(a) Write a differential equation that is satisfied by y.

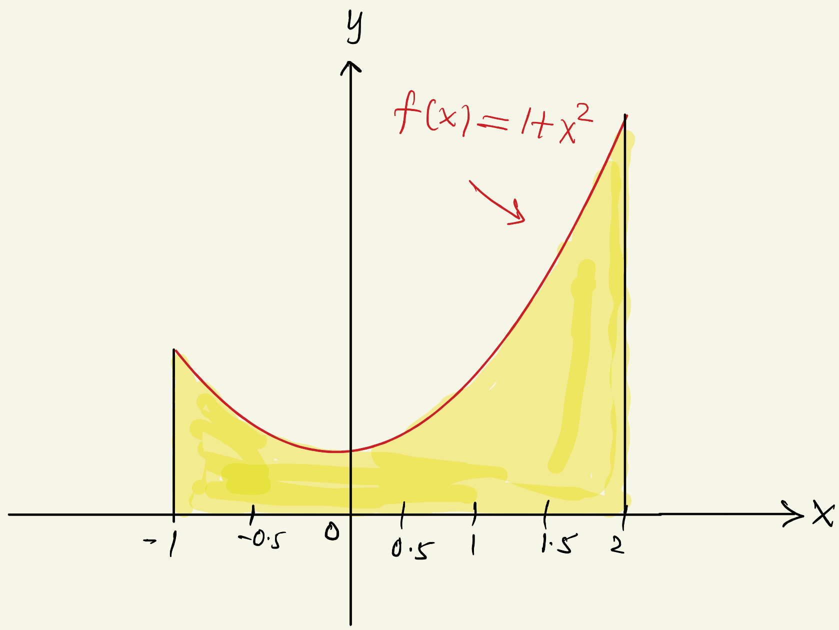
(b) Solve the differential equation in part (a).

(c) A small town has 4500 inhabitants. At 8 AM, 360 people have heard a rumor. By noon half the town has heard it. At what time will 90 % of the population have heard the rumor?

Solution

Chap-5

① $f(x) = 1+x^2, \; a=-1, \; b=2, \; \Delta x = \frac{2-(-1)}{6} = 0.5.$



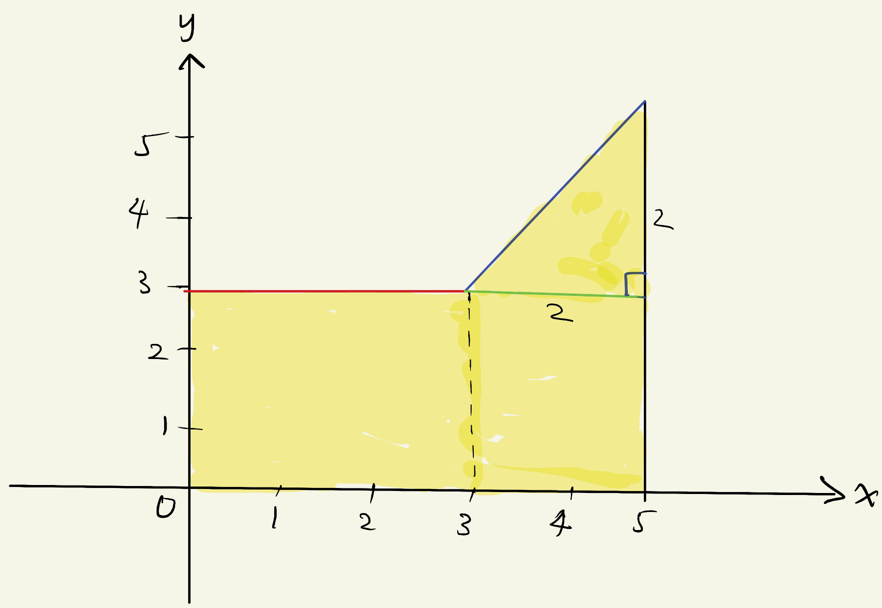
$$R_6 = (0.5)\left[f(-0.5) + f(0) + f(0.5) + f(1) + f(1.5) + f(2)\right]$$

$$= (0.5)\left[1.25 + 1 + 1.25 + 2 + 3.25 + 5\right]$$

$$= (0.5)(13.75)$$

$$= 6.875$$

② If $f(x) = \begin{cases} 3 & \text{for } x < 3 \\ x & \text{for } x \geq 3 \end{cases}$, then $\int_0^5 f(x)dx$ can be interpreted as the area of the shaded region, which consists of a 5-by-3 rectangle surmounted by an isosceles right triangle whose legs have length 2.



