots, n$

Step 3: smoothness test and quasi - exponential test：

Smooth than $\rho(k) = \dfrac{x^{(0)}(k)}{x^{(1)}(k-1)}$, $k = 3, 4, \cdots, n$

Scale comparison. . $\sigma(k) = \dfrac{x^{(0)}(k)}{x^{(0)}(k-1)}$, $k = 3, 4, \cdots, n$

If the sequence $x^{(1)}(k)$ satisfies $\dfrac{\rho(k+1)}{\rho(k)} < 1$ and $\rho(k) \in [0, \varepsilon], \varepsilon < 0.5$ the sequence $x^{(1)}(k)$ is called smooth sequence, if scale comparison satisfies $\sigma(k) \in (1, 1.5)$, it is said to have a quasi-exponential law.

Step 4: construct the sequence adjacent to the mean value:

$$Z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1), k = 1, 2, \cdots, n;$$

Step 5: establish the grey differential equation: $x^{(0)}(k) + aZ^{(1)}(k) = b$, $k = 2, 3, \cdots, n$

Set the constant term vector y_n and the summation matrix B

$$B = \begin{pmatrix} -\frac{1}{2}\left[x^{(1)}(1) + x^{(1)}(2)\right] & 1 \\ -\frac{1}{2}\left[x^{(1)}(2) + x^{(1)}(3)\right] & 1 \\ \cdots & \cdots \\ -\frac{1}{2}\left[x^{(1)}(n-1) + x^{(1)}(n)\right] & 1 \end{pmatrix}$$

$$y_n = \left\{x^{(0)}(2), x^{(0)}(3), \cdots x^{(0)}(n)\right\}^T$$

The least squares estimation is performed for the parameters a and b: $a = \begin{bmatrix} a \\ b \end{bmatrix} = \left(B^T B\right)^{-1} B^T y_n$

To sum up, we built the model $x^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right) e^{-ak} + \frac{b}{a}$

Solution of the model

Based on the analysis and establishment of the above model, we used MATLAB to solve, and obtained the calculation result of the model, namely, the aging rate in the next five years, as shown in Tip 2:

Tip. 4.1.2 predicted aging rate from 2019 to 2023

year	Aging rate /%
2019	11.2814
2020	11.7889
2021	12.3192
2022	12.8734
2023	13.4525

The rationality analysis of the model

We compared the data obtained by the model with the real aging rate from 2010 to 2018, which proved that the model had high accuracy and could be used for aging prediction. Please see Fig 1 for the curve drawing. Please refer to the attachment for the operation program.

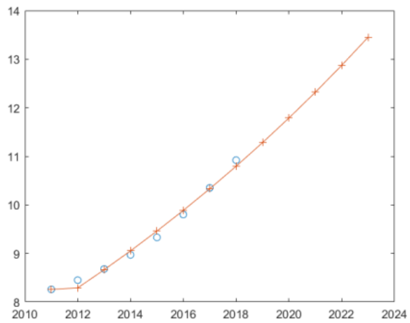

Fig. 4.1.1 function curve and actual value analysis

Prediction model of medical needs

Model building

First of all, we analyzed the factors that may affect medical needs. According to the analysis results, we collected the data of aging proportion, per capita income of residents and average GDP from 2013 to 2018, as shown in Tip 3:

Tip. 4.1.3 Recent medical demand may influence factor data

years	Proportion of elderly population	Per capita income（Ten thousand yuan）	GDP（Ten thousand yuan）
2018	0.11937	2.8228	6.4644
2017	0.11385	2.5974	5.9201
2016	0.10854	2.3821	5.368
2015	0.10465	2.1966	5.0028

2014	0.10056	2.0167	4.7005
2013	0.09672	1.8311	4.3684

According to the data in the above table, we used SPSS to curve fit the relationships among the three groups: year-aging ratio, year-resident per capita income, year-GDP. Through the fitting curve result and equation obtained, the index value of influencing factors in 2019-2025 is predicted. The prediction equation and fitting curve are as follows:

