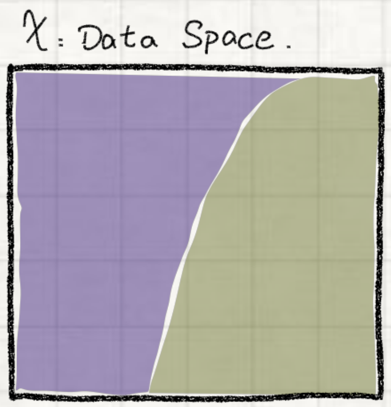
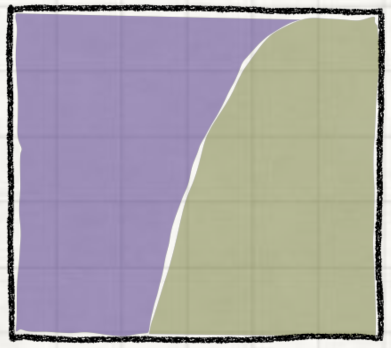
BIAS VARIANCE BOOTSTRAPPING AND BOOSTING

REVIEW

REVIEW



REVIEW



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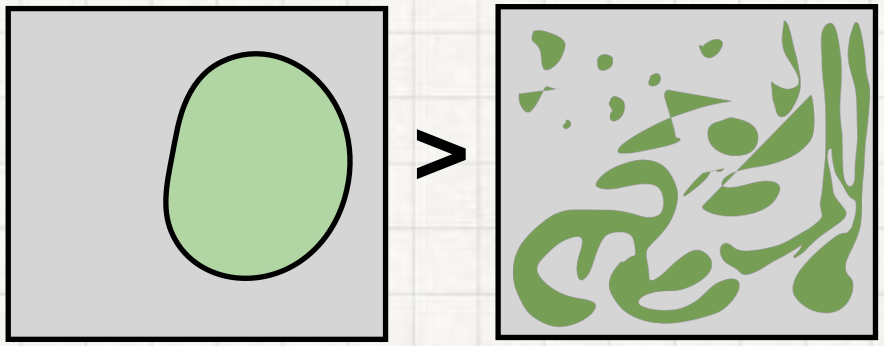

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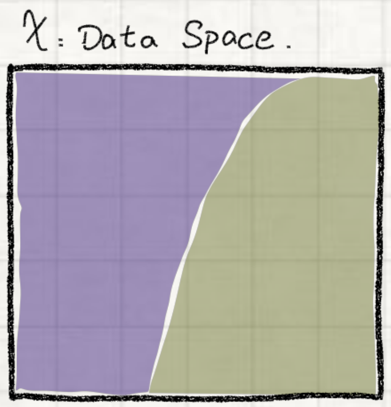

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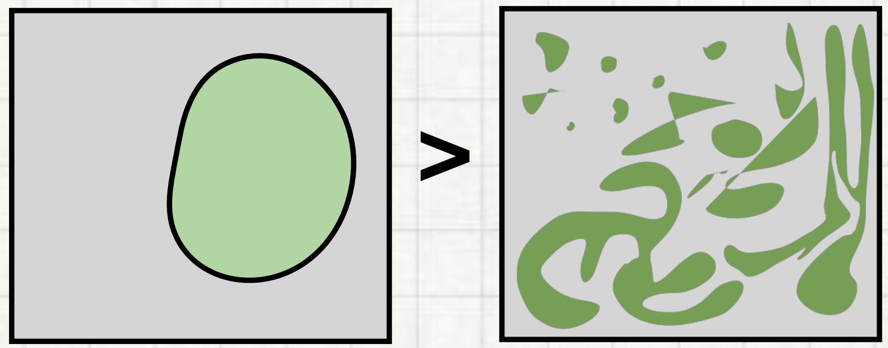
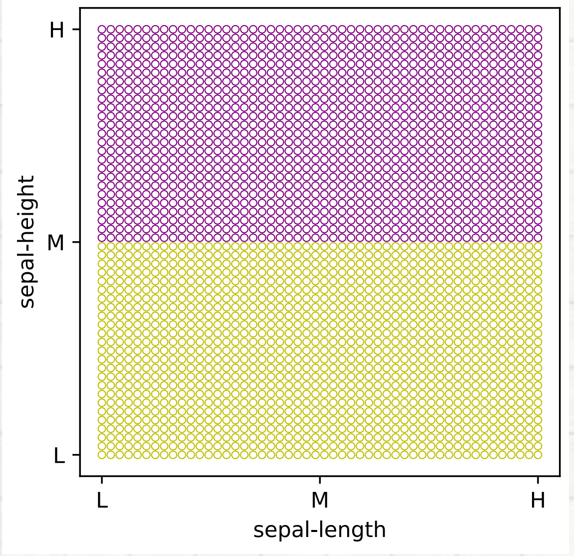
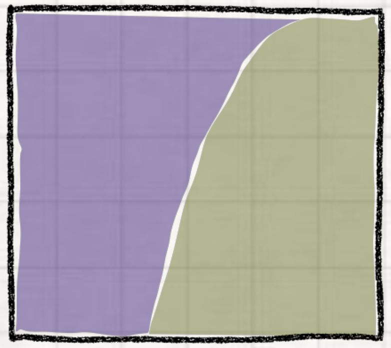




REVIEW

χ: Data Space.



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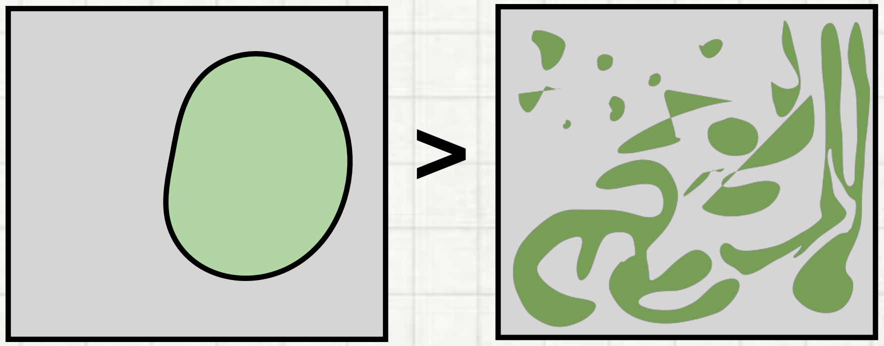

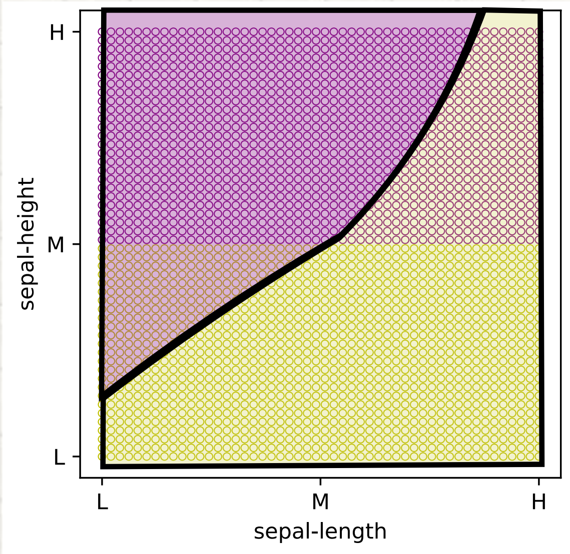

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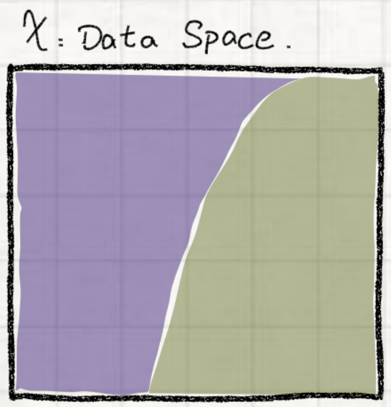

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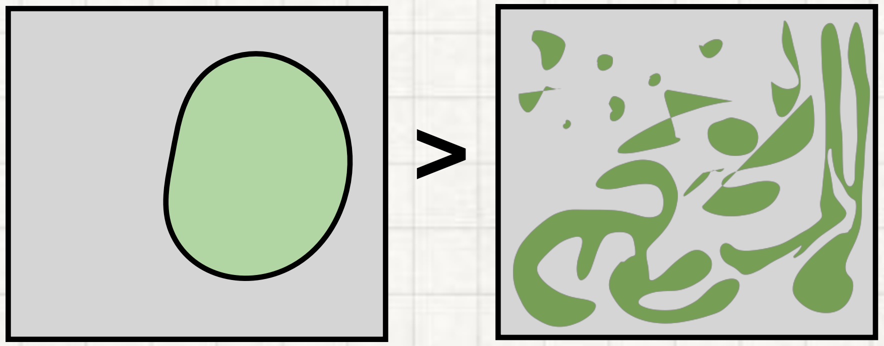
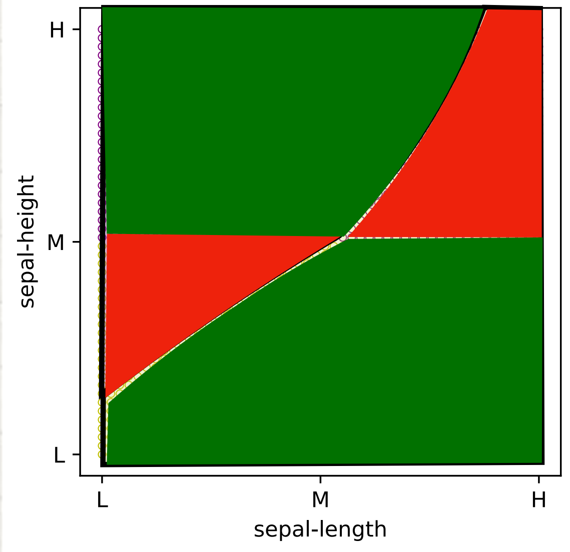
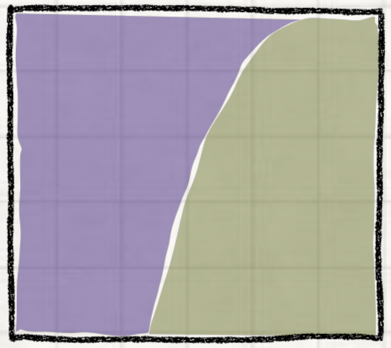




REVIEW

χ: Data Space.