Thus,

$$\int_0^5 f(x)dx = (5)(3) + \frac{1}{2}(2)(2) = 15 + 2 = 17.$$

③ We apply First Fundamental Theorem of calculus.

$$\frac{d}{dx}\int_a^x f(t)dt = f(x), \quad \frac{d}{dt}\int_a^{h(x)} f(t)\,dt = f(h(x)) \cdot h'(x).$$

(a) $$\frac{d}{dx}\int_0^x e^t\,dt = e^x.$$

(b) $$\frac{d}{dx}\int_{2x}^{x^3} \ln\left(\sqrt{t^3-2}\right)dt = \frac{d}{dx}\left[\int_{2x}^{2}\ln\left(\sqrt{t^3-2}\right)dt + \int_2^{x^3}\ln\left(\sqrt{t^3-2}\right)dt\right]$$

$$= \frac{d}{dx}\left[\int_2^{x^3}\ln\left(\sqrt{t^3-2}\right)dt - \int_2^{2x}\ln\left(\sqrt{t^3-2}\right)dt\right]$$

$$= 3x^2\ln\left(\sqrt{x^9-2}\right) - 2\ln\left(\sqrt{8x^3-2}\right)$$

(c) $$\frac{d}{dx}\int_2^{\sin(6x)}\sqrt{t^2+4}\,dt = \sqrt{(\sin(6x))^2+4}\cdot\cos(6x)\cdot 6$$

$$= 6\cos(6x)\sqrt{\sin^2(6x)+4}$$

(4) By the First Fundamental Theorem of calculus,

$$\frac{d}{dx}\left(\int_1^x f(t)dt = (x-1)e^{2x} + \int_1^x e^{-t}f(t)\right)$$

$$\Rightarrow \quad f(x) = (x-1)\cdot 2e^{2x} + e^{2x} + e^{-x}f(x)$$

$$\Rightarrow \quad f(x) = \frac{(2x-1)e^{2x}}{1-e^{-x}} = \frac{(2x-1)e^{3x}}{e^x-1}$$

(5) $\int_2^8 f(x)dx = \int_2^4 f(x)dx + \int_4^8 f(x)dx$ $\quad \int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, \ a<c<b$

$\Rightarrow \int_4^8 f(x)dx = \int_2^8 f(x)dx - \int_2^4 f(x)dx = 7.3 - 5.9 = 1.4.$

(6) $\int_0^9 [2f(x) + 3g(x)]dx = 2\int_0^9 f(x)dx + 3\int_0^9 g(x)dx = (2)(37) + (3)(16)$

$= 2(37) + 3(16)$

$= 122$

Chap-6

(7) $x^2 - 4 = -x^2 - 2x \Rightarrow x = -2, 1.$ Thus,

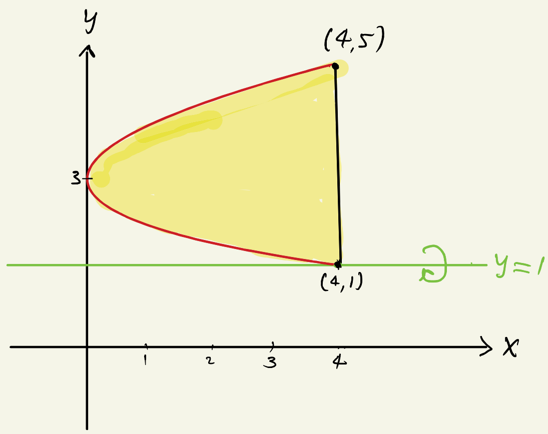
$$\text{Area} = \int_{-3}^{-2} \left[x^2 - 4 - (-x^2 - 2x)\right]dx + \int_{-2}^{1} \left[-x^2 - 2x - (x^2 - 4)\right]dx$$

$$= \int_{-3}^{-2} (2x^2 + 2x - 4)dx + \int_{-2}^{1} (-2x^2 - 2x + 4)dx$$

$$= \left[\frac{2}{3}x^3 + x^2 - 4x\right]_{-3}^{-2} + \left[-\frac{2}{3}x^3 - x^2 + 4x\right]_{-2}^{1}$$