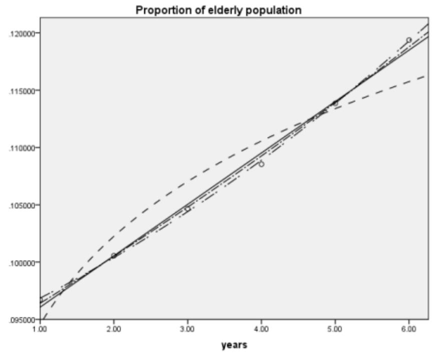

Fig 4.1.2 year -- aging ratio

The corresponding equation is: $Y_1 = 0.0002x_1^2 + 0.003x_1 + 0.094$

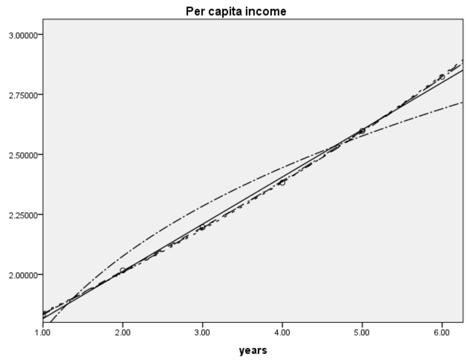

Fig. 4.1.3 year -- per capita income

The corresponding equation is: $Y_2 = 0.006x_2^2 + 0.154x_2 + 1.676$

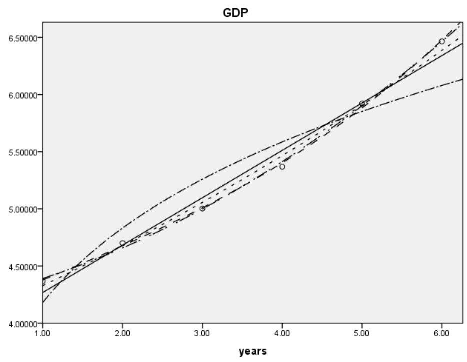

Fig.4.1.4 year-GDP

The corresponding equation is: $Y_3 = 0.037x_3^2 + 0.157x_3 + 4.197$

According to the equation we established and the graph trend, we predicted the trend of these three influencing factors in the next seven years. The results are shown in Tip 4:

Tip 4.1.4 Trends in influencing factors of medical demand over the next seven years

year	GDP	Per capita income	Aging ratio
7（2019）	7.109	3.048	0.1248
8（2020）	7.821	3.292	0.1308
9（2021）	8.607	3.548	0.1372
10（2022）	9.467	3.816	0.144
11（2023）	10.401	4.096	0.1512
12（2024）	11.409	4.388	0.1588
13（2025）	12.491	4.692	0.1668

We established a multiple linear regression model based on the predicted possible index value, and further predicted the medical demand in 2019-2025. In general, the formula of multiple linear regression analysis was as follows: $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \cdots + \beta_n x_n$

When SPSS is used to analyze data, the system will automatically complete parameter estimation and calculate the final analysis value according to the given variables. Where, the formula of the tested statistic is shown as $F = \frac{R^2 / p}{\left(1-R^2\right)/\left(n-p-1\right)}$, It can be seen that the higher the goodness of fit of the regression equation is, the more significant the significance test of the equation will be.

Solution of the model

Multiple linear regression analysis was conducted by SPSS, and it was concluded that changes in the ratio of elderly population and years had no substantial impact on medical demand. Therefore, it was omitted, and the multiple relationship between medical demand level and residents' per capita income and GDP was obtained as follows: $Z = 16.665\text{-}16.551 y_1 + 4.936 y_2$

The operation results of the above model are shown in Tip 5:

Tip 4.1.5 Running result of equation

y_1	3.048	3.292	3.548	3816	4.096	4.388	4.692
y_2	7.109	7.821	8.607	9.467	10.401	11.409	12.491
Z	1.307576	0.783564	0.426204	0.235496	0.21144	0.354036	0.663284

Among them, Z stands for medical fund requirements. It can be found that the trend of Z is not straight up or down, but gradually up and down with time migration. It shows that the need for health care funding may also be influenced by the political environment. Moreover, the significance values of the three coefficients obtained by SPSS were 0.121, 0.149 and 0.156, respectively, which were all significantly higher than 0.05, which could prove the existence of improper sampling or other factors.

The rationality test of the model

The standard p-p diagram of regression standardized residual was drawn as shown in figure 5. It can be found from the graph that the graph is a 45 °straight line from the bottom left to the top right, indicating that the sample distribution basically conforms to the standard of normal distribution, which further proves the accuracy of prediction.

