

$$f(x) = 42.3x_1^2 + 3.4x_2^2 + 5.2x_3^2 - 2.8x_1x_2 - 5.6x_1x_3 - 2.6x_2x_3 + 2x_1 + 1x_2 - 1x_3 + 12$$

Using vertex form,

Q1. Find $\|\bar{b}\|^2$

Q2. Find c .

Q3. Find the coordinate \bar{x} of $\bar{v} = \begin{bmatrix} 0.6 \\ 0.2 \end{bmatrix}$ using $A = \begin{bmatrix} 1.1 \\ -2.2 \end{bmatrix}$

$$A^T(A\bar{x} - \bar{v}) = 0. \quad \bar{x} = (A^T A)^{-1} A^T \bar{v} = \left(\begin{bmatrix} 1.1 & -2.2 \end{bmatrix} \begin{bmatrix} 1.1 & -2.2 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1.1 & -2.2 \end{bmatrix} \begin{bmatrix} 0.6 \\ 0.2 \end{bmatrix}.$$

Q4. MCQ. Is the scatter matrix with real entries diagonalizable?

Yes/ No/ additional information required.

Q5. MCQ. What is the main application of the Discrete Cosine Transform?

Image Compression/ Edge Detection/ Eigen Faces/ Whitening

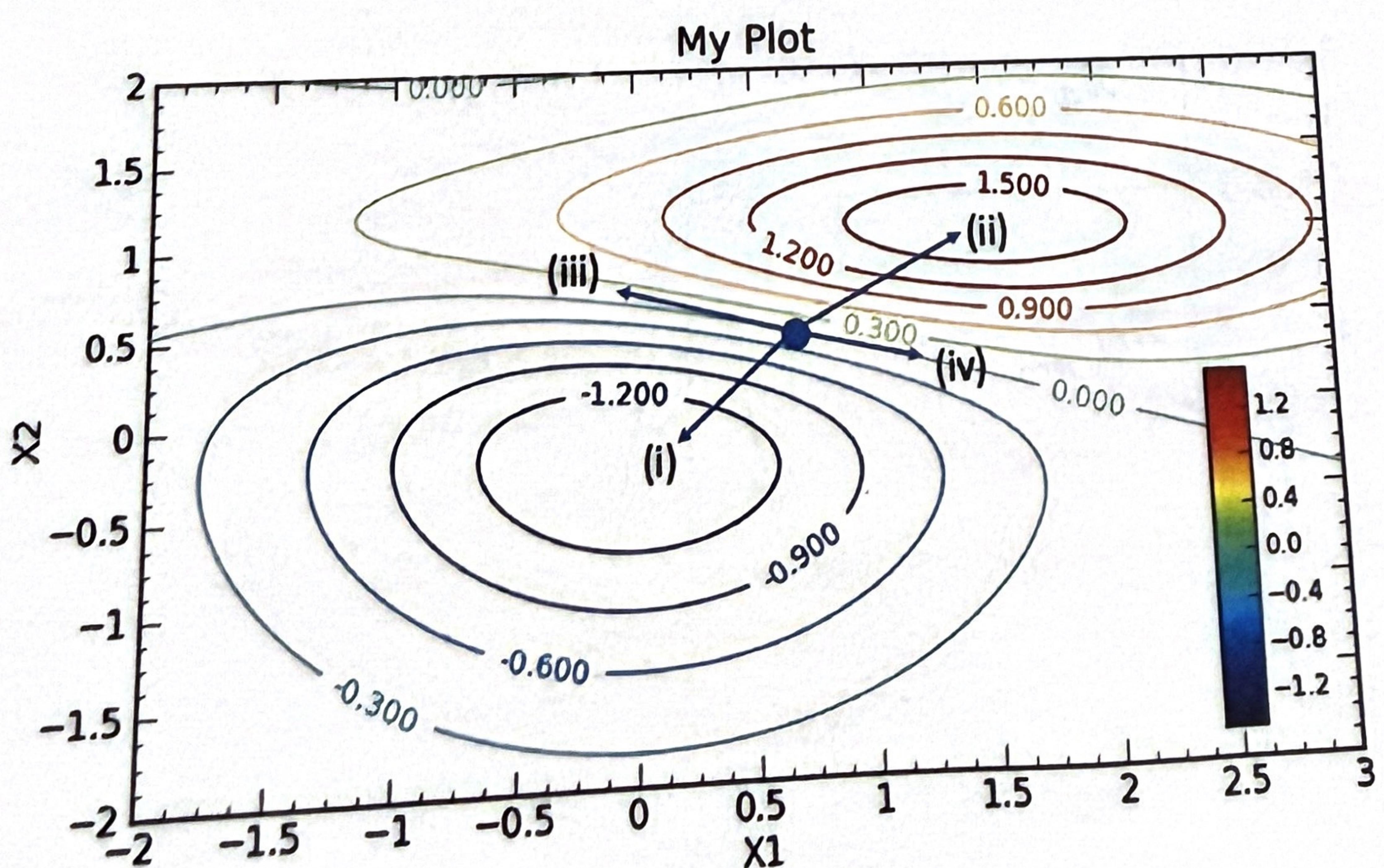
Q6. MCQ. Given the Eigenvalues of a Hessian matrix for $f(\bar{x})$ at \bar{x} being 0.5, 0.1, 4.2, $f'(\bar{x}) = -1.5$, which of the following is correct?