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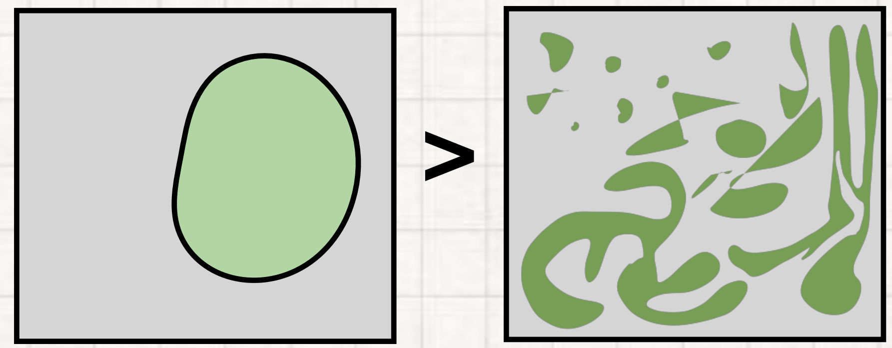
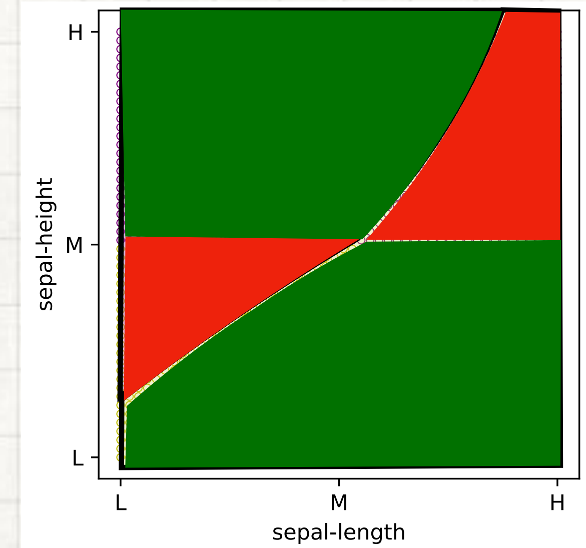
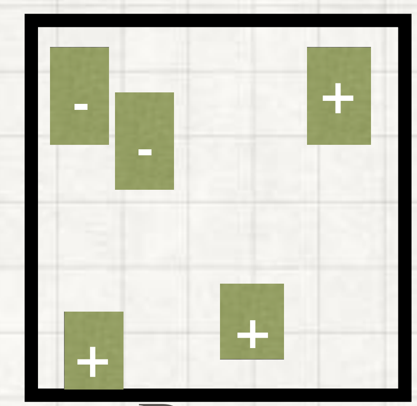
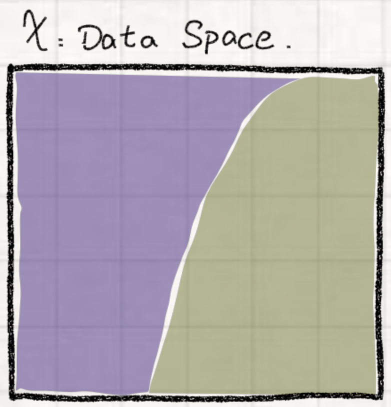






D_{train}

REVIEW



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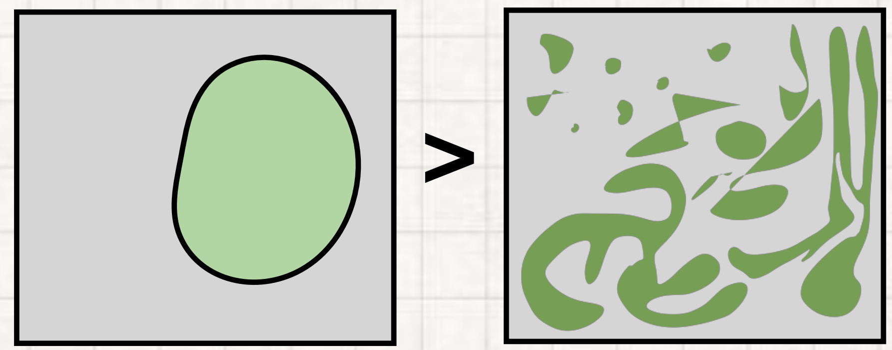
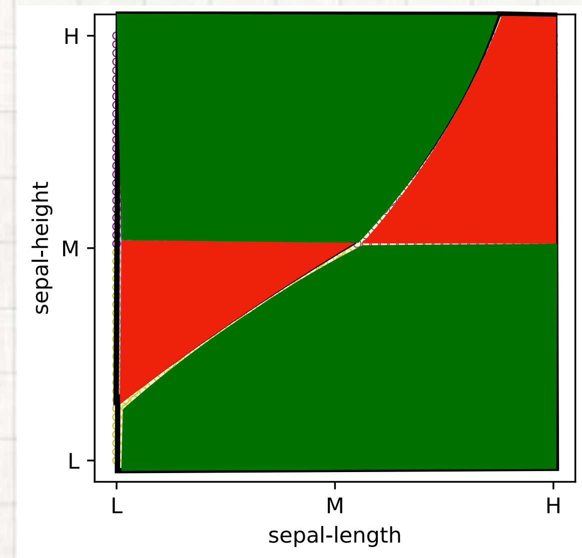
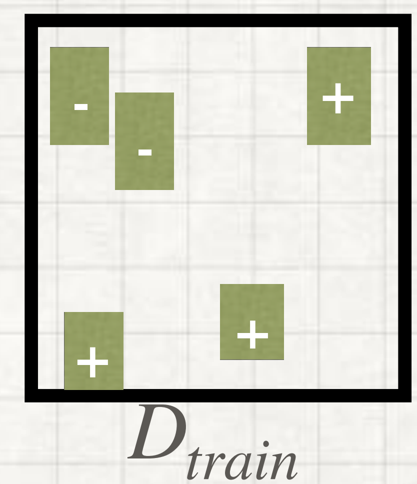
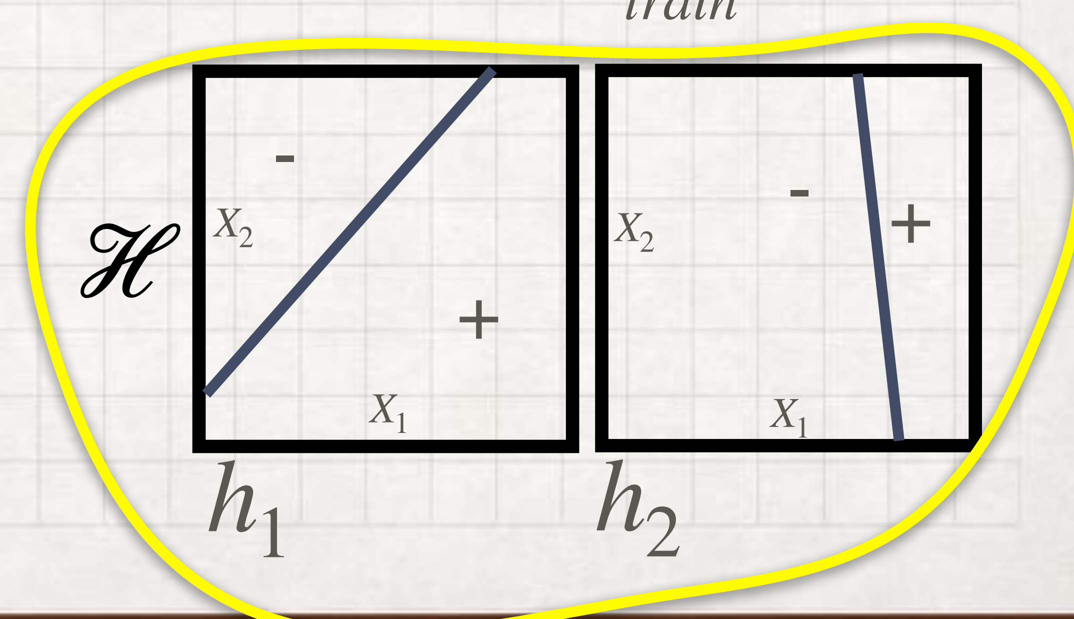


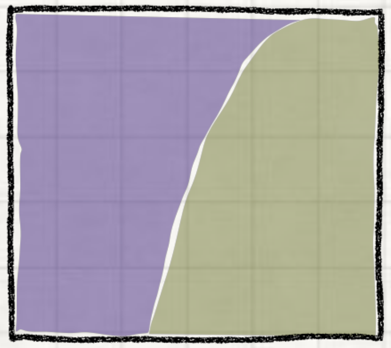






REVIEW

χ: Data Space.



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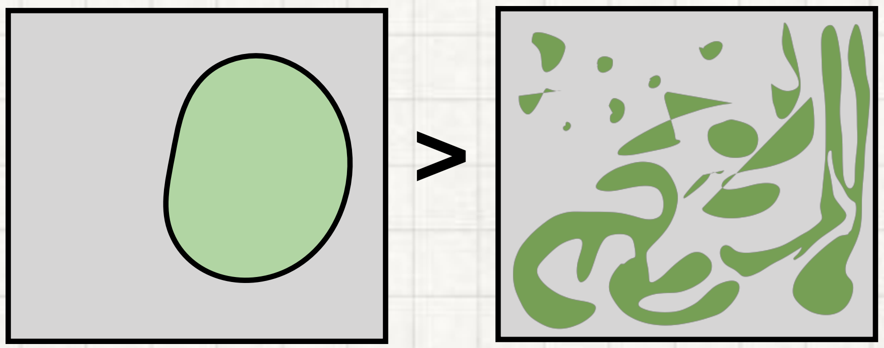