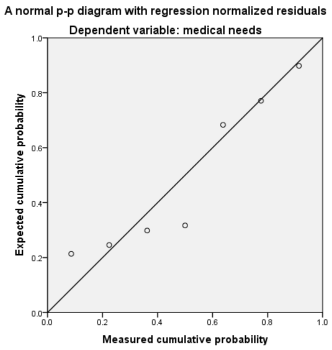

Fig 4.1.5 P-P

4.2 Model establishment and solution of problem 2

Model building

Taking Liaoning province as an example, we use the principal component analysis method to comprehensively evaluate the overall development of the whole province through the data obtained from the health, family planning and population health report. We used $\eta_1, \eta_2, \cdots, \eta_5$ to represent eye diseases, hypertension, diabetes, dyslipidemia and malignant tumors. Use $i = 1, 2, \cdots, 5$ to represent the year 2012, 2013, 2014, 2015 and 2016 respectively. The values of $\eta_1, \eta_2, \cdots, \eta_5$ in year i are denoted as $[a_{i1}, a_{i2}, \cdots, a_{i5}]$. Structure matrix $A = (a_{ij})_{5\times5}$.

Tip 4.2.1 The incidence of five diseases in liaoning in recent years

year	eye diseases	hypertension	diabetes	dyslipidemia	malignant tumors
2016	59.03%	37.91%	11.25%	38.98%	0.37%
2015	58.48%	25.20%	8.29%	38.11%	0.38%
2014	56.38%	25.85%	12.73%	37.15%	0.35%
2013	54.77%	27.90%	11.89%	45.32%	0.33%
2012	55.10%	25.20%	7.90%	51.60%	0.26%

The evaluation steps based on principal component analysis are as follows:

1) Standardized processing of original data. Convert each index value a_{ij} into standardized index $\tilde{a}_{ij}$

$$\tilde{a}_{ij}=\frac{a_{ij}-\mu_j}{s_j},i=1,2,3,4,5,\ j=1,2,3,4,5,$$

,

In the formula, $\mu_j=\frac{1}{5}\sum_{i=1}^{5}a_{ij};s_j=\sqrt{\frac{1}{5-1}\sum_{i=1}^{5}(a_{ij}-\mu_j)^2},\ j=1,2,3,4,5,$

That is, μ_j,s_j is the sample mean and sample standard deviation of the j index.

Correspondingly, $\tilde{\eta}_j=\frac{\eta_i-\mu_j}{s_j},\ j=1,2,3,4,5$ is called standardized indicator variable.

2) Calculate correlation coefficient matrix. The correlation coefficient matrix is $R=(r_{ij})_{5\times5}$

$$r_{ij}=\frac{\sum_{k=1}^{5}\tilde{a}_{ki}\cdot\tilde{a}_{kj}}{5-1},i,\ j=1,2,3,4,5,$$

In the formula, $r_{ii}=1;r_{ij}=r_{ji};r_{ij}$ is the correlation coefficient between index i and index j.

3) Calculate eigenvalues and eigenvectors. Calculate the eigenvalue $\lambda_1\geq\lambda_2\geq\cdots\geq\lambda_5\geq0$ of the correlation coefficient matrix. And the corresponding normalized eigenvector a $u_1,u_2,\cdots u_5$, Five new index variables are composed of eigenvectors:

$$\begin{gathered}\varepsilon_1=u_{11}\eta_1+u_{21}\eta_2+\cdots+u_{51}\eta_5,\\ \varepsilon_2=u_{12}\eta_1+u_{22}\eta_2+\cdots+u_{52}\eta_5,\\ \vdots\\ \varepsilon_5=u_{15}\eta_1+u_{25}\eta_2+\cdots+u_{55}\eta_5,\end{gathered}$$

In the formula: ε_1 is the first principal component; ε_2 is the second principal component; … ; ε_5 is the fifth principal component.

4) Select $p(p\leq5)$ principal components and calculate the comprehensive evaluation value.

a) The information contribution rate and cumulative contribution rate of eigenvalue $\lambda_j(j=1,2,...5)$ were calculated. Call the information contribution rate of component $b_j = \frac{\lambda_j}{\sum_{k=1}^{5}\lambda_k}, j = 1,2,...,5$ and called the cumulative contribution rate of

component ε_1 , ε_2 ,... ε_p of $\alpha_p = \frac{\sum_{k=1}^{p}\lambda_k}{\sum_{k=1}^{5}\lambda_k}$. When α_p is close to

1（$\alpha_p = 085, 0.90, 0.95$）, the first b index variables are selected as the

p($\varepsilon_1, \varepsilon_2, ... \varepsilon_p$)principal components instead of the original 5 index variables, so

that the p principal components can be comprehensively analyzed.

b) Calculated composite score. In the formula: b_j is the information contribution rate of the j principal component, which can be evaluated according to the comprehensive score.