$= \frac{38}{3}$ Square unit

8



$$\text{Volume} = \int_1^5 2\pi (y-1)\left[4-(y-3)^2\right] dy$$

$$= 2\pi \int_1^5 (y-1)(-y^2+6y-5)\, dy$$

$$= 2\pi \int_1^5 (-y^3+7y^2-11y+5)\, dy$$

$$= 2\pi \left[-\frac{1}{4}y^4 + \frac{7}{3}y^3 - \frac{11}{2}y^2 + 5y\right]_1^5$$

$$= 2\pi \left(\frac{275}{12} - \frac{19}{12}\right)$$

$$= \frac{128}{3}\pi \quad \text{cub unit}$$

9 (i) $f(x) = \frac{1}{x}$, $a = 1$, $b = 3$. Thus,

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{3-1}\int_1^3 \frac{1}{x}\,dx = \frac{1}{2}\left[\ln x\right]_1^3 = \frac{1}{2}(\ln 3 - \ln 1) = \frac{\ln 3}{2}.$$

(ii) Set $f(c) = f_{ave}$.

Thus, $\frac{1}{c} = \frac{\ln 3}{2}$

$\therefore\ c = \frac{2}{\ln 3}$

Chap-7

10 (a) $$|1-x^2| = \begin{cases} 1-x^2 & \text{if } 1-x^2 \geq 0 \text{ i.e. } -1 \leq x \leq 1 \\ x^2-1 & \text{if } 1-x^2 < 0 \text{ i.e } x<-1 \text{ or } x>1 \end{cases}.$$

Hence,

$$\int_{-2}^{2} |1-x^2|\,dx = \int_{-2}^{-1} (x^2-1)\,dx + \int_{-1}^{1} (1-x^2)\,dx + \int_{1}^{2} (x^2-1)\,dx$$

$$= \left[\frac{x^3}{3}-x\right]_{-2}^{-1} + \left[x-\frac{x^3}{3}\right]_{-1}^{1} + \left[\frac{x^3}{3}-x\right]_{1}^{2}$$

$$= 4$$

(b) $$f(x) = \max(x, x^2) = \begin{cases} x & \text{if } 0 \leq x \leq 1 \\ x^2 & \text{if } x>1 \end{cases}.$$ Hence,

$$\int_0^2 f(x)\,dx = \int_0^2 \max(x,x^2)\,dx$$

$$= \int_0^1 x\,dx + \int_1^2 x^2\,dx$$

$$= \left[\frac{x^2}{2}\right]_0^1 + \left[\frac{x^3}{3}\right]_1^2$$

$$= \left(\frac{1}{2}-0\right) + \left(\frac{8}{3}-\frac{1}{3}\right) = \frac{17}{6}.$$

(c) $\int_1^2 \sqrt{x+1}\,dx = ?$

Let $u = x+1$
$du = dx$

x	1	2
u	2	3

Hence,

$$\int_1^2 \sqrt{x+1}\,dx = \int_2^3 \sqrt{u}\,du = \int_2^3 u^{\frac{1}{2}}\,du$$

$$= \left[\frac{2}{3}u^{3/2}\right]_2^3$$

$$= \frac{2}{3}\left[3^{3/2} - 2^{3/2}\right]$$

$$= \frac{2\left(3\sqrt{3} - 2\sqrt{2}\right)}{3}.$$

(d) Let $u = 30 - x^{3/2}$

$du = -\frac{3}{2}\sqrt{x}\,dx$

$\sqrt{x}\,dx = -\frac{2}{3}du$

$x = 0 \Rightarrow u = 30$

$x = 1 \Rightarrow u = 29$

$$\int_0^1 \frac{\sqrt{x}\,dx}{(30-x^{3/2})^2} = \int_{30}^{29} \frac{-\frac{2}{3}du}{u^2} = -\frac{2}{3}\int_{30}^{29} \frac{1}{u^2}\,du$$

$$= -\frac{2}{3}\left[-\frac{1}{u}\right]_{30}^{29} = -\frac{2}{3}\left[-\frac{1}{29} - \left(-\frac{1}{30}\right)\right] = \frac{1}{1305}.$$

