Accelerating Dynamic Graph Analytics on GPUs

Mo Sha, Yuchen Li, Bingsheng He, Kian-Lee Tan
School of Computing, National University of Singapore, Singapore
{sham, liyuchen, hebs, tankl}@comp.nus.edu.sg

ABSTRACT

As graph analytics often involves compute-intensive operations, GPUs have been extensively used to accelerate the processing. However, in many applications such as social networks, cyber security, and fraud detection, their representative graphs evolve frequently and one has to perform a rebuild of the graph structure on GPUs to incorporate the updates. Hence, rebuilding the graphs becomes the bottleneck of processing high-speed graph streams. In this paper, we propose a GPU-based dynamic graph storage scheme to support existing graph algorithms easily. Furthermore, we propose parallel update algorithms to support efficient stream updates so that the maintained graph is immediately available for high-speed analytic processing on GPUs. Our extensive experiments with three streaming applications on large-scale real and synthetic datasets demonstrate the superior performance of our proposed approach.

PVLDB Reference Format:
M. Sha, Y. Li, B. He, and K.-L. Tan. Accelerating Dynamic Graph Analytics on GPUs. *PVLDB*, 11(1): 107-120, 2017.
DOI: https://doi.org/10.14778/3136610.3136619

1. INTRODUCTION

Due to the rising complexity of data generated in the big data era, graph representations are used ubiquitously. Massive graph processing has emerged as the de facto standard of analytics on web graphs, social networks (e.g., Facebook and Twitter), sensor networks (e.g., Internet of Things) and many other application domains which involve high-dimensional data (e.g., recommendation systems). These graphs are often highly dynamic: network traffic data averages 10^9 packets/hour/router for large ISPs [23]; Twitter has 500 million tweets per day [41]. Since real-time analytics is fast becoming the norm [27, 12, 36, 44], it is critical for operations on dynamic massive graphs to be processed efficiently.

Dynamic graph analytics has a wide range of applications. Twitter can recommend information based on the up-to-date TunkRank (similar to PageRank) computed based on a dynamic attention graph [14] and cellular network operators can fix traffic hotspots in their networks as they are detected [28]. To achieve real-time performance, there is a growing interest to offload graph analytics to GPUs due to its much stronger arithmetical power and higher memory bandwidth compared with CPUs [45]. Although existing solutions, e.g. Medusa [58] and Gunrock [50], have explored GPU graph processing, we are aware of only one work [30] that considers a dynamic graph scenario which is a major gap for running analytics on GPUs. In fact, a delay in updating a dynamic graph may lead to undesirable consequences. For instance, consider an online travel insurance system that detects potential frauds by running ring analysis on profile graphs built from active insurance contracts [5]. Analytics on an outdated profile graph may fail to detect frauds which can cost millions of dollars. However, updating the graph will be too slow for issuing contracts and processing claims in real time, which will severely influence legitimate customers' user experience. This motivates us to develop an update-efficient graph structure on GPUs to support dynamic graph analytics.

There are two major concerns when designing a GPU-based dynamic graph storage scheme. First, the proposed storage scheme should handle both insertion and deletion operations efficiently. Though processing updates against insertion-only graph stream could be handled by reserving extra spaces to accommodate updates, this naïve approach fails to preserve the locality of the graph entries and cannot support deletions efficiently. Considering a common sliding window model on a graph edge stream, each element in the stream is an edge in a graph and analytic tasks are performed on the graph induced by all edges in the up-to-date window [51, 15, 17]. A naïve approach needs to access the entire graph in the sliding window to process deletions. This is obviously undesirable against high-speed streams. Second, the proposed storage scheme should be general enough for supporting existing graph formats on GPUs so that we can easily reuse existing static GPU graph processing solutions for graph analytics. Most large graphs are inherently *sparse*. To maximize the efficiency, existing works [6, 33, 32, 30, 53, 24] on GPU sparse graph processing rely on optimized data formats and arrange the graph entries in certain sorted order, e.g. CSR [33, 6] sorts the entries by their row-column ids. However, to the best of our knowledge, no schemes on GPUs can support efficient updates and maintain a sorted graph format at the same time, other than a rebuild. This motivates us to design an update-efficient

Proceedings of the VLDB Endowment, Vol. 11, No. 1

DOI: https://doi.org/10.14778/3136610.3136619

sparse graph storage scheme on GPUs while keeping the locality of the graph entries for processing analytics instantly.

In this paper, we introduce a GPU-based dynamic graph analytic framework followed by proposing the dynamic graph storage scheme on GPUs. Our preliminary study shows that a cache-oblivious data structure, i.e., Packed Memory Array (PMA [10, 11]), can potentially be employed for maintaining dynamic graphs on GPUs. PMA, originally designed for CPUs [10, 11], maintains sorted elements in a partially contiguous fashion by leaving gaps to accommodate fast updates with a constant bounded gap ratio. The simultaneously sorted and contiguous characteristic of PMA nicely fits the scenario of GPU streaming graph maintenance. However, the performance of PMA degrades when updates occur in locations which are close to each other, due to the unbalanced utilization of reserved spaces. Furthermore, as streaming updates often come in batches rather than one single update at a time, PMA does not support parallel insertions and it is non-trivial to apply PMA to GPUs due to its intricate update patterns which may cause serious thread divergence and uncoalesced memory access issues on GPUs.

We thus propose two GPU-oriented algorithms, i.e. GPMA and GPMA+, to support efficient parallel batch updates. GPMA explores a lock-based approach which becomes increasingly popular due to the recent GPU architectural evolution for supporting atomic operations [18, 29]. While GPMA works efficiently for the case where few concurrent updates conflict, e.g., small-size update batches with random updating edges in each batch, there are scenarios where massive conflicts occur and hence, we propose a lock-free approach, i.e. GPMA+. Intuitively, GPMA+ is a bottom-up approach by prioritizing updates that occur in similar positions. The update optimizations of our proposed GPMA+ are able to maximize coalesced memory access and achieve linear performance scaling w.r.t the number of computation units on GPUs, regardless of the update patterns.

The contributions of this paper are summarized as follows:

- We introduce a framework for GPU dynamic graph analytics and propose, the first of its kind, a GPU dynamic graph storage scheme to pave the way for real-time dynamic graph analytics on GPUs.
- We devise two GPU-oriented parallel algorithms: GPMA and GPMA+, to support efficient updates against high-speed graph streams.
- We conduct extensive experiments to show the performance superiority of GPMA and GPMA+. In particular, we design different update patterns on real and synthetic graph streams to validate the update efficiency of our proposed algorithms against their CPU counterparts as well as the GPU rebuild baseline. In addition, we implement three real world graph analytic applications on the graph streams to demonstrate the efficiency and broad applicability of our proposed solutions. In order to support larger graphs, we extend our proposed formats to multiple GPUs and demonstrate the scalability of our approach with multi-GPU systems.

The remainder of this paper is organized as follows. The related work is discussed in Section 2. Section 3 presents a general workflow of dynamic graph processing on GPUs. Subsequently, we describe GPMA and GPMA+ in Sections 4-5 respectively. Section 6 reports results of a comprehensive experimental evaluation. We conclude the paper and discuss some future works in Section 7.

2. RELATED WORK

In this section, we review related works in three different categories as follows.

2.1 Graph Stream Processing

Over the last decade, there has been an immense interest in designing efficient algorithms for processing massive graphs in the data stream model (see [36] for a detailed survey). This includes the problems of PageRank-styled scores [39], connectivity [21], spanners [20], counting subgraphs e.g. triangles [48] and summarization [46]. However, these works mainly focus on the theoretical study to achieve the best approximation solution with linear bounded space. Our proposed methods can incorporate existing graph stream algorithms with ease as our storage scheme can support most graph representations used in existing algorithms. Many systems have been proposed for streaming data processing, e.g. Storm [47], Spark Streaming [55], Flink [1]. Attracted by its massively parallel performance, several attempts have successfully demonstrated the advantages of using GPUs to accelerate data stream processing [49, 57].

However, the aforementioned systems focus on general stream processing and lack support for graph stream processing. Stinger [19] is a parallel solution to support dynamic graph analytics on a single machine. More recently, Kineograph [14], CellIQ [28] and GraphTau [27] are proposed to address the need for general time-evolving graph processing under the distributed settings. However, to our best knowledge, existing works focusing on CPU-based time-evolving graph processing will be inefficient on GPUs, because CPU and GPU are two architectures with different design principles and performance concerns in the parallel execution. We are aware of only one work [30] that explores the direction of using GPUs to process real-time analytics on dynamic graphs. However, this work only supports insertions and lacks an efficient indexing mechanism.