Solution of the model

By using the *Matlab*, the characteristic roots and their contribution rates of the correlation coefficient matrix can be obtained, and the cumulative contribution rates of the first three characteristic roots can reach more than 93%. The first three principal components are selected for comprehensive evaluation. The eigenvectors corresponding to the first three eigenroots are shown in the Tip 2.

Tip 4.2.2 The eigenvector corresponding to the eigenroot

	H 1	H 2	H 3	H 4	H 5
The first eigenvector	0.4840	-0.4990	0.1230	-0.4010	0.5837
The second eigenvector	0.3507	0.0769	0.8647	0.1051	-0.3351
The third eigenvector	0.2573	0.8599	-0.0024	-0.1659	0.4084

The three principal components can be obtained as follows

$$\varepsilon_1 = 0.4840\tilde{\mu}_1 - 0.4990\tilde{\mu}_2 + 0.1230\tilde{\mu}_3 - 0.4010\tilde{\mu}_4 + 0.5837\tilde{\mu}_5$$

$$\varepsilon_2 = 0.3507\tilde{\mu}_1 + 0.0769\tilde{\mu}_2 + 0.8647\tilde{\mu}_3 + 0.1051\tilde{\mu}_4 - 0.3351\tilde{\mu}_5$$

$$\varepsilon_3 = 0.2573\tilde{\mu}_1 + 0.8599\tilde{\mu}_2 - 0.0024\tilde{\mu}_3 - 0.1659\tilde{\mu}_4 + 0.4084\tilde{\mu}_5$$

Based on the three principal component contribution rates, a principal component comprehensive evaluation model was constructed.

$$Z = 0.6038\varepsilon_1 + 0.2159\varepsilon_2 + 0.1662\varepsilon_3$$

By substituting the three principal component values of various diseases into the above equation, the ranking and comprehensive evaluation results of common diseases in Liaoning province can be obtained, as shown in the Tip 3.

Tip 4.2.3 Evaluation of five diseases

disease	eye diseases	hypertension	diabetes	dyslipidemia	malignant tumors
ranking	1	3	2	4	5
evaluation	1.3447	0.4325	0.0467	-0.1924	-1.6314

4.3 Model establishment and solution of problem 3

Model building

We regard doctor diagnosis as a service and each doctor as a service desk, and the service process is shown in Fig. 1:

Fig.4.3.1 Queuing theory model

Assuming that customers arrive individually, the interval of successive arrivals obeys an exponential distribution with parameter λ.There are a total of s service desks in the system. The service time of each service desk is independent of each other and obeys the negative exponential distribution with the parameter of μ. When the patients arrive, if there is a vacant service desk, they will immediately accept the service. Otherwise, they will queue up and wait. Considering the particularity of the hospital, the waiting time is set as infinite without considering the loss. Therefore, there are two ways of queuing. One is that patients arrive in a row, and then seek the free service desk to see a doctor. That is to say, single queue and multiple service desk model $M/M/S/\infty$The other is that patients can queue at any window, which is multi-queue multi-service desk model, so s $M/M/1/\infty$ models can be obtained. The following is our analysis and comparison of the two models. The following is our analysis and comparison of the two models.

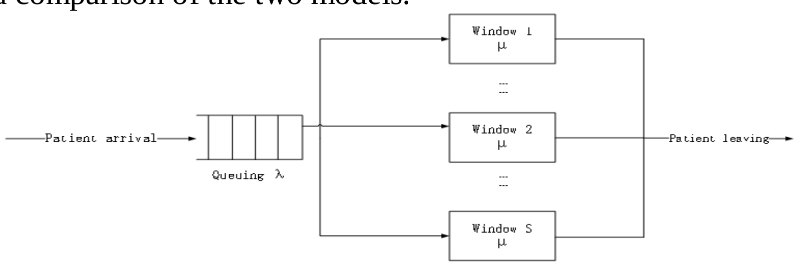

Fig.4.3.2 $M/M/S/\infty$

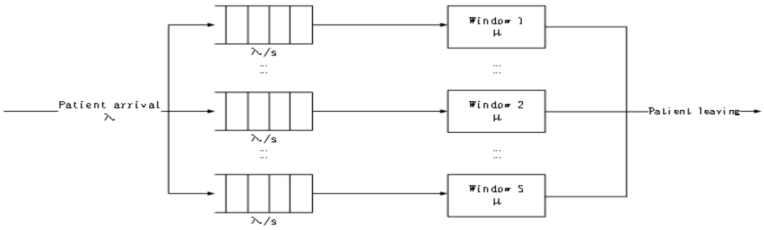

Fig.4.3.3 $M/M/1/\infty$

1) A reference to the process of birth and death

In the queuing theory, N(t) is used to represent the number of customers in the system at time t, "sheng" is used to represent the arrival of patients, and "extinction" is used to represent the departure of patients. For the patient queuing process, $\{N(t), t \geq 0\}$ is a special random process-birth and death process. According to the principle of "inflow = outflow" of the system under statistical equilibrium, the equilibrium equation under any state can be obtained as shown in the Tip.1:

Tip.4.3.4 Balance equation

n	inflow	outflow
0	$\mu_1 p_1$	$\lambda_0 p_0$
1	$\lambda_0 p_0 + \mu_2 p_2$	$(\lambda_1 + \mu_1)\ p_1$
2	$\lambda_1 p_1 + \mu_3 p_3$	$(\lambda_2 + \mu_2)\ p_2$
……	……	……
n	$\lambda_{n-1} p_{n-1} + \mu_{n+1} p_{n+1}$	$(\lambda_n + \mu_n)\ p_n$
……	……	……

From the above formula can be derived $p_n = C_n p_0\ n=1,2,\cdots\cdots$

Among them $C_n = \frac{\lambda_{n-1}\lambda_{n-2}\cdots\lambda_0}{\mu_n\mu_{n-1}\cdots\mu_1} p_0\ n=1,2,\cdots\cdots$

Because of the probability distribution $\sum_{n=0}^{\infty} p_n = 1\ p_0 + \sum_{n=1}^{\infty} C_n p_0 = 1$ .Get $p_0 = \frac{1}{1+\sum_{n=1}^{\infty} C_n}$.

2) The model $M/M/S/\infty$.

Under the premise of stationary system, $p_n = \{N = n\}\ (n = 0,1,2\cdots\cdots)$ is the probability distribution of captain N after the system reaches an equilibrium state.

$$\lambda_n = \lambda,\ \mu_n = \begin{cases} n\mu, n = 1,2,\cdots,s, \\ s\mu, n = s, s+1,\cdots。 \end{cases} n=1,2,\cdots\cdots$$

Where $\rho_s < 1$, the index of queuing system can be deduced from the process of birth and death.

$$C_n = \begin{cases} \frac{(\lambda/\mu)^n}{n!}, n = 1,2,\cdots,s, \\ \frac{(\lambda/\mu)^s}{s!}\left(\frac{\lambda}{s\mu}\right)^{n-s} = \frac{(\lambda/\mu)^n}{s!s^{n-s}}, n \geq s. \end{cases}$$

The probability that the service desk is idle is $p_0 = \left[\sum_{n=0}^{s-1} \frac{\rho^n}{n!} + \frac{\rho^s}{s!(1-\rho_s} \right]^{-1}$。

The probability that the patient has to wait is $c(s,\rho) = \sum_{n=s}^{\infty} p_n = \frac{\rho}{s!(1-\rho_s)} p_0$

Average queue length (person) is $L_q = \sum_{n=s+1}^{\infty} (n-s) p_n = \frac{c(s,\rho)\rho_s}{1-\rho_s}$

The average number of busy service desks is $\overline{s} = \rho \sum_{n=0}^{s-1} np_n + s \sum_{n=s}^{\infty} p_n = \rho$

The average waiting time is $W_q = \frac{L_q}{\lambda}$

3) The model $M/M/1/\infty$

The original $M/M/S/\infty$ is a queuing system composed of s subsystems. Each of them conforms to the distribution rule of $M/M/1/\infty$, and the average arrival rate of each system is $\lambda_1 = \lambda_2 = \cdots \lambda_s = \frac{\lambda}{s}$.

From the index of $M/M/1/\infty$, we can deduce the index of s subsystemThe probability that each subsystem is idle $p_0' = 1 - \frac{\rho}{s} = 1 - \rho_s$.

Average queue length (person) is $L_q' = \frac{\lambda^2}{\mu(\mu - X)} = \frac{(\frac{\lambda}{s})^2}{\mu(\mu - \frac{\lambda}{s})} = \frac{\rho_s}{(\frac{1}{\rho_s} - 1)}$.