11 (a)

$$\int \frac{(x-1)^3}{\sqrt{x}}\,dx = \int \frac{x^3-3x^2+3x-1}{\sqrt{x}}\,dx$$

$$= \int \left(x^{5/2} - 3x^{3/2} + 3x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right)dx$$

$$= \frac{2}{7}x^{7/2} - \frac{6}{7}x^{5/2} + 2x^{3/2} - 2x^{\frac{1}{2}} + C$$

(b)

$$\int \frac{x^4}{1+x^2}\,dx = \int \left(\frac{x^4-1}{1+x^2} + \frac{1}{1+x^2}\right)dx$$

$$= \int \left(x^2-1 + \frac{1}{1+x^2}\right)dx$$

$$= \frac{x^3}{3} - x + \tan^{-1}x + C$$

(c)

$$\int \frac{e^x(1+x)}{\cos^2(xe^x)}\,dx = \int \frac{du}{\cos^2 u} \leftarrow$$

let $u = xe^x$

$du = (xe^x + e^x)\,dx$

$= e^x(1+x)\,dx$

$$= \int \sec^2 u\,du$$

$$= \tan u + C$$

$$= \tan(xe^u) + C$$

(d)

$$\int e^{e^x+x}\,dx = \int e^{e^x} \cdot e^x\,dx \leftarrow$$

let $u = e^x$

$du = e^x dx$

$$= \int e^u\,du$$

$$= e^u + c$$

$$= e^{e^x} + c$$

(e) Let $u = x^3$ $\quad dv = \cos x\, dx$

$du = 3x^2$ $\quad v = \sin x$

$$\int x^3 \cos x\, dx = x^3 + 3x^2 \cos x - 6x \sin x - 6\cos x + C.$$

(f) $$\int \frac{x^2\, dx}{(x^2+x-2)(x-3)} = \int \frac{x^2\, dx}{(x-1)(x+2)(x-3)}.$$

We write the integrand as a sum of simpler partial fractions:

$$\frac{x^2}{(x^2+x-2)(x-3)} = \frac{x^2}{(x-1)(x+2)(x-3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3},$$

where $$A = \frac{1^2}{(1+2)(1-3)} = -\frac{1}{6},$$

$$B = \frac{2^2}{(-2-1)(-2-3)} = \frac{4}{15},$$

$$C = \frac{3^2}{(3-1)(3+2)} = \frac{9}{10}.$$

So, $$\frac{x^2}{(x-1)(x+2)(x-3)} = \frac{-\frac{1}{6}}{x-1} + \frac{\frac{4}{15}}{x+2} + \frac{\frac{9}{10}}{x-3}$$

Hence, using integration by partial fractions, we have

$$\int \frac{x^2\,dx}{(x^2+x-2)(x-3)} = -\frac{1}{6}\int \frac{dx}{x-1} + \frac{4}{15}\int \frac{dx}{x+2} + \frac{9}{10}\int \frac{dx}{x-3}$$

$$= -\frac{1}{6}\ln|x-1| + \frac{4}{15}\ln|x+2| + \frac{9}{10}\ln|x-3| + c.$$

(g) $$\int \frac{dx}{\sqrt{e^{2x}-1}} = \int \frac{\tan\theta\,d\theta}{\sqrt{\sec^2\theta - 1}} \longleftarrow$$

let $e^x = 1\cdot\sec\theta = \sec\theta \Rightarrow \theta = \sec^{-1}(e^x)$

$e^{2x} = \sec^2\theta$

$e^x dx = \sec\theta\tan\theta\,d\theta$

$dx = \tan\theta\,d\theta$

$$= \int d\theta$$

$$= \theta + c$$

$$= \sec^{-1}(e^x) + c$$

12 (a) $$\int_1^\infty 3e^{1-x}dx = \lim_{t\to\infty}\int_1^t 3e\cdot e^{-x}dx$$

$$= \lim_{t\to\infty}\left[-3e\cdot e^{-x}\right]_1^t$$

$$= (-3e)\lim_{t\to\infty}\left(e^{-t} - e^{-1}\right)$$

$$= (-3e)\left(-\frac{1}{e}\right)$$

$$= 3$$

(b) $$\int_e^\infty \frac{1}{x(\ln x)^2}dx = \int_e^\infty \frac{1}{(\ln x)^2}\cdot\frac{1}{x}\,dx$$