2.2 Graph Analytics on GPUs

Graph analytic processing is inherently data- and compute-intensive. Massively parallel GPU accelerators are powerful to achieve supreme performance of many applications. Compared with CPU, which is a general-purpose processor featuring large cache size and high single core processing capability, GPU devotes most of its die area to a large number of simple Arithmetic Logic Units (ALUs), and executes code in a SIMT (Single Instruction Multiple Threads) fashion. With the massive amount of ALUs, GPU offers orders of magnitude higher computational throughput than CPU in applications with ample parallelism. This leads to a spectrum of works which explore the usage of GPUs to accelerate graph analytics and demonstrate immense potentials. Examples include breath-first search (BFS) [33], subgraph query [32], PageRank [6] and many others. The success of deploying specific graph algorithms on GPUs motivates the design of general GPU graph processing systems like Medusa [58] and Gunrock [50]. However, the aforementioned GPU-oriented graph algorithms and systems assume static graphs. To handle dynamic graph scenario, existing works have to perform a rebuild on GPUs against each single update. DCSR [30] is the only solution, to the best of our knowledge, which is designed for insertion-only scenarios as it is based on linked edge block and rear appending technique. However, it does not support deletions or effi-

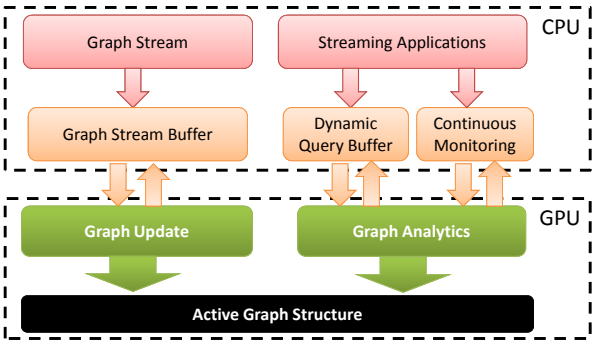


Figure 1: The dynamic graph analytic framework

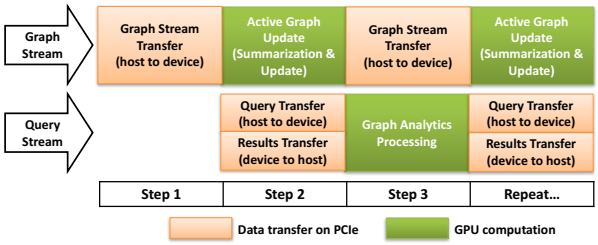


Figure 2: Asynchronous streams

cient searches. We propose GPMA to enable efficient dynamic graph updates (i.e. insertions and deletions) on GPUs in a fine-grained manner. In addition, existing graph analytics and systems optimized for GPUs can replace their storage layers directly with ease since the fundamental graph storage schemes used in existing works can be directly implemented on top of our proposed storage scheme.

2.3 Storage Formats on GPUs

Sparse matrix representation is a popular choice for storing large graphs on GPUs [3, 2, 58, 50]. The Coordinate Format [16] (COO) is the simplest format which only stores non-zero matrix entries by their coordinates with values. COO sorts all the non-zero entries by the entries' row-column key for fast entry accesses. CSR [33, 6] compresses COO's row indices into an offset array to reduce the memory bandwidth when accessing the sparse matrix. To optimize matrices with different non-zero distribution patterns, there exists many customized storage formats proposed, e.g., Block COO [52] (BCCOO), Blocked Row-Column [7] (BRC) and Tiled COO [53] (TCOO). Existing formats require to maintain a certain sorted order of their storage base units according to the unit's position in the matrix, e.g. entries for COO and blocks for BCCOO, and still ensure the locality of the units. As mentioned previously, few prior schemes can handle efficient sparse matrix updates on GPUs. To the best of our knowledge, PMA [10, 11] is a common structure which maintains a sorted array in a contiguous manner and supports efficient insertions/deletions. However, PMA is designed for CPU and no concurrent updating algorithm is ever proposed. Thus, we are motivated to propose GPMA and GPMA+ for supporting efficient concurrent updates on all existing storage formats.

3. A DYNAMIC FRAMEWORK ON GPUS

To address the need for real-time dynamic graph analytics, we offload the tasks of concurrent dynamic graph maintenance and its corresponding analytic processing to GPUs. In this section, we introduce a general GPU dynamic graph analytic framework. The design of the framework takes into account two major concerns: the framework should not only handle graph updates efficiently but also support existing GPU-oriented graph analytic algorithms without forfeiting their performance.

Model. We adopt a common sliding window graph stream model [36, 28, 46]. The sliding window model consists of an unbounded sequence of elements $(u, v)_t$[1] which indicates the edge (u, v) arrives at time t, and a sliding window which keeps track of the most recent edges. As the sliding window moves with time, new edges in the stream are inserted into the window and expiring edges are deleted. In real world applications, the sliding window of a graph stream can be used to monitor and analyze fresh social actions that appear on Twitter [51] or the call graph formed by the most recent CDR data [28]. In this paper, we focus on how to handle edge streams but our proposed scheme can also handle the dynamic *hyper graph* scenario with hyper edge streams.

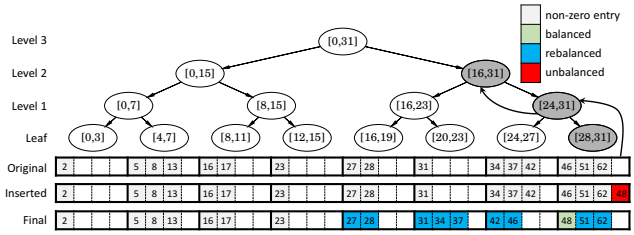
Apart from the sliding window model, the graph stream model which involves explicit insertions and deletions (e.g., a user requests to add or delete a friend in the social network) is also supported by our scheme as the proposed dynamic graph storage structure is designed to handle random update operations. That is, our system supports two kinds of updates, *implicit* ones generated from the sliding window mechanism and *explicit* ones generated from upper level applications or users.

The overview of the dynamic graph analytic framework is presented in Figure 1. Given a graph stream, there are two types of streaming tasks supported by our framework. The first type is the ad-hoc queries such as neighborhood and reachability queries on the graph which is constantly changing. The second type is the monitoring tasks like tracking PageRank scores. We present the framework by illustrating how to handle the graph streams and the corresponding queries while hiding data transfer between CPU and GPU, as follows:

Graph Streams. The graph stream buffer module batches the incoming graph streams on the CPU side (host) and periodically sends the updating batches to the graph update module located on GPU (device). The graph update module updates the "active" graph stored on the device by using the batch received. The "active" graph is stored in the format of our proposed GPU dynamic graph storage structure. The details of the graph storage structure and how to update the graph efficiently on GPUs will be discussed extensively in later sections.

Queries. Like the graph stream buffer, the dynamic query buffer module batches ad-hoc queries submitted against the stored active graph, e.g., queries to check the dynamic reachability between pairs of vertices. The tracking tasks will also be registered in the continuous monitoring module, e.g., tracking up-to-date PageRank. All ad-hoc queries and monitoring tasks will be transferred to the graph analytic module for GPU accelerated processing. The analytic module

[1]Our proposed framework handles both directed and undirected edges.



	Leaf	Level 1	Level 2	Level 3
segment size	4	8	16	32
density lower bound ρ	0.08	0.19	0.29	0.40
density upper bound τ	0.92	0.88	0.84	0.80
min # of entries	1	2	4	8
max # of entries	3	6	12	24

Figure 3: PMA insertion example (Left: PMA for insertion; Right: predefined thresholds)

interacts with the active graph to process the queries and the tracking tasks. Subsequently, the query results will be transferred back to the host. As most existing GPU graph algorithms use optimized array formats like CSR to accelerate the performance [18, 29, 35, 53], our proposed storage scheme provides an interface for storing the array formats. In this way, existing algorithms can be integrated into the analytic module with ease. We describe the details of the integration in Section 4.2.

Hiding Costly PCIe Transfer. Another critical issue on designing GPU-oriented systems is to minimize the data transfer between the host and the device through PCIe. Our proposed batching approach allows overlapping data transfer by concurrently running analytic tasks on the device. Figure 2 shows a simplified schedule with two asynchronous streams: graph streams and query streams respectively. The system is initialized at Step 1 where the batch containing incoming graph stream elements is sent to the device. At Step 2, while PCIe handles bidirectional data transfer for previous query results (device to host) and freshly submitted query batch (host to device), the graph update module updates the active graph stored on the device. At Step 3, the analytic module processes the received query batch on the device and a new graph stream batch is concurrently transferred from the host to the device. It is clear to see that, by repeating the aforementioned process, all data transfers are overlapped with concurrent device computations.

4. GPMA DYNAMIC GRAPH PROCESSING

To support dynamic graph analytics on GPUs, there are two major challenges discussed in the introduction. The first challenge is to maintain the dynamic graph storage in the device memory of GPUs for efficient update as well as compute. The second challenge is that the storage strategy should show its good compatibility with existing graph analytic algorithms on GPUs.

In this section, we discuss how to address the challenges with our proposed scheme. First, we introduce GPMA for GPU resident graph storage to simultaneously achieve update and compute efficiency (Section 4.1). Subsequently, we illustrate GPMA's generality in terms of deploying existing GPU based graph analytic algorithms (Section 4.2).

4.1 GPMA Graph Storage on GPUs

In this subsection, we first discuss the design principles our proposed dynamic graph storage should follow. Then we introduce how to implement our proposal.

Design Principles. The proposed graph storage on GPUs should take into account the following principles:

- The proposed dynamic graph storage should efficiently support a broad range of updating operations, including *insertions*, *deletions* and *modifications*. Furthermore, it should have a good locality to accommodate the highly parallel memory access characteristic of GPUs, in order to achieve high memory efficiency.
- The physical storage strategy should support common logical storage formats. Existing graph analytic solutions on GPUs based on such formats can be adapted easily.

Background of PMA. GPMA is primarily motivated by a novel structure, Packed Memory Array (PMA [10, 11]), which is proposed to maintain sorted elements in a partially continuous fashion by leaving gaps to accommodate fast updates with a bounded gap ratio. PMA is a self-balancing binary tree structure. Given an array of N entries, PMA separates the whole memory space into *leaf segments* with $O(\log N)$ length and defines *non-leaf segments* as the space occupied by their descendant segments. For any segment located at height i (leaf height is 0), PMA designs a way to assign the lower and upper bound density thresholds for the segment as ρ_i and τ_i respectively to achieve $O(\log^2 N)$ amortized update complexity. Once an insertion/deletion causes the density of a segment to fall out of the range defined by (ρ_i, τ_i), PMA tries to adjust the density by re-allocating all elements stored in the segment's parent. The adjustment process is invoked recursively and will only be terminated if all segments' densities fall back into the range defined by PMA's density thresholds. For an ordered array, modifications are trivial. Therefore, we mainly discuss insertions because deletions are the dual operation of insertions in PMA.

