Longest Common Subsequence

Problem Statement

S1 = "AGGTAYB"
S2 = "GXTXAYB"

Expected Result = 5

Longest Common Subsequence

- GTAYB = 5
- TAYB
- AYB
- YB
- B

3 3 ~~4~~ ~~5~~ ~~6~~

S1 = ~~7~~(m)
S2 = ~~7~~(n)

~~6~~ ~~5~~ 2

	0	1	2	3	4	5	6
S1 →	A	G	G	T	A	Y	B

	0	1	2	3	4	5	6
S2 →	G	X	T	X	A	Y	B

LCS(S1, S2, m, n) {

Base case condition

if (m==0 || m==0) {

return 0;

}

1

Exact match

if (S1.charAt(m-1) == S2.charAt(n-1)) {

2 return 1 + LCS(S1, S2, m-1, n-1);

}

else {

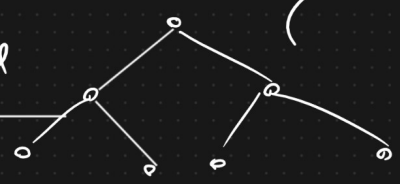
No Match

return max(LCS(S1, S2, m-1, n),

3 LCS(S1, S2, m, n-1));

}

1 choices



Optimal Substructure

2 Overlapping Subproblem

S1.length

S2.length

Time complexity → $O(2^{m*n})$

Recursive tree

$LCS(s_1, s_2, 7, 7)$ → 5

4

$5 = 1 + LCS(s_1, s_2, 6, 6)$

3

$4 = 1 + LCS(s_1, s_2, 5, 5)$

2

$3 = 1 + LCS(s_1, s_2, 4, 4)$

1 $LCS(s_1, s_2, \overset{m}{3}, \overset{n}{4})$

2 $LCS(s_1, s_2, 4, 3)$

1 $LCS(s_1, s_2, \overset{m}{2}, \overset{n}{4})$

1 $LCS(s_1, s_2, \overset{m}{3}, \overset{n}{3})$

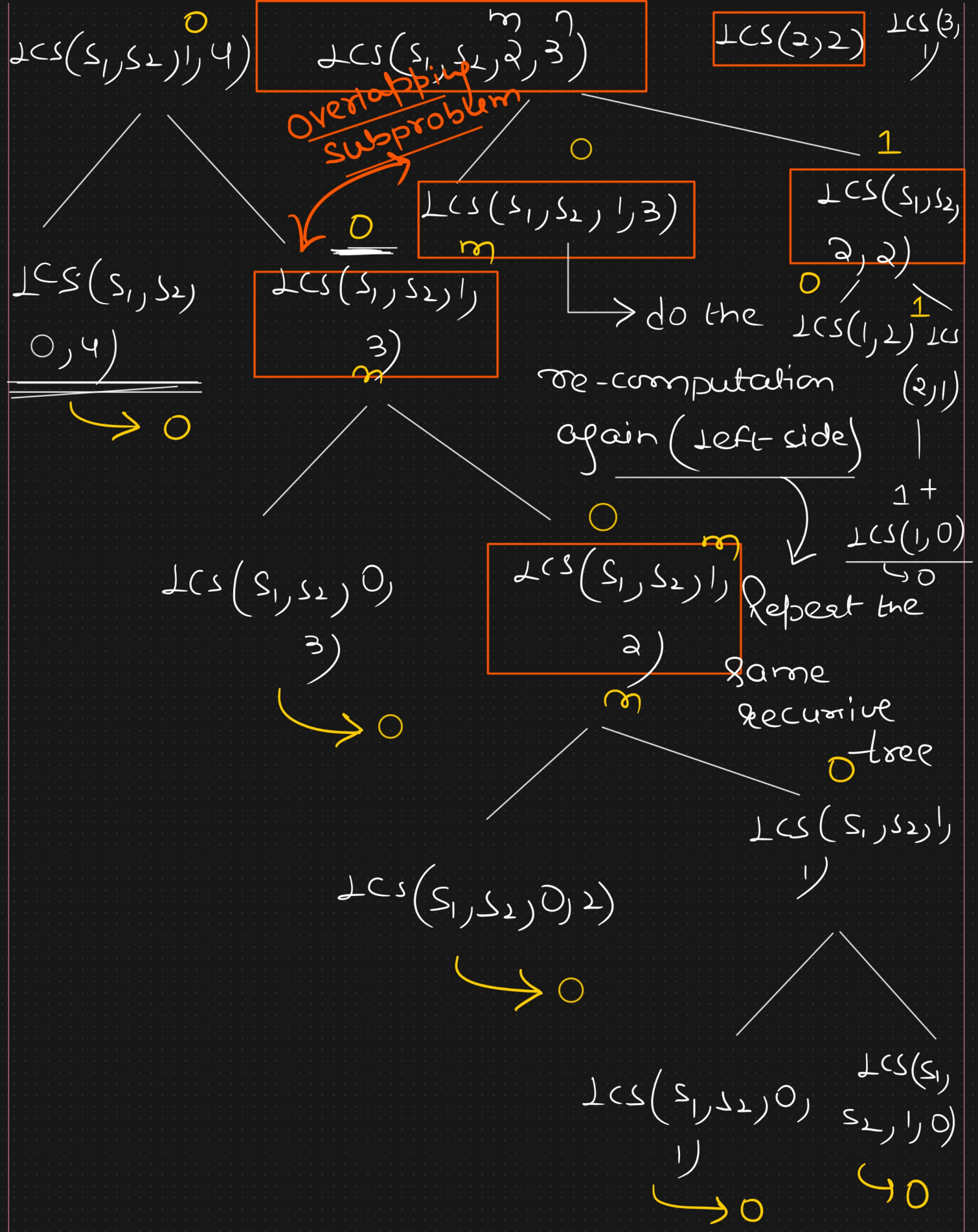
0 $1 + LCS(2, 0)$

1 $\boxed{LCS(\overset{m}{2}, \overset{n}{3})}$

1 $\boxed{LCS(3, 2)}$

1

1


LCS(S1,S2,1,4)
0
LCS(S1,S2,2,3)
m n
LCS(2,2)
LCS(3,1)
Overlapping subproblem
LCS(S1,S2,0,4)
0
LCS(S1,S2,1,3)
m
0
LCS(S1,S2,2,2)
1
0
LCS(1,2)
1
LCS(2,1)
1 +
LCS(1,0)
0
do the re-computation again (left-side)
Repeat the same recursive tree
LCS(S1,S2,1,3)
m
LCS(S1,S2,0,3)
0
LCS(S1,S2,1,2)
m
0
LCS(S1,S2,0,2)
0
LCS(S1,S2,1,1)
0
LCS(S1,S2,0,1)
0
LCS(S1,S2,1,0)
0

Right subtree

$LCS(s_1, s_2, 4, 3) \Longrightarrow 2$

$\downarrow$

1

$1 + LCS(s_1, s_2, 3, 2)$

$\hookrightarrow$ Re-computation

(above subtree)

012345

String s1 = "AGGTAB";

String s2 = "GXTXAYB";

0123456

$m = 6$

$n = 7$

Tabulation

m \ n		G	X	T	X	A	Y	B
	0	1	2	3	4	5	6	7
	0	0	0	0	0	0	0	0
A	1	0	0	0	0	1	1	1
G	2	0	1	1	1	1	1	1
G	3	0	1	1	1	1	1	1
T	4	0	1	1	2	2	2	2
A	5	0	1	1	2	2	3	3
B	6	0	1	1	2	2	3	3

G T A B

(Diagonal Linkage)

Longest Common Subsequence