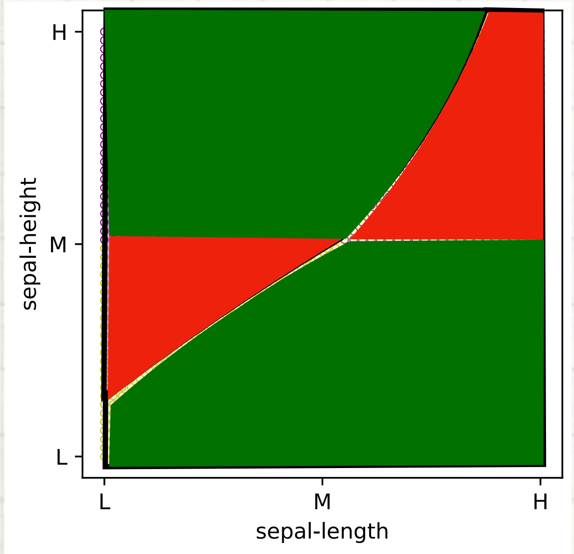

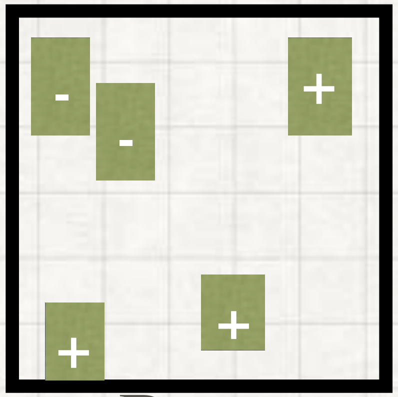

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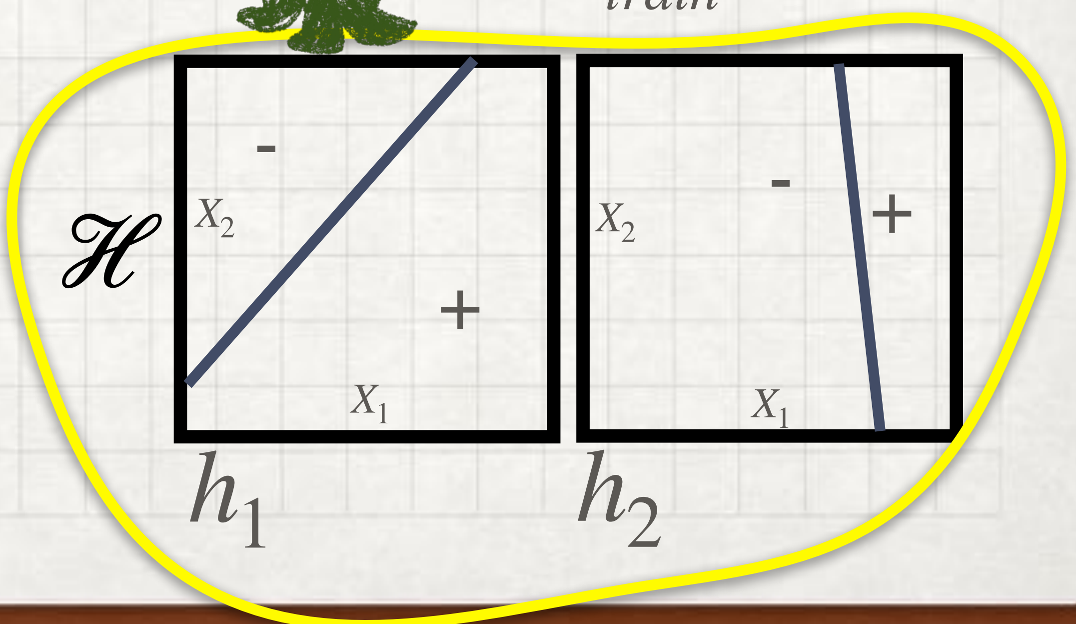

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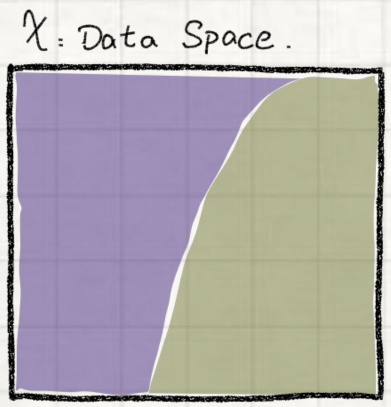

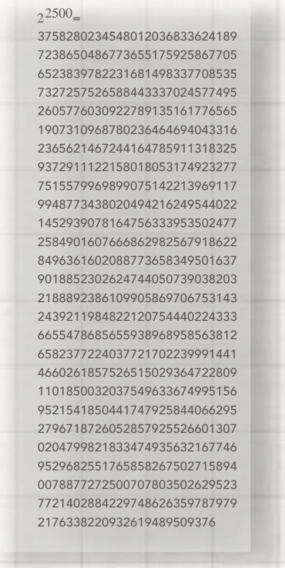

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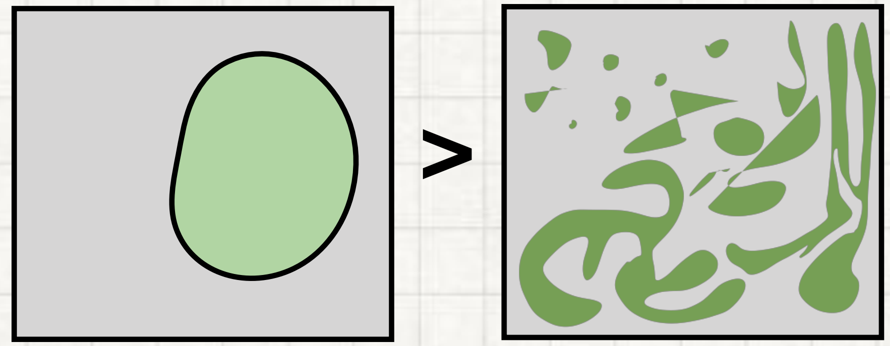
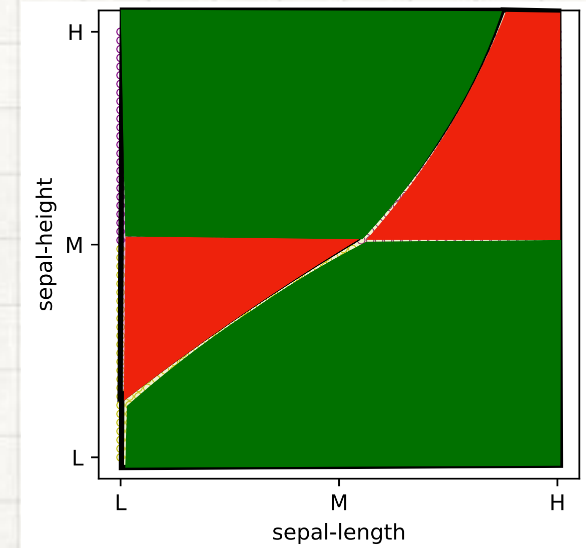
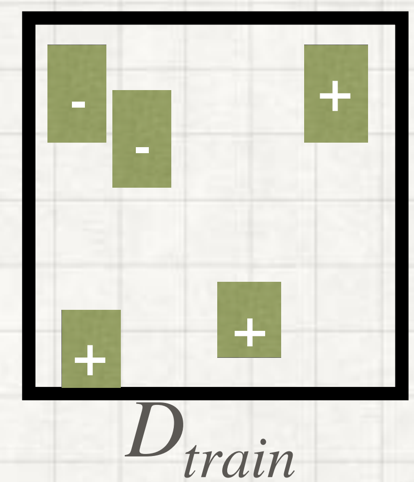
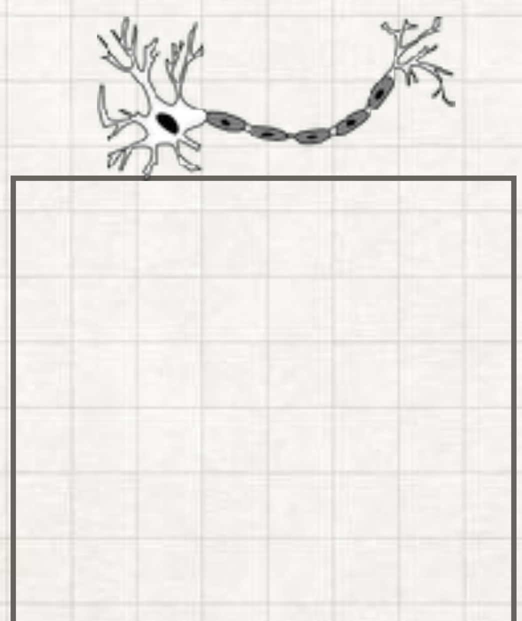
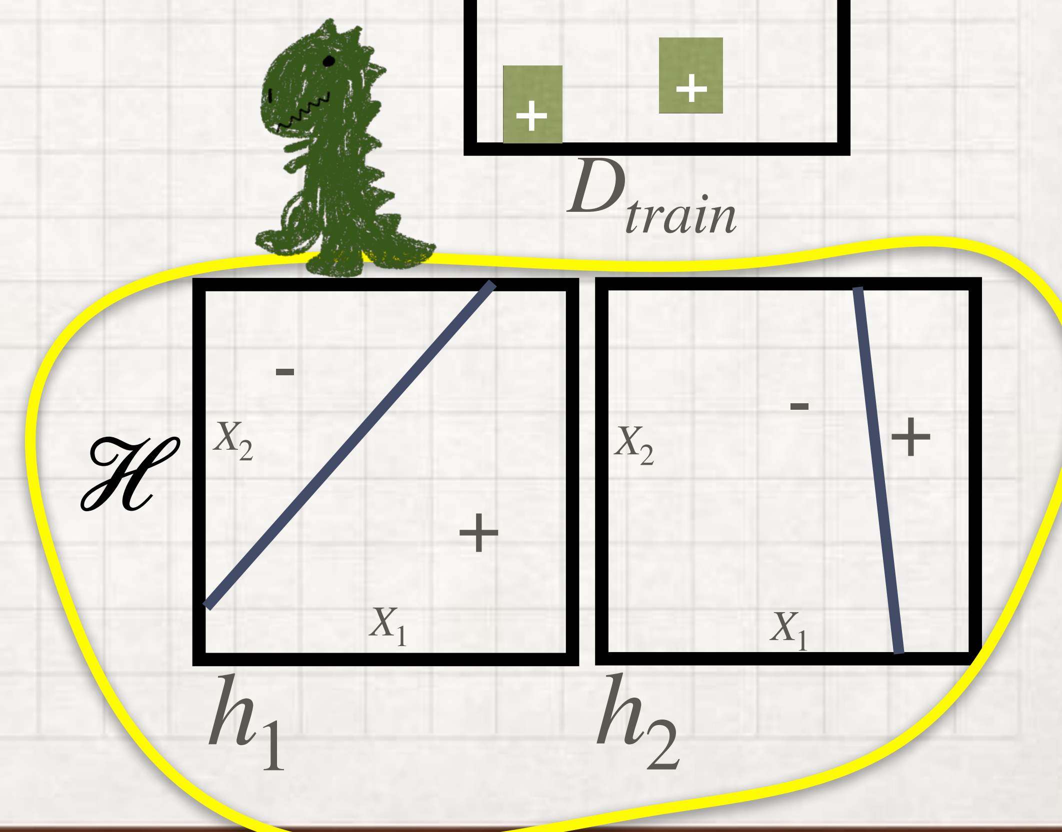