EXAMPLE 1. *Figures 3 presents an example for PMA insertion. Each segment is uniquely identified by an interval (starting and ending position of the array) displayed in the corresponding tree node, e.g., the root segment is* ***segment-[0,31]*** *as it covers all 32 spaces. All values stored in PMA are displayed in the array. The table in the figure shows predefined parameters including the segment size, the assignment of density thresholds (ρ_i, τ_i) and the corresponding minimum and maximum entry sizes at different heights of the tree. We use these setups as a running example throughout the paper. To insert an entry. i.e. 48, into PMA, the corresponding leaf segment is firstly identified by a binary search, and the new entry is placed at the rear of leaf segment. The insertion causes the density (=4) of the leaf segment to exceed the threshold (τ=3). Thus, we need to identify the nearest ancestor segment which can accommodate the insertion without violating the thresholds, i.e., the* ***segment-[16,31]****. Finally, the insertion is completed by re-dispatching all entries evenly in* ***segment-[16,31]****.*

LEMMA 1 ([10, 11]). *The amortized update complexity of PMA is proved to be $O(\log^2 N)$ in the worst case and $O(\log N)$ in the average case.*

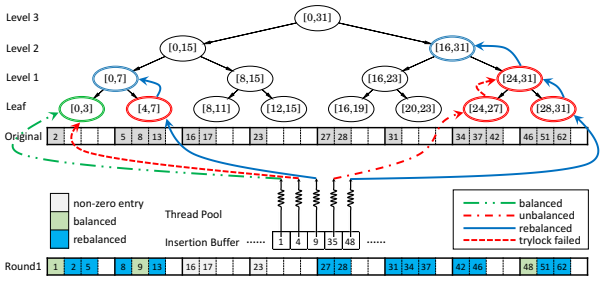


Figure 4: GPMA concurrent insertions

It is evident that PMA could be employed for dynamic graph maintenance as it maintains sorted elements efficiently with high locality on CPU. However, the update procedure described in [11] is inherently sequential and no concurrent algorithms have been proposed. To support batch updates of edge insertions and deletions for efficient graph stream analytic processing, we devise GPMA to support concurrent PMA updates on GPUs. Note that we focus on the insertion process for a concise presentation because the deletion process is a dual process w.r.t. the insertion process in PMA.

Concurrent Insertions in GPMA. Motivated by PMA on CPUs, we propose GPMA to handle a batch of insertions concurrently on GPUs. Intuitively, GPMA assigns an insertion to a thread and concurrently executes PMA algorithm for each thread with a lock-based approach to ensure consistency. More specifically, all leaf segments of insertions are identified in advance, and then each thread checks whether the inserted segments still satisfy their thresholds from bottom to top. For each particular segment, it is accessed in a mutually exclusive fashion. Moreover, all threads are synchronized after updating all segments located at the same tree height to avoid possible conflicts as segments at a lower height are fully contained in the segments at a higher level.

Algorithm 1 presents the pseudocode for GPMA concurrent insertions. We highlight the lines added to the original PMA update algorithm in order to achieve concurrent update of GPMA. As shown in line 2, all entries in the insertion set are iteratively tried until all of them take effect. For each iteration shown in line 9, all threads start at the leaf segments and attempt the insertions in a bottom-up fashion. If a particular thread fails the mutex competition in line 11, it aborts immediately and waits for the next attempt. Otherwise, it inspects the density of the current segment. If the current segment does not satisfy the density requirement, it will try the parent segment in the next loop iteration (lines 13-14). Once an ancestor segment is able to accommodate the insertion, it merges the new entry in line 16 and the entry is removed from the insertion set. Subsequently, the updated segment will re-dispatch all its entries evenly and the process is terminated.

Example 2. *Figure 4 illustrates an example with five insertions, i.e.* $\{1, 4, 9, 35, 48\}$*, for concurrent GPMA insertion. The initial structure is the same as in Example 1. After identifying the leaf segment for insertion, threads responsible for* ***Insertion-1*** *and* ***Insertion-4*** *compete for the same leaf segment. Assuming* ***Insertion-1*** *succeeds in getting the mutex,* ***Insertion-4*** *is aborted. Due to enough free space of the segment,* ***Insertion-1*** *is successfully inserted. Even though there is no leaf segment competition for* ***Insertions-9,35,48****, they should continue to inspect the corresponding parent segments because none of the left segments satisfy the density requirement after the insertions.* ***Insertions-35,48*** *still compete for the same level-1 segment and* ***Insertion-48*** *wins. For this example, three of the insertions are successful and the results are shown in the bottom of Figure 4.* ***Insertions-4,35*** *are aborted in this iteration and will wait for the next attempt.*

Algorithm 1 GPMA Concurrent Insertion

```
1:  procedure GPMAInsert(Insertions I)
2:      while I is not empty do
3:          parallel for i in I
4:              Seg s ← BinarySearchLeafSegment(i)
5:              TryInsert(s, i, I)
6:          synchronize
7:          release locks on all segments

8:  procedure TryInsert(Seg s, Insertion i, Insertions I)
9:      while s ≠ root do
10:         synchronize
11:         if fails to lock s then
12:             return                          ▷ insertion aborts
13:         if (|s| + 1)/capacity(s) ≥ τ then
14:             s ← parent segment of s
15:         else
16:             Merge(s, i)
17:             re-dispatch entries in s evenly
18:             remove i from I
19:             return                          ▷ insertion succeeds
20:     double the space of the root segment
```

4.2 Adapting Graph Algorithms to GPMA

Existing graph algorithms often use sparse matrix format to store the graph entries since most large graphs are naturally sparse[5]. Although many different sparse storage formats have been proposed, most of the formats assume a specific order to organize the non-zero entries. These formats enforce the order of the graph entries to optimize their specific access patterns, e.g., row-oriented (COO[2]), diagonal-oriented (JAD), and block-/tile-based (BCCOO, BRC and TCOO). It is natural that the ordered graph entries can be projected into an array and these similar formats can be supported by GPMA easily. Among all formats, we choose CSR as an example to illustrate how to adapt it to GPMA.

CSR as a case study. CSR is most widely used by existing algorithms on sparse matrices or graphs. CSR compresses COO's row indices into an offset array, which contributes to reducing the memory bandwidth when accessing the sparse matrix, and achieves a better workload estimation for skewed graph distribution (e.g., power-law distribution). The following example demonstrates how to implement CSR on GPMA.

Example 3. *In Figure 5, we have a graph of three vertices and six edges. The number on each edge denotes the weight of the corresponding edge. The graph is represented as a sparse matrix and is further transformed to the CSR format shown in the upper right. CSR sorts all non-zero entries in the row-oriented order, and compresses row indices*

[2]Generally, COO means ordered COO and it can also be column-oriented.

Algorithm 2 Breadth-First Search

```
1: procedure BFS(Graph G, Vertex s)
2:     for each vertex u ∈ G.V − {s} do
3:         u.visited = false
4:     Q ← φ
5:     s.visited ← true
6:     ENQUEUE(Q, s)
7:     while Q ≠ φ do
8:         u ← DEQUEUE(Q)
9:         for each v ∈ G.Adj[u] do
10:            if IsEntryExist(v) then
11:                if v.visited = false then
12:                    v.visited ← true
13:                    ENQUEUE(v)
```

Algorithm 3 GPU-based BFS Neighbour Gathering

```
1: procedure GATHER(Vertex frontier, Int csrOffset)
2:     {r, rEnd} ← csrOffset[frontier, frontier + 1]
3:     for (i ← r+threadId; i<rEnd; i+=threadNum) do
4:         if IsEntryExist(i) then ParallelGather(i)
```

into intervals as a row offset array. The lower part denotes the GPMA representation of this graph. In order to maintain the row offset array without synchronization among threads, we add a guard entry whose column index is ∞ during concurrent insertions. That is to say, when the guard is moved, the corresponding element in row offset array will change.

Given a graph stored on GPMA, the next step is to adapt existing graph algorithms to GPMA. In particular, how existing algorithms access the graph entries stored on GPMA is of vital importance. As for the CSR example, most algorithms access the entries by navigating through CSR's ordered array[18, 29, 35, 53]. We note that a CSR stored on GPMA is also an array which has bounded gaps interleaved with the graph entries. Thus, we are able to efficiently replace the operations of arrays with the operations of GPMA. We will demonstrate how we can do this replacement as follows.

Algorithm 2 illustrates the pseudocode of the classic BFS algorithm. We should pay attention to line 10, which is highlighted. Compared with the raw adjacency list, the applications based on GPMA need to guarantee the current vertex being traversed is a valid neighbour instead of an invalid space in GPMA's gap.

Algorithm 2 provides a high-level view for GPMA adaption. Furthermore, we present how it adapts GPMA in the parallel GPU environment with some low-level details. Algorithm 3 is the pseudocode of the *Neighbour Gathering* parallel procedure, which is a general primitive for most GPU-based vertex-centric graph processing models [37, 18, 22]. This primitive plays a role similar to line 10 of Algorithm 2 but in a parallel fashion in accessing the neighbors of a particular vertex. When traversing all neighbours of frontiers, *Neighbour Gathering* follows the SIMT manner, which means that there are *threadNum* threads as a group assigned to one of the vertex frontier and the procedure in Algorithm 3 is executed in parallel. For the index range (in the CSR on GPMA) of the current frontier given by csrOffset (shown in line 2), each thread will handle the corresponding tasks according to its *threadId*. For GPU-based BFS, the visited labels of neighbours for all frontiers will not be judged immediately after the neighbours are accessed. Instead, they will be compacted to contiguous memory in advance for higher memory efficiency. Similarly, we can also check for the existence of entries for other graph applications to adapt them to GPMA. To summarize, GPMA can be adapted to common graph analytic applications which are implemented in different representation and execution models, including matrix-based (e.g., PageRank), vertex-centric (e.g., BFS) and edge-centric (e.g., Connected Component).