Local minimum/ local maximum/ saddle point/ additional information required.

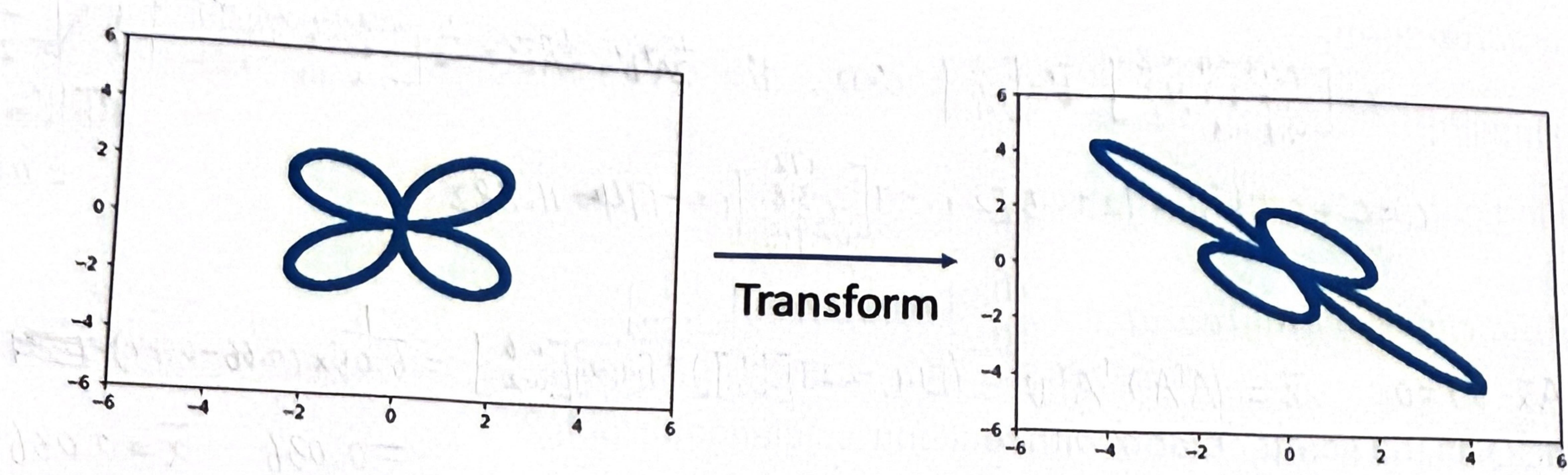
Q7. MCQ. Given the Eigenvalues of a Hessian matrix for $f(\bar{x})$ at \bar{x} being 6.3, 7.9, -3.2, $f'(\bar{x}) = 0$, which of the following is correct?

Local minimum/ local maximum/ saddle point/ additional information required.

Q8. MCQ. Predict the direction of the gradient based on the contour plot below:



Q9. MCQ



Given datapoints on the left figure,
Perform transformation $A = V\Delta V^T$,

$$\text{where } V = \begin{bmatrix} \cos(45^\circ) & \sin(45^\circ) \\ -\sin(45^\circ) & \cos(45^\circ) \end{bmatrix}$$