Let $u = \ln x$

$du = \frac{1}{x}dx$

$x = e \Rightarrow u = 1$

$x\to\infty \Rightarrow u\to\infty$

$$= \int_1^\infty \frac{1}{u^2}du \longleftarrow \int_1^\infty \frac{1}{x^p}dx, \quad p = 2 > 1$$

$$= \frac{1}{2-1}$$

$$= 1$$

Chap-8

13 $a=0$, $b=1$, and

$$y = \frac{4\sqrt{2}}{3} x^{3/2} - 1$$

$$\frac{dy}{dx} = \frac{4\sqrt{2}}{3} \cdot \frac{3}{2} x^{\frac{1}{2}} = 2\sqrt{2}\, x^{\frac{1}{2}}$$

$$\left(\frac{dy}{dx}\right)^2 = \left(2\sqrt{2}\, x^{\frac{1}{2}}\right)^2 = 8x.$$

Thus, the length L of the curve $y = \frac{4\sqrt{2}}{3} x^{3/2} - 1$ from $x=0$ to $x=1$ is

$$L = \int_0^1 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_0^1 \sqrt{1+8x}\, dx$$

$u = 1+8x$

$du = 8dx$

$$= \frac{1}{8} \cdot \frac{2}{3} \cdot (1+8x)^{3/2} \Big|_0^1$$

$$= \frac{13}{6}$$

14 $\quad y = x^3, \quad a = 0, \; b = \frac{1}{2},$

$$\frac{dy}{dx} = 3x^2 dx$$

$$\left(\frac{dy}{dx}\right)^2 = 9x^4$$

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+9x^4}$$

$$\therefore \quad S = \int_a^b 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_0^{\frac{1}{2}} 2\pi\, x^3 \sqrt{1+9x^4}\, dx$$

$$= 2\pi \cdot \frac{1}{36} \cdot \frac{2}{3} \left(1+9x^4\right)^{3/2} \Big|_0^{\frac{1}{2}}$$

$u = 1+9x^4$

$du = 36\, x^3 dx$

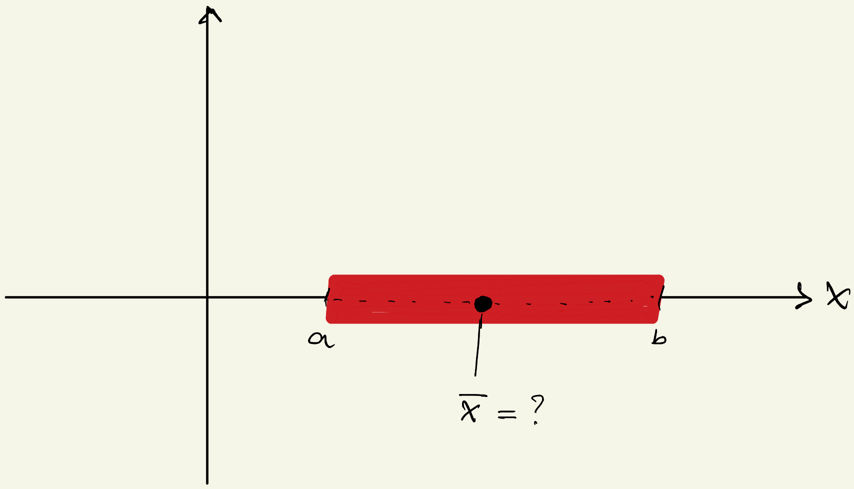
$\frac{du}{36} = x^3 dx$

$$= \frac{\pi}{27}\left[\left(1+\frac{9}{16}\right)^{3/2} - 1\right]$$

$$= \frac{\pi}{27}\left[\left(\frac{25}{16}\right)^{3/2} - 1\right]$$

$$= \frac{61\pi}{1728}$$

15



We model the strip as a portion of the x-axis from $x=a$ to $x=b$ as shown in the figure above.

density : $\delta(x) = \delta$ (constant).

Moment : $M_0 = \int_a^b x\,\delta(x)\,dx = \delta \int_a^b x\,dx = \delta \left[\frac{x^2}{2}\right]_a^b = \frac{\delta}{2}(b^2-a^2)$.

Mass : $M = \int_a^b \delta(x)\,dx = \delta \int_a^b dx = \delta\,[x]_a^b = \delta(b-a)$.