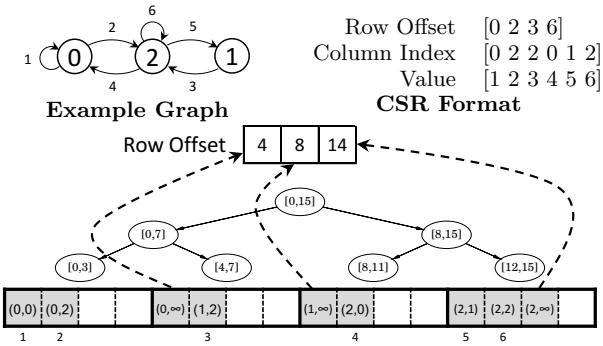


Figure 5: GPMA based on CSR

5. GPMA+: GPMA OPTIMIZATION

Although GPMA can support concurrent graph updates on GPUs, the update algorithm is basically a lock-based approach and can suffer from serious performance issue when different threads compete for the same lock. In this section, we propose a lock-free approach, i.e. GPMA+, which makes full utilization of GPU's massive multiprocessors. We carefully examine the performance bottleneck of GPMA in Section 5.1. Based on the issues identified, we propose GPMA+ for optimizing concurrent GPU updates with a lock-free approach in Section 5.2.

5.1 Bottleneck Analysis

The following four critical performance issues are identified for GPMA:

- **Uncoalesced Memory Accesses**: Each thread has to traverse the tree from the root segment to identify the corresponding leaf segment to be updated. For a group of GPU threads which share the same memory controller (including access pipelines and caches), memory accesses are uncoalesced and thus, cause additional IO overheads.
- **Atomic Operations for Acquiring Lock**: Each thread needs to acquire the lock before it can perform the update. Frequently invoking atomic operations for acquiring locks will bring huge overheads, especially for GPUs.
- **Possible Thread Conflicts**: When two threads conflict on a segment, one of them has to abort and wait for the next attempt. In the case where the updates occur on segments which are located proximately, GPMA will end up with low parallelism. As most real world large graphs have the power law property, the effect of thread conflicts can be exacerbated.
- **Unpredictable Thread Workload**: Workload balancing is another major concern for optimizing concurrent algorithms [45]. The workload for each thread in GPMA is unpredictable because: (1) It is impossible to obtain the last non-leaf segment traversed by each thread in advance;

(2) The result of lock competition is random. The unpredictable nature triggers the imbalanced workload issue for GPMA. In addition, threads are grouped as warps on GPUs. If a thread has a heavy workload, the remaining threads of the same warp are idle and cannot be re-scheduled.

5.2 Lock-Free Segment-Oriented Updates

Based on the discussion above, we propose GPMA+ to lift all bottlenecks identified. The proposed GPMA+ does not rely on lock mechanism and achieves high thread utilization simultaneously. Existing graph algorithms can be adapted to GPMA+ in the same manner as GPMA.

Compared with GPMA, which handles each update separately, GPMA+ concurrently processes updates based on the segments involved. It breaks the complex update pattern into existing concurrent GPU primitives to achieve maximum parallelism. There are three major components in the GPMA+ update algorithm:

(1) The updates are first sorted by their keys and then dispatched to GPU threads for locating their corresponding leaf segments according to the sorted order.
(2) The updates belonging to the same leaf segment are grouped for processing and GPMA+ processes the updates level by level in a bottom-up manner.
(3) At any particular level, we leverage GPU primitives to invoke all computing resources for segment updates.

We note that, the issue of *uncoalesced memory access* in GPMA is resolved by component (1) as the updating threads are sorted in advance to achieve similar traversal paths. Component (2) completely avoids the use of locks, which solves the problem of *atomic operations* and *thread conflicts*. Finally, component (3) makes use of GPU primitives to achieve *workload balancing* among all GPU threads.

We present the pseudocode for GPMA+'s segment-oriented insertion in the procedure GpmaPlusInsertion of Algorithm 4. Note that, similar to Section 4 (GPMA), we focus on presenting the insertions for GPMA+ and the deletions could be naturally inferred. The inserting entries are first sorted by their keys in line 2 and the corresponding segments are then identified in line 3. Given the update set U, GPMA+ processes updating segments level by level in lines 4-15 until all updates are executed successfully (line 11). In each iteration, UniqueInsertion in line 7 groups update entries belonging to the same segments into unique segments, i.e., S^*, and produces the corresponding index set I for quick accesses of update entries located in a segment from S^*. As shown in lines 19-20, UniqueSegments only utilizes standard GPU primitives, i.e. RunLenghtEncoding and ExclusiveScan. RunLenghtEncoding compresses an input array by merging runs of an element into a single element. It also outputs a count array denoting the length of each run. ExclusiveScan calculates, for each entry e in an array, the sum of all entries before e. Both primitives have very efficient parallelized GPU-based implementation which makes full utilization of the massive GPU cores. In our implementation, we use the NVIDIA CUB library [4] for these primitives. Given a set of unique updating segments, TryInsert+ first checks if a segment s has enough space for accommodating the updates by summing the valid entries in s (CountSegment) and the number of updates in s (CountUpdatesInSegment). If the density threshold is satisfied, the updates will be materialized by merging the inserting entries with existing entries in the segment (as shown in line 26). Subsequently, all entries in the segment will be re-dispatched to balance the densities. After TryInsert+, the algorithm will terminate if there are no entries to be updated. Otherwise, GPMA+ will advance to higher levels by setting all remaining segments to their parent segments (lines 12-15). The following example illustrates GPMA+'s segment-oriented updates.

Example 4. *Figure 6 illustrates an example for GPMA+ insertions with the same setup as in example 2. The left part is GPMA+'s snapshots in different rounds during this batch of insertions. The right part denotes the corresponding array information after the execution of each round. Five insertions are grouped into four corresponding leaf segments (denoted in different colors and their starting positions).*

For the first iteration at the leaf level, ***Insertions-1,4*** *of the first segment (denoted as red) are merged into the corresponding leaf segment, then its success flag is marked and will not be considered in the next round. The remaining intervals fail in this iteration and their corresponding segments will upgrade to their parent segments. It should be noted that the purple and the orange grids belong to the same parent segment and therefore, will be merged and then dispatched to their shared parent segment (as shown in Round 1). In this round, both segments (denoted as yellow and purple) cannot satisfy the density threshold, and their successful flags are not checked. In Round 2, both update segments can be merged by the corresponding insertions and no update segments will be considered in the next round since all of them are flagged.*

In Algorithm 4, TryInsert+ is the most important function as it handles all the corresponding insertions with no conflicts. Moreover, it achieves a balanced workload for each concurrent task. This is because GPMA+ handles the updates level by level and each segment to be updated in a particular level has exactly the same capacity. However, segments at different levels have different capacities. Intuitively, the probability of updating a segment with a larger size (a segment closer to the root) is much lower than that of a segment with a smaller size (a segment closer to the leaf). To optimize towards the GPU architecture, we propose the following optimization strategies for TryInsert+ for segments with different sizes.

- **Warp-Based**: For a segment with entries not larger than the warp size, the segment will be handled by a warp. Since all threads in the same warp are tied together and warp-based data is held by registers, updating a segment by a warp does not require explicit synchronization and will obtain superior efficiency.
- **Block-Based**: For a segment of which the data can be loaded in GPU's shared memory, block-based approach is chosen. Block-based approach executes all updates in the shared memory. As shared memory has much larger size than warp registers, block-based approach can handle large segments efficiently.
- **Device-Based**: For a segment with a size larger than the size of the shared memory, we handle them via global memory and rely on kernel synchronization. Device-based approach is slower than the two approaches above, but it has much less restriction on memory size (less than device memory amount) and is not invoked frequently.

We refer interested readers to the appendix of the extended technical report [42] for the detailed algorithm of the optimizations above.