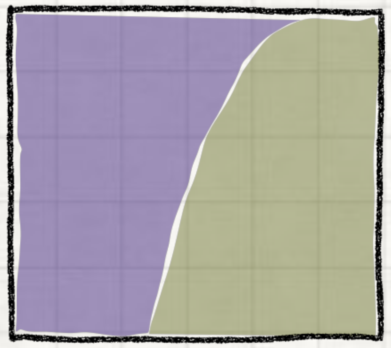


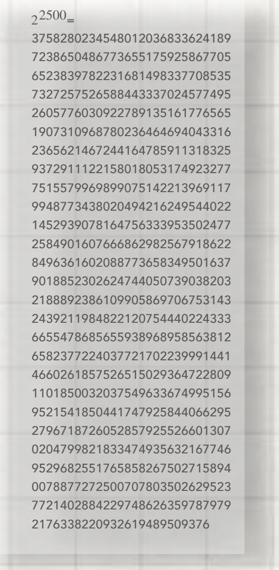


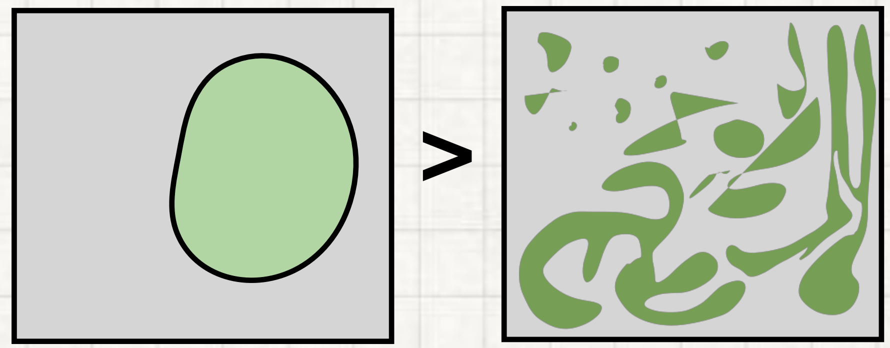


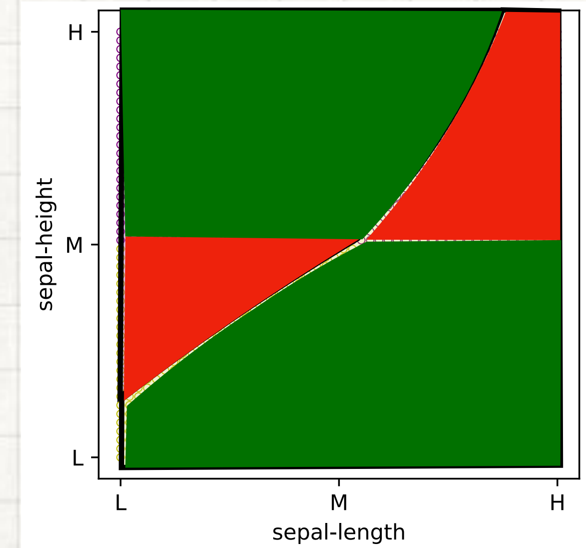
REVIEW

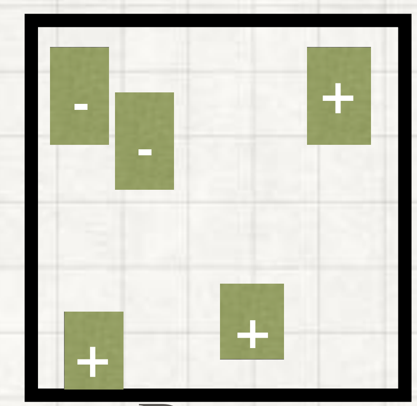
χ: Data Space.











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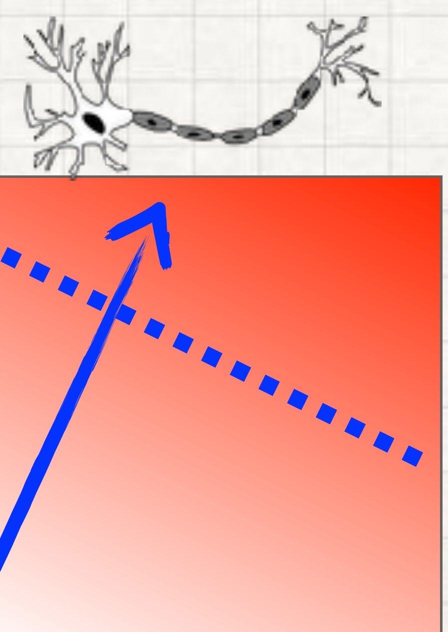
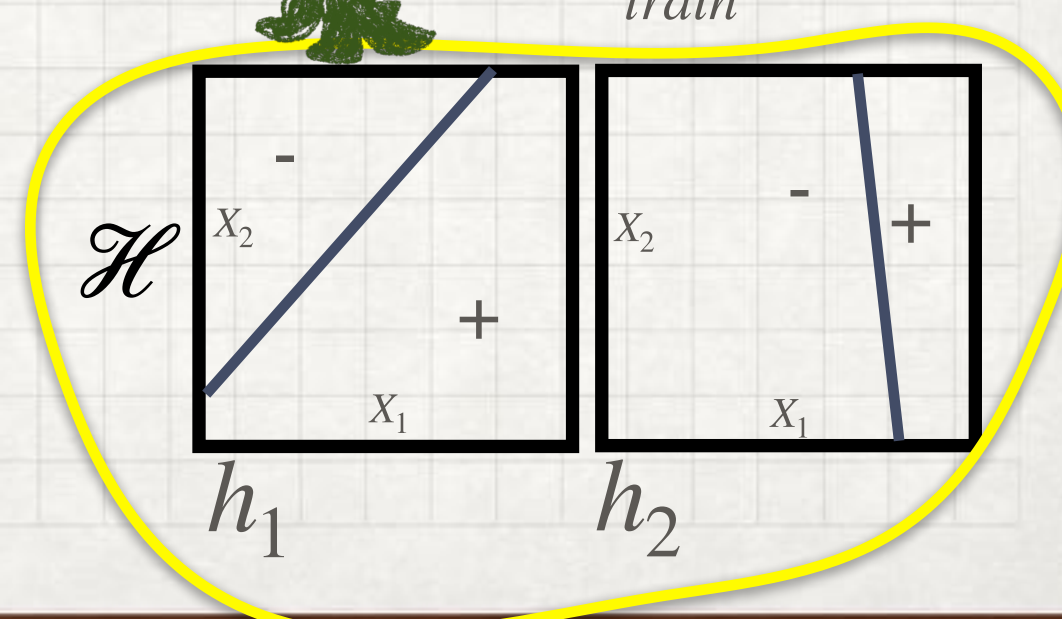
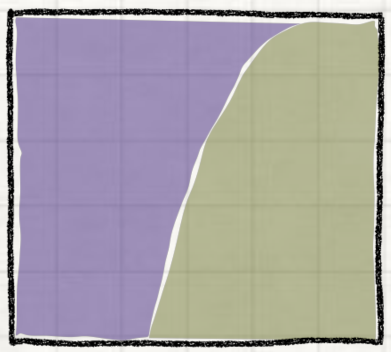




REVIEW

$\mathcal{X}$: Data Space.



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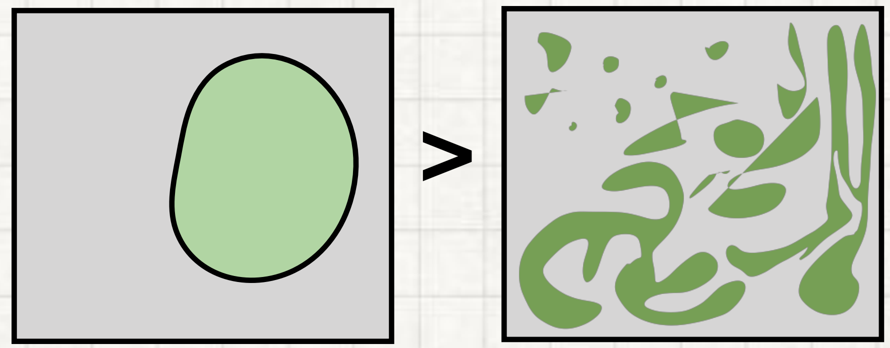
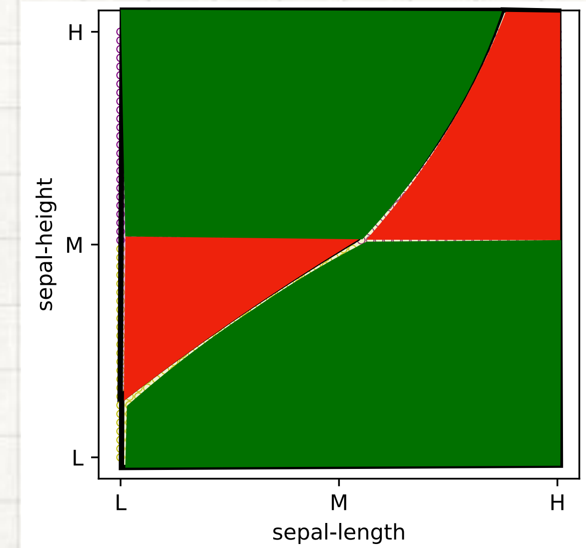