Thus,

$$\bar{x} = \frac{M_0}{M} = \frac{\frac{\delta}{2}(b^2-a^2)}{\delta(b-a)}$$

$$= \frac{\frac{1}{2}(b-a)(b+a)}{b-a}$$

$$= \frac{a+b}{2}$$

Chap-9

16 (a) $x(1+y^2)\,dx = y(1+x^2)\,dy$

$$\Rightarrow \frac{x}{1+x^2}\,dx = \frac{y}{1+y^2}\,dy \longleftarrow \text{Separable}$$

$$\Rightarrow \int \frac{x}{1+x^2}\,dx = \int \frac{y}{1+y^2}\,dy$$

$$\Rightarrow \frac{1}{2}\int \frac{2x}{1+x^2}\,dx = \frac{1}{2}\int \frac{2y}{1+y^2}\,dy$$

$$\Rightarrow \frac{1}{2}\ln(1+x^2) = \frac{1}{2}\ln(1+y^2) + c$$

$$\Rightarrow \ln(1+x^2)^{\frac{1}{2}} - \ln(1+y^2)^{\frac{1}{2}} = c$$

$$\Rightarrow \ln\left(\frac{1+x^2}{1+y^2}\right)^{\frac{1}{2}} = c$$

$$\Rightarrow \frac{1+x^2}{1+y^2} = e^{2c}$$

$$\Rightarrow \frac{1+x^2}{1+y^2} = k, \qquad k = e^{2c}$$

(b) $\dfrac{dy}{dx} = \dfrac{1}{e^{x+y}+y^2e^x} = \dfrac{1}{e^x\cdot e^y+y^2e^x} = \dfrac{1}{e^x(e^y+y^2)}$

$\Rightarrow (e^y+y^2)\,dy = \dfrac{1}{e^x}\,dx$ ⟵ Separable

$\Rightarrow \int(e^y+y^2)\,dy = \int e^{-x}dx$

$\Rightarrow e^y+\dfrac{y^3}{3} = -e^{-x}+c$

(c) $\dfrac{dy}{dx} = \dfrac{1-y}{1+x} \Leftrightarrow \dfrac{dy}{1-y} = \dfrac{dx}{1+x}$ ⟵ Separable

$\Rightarrow \int\dfrac{dy}{1-y} = \int\dfrac{dx}{1+x}$

$\Rightarrow -\ln|1-y| = \ln|1+x|+c$

$\Rightarrow \ln|1+x|+\ln|1-y| = C$

$\Rightarrow \ln|(1+x)(1-y)| = C$

$\Rightarrow (1+x)(1-y) = \pm e^C = k$

(d) $x\dfrac{dy}{dx}+y = \dfrac{1}{x}$

$\Rightarrow \dfrac{dy}{dx}+\dfrac{1}{x}y = \dfrac{1}{x^2}$ ⟵ FOLDE : $P(x)=\dfrac{1}{x}$, $Q(x)=\dfrac{1}{x^2}$.

$$I(x) = e^{\int \frac{1}{x} dx} = e^{\ln|x|} = x, \quad x > 0.$$

$$y = \frac{1}{x}\left[\int x \cdot \frac{1}{x^2}\, dx + C\right]$$

$$= \frac{1}{x}\left[\int \frac{1}{x}\, dx + c\right]$$

$$= \frac{1}{x}\left(\ln x + C\right), \quad x > 0$$

$$= \frac{\ln x + C}{x}$$

$$y(1) = 2$$

$$\Rightarrow \frac{\ln 1 + c}{1} = 2$$

$$\Rightarrow c = 2$$

Thus,

$$y = \frac{\ln x + 2}{x}.$$

(17) Let $Q(t)$ denote the number of army personnel who had contracted the flu after t days.

Then,

$$\frac{dQ}{dt} = kQ(5000-Q)$$

where k is a positive constant of proportionality.

$$\frac{dQ}{dt} = kQ(5000-Q)$$

$$\Rightarrow \frac{dQ}{Q(5000-Q)} = k\,dt \leftarrow \text{Separable}$$

$$\Rightarrow \int \frac{dQ}{Q(5000-Q)} = \int k\,dt$$

$$\Rightarrow \int \frac{1}{5000}\left[\frac{1}{Q} + \frac{1}{5000-Q}\right] dQ = \int k\,dt$$

$$\Rightarrow \ln|Q| - \ln|5000-Q| = 5000kt + C$$

$$\Rightarrow \ln\left|\frac{Q}{5000-Q}\right| = 5000kt + C$$

$$\Rightarrow \left|\frac{Q}{5000-Q}\right| = e^{5000kt + c} = e^{c} \cdot e^{5000kt}$$