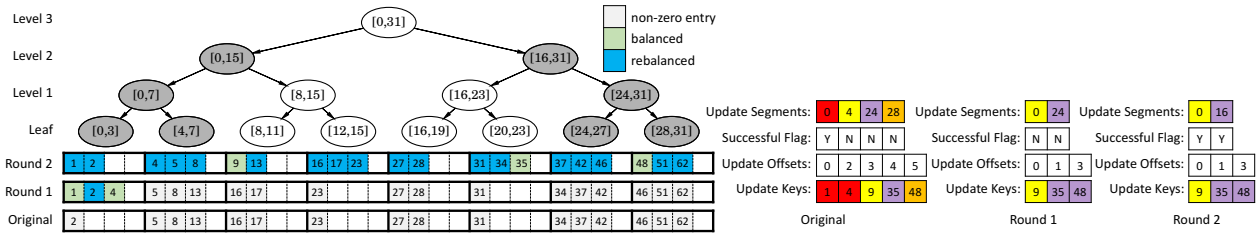


Figure 6: GPMA+ concurrent insertions (best viewed in color)

Algorithm 4 GPMA+ Segment-Oriented Insertion

```
1: procedure GpmaPlusInsertion(Updates U)
2:     Sort(U)
3:     Segs S ← BinarySearchLeafSegments(U)
4:     while root segment is not reached do
5:         Indices I ← ∅
6:         Segs S* ← ∅
7:         (S*, I) ← UniqueSegments(S)
8:         parallel for s ∈ S*
9:             TryInsert+(s, I, U)
10:        if U = ∅ then
11:            return
12:        parallel for s ∈ S
13:            if s does not contain any update then
14:                remove s from S
15:            s ← parent segment of s
16:    r ← double the space of the old root segment
17:    TryInsert+(r, ∅, U)

18: function UniqueSegments(Segs S)
19:    (S*, Counts) ← RunLengthEncoding(S)
20:    Indices I ← ExclusiveScan(Counts)
21:    return (S*, I)

22: procedure TryInsert+(Seg s, Indices I, Updates U)
23:    n_s ← CountSegment(s)
24:    U_s ← CountUpdatesInSegment(s,I,U)
25:    if (n_s + |U_s|)/capacity(s) < τ then
26:        Merge(s, U_s)
27:        re-dispatch entries in s evenly
28:        remove U_s from U
```

Theorem 1. *Given there are K computation units in the GPU, the amortized update performance of GPMA+ is $O(1+\frac{log^2 N}{K})$, where N is the maximum number of edges in the dynamic graph.*

Proof. Let X denote the set of updating entries contained in a batch. We consider the case where $|X| \geq K$ as it is rare to see $|X| < K$ in real world scenarios. In fact, our analysis works for cases where $|X| = O(K)$. The total update complexity consists of three parts: (1) sorting the updating entries; (2) searching the position of the entries in GPMA; (3) inserting the entries. We study these three parts separately below.

For part (1), the sorting complexity of $|X|$ entries on the GPU is $O(\frac{|X|}{K})$ since parallel radix sort is used (keys in GPMA are integers for storing edges). Then, the amortized sorting complexity is $O(\frac{|X|}{K})/|X| = O(1)$.

For part (2), the complexity of concurrently searching $|X|$ entries on GPMA is $O(\frac{|X|\cdot logN}{K})$ since each entry is assigned to one thread and the depth of traversal is the same for one thread (GPMA is a balanced tree). Thus, the amortized searching complexity is $O(\frac{|X|\cdot logN}{K})/|X| = O(\frac{logN}{K})$.

For part (3), we need to conduct a slightly complicated analysis. We denote the total insertion complexity of X with GPMA+ as $c^X_{\text{GPMA+}}$. As GPMA+ is updated level by level, $c^X_{\text{GPMA+}}$ can be decomposed into: $c^X_{\text{GPMA+}} = c_0 + c_1 + ... + c_h$ where h is the height of the PMA tree.

Given any level i, let z_i denote the number of segments to be updated by GPMA+. Since all segments at level i have the same size, we denote p_i as the sequential complexity to update any segment $s_{i,j}$ at level i (TryInsert+ in Algorithm 4). GPMA+ evenly distributes the computing resources to each segment. As processing each segment only requires a constant number of scans on the segment by GPU primitives, the complexity for GPMA+ to process level i is $c_i = \frac{p_i \cdot z_i}{K}$. Thus we have:

$$c^X_{\text{GPMA+}} = \sum_{i=0,..,h} \frac{p_i \cdot z_i}{K} \leq \frac{1}{K} \sum_{x \in X} c^x_{\text{PMA}}$$

where c^x_{PMA} is the sequential complexity for PMA to process the update of a particular entry $x \in X$. The inequality holds because for each segment updated by GPMA+, it must be updated at least once by a sequential PMA process. With Lemma 1, we have $c^x_{\text{PMA}} = O(log^2 N)$ and thus $c^X_{\text{GPMA+}} = O(\frac{|X|\cdot log^2 N}{K})$. Then the amortized complexity to update one single entry under the GPMA scheme naturally follows as $O(1 + \frac{log^2 N}{K})$.

Finally, we conclude the proof by combining the complexities from all three parts. □

Theorem 1 proves that the speedups of GPMA+ over sequential PMA is linear to the number of processing units available on GPUs, which showcases the theoretical scalability of GPMA+.

6. EXPERIMENTAL EVALUATION

In this section, we present the experimental evaluation of our proposed methods. First, we present the setup of the experiments. Second, we examine the update costs of different schemes for maintaining dynamic graphs. Finally, we implement three different applications to show the performance and the scalability of the proposed solutions.

6.1 Experimental Setup

Datasets. We collect two real world graphs (Reddit and Pokec) and synthesize two random graphs (Random and Graph500) to test the proposed methods. The datasets are

Table 1: Experimented Graph Algorithms and the Compared Approaches

Compared Approaches	Graph Container	BFS	ConnectedComponent	PageRank
CPU Approaches	AdjLists	Standard Single Thread Algorithms		
	PMA [10, 11]			
	Stinger [19]	Stinger built-in Parallel Algorithms		
GPU Approaches	cuSparseCSR [3]	D. Merrill et al.[37]	J. Soman et al.[43]	CUSP SpMV [2]
	GPMA/GPMA+			

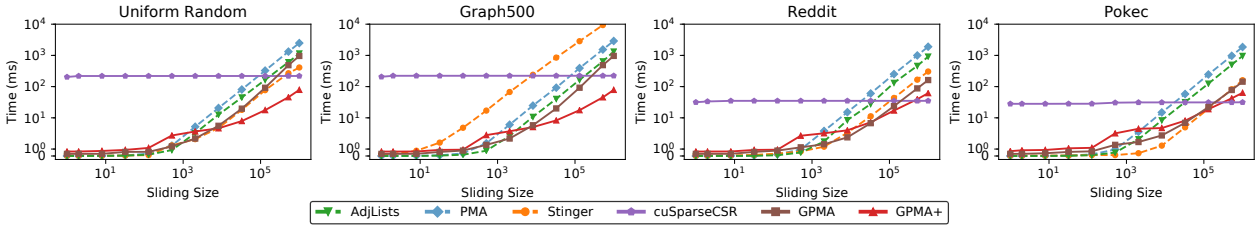


Figure 7: Performance comparison for updates with different batch sizes. The dashed lines represent CPU-based solutions whereas the solid lines represent GPU-based solutions.

Table 2: Statistics of Datasets

Datasets	$\|V\|$	$\|E\|$	$\|E\|/\|V\|$	$\|E_s\|$	$\|E_s\|/\|V\|$
Reddit	2.61M	34.4M	13.2	17.2M	6.6
Pokec	1.60M	30.6M	19.1	15.3M	9.6
Graph500	1.00M	200M	200	100M	100
Random	1.00M	200M	200	100M	100

described as follows and their statistics are summarized in Table 2.

- Reddit is an online forum where user actions include post and comment. We collect all comment actions from a public resource[3]. Each comment of a user b to a post from another user a is associated with an edge from a to b, and the edge indicates an action of a has triggered an action of b. As each comment is labeled with a timestamp, it naturally forms a dynamic influence graph.
- Pokec is the most popular online social network in Slovakia. We retrieve the dataset from SNAP [31]. Unlike other online datasets, Pokec contains the whole network over a span of more than 10 years. Each edge corresponds to a friendship between two users.
- Graph500 is a synthetic dataset obtained by using the Graph500 RMAT generator [38] to synthesize a large power law graph.
- Random is a random graph generated by the Erdős-Renyi model. Specifically, given a graph with n vertices, the random graph is generated by including each edge with probability p. In our experiments, we generate a Erdős-Renyi random graph with 0.02% of non-zero entries against a full clique.

Stream Setup. In our datasets, Reddit has a timestamp on every edge whereas the other datasets do not possess timestamps. As commonly used in existing graph stream algorithms [56, 54, 39], we randomly set the timestamps of all edges in the Pokec, Graph500 and Random datasets. Then, the graph stream of each dataset receives the edges with increasing timestamps.

For each dataset, a dynamic graph stream is initialized with a subgraph consisting of the dataset's first half of its total edges according to the timestamps, i.e., E_s in Table 2 denotes the initial edge set of a dynamic graph before the stream starts. To demonstrate the update performance of both insertions and deletions, we adopt a sliding window setup where the window contains a fixed number of edges. Whenever the window slides, we need to update the graph by deleting expired edges and inserting arrived edges until there are no new edges left in the stream.

[3]https://www.kaggle.com/reddit/reddit-comments-may-2015

Applications. We conduct experiments on three most widely used graph applications to showcase the applicability and the efficiency of GPMA+.

- **BFS** is a key graph operation which is extensively studied in previous works on GPU graph processing [25, 34, 13]. It begins with a given vertex (or *root*) of an unweighted graph and iteratively explores all connected vertices. The algorithm will assign a minimum distance away from the root vertex to every visited vertex after it terminates. In the streaming scenario, after each graph update, we select a random root vertex and perform BFS from the root to explore the entire graph.
- **Connected Component** is another fundamental algorithm which has been extensively studied under both CPU [26] and GPU [43] environment. It partitions the graph in the way that all vertices in a partition can reach the others in the same partition and cannot reach vertices from other partitions. In the streaming context, after each graph update, we run the ConnectedComponent algorithm to maintain the up-to-date partitions.
- **PageRank** is another popular benchmarking application for large scale graph processing. Power iteration method is a standard method to evaluate the PageRank where the Sparse Matrix Vector Multiplication (SpMV) kernel is recursively executed between the graph's adjacency matrix and the PageRank vector. In the streaming scenario, whenever the graph is updated, the power iteration is invoked and it obtains the up-to-date PageRank vector by operating on the updated graph adjacency matrix and the PageRank vector obtained in the previous iteration. In our experiments, we follow the standard setup by setting the damping factor to 0.85 and we terminate the power iteration once the 1-norm error is less than 10^{-3}.