$\mathcal{X} \to \mathcal{Y}$

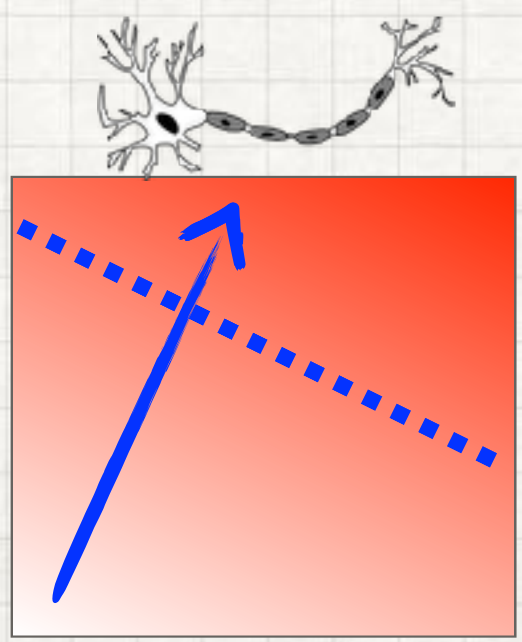
$\{(x_1, y_1), (x_2, y_2), \dots\}$

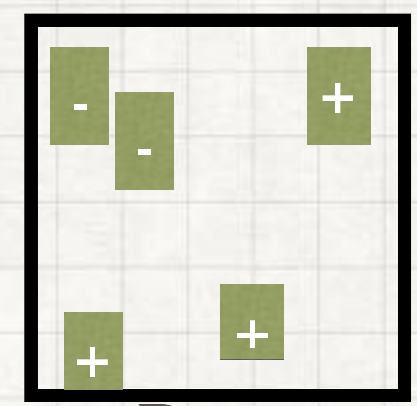
Loss

Algorithm

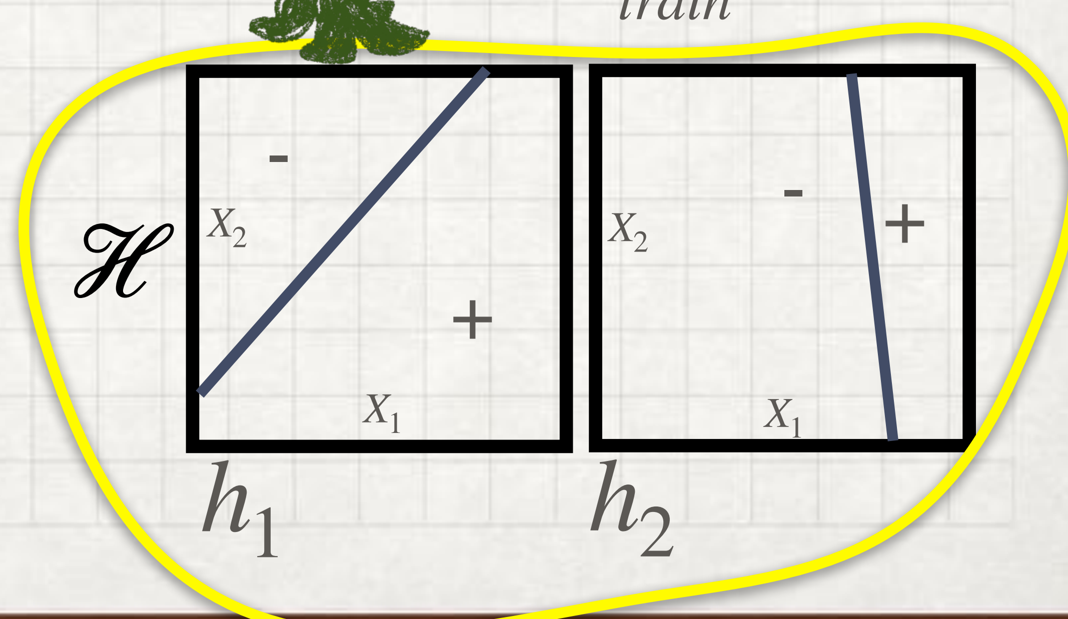
$h^* \in \mathcal{H}$

$\mathcal{H} : \{h_1, h_2, \dots, \} : x \to y$



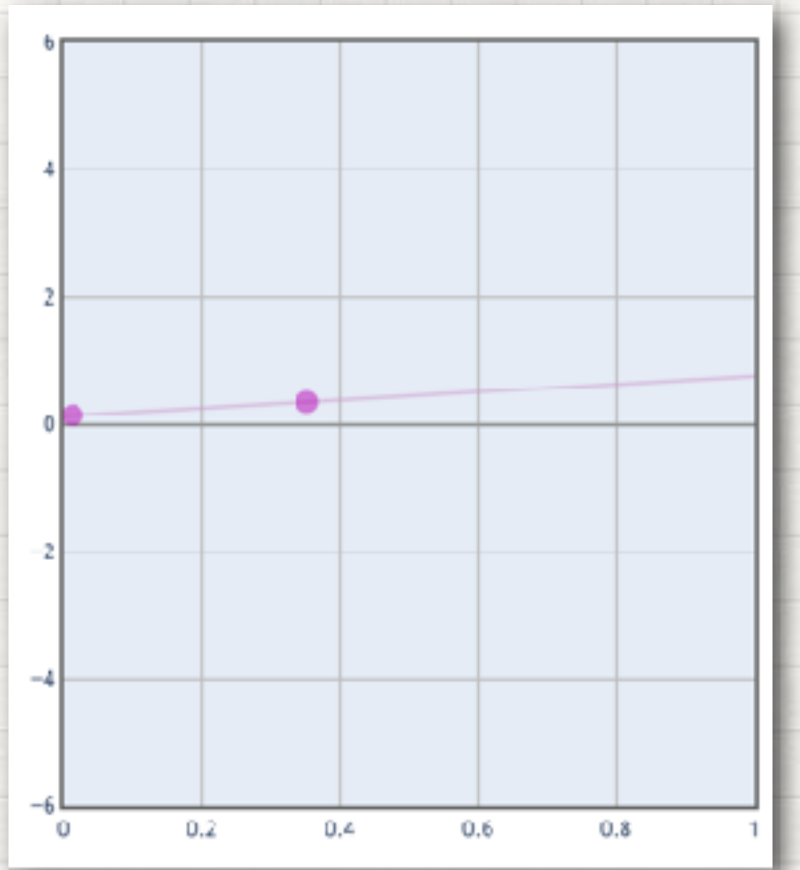


D_{train}



UNCERTAINTY IN LEARNING

UNCERTAINTY IN LEARNING



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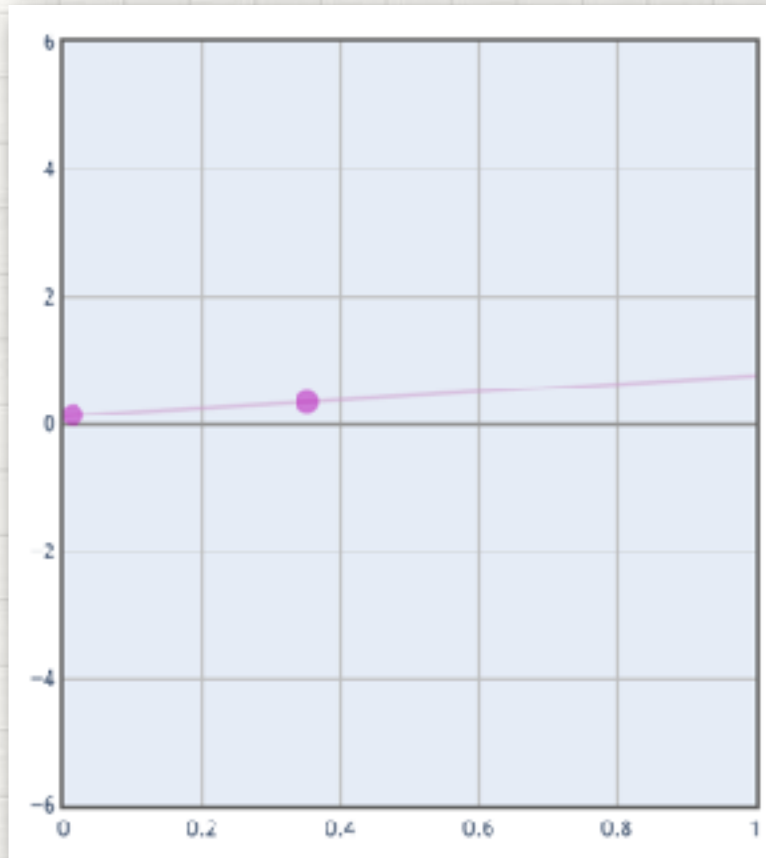

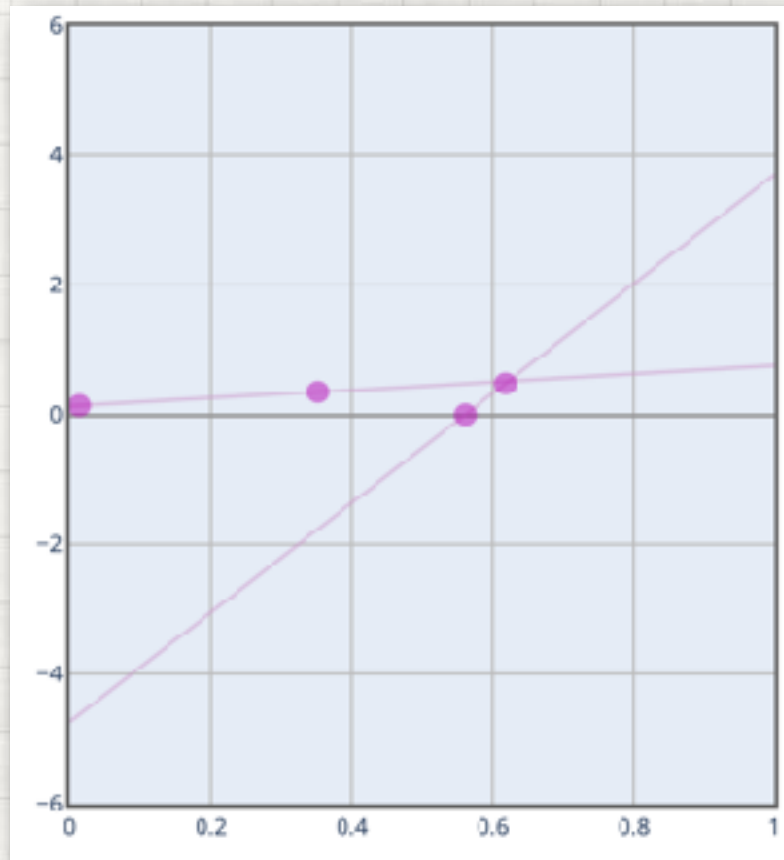

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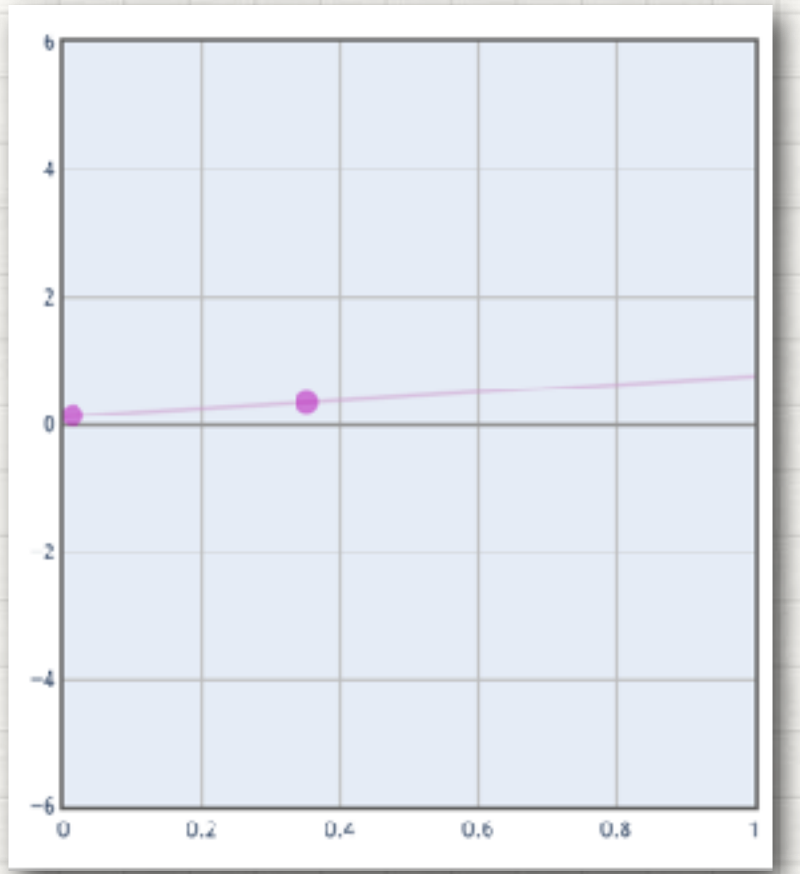