The average waiting time is $W_q = \frac{Lq'}{\frac{\lambda}{s}} = \frac{sLq'}{\lambda}$

4) Comparison of the two models

Generally speaking, the shorter the average waiting time of the queue, the more reasonable the queuing mode of the system will be. Therefore, let W_{q1} and W_{q2} be the average waiting time of single-queue multi-service desk and multi-queue multi-service desk respectively. $W_{q1} = \frac{c(s,\rho)\rho_s}{(1-p_s)\lambda}$ $W_{q2} = \frac{\rho_s^2 s}{(1-\rho_s)\lambda}$

Using the comparative method $\frac{W_{q1}}{W_{q2}} = \frac{c(s,\rho)}{p_s s} = \frac{c(s,\rho)}{\rho}$,

$c(s,\rho)$ is the probability that the patient has to wait so $c(s,\rho) < 1$,

ρ is the average of busy information desks .So $\rho > 1$, $\frac{1}{\rho} < 1$

So the $\frac{W_{q1}}{W_{q2}} < 1$ that you get is the same thing as the $W_{q1} < W_{q2}$

Solution of the model

To sum up, the first model has less queuing time. Relevant medical institutions can combine multiple queues into one queue, that is, multiple input customer sources are mixed into one input customer source, so as to avoid the situation that one service desk is idle and the queue of another service desk is prolonged due to queuing, so that the system can make more efficient use of the service capacity of the system. It Can effectively balance the load capacity of the service desk, so that you can get a shorter queue waiting time.

4.4 Model establishment and solution of problem 4

Model building

The establishment of lotka-volterra population competition model

For public hospitals and private hospitals, when they operate alone, the number changes obey the law.

$$\frac{dx}{dt}=r_1x(1-\frac{x}{N_1}) \quad , \quad \frac{dy}{dt}=r_1y(1-\frac{y}{N_1})$$

When both types of hospitals are operating at the same time, the hindrance of a to b's growth is proportional to the number of a, and b has the same effect on a.

$$\frac{dx}{dt}=r_1x(1-\frac{x}{n_1}-s_1\frac{y}{n_2}) \quad , \quad \frac{dy}{dt}=r_2y(1-\frac{y}{n_2}-s_2\frac{x}{n_1})$$

In the formula, $x(t)$ and $y(t)$ are respectively the number of public hospitals and private hospitals. r_1, r_2 are their inherent growth rates. n_1, n_2 is their maximum capacity. The meaning of s_1 is that for the medical resources of public hospitals, the number of medical patients of private hospitals (relative to n_2) is s_1 times of that of public hospitals (relative to n_1), and the same is true of s_2.

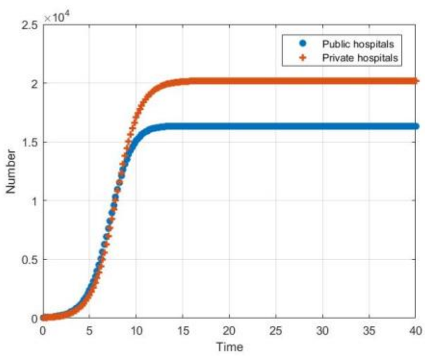

Fig.4.4.1 Chart of growth in hospital competition

According to the analysis in Fig.1 with the increase of time, public hospitals and private hospitals have an obvious linear change trend, but when they reach a certain

number, they all have a fast linear slowing down trend, but the slowing down of public hospitals occurs earlier. Finally, the number of private hospitals exceeds the number of public hospitals, and the two keep the number stable and coexist with each other. The result is that the increase rate of the number of public hospitals is higher than that of private hospitals in the early stage, which inhibits the increase of private hospitals. Later, the need to cooperate with private hospitals led to an increase in the number of private hospitals, which in turn inhibited the increase of public hospitals.
Hierarchical analysis is used as shown in Fig.2 to analyze the competition between public and private hospitals and seek to realize diversified medical services.

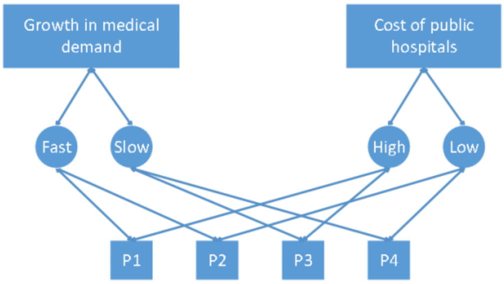

Fig.4.4.2 Analytic hierarchy

There are four competitive scenarios:

p_1 occupies part of the market and shares the peak medical demand

p_2 dominates the market for specific diseases

p_3 share the pressure of high demand and low price

p_4 public hospitals prevent private hospitals from entering the market

Solution of the model

(1) when the demand for medical care is growing fast, the cost of public hospitals is high. At this time, private hospitals share the peak demand for medical care and relieve the medical strain.
(2) when the demand for medical care is growing fast, the cost of public hospitals is low. At this time, private hospitals can play an important role in the field of special diseases, especially in the field where public hospitals have a small market share.
(3) when the growth of medical demand is slow, the cost of public hospitals is high. At this time, private hospitals can share people's demand for low-cost drugs.
(4)When the growth of medical demand is slow, the cost of public hospitals is high. Private hospitals are in a weak position, and public hospitals actively organize private hospitals to enter the market.