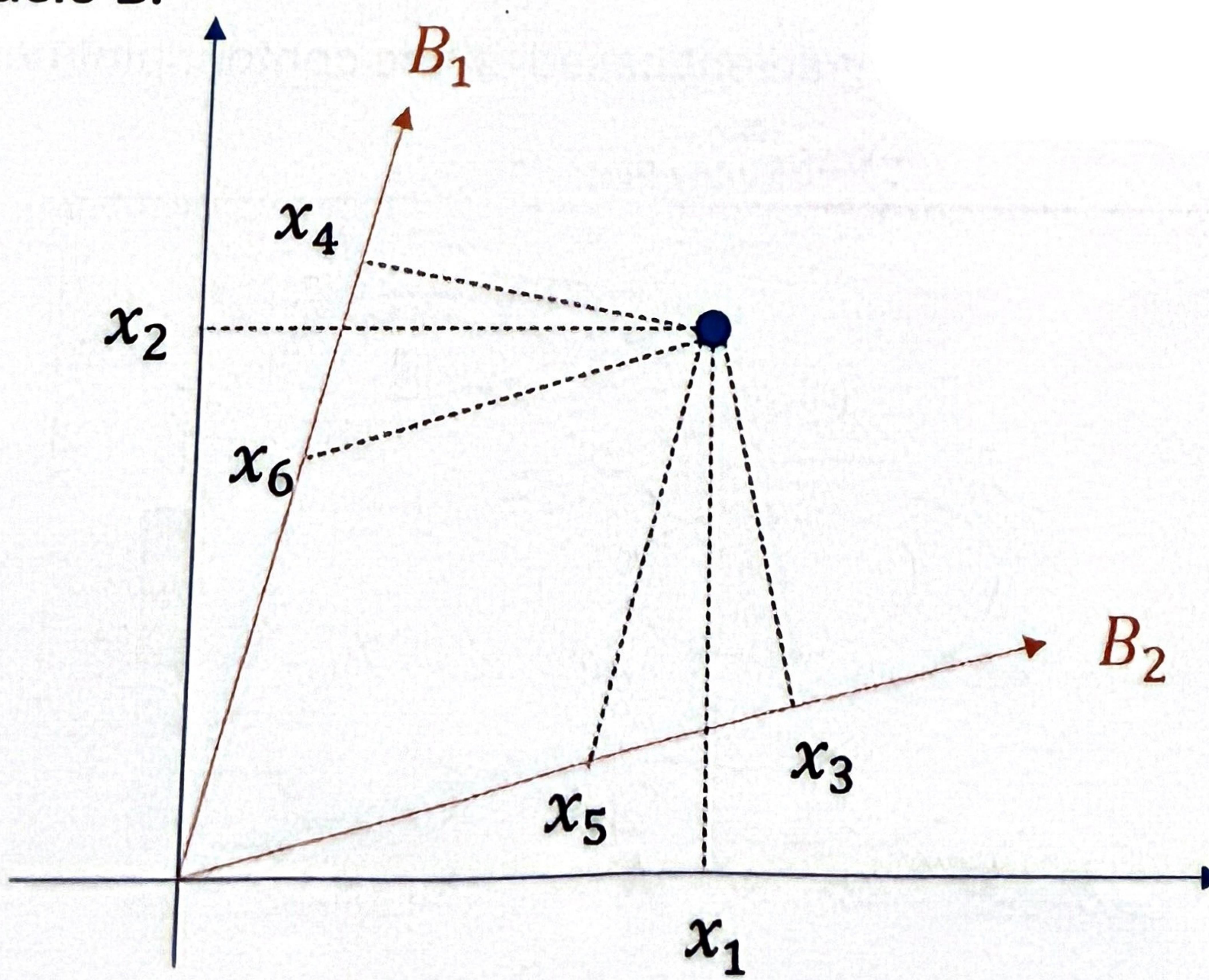
$$\Delta = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

Identify the values for a and b.

- $a = 2, b = 0.5$
- $a = 0.5, b = 2$
- $a = -0.5, b = 2$
- $a = 2, b = 2$

Q10. Prepare your answer as row vector:

Identify the coordinates of the blue dot in the new basis B.



Practical Test 1

Student ID: _____ Name: _____

Given matrix D,

$$D = \begin{bmatrix} 1.4 & 5.5 \\ 2.4 & -1.5 \\ 6.6 & 2.1 \\ 1.4 & 5.2 \\ 2.4 & -8 \end{bmatrix}$$

Q1 Find the co-variance matrix after demean (matrix, do not use np.cov()).

Q2 Find the eigenvector with the largest eigenvalue (column vector).

Given a quadratic function, and vertex form $(\bar{x} - \bar{b})^T A (\bar{x} - \bar{b}) + c$
 $f(\bar{x}) = 6.3x_1^2 + 12x_2^2 + 6.2x_3^2 - 2.8x_1x_2 - 1.2x_1x_3 - 4.2x_2x_3$
 $+ 2.3x_1 + 1.5x_2 - 2x_3 + 9$

Q3 Find the A in vertex form (matrix).

Q4 Find the \bar{b} in vertex form (column vector).

Q5 Find the c in vertex form (scalar).