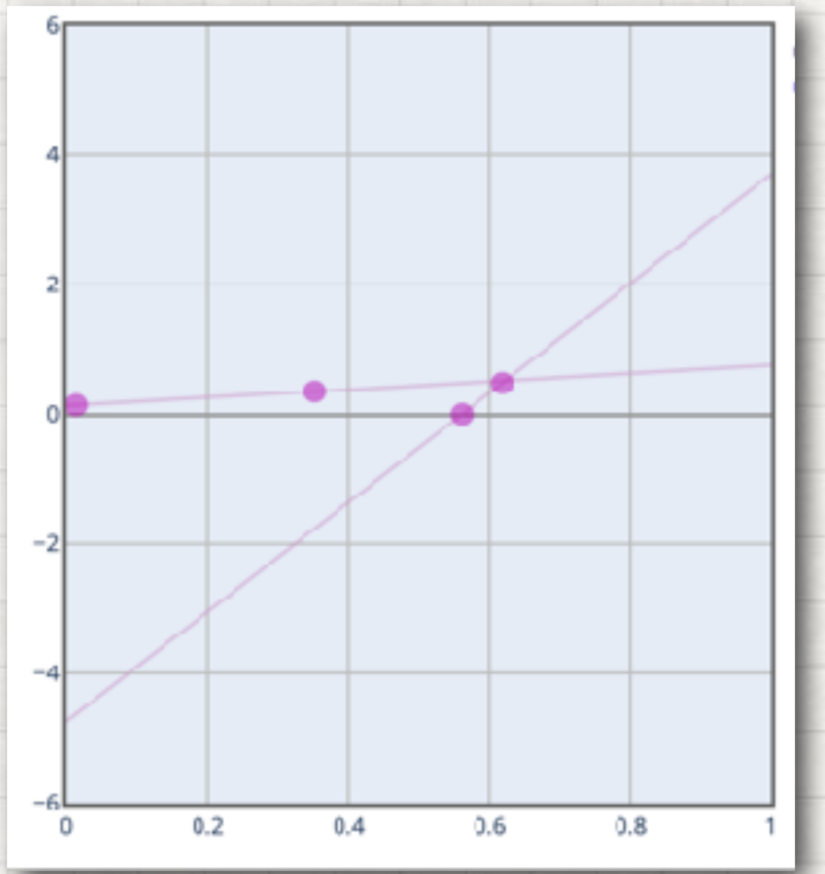

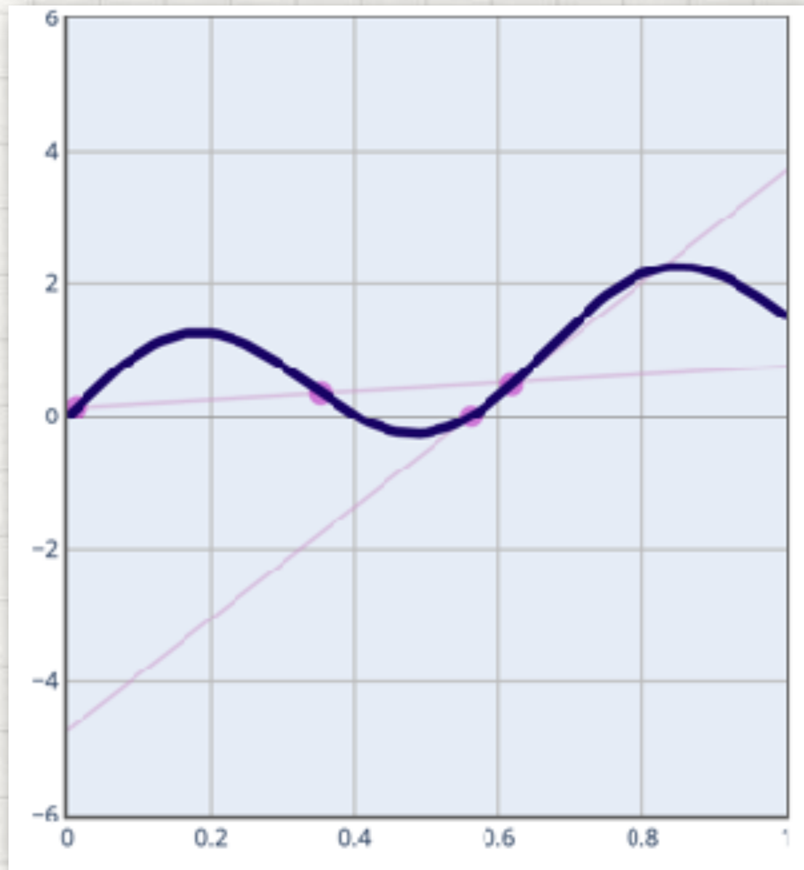

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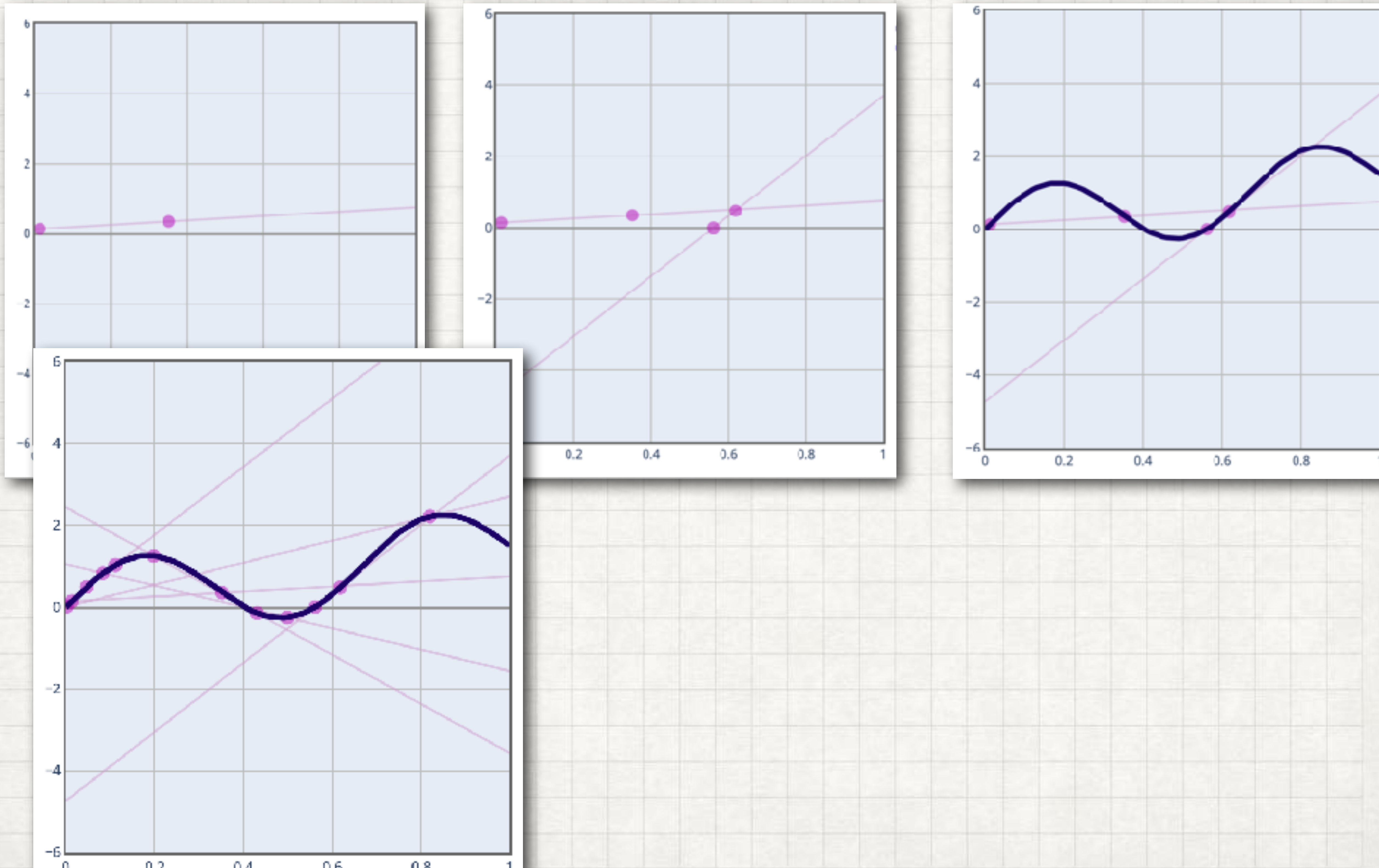
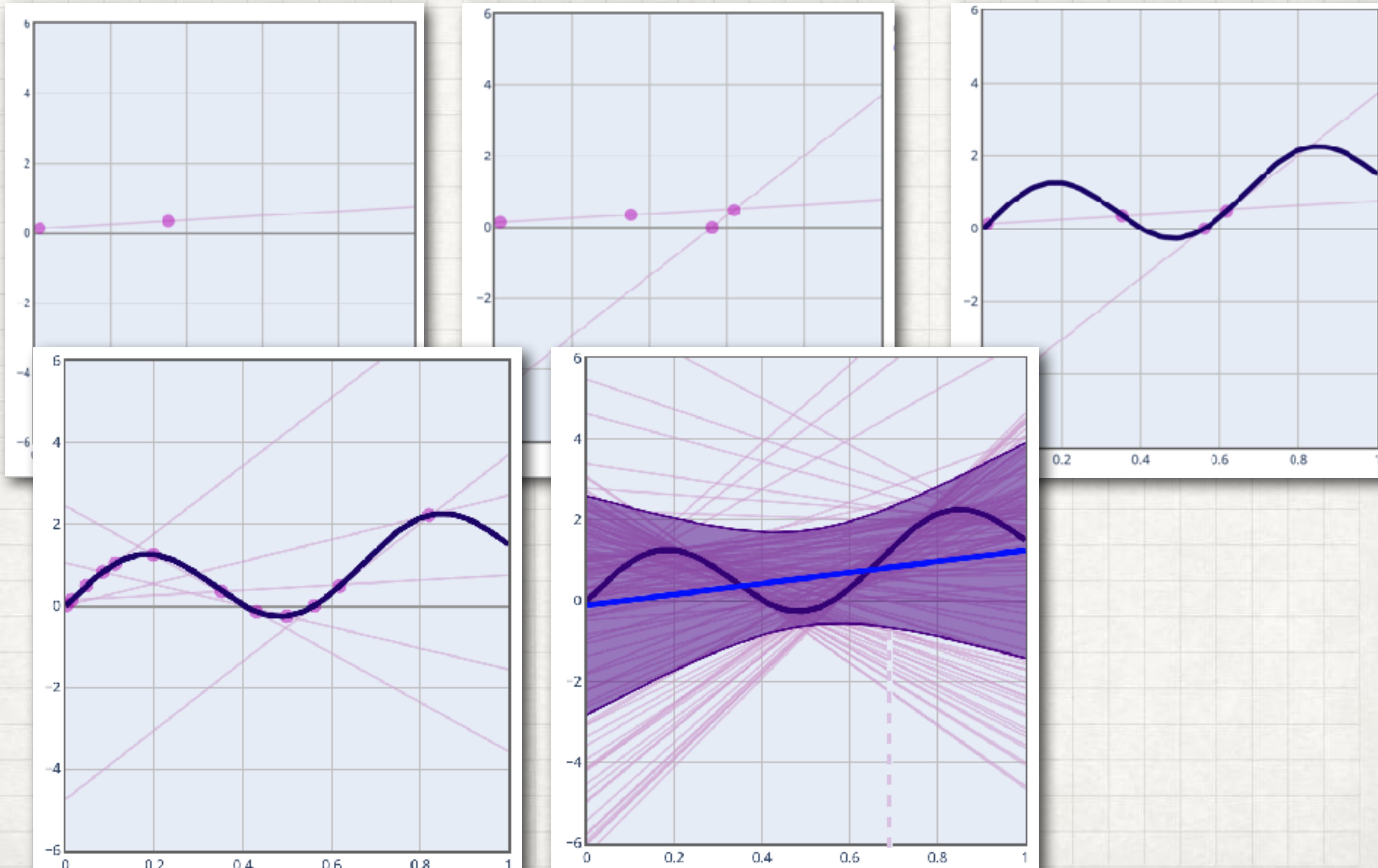
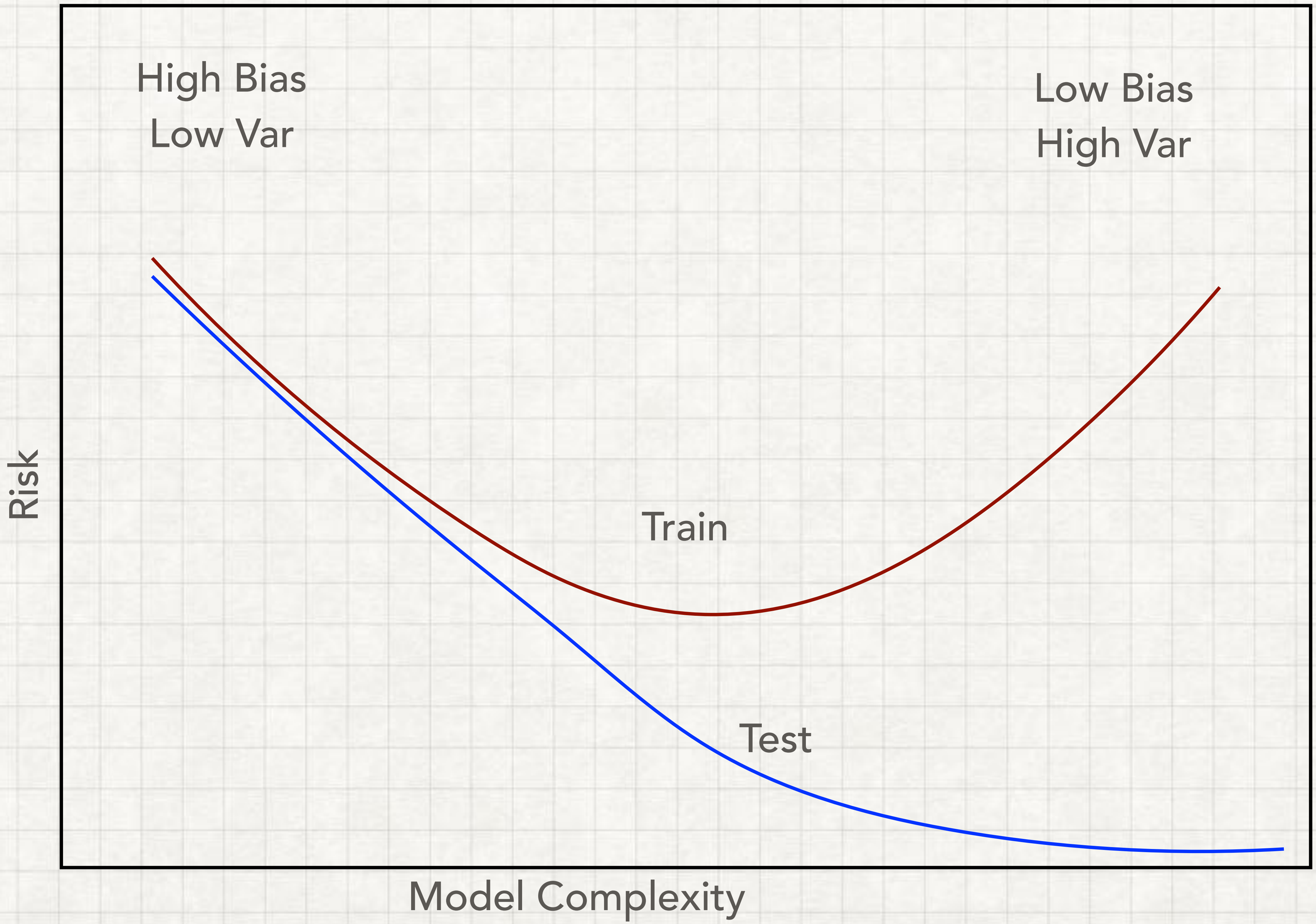