These three applications have different memory and computation requirements. BFS requires little computation but

performs frequent random memory accesses, and PageRank using SpMV accesses the memory sequentially and it is the most compute-intensive task among all three applications.

Maintaining Dynamic Graph. We adopt the CSR [33, 6] format to represent the dynamic graph maintained. Note that all approaches proposed in the paper are not restricted to CSR but general enough to incorporate any popular representation formats like COO [16], JAD [40], HYB [9, 35] and many others. To evaluate the update performance of our proposed methods, we compare different graph data structures and respective approaches on both CPUs and GPUs.

- AdjLists (CPU). AdjLists is a basic approach for CSR graph representation. As the CSR format sorts all entries according to their row-column indices, we implement AdjLists with a vector of $|V|$ entries for $|V|$ vertices and each entry is a RB-Tree to denote all (out)neighbors of each vertex. The insertions/deletions are operated by TreeSet insertions/deletions.
- PMA (CPU). We implement the original CPU-based PMA and adopt it for the CSR format. The insertions/deletions are operated by PMA insertions/deletions.
- Stinger (CPU). We compare the graph container structure used in the state-of-the-art CPU-based parallel dynamic graph analytic system, Stinger [19]. The updates are handled by the internal logic of Stinger.
- cuSparseCSR (GPU). We also compare with the GPU-based CSR format used in the NVIDIA cuSparse library [3]. The updates are executed by calling the rebuild function in the cuSparse library.
- GPMA/GPMA+. These are our proposed approaches. Although insertions and deletions could be handled similarly, in the sliding window models where the numbers of insertions and deletions are often equal, the lazy deletions can be performed via marking the location as deleted without triggering the density maintenance and recycling for new insertions.

Note that we do not compare with DCSR [30] because, as discussed in Section 2.2, the scheme can neither handle deletions nor support efficient searches, which makes it incomparable to all schemes proposed in this paper.

To validate if using the dynamic graph format proposed in this paper affects the performance of graph algorithms, we implement the state-of-the-art GPU-based algorithms on the CSR format maintained by GPMA/GPMA+ as well as cuSparseCSR. Meanwhile, we invoke Stinger's built-in APIs to handle the same workloads of the graph algorithms, which are considered as the counterpart of GPU-based approaches in highly parallel CPU environment. Finally, we implement the standard single-threaded algorithms for each application in AdjLists and PMA as baselines for thorough evaluation. The details of all compared solutions for each application is summarized in Table 1.

Experimental Environment. All algorithms mentioned in the remaining part of this section are implemented with CUDA 7.5 and GCC 4.8.4 with -O3 optimization. All experiments except Stinger run on a CentOS server which has Intel(R) Core i7-5820k (6-cores, 3.30GHz) with 64GB main memory and three GeForce TITAN X GPUs (each has 12GB device memory), connected with PCIe v3.0. Stinger baselines run on a multi-core server which is deployed 4-way Intel(R) Xeon(R) CPU E7-4820 v3 (40-cores, 1.90GHz) with 128GB main memory.

6.2 The Performance of Handling Updates

In this subsection, we compare the update costs for different update approaches. As previously mentioned, we start with the initial subgraph consisting of each dataset's first half of total edges. We measure the average update time where the sliding window iteratively shifts for a batch of edges. To evaluate the impact of update batch sizes, the batch size is set to range from one edge and exponentially grow to one million edges with base two. Figure 7 shows the average latency for all approaches with different sliding batch sizes. Note that the x-axis and y-axis are plotted in log scales. We have also tested sorted graph streams to evaluate extreme cases. We omit the detailed results due to limited space and interested readers are referred to [42].

We observe that, PMA-based approaches are very efficient in handling updates when the batch size is small. As batch size becomes larger, the performance of PMA and GPMA quickly degrades to the performance of simple rebuild. Although GPMA achieves better performance than GPMA+ for small batches since the concurrent updating entries are unlikely to conflict, thread conflicts become serious for larger batches. Due to its lock-free characteristic, GPMA+ shows superior performance over PMA and GPMA. In particular, GPMA+ has speedups of up to 20.42x and 18.30x against PMA and GPMA respectively. Stinger shows impressive update performance in most cases as Stinger efficiently updates its dynamic graph structure in a parallel fashion and the code runs on a powerful multi-core CPU system. For now, a multi-core CPU system is considered more powerful than GPUs for pure random data structure maintenance but costs more (in our experimental setup, our CPU server costs more than 5 times that of the GPU server). Moreover, we also note that, Stinger shows extremely poor performance in the Graph500 dataset. According to the previous study [8], the phenomenon is due to the fact that Stinger holds a fixed size of each edge block. Since Graph500 is a heavily skewed graph as the graph follows the power law model, the skewness causes severe performance deficiency in the utilization of memory for Stinger.

We observe the sharp increase for GPMA+ performance curves occur when the batch size is 512. This is because the multi-level strategy is used in GPMA+ (which is mentioned in Section 5.2) and shared-memory constraint cannot support batch size which is more than 512 on our hardware. Finally, the experiments show that, GPMA is faster than GPMA+ when the update batch is smaller and leads to few thread conflicts, because the GPMA+ logic is more complicated and includes overheads by a number of kernel calls. However, using GPMA only benefits when the update batch is extremely small and the performance gain in such extreme case is also negligible compared with GPMA+. Hence, we can conclude that GPMA+ shows its stability and efficiency across different update patterns compared with GPMA, and we will only show the results of GPMA+ in the remaining experiments.

6.3 Application Performance

As previously mentioned, all compared application-specific approaches are summarized in Table 1. We find that integrating GPMA+ into an existing GPU-based implementation requires little modification. The main one is in transforming the array operations in the original implementation to the operations on GPMA+, as presented in Section 4.2. The intentions of this subsection are two-fold. First, we test if using

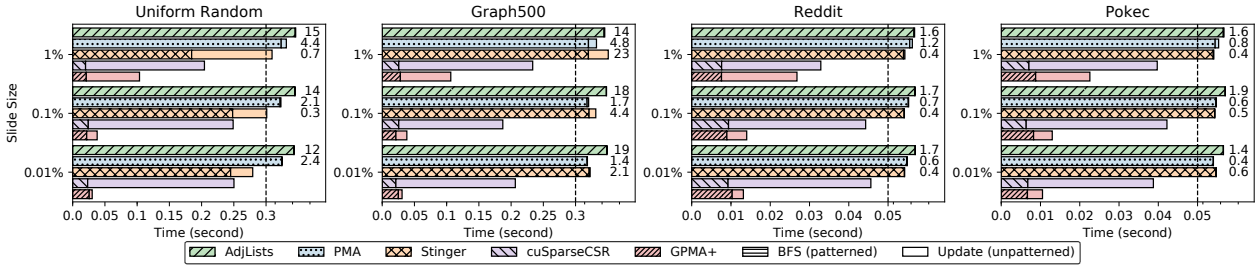


Figure 8: Streaming BFS

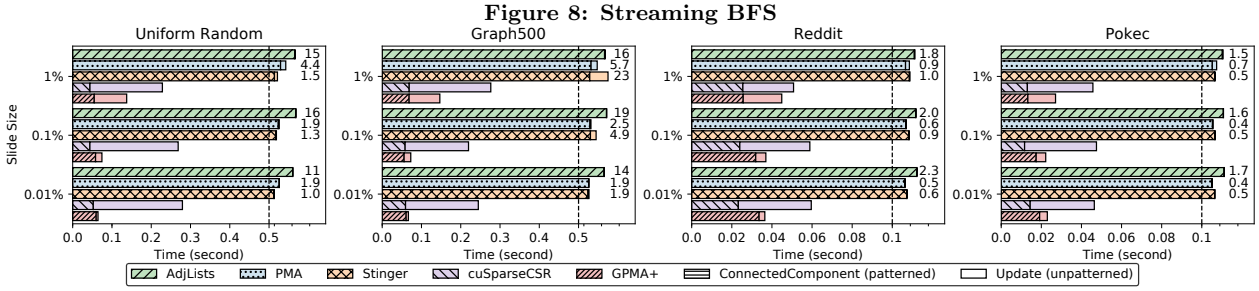


Figure 9: Streaming Connected Component

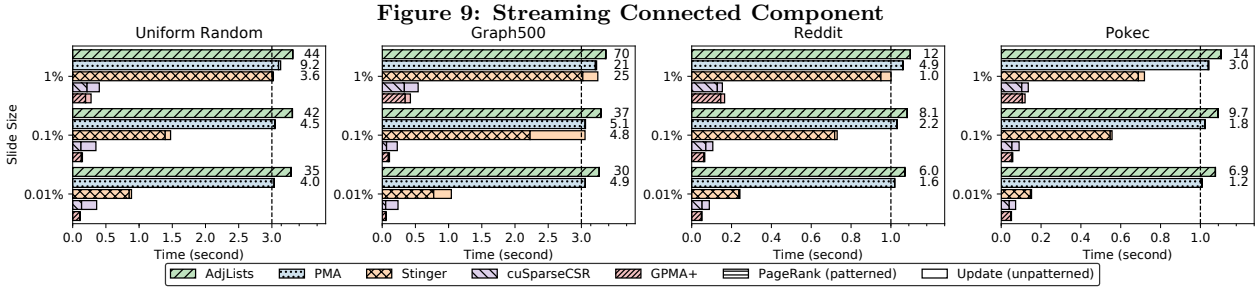


Figure 10: Streaming PageRank

the PMA-like data structure to represent the graph brings significant overheads for the graph algorithms. Second, we demonstrate how the update performance affects the overall efficiency of dynamic graph processing.