Therefore, the optimal cooperation and competition strategy: on the one hand, public hospitals continue to develop basic medical services and strive to reduce the cost of public hospitals. Private hospitals actively cooperate with public hospitals to fill beds in public hospitals. Achieve patient circulation and sharing. At the same time, private hospitals can seize the market in the field of special diseases.

4.5 Model establishment and solution of problem 5

At present, China's medical management and security system structure has been completed. During the period of the 14th five-year plan, the construction of the system has entered the stage of "standardization, refinement and intellectualization". It is of great significance for medical institutions to solve the phenomenon of crowded medical queue, strengthen the reform of medical institutions, and form a mechanism for medical institutions to raise quality and control fees.

4.5.1 Full coverage of medical insurance for lawful residents

According to the analysis of multiple linear regression model, during the period of 145th five-year plan, it is estimated that the per capita medical fund demand is about 6000~7000 yuan. Relevant departments should promote and guide residents to participate in insurance, basically realize full coverage, and record, supplement and perfect the medical insurance of all kinds of personnel. We will make medical insurance benefits accessible to all, and ensure that key groups such as micro, small and medium-sized enterprises, people with flexible employment, and migrant workers participate in the insurance program.

4.5.2The establishment of intelligent medical security system

Registration difficulty and long queue time have become a common problem for large medical institutions to build an intelligent medical security system
Providing more instruments and equipment for patients according to the actual situation can make medical security standardized, information alized and intelligent construction and application, and provide better nursing services for medical staff. According to our analysis of the model, the relevant medical institutions can combine multiple queue is a queue, the multiple input source of customer confusion as an input source of customers, to avoid a free service desk and another service desk for queue to extend, can be more efficient use of the system's ability to service. Public hospitals cooperate with private hospitals to make up for their shortcomings.

4.5.3 Rational allocation of limited resources

Different areas have different common diseases and different subjects have different medical personnel. Optimizing configuration and providing more convenient services for medical staff are the most competitive means for medical institutions. Eye diseases, hypertension, diabetes, dyslipidemia and malignant tumors are the five common diseases in Liao Ning province investigated by us. Through our modeling analysis, it can be concluded that relevant departments should set up more ophthalmic facilities, reasonably allocate limited resources, and constantly improve the satisfaction of medical staff.

The hospital will give play to the subjective initiative to compete and develop, and turn the passive into the active. The 14th Five-Year Plan is a key period for deepening the reform of the linkage among the three medical institutions in China. Medical institutions should reasonably plan the medical management mechanism according to the local actual situation, constantly improve the quality of medical services, and bring health to every corner.

5.Strengths and Weakness

5.1 The first question

Advantages: grey system theory does not have excessive requirements on the number of samples, nor does it need typical distribution rules. It is suitable for making a better prediction of the future aging trend, a variable with more uncertain factors. A phenomenon is often associated with multiple factors. It is more effective to predict or estimate the dependent variable by the optimal combination of multiple independent variables than to predict or estimate only one independent variable, and more reasonable to obtain the future medical fund needs.
Disadvantages: sometimes in regression analysis, which factor to choose and which expression to use for the factor is only a speculation, which leads to the untestability of some factors and makes regression analysis limited in some cases.

5.2 The second question

Advantages: can eliminate the interaction between various diseases, reduce the workload of disease type selection. In the comprehensive evaluation function, the weight of each principal component is its contribution rate, which reflects the proportion of the original information contained in the principal component in the total information Weights are objective and reasonable. It overcomes the defects of some evaluation methods.
Disadvantages: the interpretation of principal components is generally more or less fuzzy, not the original variable meaning nanometer clear, accurate, this is the process of variable dimensionality reduction.The price of not getting angry.

5.3 The third question

Advantages: the comparison of this model has high single-precision, and the discussion of queuing in multi-service desk is very clear. In the case of any $\rho_s < 1$, the system is stationary.
Disadvantages: conclusions use is small and does not take into account the limitations of emergency patients and patient waiting time.