UNCERTAINTY IN LEARNING



BIAS VS VARIANCE

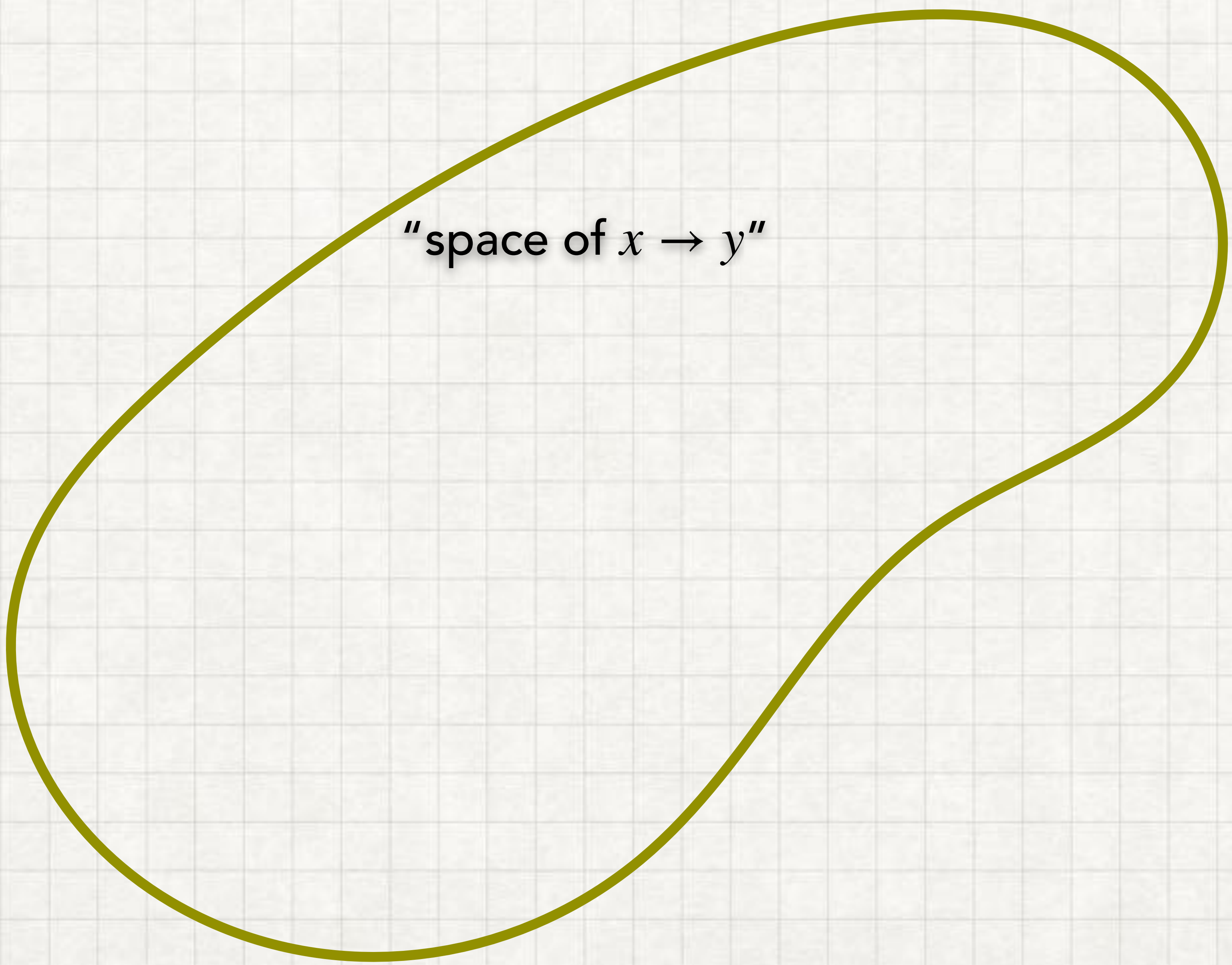


MOD1: INTUITIVE INTERPRETATION OF $\mathcal{H}$ FAMILY AND RISK

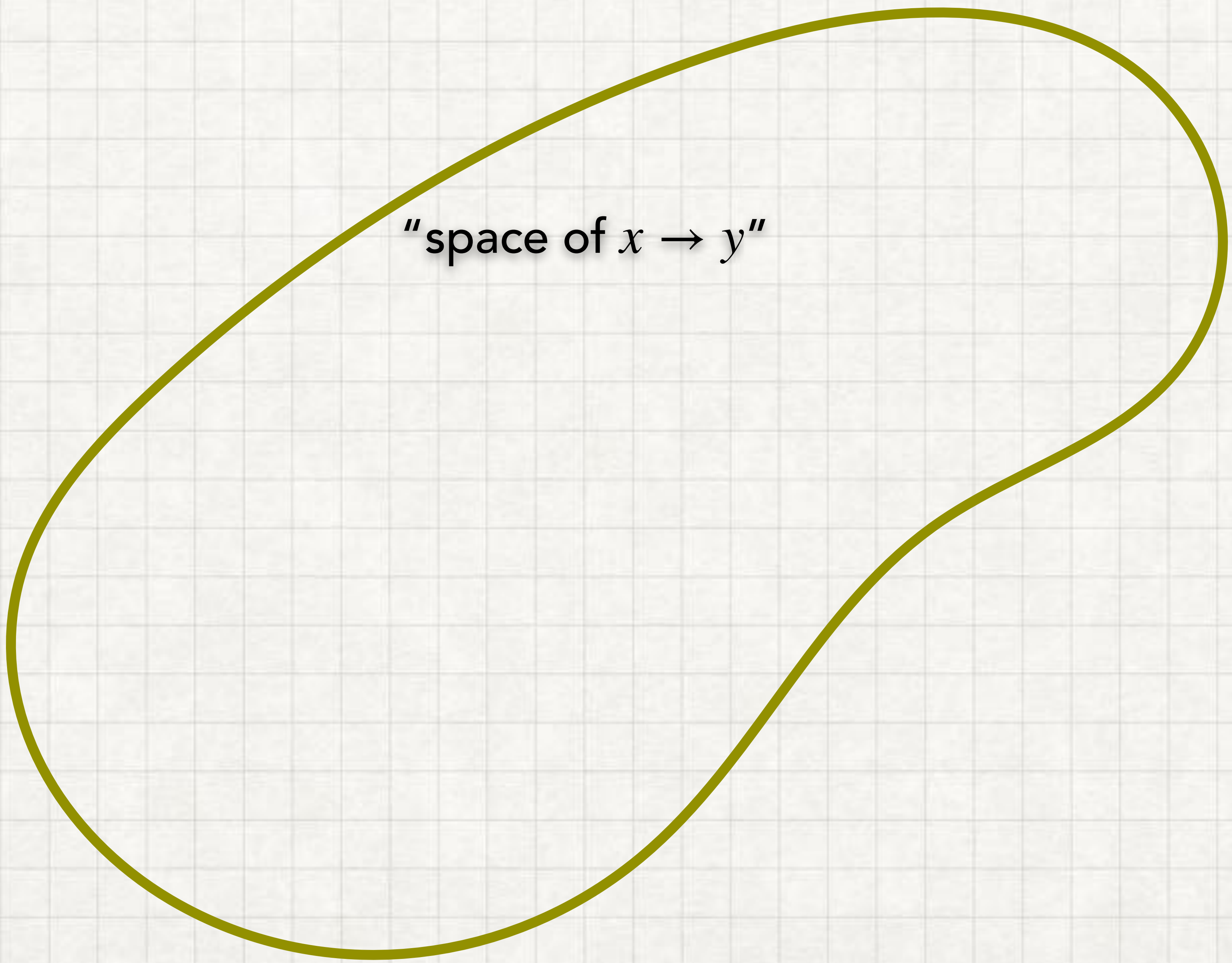
ERROR/RISK IN FUNCTIONAL SPACE

"space of $x \to y$"

ERROR/RISK IN FUNCTIONAL SPACE



FUNCTIONAL SPACE?

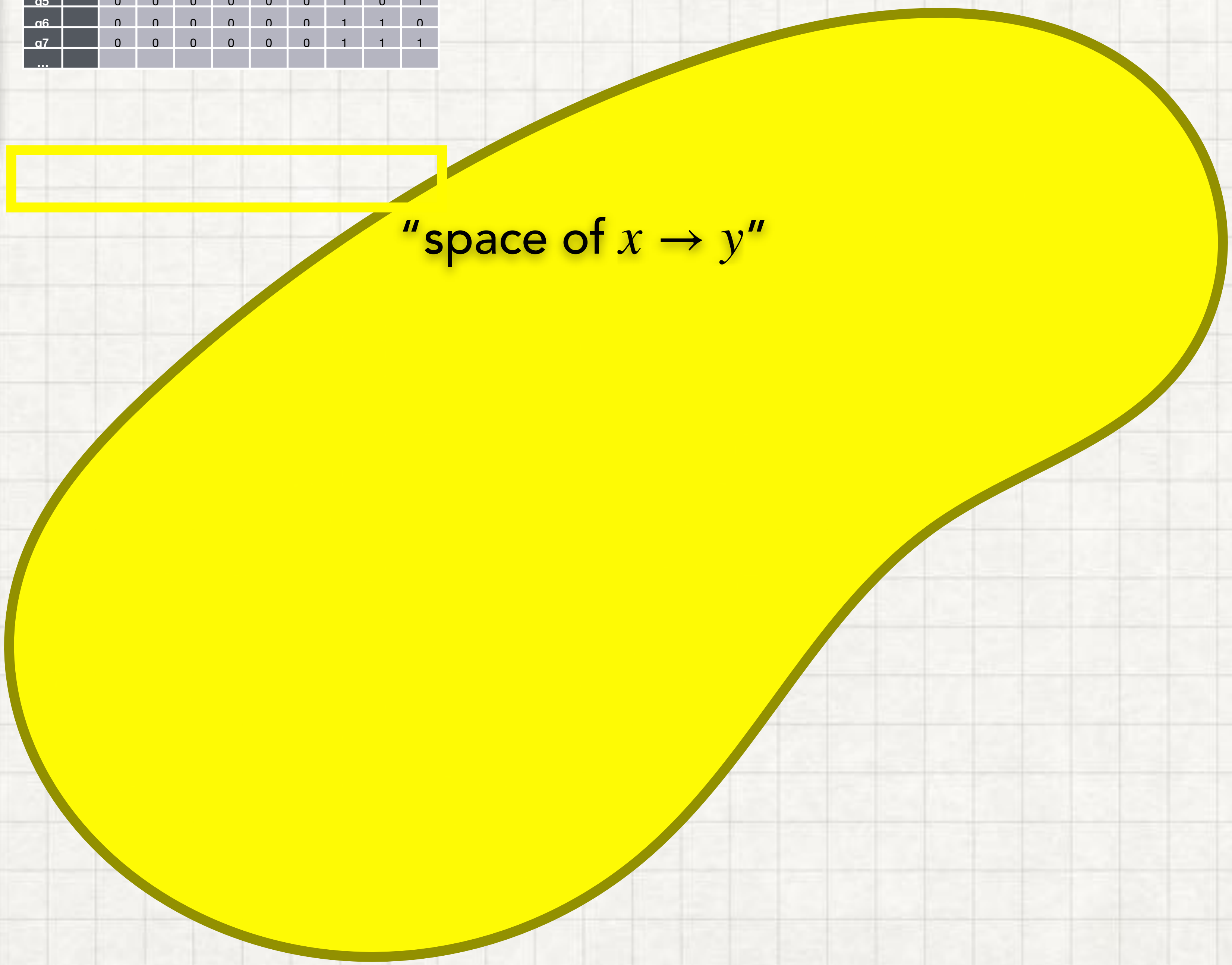


FUNCTIONAL SPACE?

- $2^{2500}=$

375828023454801203683362418972386504867736551759258677056523839782231681498337708535732725752658844333702457749526057760309227891351617765651907310968780236464694043316236562146724416478591131832593729111221580180531749232777515579969899075142213969117994877343802049421624954402214529390781647563339535024772584901607666862982567918622849636160208877365834950163790188523026247440507390382032