In the remaining part of this section, we present the performance of different approaches by showing their average elapsed time to process a shift of the sliding window with three different batch sizes, i.e., the batches contain 0.01%, 0.1% and 1% edges of the respective dataset. We have also tested the graph stream with explicit random insertions and deletions for all applications as an extended experiment. We omit the detailed results here since they are similar to the results of the sliding window model and we refer interested readers to [42]. We distinguish the time spent on updates and analytics with different patterns among all figures.

BFS Results: Figure 8 presents the results for BFS. Although processing BFS only accesses each edge in the graph once, it is still an expensive operation because BFS can potentially scan the entire graph. This has led to the observation that CPU-based approach takes significant amount of time for BFS computation whereas the update time is comparatively negligible. Thanks to the massive parallelism and high memory bandwidth of GPUs, GPU-based approaches are much more efficient than CPU-based approaches for BFS computation as well as the overall performance. For the cuSparseCSR approach, the rebuild process is the bottleneck as the update needs to scan the entire group multiple times. In contrast, GPMA+ takes much shorter time for the update and has nearly identical BFS performance compared with cuSparseCSR. Thus, GPMA+ dominates the comparisons in terms of the overall processing efficiency.

We have also tested our framework in terms of hiding data transfer over PCIe by using asynchronous streams to concurrently perform GPU computation and PCIe transfer. In Figure 11, we show the results when running concurrent execution by using the GPMA+ approach. The data transfer consists of two parts: sending graph updates and fetching updated distance vector (from the query vertex to all other vertices). It is clear from the figure that, under any circumstances, sending graph updates is overlapped by GPMA+ update processing and fetching the distance vector is overlapped by BFS computation. Thus, the data transfer is completely hidden in the concurrent streaming scenario. As the observations remain similar in other applications, we omit their results and explanations, and the details can be found in the appendix of our extended technical report [42].

Connected Component Results: Figure 9 presents the results for running ConnectedComponent on the dynamic graphs. The results show different performance patterns compared with BFS as ConnectedComponent takes more time in processing which is caused by a number of graph traversal passes to extract the partitions. Meanwhile, the update cost remains the same. Thus, GPU-based solutions enhance their performance superiority over CPU-based solutions. Nevertheless, the update process of cuSparseCSR is still expensive compared with the time spent on Connected-

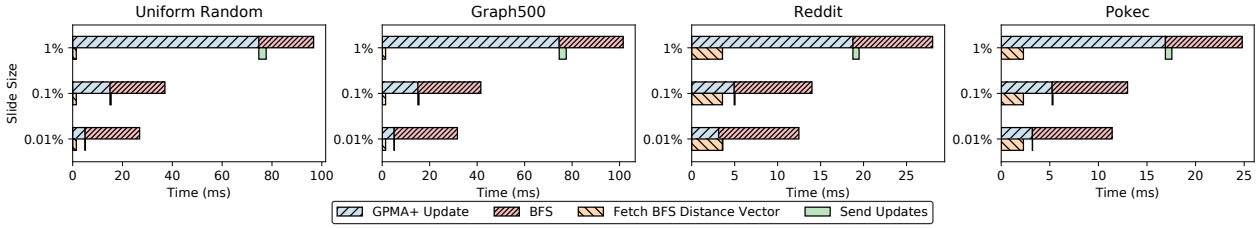


Figure 11: Concurrent data transfer and BFS computation with asynchronous stream

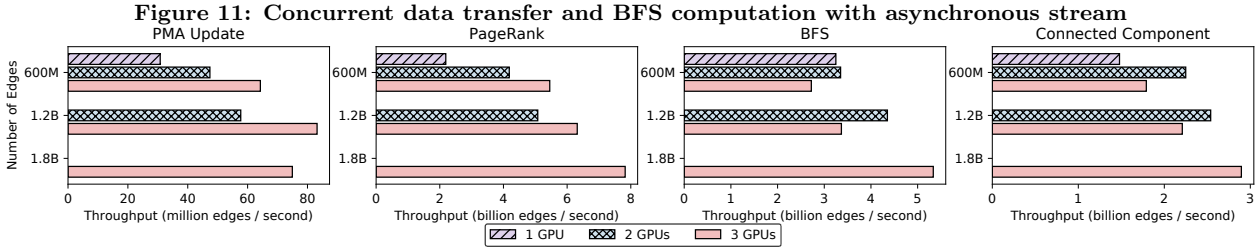


Figure 12: Multi-GPU performance on different sizes of `Graph500` datasets

`Component`. `GPMA+` is very efficient in processing the updates. Although we have observed that, in the `Reddit` and the `Pokec` datasets, `GPMA+` shows some discrepancies for running the graph algorithm against `cuSparseCSR` due to the "holes" introduced in the graph structure, the discrepancies are insignificant considering the huge performance boosts for updates. Thus, `GPMA+` still dominates the rebuild approach for overall performance.

PageRank Results: Figure 10 presents the results for `PageRank`. `PageRank` is a compute-intensive task where the `SpMV` kernel is iteratively invoked on the entire graph until the PageRank vector converges. The pattern follows from previous results: CPU-based solutions are dominated by GPU-based approaches because iterative `SpMV` is a more expensive process than `BFS` and `ConnectedComponent`, and GPU is designed to handle massively parallel computation like `SpMV`. Although `cuSparseCSR` shows inferior performance compared with `GPMA+`, the improvement brought by `GPMA+`'s efficient update is not as significant as that in previous applications since the update costs are small compared with the cost of iterative `SpMV` kernel calls. Nevertheless, the dynamic structure of `GPMA+` does not affect the efficiency of the `SpMV` kernel and `GPMA+` outperforms other approaches in all experiments.

6.4 Scalability

`GPMA` and `GPMA+` can also be extended to multiple GPUs to support graphs with a size larger than the device memory of one GPU. To showcase the scalability of our proposed framework, we implement the multi-GPU version of `GPMA+` and then carry out experiments of the aforementioned graph analytic applications.

We generate three large datasets using `Graph500` with increasing numbers of edges (600 Million, 1.2 Billion and 1.8 Billion) and conduct the same performance experiments in section 6.3 with 1% slide size, on 1, 2 and 3 GPUs respectively. We evenly partition graphs according to the vertex index and synchronize all devices after each iteration. For a fair comparison among different datasets, we use throughput as our performance metric. The experimental results of `GPMA+` updates and application performance are illustrated in Figure 12. We do not compare with `Stinger` because in this subsection, we focus on the evaluation on the scalability of `GPMA+`. The memory consumption of `Stinger` exceeds our machine's 128GB main memory based on its default configuration in the standalone mode.

Multiple GPUs can extend the memory capacity so that analytics on larger graphs can be executed. According to Figure 12, the improvement in terms of throughput for multiple GPUs behaves differently in various applications. For `GPMA+` update and `PageRank`, we achieve a significant improvement with more GPUs, because their workloads between communications are relatively compute-intensive. For `BFS` and `ConnectedComponent`, the experimental results demonstrate a tradeoff between overall computing power and communication cost with increasing number of GPUs, as these two applications incur larger communication cost. Nevertheless, multi-GPU graph processing is an emerging research area and more effectiveness optimizations are left as future work. Overall, this set of preliminary experiments shows that our proposed scheme is capable of supporting large scale dynamic graph analytics.

7. CONCLUSION & FUTURE WORK

In this paper, we address how to dynamically update the graph structure on GPUs in an efficient manner. First, we introduce a GPU dynamic graph analytic framework, which enables existing static GPU-oriented graph algorithms to support high-performance evolving graph analytics. Second, to avoid the rebuild of the graph structure which is a bottleneck for processing dynamic graphs on GPUs, we propose `GPMA` and `GPMA+` to support incremental dynamic graph maintenance in parallel. We prove the scalability and complexity of `GPMA+` theoretically and evaluate the efficiency through extensive experiments. As future work, we would like to explore a hybrid CPU-GPU approach for dynamic graph processing and more effective optimizations for involved applications.

8. ACKNOWLEDGEMENT

The project is partially supported by a MoE Tier 2 grant (MOE2017-T2-1-141) in Singapore. Bingsheng's work is in part supported by a MoE AcRF Tier 1 grant (T1 251RES1610) in Singapore.

9. REFERENCES

[1] Apache flink. https://flink.apache.org/. Accessed: 2016-10-18.

[2] Cusp library. https://developer.nvidia.com/cusp. Accessed: 2017-03-25.

[3] cusparse. https://developer.nvidia.com/cusparse. Accessed: 2016-11-09.

[4] CUDA UnBound (CUB) library. https://nvlabs.github.io/cub/, 2015.

[5] L. Akoglu, H. Tong, and D. Koutra. Graph based anomaly detection and description: a survey. *Data Min. Knowl. Discov.*, 29(3):626–688, 2015.

[6] A. Ashari, N. Sedaghati, J. Eisenlohr, S. Parthasarathy, and P. Sadayappan. Fast sparse matrix-vector multiplication on gpus for graph applications. In *SC*, pages 781–792, 2014.

[7] A. Ashari, N. Sedaghati, J. Eisenlohr, and P. Sadayappan. An efficient two-dimensional blocking strategy for sparse matrix-vector multiplication on gpus. In *ICS*, pages 273–282, 2014.

[8] D. A. Bader, J. Berry, A. Amos-Binks, D. Chavarría-Miranda, C. Hastings, K. Madduri, and S. C. Poulos. Stinger: Spatio-temporal interaction networks and graphs (sting) extensible representation. *Georgia Institute of Technology, Tech. Rep*, 2009.

