Final Extra Exercises-BSC121

Chapter 5

(1) Estimate the area under the graph of $f(x) = 1 + x^2$ from $x = -1$ to $= 2$ using six rectangles and right endpoints.

(2) Find $\int_0^5 f(x)dx$, where $f(x) = \begin{cases} 3 & \text{for } x < 3 \\ x & \text{for } x \geq 3. \end{cases}$

(3) Find the following derivatives.

(a) $\dfrac{d}{dx} \displaystyle\int_0^x e^t \; dt.$

(b) $\dfrac{d}{dx} \displaystyle\int_{2x}^{x^3} \ln\left(\sqrt{t^3 - 2}\right) \; dt.$

(c) $\dfrac{d}{dx} \displaystyle\int_2^{\sin(6x)} \sqrt{t^2 + 4} \; dt.$

(4) If $\displaystyle\int_1^x f(t) \; dt = (x-1)\, e^{2x} + \int_1^x e^{-t} f(t) \; dt$ for all x, then find a formula for $f(x)$.

(5) If $\displaystyle\int_2^8 f(x) \; dx = 7.3$ and $\displaystyle\int_2^4 f(x) \; dx = 5.9$, then find $\displaystyle\int_4^8 f(x) \; dx$.

(6) If $\displaystyle\int_0^9 f(x) \; dx = 37$ and $\displaystyle\int_0^9 g(x) \; dx = 16$, then find $\displaystyle\int_0^9 [2f(x) + 3g(x)] \; dx$.

Chapter 6

(7) Find the total area of the shaded region in **Figure 1**.

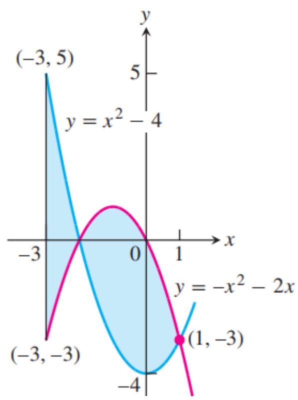

Figure 1

(8) Use the method of cylindrical shells to find the volume of the solid generated by rotating the region bounded by the curve $x = (y-3)^2$ and the line $x = 4$ about the line $y = 1$.

(9) Given $f(x) = \dfrac{1}{x}$.

(i) Find the average values of f on the interval $[1, 3]$.

(ii) Find all the values of c that satisfy the Mean Value Theorem for integrals for f on the interval $[1, 3]$.

Chapter 7

(10) Evaluate the following definite integrals.

(a) $\displaystyle\int_{-2}^{2} |1 - x^2| \, dx.$

(b) $\displaystyle\int_{0}^{2} f(x) \, dx$, where $f(x) = \max\left(x,\ x^2\right)$.

(c) $\displaystyle\int_{1}^{2} \sqrt{x+1} \, dx.$

(d) $\displaystyle\int_{0}^{1} \frac{\sqrt{x} \, dx}{\left(30 - x^{3/2}\right)^2}.$

(11) Find the following indefinite integrals.

(a) $\displaystyle\int \frac{(x-1)^3}{\sqrt{x}}\ dx.$

(b) $\displaystyle\int \frac{x^4}{1+x^2}\ dx.$

(c) $\displaystyle\int \frac{e^x(1+x)}{\cos^2(xe^x)}\ dx.$

(d) $\displaystyle\int e^{(e^x+x)}\ dx.$

(e) $\displaystyle\int x^3 \cos x\ dx.$

(f) $\displaystyle\int \frac{x^2\ dx}{(x^2+x-2)(x-3)}.$

(g) $\displaystyle\int \frac{dx}{\sqrt{e^{2x}-1}}.$

(12) Evaluate the following improper integrals.

(a) $\displaystyle\int_1^\infty 3e^{1-x}\ dx.$

(b) $\displaystyle\int_e^\infty \frac{1}{x(\ln x)^2}\ dx.$

Chapter 8

(13) Find the length of the curve $y = \dfrac{4\sqrt{2}}{3}\ x^{\frac{3}{2}} - 1$, $0 \le x \le 1$.

(14) Find the area of the surface generated by revolving the curve $y = x^3$, $0 \le x \le \dfrac{1}{2}$, about the x-axis.

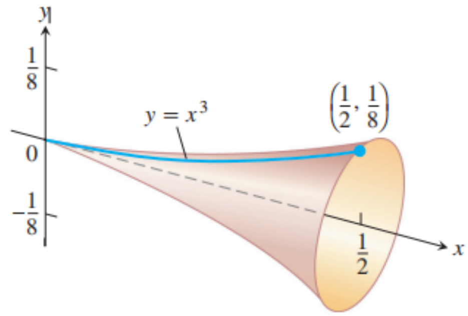

Figure 2

(15) Show that the center of mass of a straight, thin strip or rod of constant density lies halfway between its two ends.

Chapter 9

(16) Solve the following differential equations.

(a) $x\left(1+y^2\right)dx = y\left(1+x^2\right)dy$.

(b) $\dfrac{dy}{dx} = \dfrac{1}{e^{x+y}+y^2e^x}$.

(c) $\dfrac{dy}{dx} = \dfrac{1-y}{1+x}$.

(d) $x\dfrac{dy}{dx} + y = \dfrac{1}{x}, \qquad x > 0, \qquad y(1) = 2.$

(17) During a flu epidemic, 5% of the 5000 army personnel stationed at Fort MacArthur had contracted influenza at time $t = 0$. Furthermore, the rate at which they were contracting influenza was jointly proportional to the number of personnel who had already contracted the disease and the noninfected population. If 20% of the personnel had contracted the flu by the 10th day, find the number of personnel who had contracted the flu by the 13th day.

(18) One model for the spread of a rumor is that the rate of spread is proportional to the product of the fraction y of the population who have heard the rumor and the fraction who have not heard the rumor.

(a) Write a differential equation that is satisfied by y.

(b) Solve the differential equation in part (a).

(c) A small town has 4500 inhabitants. At 8 AM, 360 people have heard a rumor. By noon half the town has heard it. At what time will 90 % of the population have heard the rumor?