5.4 The fourth question

Advantages: the stability of two kinds of hospitals can be observed, analyzed and verified visually from the number curve and phase trajectory of two kinds of hospitals by using the method of combination of number and shape
Disadvantages: less environmental factors are considered, and the influence of sudden factors cannot be predicted

6. Conclusion

1. It can be seen from the grey prediction results that the aging trend is gradually on th e rise, and it is estimated that during the 14th five-year plan period, the per capita med ical demand fund will fluctuate between 5000 and 7000, which reflects that the medic al demand may be affected by other uncertain factors.
2. Taking liaoning province as an example, the main component analysis showed that eye diseases ranked first among the common diseases in liaoning province.Therefore, eye diseases will need more medical needs, the need for provincial key public hospital s to develop vigorously.
3. Relevant medical institutions can avoid one service by combining multiple queues into one queue, i.e. multiple input customer sources are mixed into one input custome r source
When the service desk is idle and the queue of the other service desk is prolonged due to queuing, the system can make more efficient use of the service capacity of the syst em.Can effectively balance the load capacity of the service desk, so that you can get a shorter queue waiting time.
4.On the one hand, public hospitals continue to develop basic medical services and try to reduce the cost of public hospitals.Private hospitals actively cooperate with public hospitals to fill beds in public hospitals.Achieve patient circulation and sharing.At the same time, private hospitals can expand the market in the field of special diseases.

References

[1] Yang Guangxia, Xiehua, etc. By SPSS data statistics and analysis from beginners to master — Beijing: tsinghua university press, 2014
[2] Ba shusong, Fang yumhao, Zhu weihao. Pension gap and countermeasures under the background of aging population in China [J]. Economics and management,2018,32(06):18-24.
[3] Liu huiying. Difficulties in competition and cooperation between public hospitals and private hospitals in China — based on field research in tianjin and chongqing [J]. Science and technology innovation guide,2013(20):214-215
[4] Si shoukui, Sun xijing.Lingou software and application: national defense industry press, 2017.5
[5] jiang qiyuan. Mathematical model (fourth edition) [M]. Higher education press,2011
[6]Wu huawen. Prediction of railway passenger volume based on unbiased grey residual theory [J/OL]. Railway transport and economy,2019(05):121-126
[7]He zirong, jiang xiaodong, Yang lizhi. Optimal control of competitive population model under individual scale difference [J]. Progress in mathematics,2015,44(03):449-458.
[8] xu huan, liu yuxiu, Yang guobin, yi xueming. Analysis of competition model of multiple hospitals based on game theory [J]. Chinese journal of social medicine,2011,28(02):91-93.
[9] zhao yantao, zhu mengrong. Development trend and difficulties and opportunities faced by private hospitals in China [J]. Medical herald,2019,38(02):276-278.

Appendix

Tip.1.1 Ranking and comprehensive evaluation results of five diseases

disease	eye diseases	hypertension	diabetes	dyslipidemia	malignant tumors
ranking	1	3	2	4	5
evaluation	1.3447	0.4325	0.0467	-0.1924	-1.6314

Aging prediction code：

```
t0 = [2011:2018];X0 = [8.26,8.45,8.68,8.97,9.33,9.8,10.35,10.92]
n = length(X0);
lambda = X0(1:n-1)./X0(2:n);
range = minmax(lambda;)
exp([-2/(n+1), 2/(n+2)])
X1 = cumsum(X0);
Z1 = (X1(1:n-1)+X1(2:n))/2
B = [-Z1, ones(n-1,1)];
Y = X0(2:n);
u = B\Y; a = u(1);
b = u(2);
k = 0:n+4;
xhat1 = (X0(1) - b/a).*exp(-a*k) + b/a;
xhat0 = [X0(1) diff(xhat1)]
plot(t0,X0, o;t0(1)+k, xhat0;-+;)
```

Population competition code：

```
function dx=fun(t,x,r1,r2,n1,n2,s1,s2)
r1=0.8;
r2=0.8;
n1=20000;
n2=22000;
s1=0.2;
s2=0.1;
dx=[r1*x(1)*(1-x(1)/n1-s1*x(2)/n2);r2*x(2)*(1-s2*x(1)/n1-x(2)/n2)];
h=0.2;
ts=[0:h:40];
x0=[50,40];
opt=odeset('reltol',1e-6,'abstol',1e-9);
[t,x]=ode45(@fun,ts,x0,opt);
jingzheng
plot(t,x(:,1),'*',t,x(:,2),'+','LineWidth',2),grid;
>> plot(x(:,1),x(:,2),'*','LineWidth',2),grid ;
```