[9] N. Bell and M. Garland. Efficient sparse matrix-vector multiplication on CUDA. Technical Report NVR-2008-004, NVIDIA Corporation, 2008.

[10] M. A. Bender, E. D. Demaine, and M. Farach-Colton. Cache-oblivious b-trees. *SIAM J. Comput.*, 35(2):341–358, 2005.

[11] M. A. Bender and H. Hu. An adaptive packed-memory array. *ACM Trans. Database Syst.*, 32(4), 2007.

[12] L. Braun, T. Etter, G. Gasparis, M. Kaufmann, D. Kossmann, D. Widmer, A. Avitzur, A. Iliopoulos, E. Levy, and N. Liang. Analytics in motion: High performance event-processing and real-time analytics in the same database. In *SIGMOD*, pages 251–264, 2015.

[13] F. Busato and N. Bombieri. Bfs-4k: an efficient implementation of bfs for kepler gpu architectures. *TPDS*, 26(7):1826–1838, 2015.

[14] R. Cheng, J. Hong, A. Kyrola, Y. Miao, X. Weng, M. Wu, F. Yang, L. Zhou, F. Zhao, and E. Chen. Kineograph: Taking the pulse of a fast-changing and connected world. In *EuroSys*, pages 85–98, 2012.

[15] M. S. Crouch, A. McGregor, and D. Stubbs. Dynamic graphs in the sliding-window model. In *European Symposium on Algorithms*, pages 337–348. Springer, 2013.

[16] H.-V. Dang and B. Schmidt. The sliced coo format for sparse matrix-vector multiplication on cuda-enabled gpus. *Procedia Computer Science*, 9:57–66, 2012.

[17] M. Datar, A. Gionis, P. Indyk, and R. Motwani. Maintaining stream statistics over sliding windows. *SIAM journal on computing*, 31(6):1794–1813, 2002.

[18] A. Davidson, S. Baxter, M. Garland, and J. D. Owens. Work-efficient parallel gpu methods for single-source shortest paths. In *Parallel and Distributed Processing Symposium, 2014 IEEE 28th International*, pages 349–359. IEEE, 2014.

[19] D. Ediger, R. McColl, E. J. Riedy, and D. A. Bader. STINGER - High performance data structure for streaming graphs. *HPEC*, 2012.

[20] M. Elkin. Streaming and fully dynamic centralized algorithms for constructing and maintaining sparse spanners. *ACM Trans. Algorithms*, 7(2):20:1–20:17, 2011.

[21] J. Feigenbaum, S. Kannan, A. McGregor, S. Suri, and J. Zhang. On graph problems in a semi-streaming model. *Theor. Comput. Sci.*, 348(2-3):207–216, 2005.

[22] Z. Fu, M. Personick, and B. Thompson. *MapGraph: A High Level API for Fast Development of High Performance Graph Analytics on GPUs*. A High Level API for Fast Development of High Performance Graph Analytics on GPUs. ACM, New York, New York, USA, June 2014.

[23] S. Guha and A. McGregor. Graph synopses, sketches, and streams: A survey. *PVLDB*, 5(12):2030–2031, 2012.

[24] W. Guo, Y. Li, M. Sha, and K.-L. Tan. Parallel personalized pagerank on dynamic graphs. *PVLDB*, 11(1), 2017.

[25] P. Harish and P. Narayanan. Accelerating large graph algorithms on the gpu using cuda. In *International Conference on High-Performance Computing*, pages 197–208. Springer, 2007.

[26] D. S. Hirschberg. Parallel algorithms for the transitive closure and the connected component problems. In *Proceedings of the eighth annual ACM symposium on Theory of computing*, pages 55–57. ACM, 1976.

[27] A. P. Iyer, L. E. Li, T. Das, and I. Stoica. Time-evolving graph processing at scale. In *Proceedings of the Fourth International Workshop on Graph Data Management Experiences and Systems*, pages 5:1–5:6, 2016.

[28] A. P. Iyer, L. E. Li, and I. Stoica. Celliq : Real-time cellular network analytics at scale. In *NSDI*, pages 309–322, 2015.

[29] R. Kaleem, A. Venkat, S. Pai, M. Hall, and K. Pingali. Synchronization trade-offs in gpu implementations of graph algorithms. In *Parallel and Distributed Processing Symposium, 2016 IEEE International*, pages 514–523. IEEE, 2016.

[30] J. King, T. Gilray, R. M. Kirby, and M. Might. Dynamic sparse-matrix allocation on gpus. In *ISC*, pages 61–80, 2016.

[31] J. Leskovec and R. Sosič. Snap: A general-purpose network analysis and graph-mining library. *TIST*, 8(1):1, 2016.

[32] X. Lin, R. Zhang, Z. Wen, H. Wang, and J. Qi. Efficient subgraph matching using gpus. In *ADC*, pages 74–85, 2014.

[33] H. Liu, H. H. Huang, and Y. Hu. ibfs: Concurrent breadth-first search on gpus. In *SIGMOD*, pages 403–416, 2016.

[34] L. Luo, M. Wong, and W.-m. Hwu. An effective gpu implementation of breadth-first search. In *DAC*, pages 52–55, 2010.

[35] M. Martone, S. Filippone, S. Tucci, P. Gepner, and M. Paprzycki. Use of hybrid recursive csr/coo data

structures in sparse matrix-vector multiplication. In *IMCSIT*, pages 327–335. IEEE, 2010.

[36] A. McGregor. Graph stream algorithms: A survey. *SIGMOD Rec.*, 43(1):9–20, 2014.

[37] D. Merrill, M. Garland, and A. Grimshaw. High-Performance and Scalable GPU Graph Traversal. *TOPC*, 1(2), 2015.

[38] R. C. Murphy, K. B. Wheeler, B. W. Barrett, and J. A. Ang. Introducing the graph 500. 2010.

[39] N. Ohsaka, T. Maehara, and K.-i. Kawarabayashi. Efficient pagerank tracking in evolving networks. In *KDD*, pages 875–884, 2015.

[40] Y. Saad. Numerical solution of large nonsymmetric eigenvalue problems. *Computer Physics Communications*, 53(1):71–90, 1989.

[41] D. Sayce. 10 billions tweets, number of tweets per day. `http://www.dsayce.com/social-media/10-billions-tweets/`. Accessed: 2016-10-18.

[42] M. Sha, Y. Li, B. He, and K.-L. Tan. Technical report: Accelerating dynamic graph analytics on gpus. *arXiv preprint arXiv:1709.05061*, 2017.

[43] J. Soman, K. Kothapalli, and P. J. Narayanan. A fast GPU algorithm for graph connectivity. *IPDPS Workshops*, 2010.

[44] M. Stonebraker, U. Çetintemel, and S. Zdonik. The 8 requirements of real-time stream processing. *ACM SIGMOD Record*, 34(4):42–47, 2005.

[45] J. A. Stratton, N. Anssari, C. Rodrigues, I.-J. Sung, N. Obeid, L. Chang, G. D. Liu, and W.-m. Hwu. Optimization and architecture effects on gpu computing workload performance. In *InPar*, pages 1–10, 2012.

[46] N. Tang, Q. Chen, and P. Mitra. Graph stream summarization: From big bang to big crunch. In *SIGMOD*, pages 1481–1496, 2016.

[47] A. Toshniwal, S. Taneja, A. Shukla, K. Ramasamy, J. M. Patel, S. Kulkarni, J. Jackson, K. Gade, M. Fu, J. Donham, N. Bhagat, S. Mittal, and D. Ryaboy. Storm@twitter. In *SIGMOD*, pages 147–156, 2014.

[48] C. E. Tsourakakis, U. Kang, G. L. Miller, and C. Faloutsos. DOULION: counting triangles in massive graphs with a coin. In *SIGKDD*, pages 837–846, 2009.

[49] U. Verner, A. Schuster, M. Silberstein, and A. Mendelson. Scheduling processing of real-time data streams on heterogeneous multi-gpu systems. In *SYSTOR*, page 7, 2012.

[50] Y. Wang, A. Davidson, Y. Pan, Y. Wu, A. Riffel, and J. D. Owens. Gunrock: A high-performance graph processing library on the gpu. *SIGPLAN Not.*, 50(8):265–266, 2015.

[51] Y. Wang, Q. Fan, Y. Li, and K.-L. Tan. Real-time influence maximization on dynamic social streams. *PVLDB*, 10(7):805–816, 2017.

[52] S. Yan, C. Li, Y. Zhang, and H. Zhou. yaspmv: yet another spmv framework on gpus. In *SIGPLAN Notices*, volume 49, pages 107–118, 2014.

[53] X. Yang, S. Parthasarathy, and P. Sadayappan. Fast sparse matrix-vector multiplication on gpus: Implications for graph mining. *PVLDB*, 4(4):231–242, 2011.

[54] Y. Yang, Z. Wang, J. Pei, and E. Chen. Tracking influential nodes in dynamic networks. *arXiv preprint arXiv:1602.04490*, 2016.

[55] M. Zaharia, T. Das, H. Li, T. Hunter, S. Shenker, and I. Stoica. Discretized streams: Fault-tolerant streaming computation at scale. In *SOSP*, pages 423–438, 2013.

[56] H. Zhang, P. Lofgren, and A. Goel. Approximate personalized pagerank on dynamic graphs. *arXiv preprint arXiv:1603.07796*, 2016.

[57] Y. Zhang and F. Mueller. Gstream: A general-purpose data streaming framework on GPU clusters. In *ICPP*, pages 245–254, 2011.

[58] J. Zhong and B. He. Medusa: Simplified graph processing on gpus. *IEEE Trans. Parallel Distrib. Syst.*, 25(6):1543–1552, 2014.