$$\Rightarrow \frac{Q}{5000-Q} = Me^{5000kt}, \qquad M = \pm e^{c}$$

$$\Rightarrow Q = \frac{5000Me^{5000kt}}{1+Me^{5000kt}}$$

$$\Rightarrow Q(t) = \frac{5000}{1+Ae^{-5000kt}}, \qquad A = \frac{1}{M}.$$

The condition that 5 % of the population had contracted influenza at $t=0$ implies that

$$Q(0) = \frac{5000}{1+A} = 250 \Rightarrow A = A = 19.$$

Therefore,

$$Q(t) = \frac{5000}{1+19e^{-5000kt}}.$$

The condition that 20 % of the population had contracted influenza by the 10^{th} day gives

$$Q(10) = \frac{5000}{1+19e^{-50,000k}} = 1000$$

$$\Rightarrow 1+19e^{-50,000k} = 5$$

$$\Rightarrow e^{-50,000k} = \frac{4}{19}$$

$$\Rightarrow -50,000k = \ln 4 - \ln 19$$

$$\Rightarrow k = -\frac{1}{50,000}\left(\ln 4 - \ln 19\right)$$

$$\approx 0.0000312 .$$

Hence,

$$Q(t) = \frac{5000}{1+19e^{-0.156t}} .$$

Also, the number of army personnel who had contracted the flu by the 13^{th} day is given by

$$Q(13) = \frac{5000}{1+19e^{-0.156(13)}} = \frac{5000}{1+19e^{-2.028}} \approx 1428$$

(or $\approx 29\%$)

18 (a) Let $y = y(t)$ be the fraction who have heard by time t. Then, the fraction who have not heard is $1-y$. The rate of change of y (we are told) is proportional to the product $y(1-y)$. Let k be the constant of proportionality. Therefore, the differential equation that is satisfied by y becomes:

$$\frac{dy}{dt} = k\,y(1-y) \quad \ldots \quad (*)$$

(b) From part (a), we have

$$\frac{dy}{dt} = k\,y(1-y)$$

$$\Rightarrow \int \frac{dy}{y(1-y)} = \int k\,dt$$

$$\Rightarrow \int \left[\frac{1}{y} + \frac{1}{1-y}\right] dy = k \int dt$$

$$\Rightarrow \ln|y| - \ln|1-y| = kt + C$$

$$\Rightarrow \ln\left|\frac{y}{1-y}\right| = kt + C$$

$$\Rightarrow \frac{y}{1-y} = Me^{kt}, \qquad M = \pm e^{C}$$

$$\Rightarrow \quad y = M(1-y)e^{kt} = Me^{kt} - Mye^{kt}$$

$$\Rightarrow \quad y + Mye^{kt} = Me^{kt}$$

$$\therefore \quad y(t) = \frac{Me^{kt}}{1+Me^{kt}} = \frac{1}{1+Ae^{-kt}}, \qquad A = \frac{1}{M}.$$

(c) The initial proportion that have heard the rumor is

$$y_0 = \frac{360}{4500} = 0.08.$$

Thus,

$$y(0) = \frac{1}{1+Ae^{-k(0)}} = 0.08 \Rightarrow \frac{1}{1+A} = 0.08$$

$$\Rightarrow 1+A = 12.5$$

$$\Rightarrow A = 11.5$$

We also know that at noon ($t = 4$ hours after 8AM), the fraction

$$y(4) = 0.5.$$

Then,

$$\frac{1}{1+11.5e^{-k(4)}} = 0.5 \Rightarrow k = 0.61.$$

Therefore,

$$y(t) = \frac{1}{1+11.5e^{-0.61t}}.$$

Now, we calculate t, so that $y(t) = 0.9$. That is

$$0.9 = \frac{1}{1+11.5e^{-0.61t}}$$

$$\Rightarrow \quad 1+11.5\,e^{-0.61t} = \frac{10}{9}$$

$$\Rightarrow \quad 11.5\,e^{-0.61t} = \frac{1}{9}$$

$$\Rightarrow \quad e^{-0.61t} = \frac{1}{103.5}$$

$$\Rightarrow \quad -0.61t = \ln 1 - \ln(103.5)$$

$$\Rightarrow \quad 0.61t = \ln(103.5)$$

$$\Rightarrow \quad t = \frac{\ln(103.5)}{0.61} = 7.6 \quad (7 \text{ hours and } 36 \text{ minutes})$$

Therefore, at 15:36 ($t = 7$ hours and 36 minutes after 8 AM), 90 % of the population have heard the rumor.