188892386109905869706753143243921198482212075444022433366554786856559389689585638126582377224037721702239991441466026185752651502936472280911018500320375496336749951569521541850441747925844066295279671872605285792552660130702047998218334749356321677469529682551765858267502715894007887727250070780350262952377214028842297486263597879792176338220932619489509376

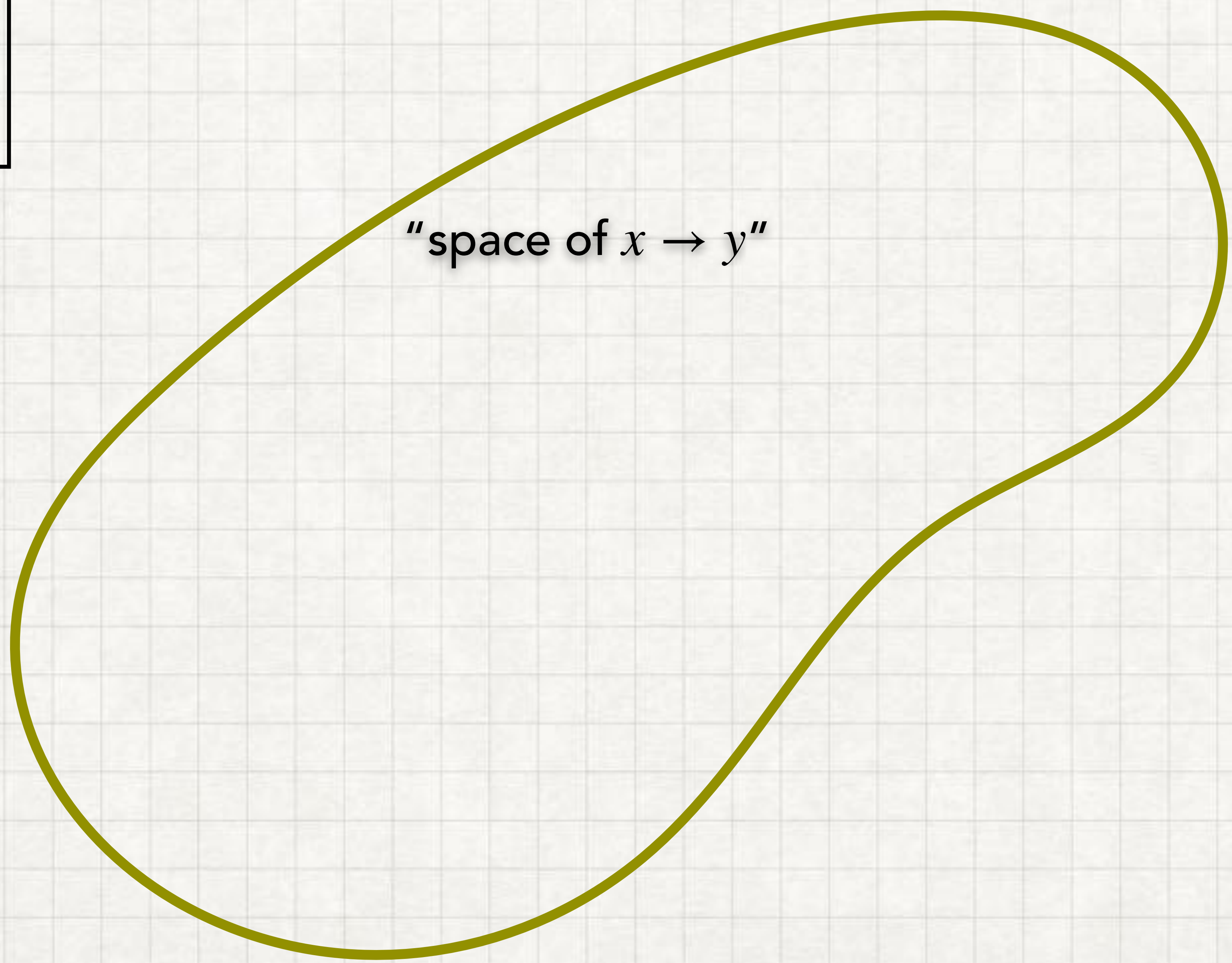
	X1	L	L	L	M	M	M	H	H	H
Y|X	X2	L	M	H	L	M	H	L	M	H
g0		0	0	0	0	0	0	0	0	0
g1		0	0	0	0	0	0	0	0	1
g2		0	0	0	0	0	0	0	1	0
g3		0	0	0	0	0	0	0	1	1
g4		0	0	0	0	0	0	1	0	0
g5		0	0	0	0	0	0	1	0	1
g6		0	0	0	0	0	0	1	1	0
g7		0	0	0	0	0	0	1	1	1
...										



FUNCTIONAL SPACE ...

$\mathcal{X}, \mathcal{Y}$

setosa / versicolor ?



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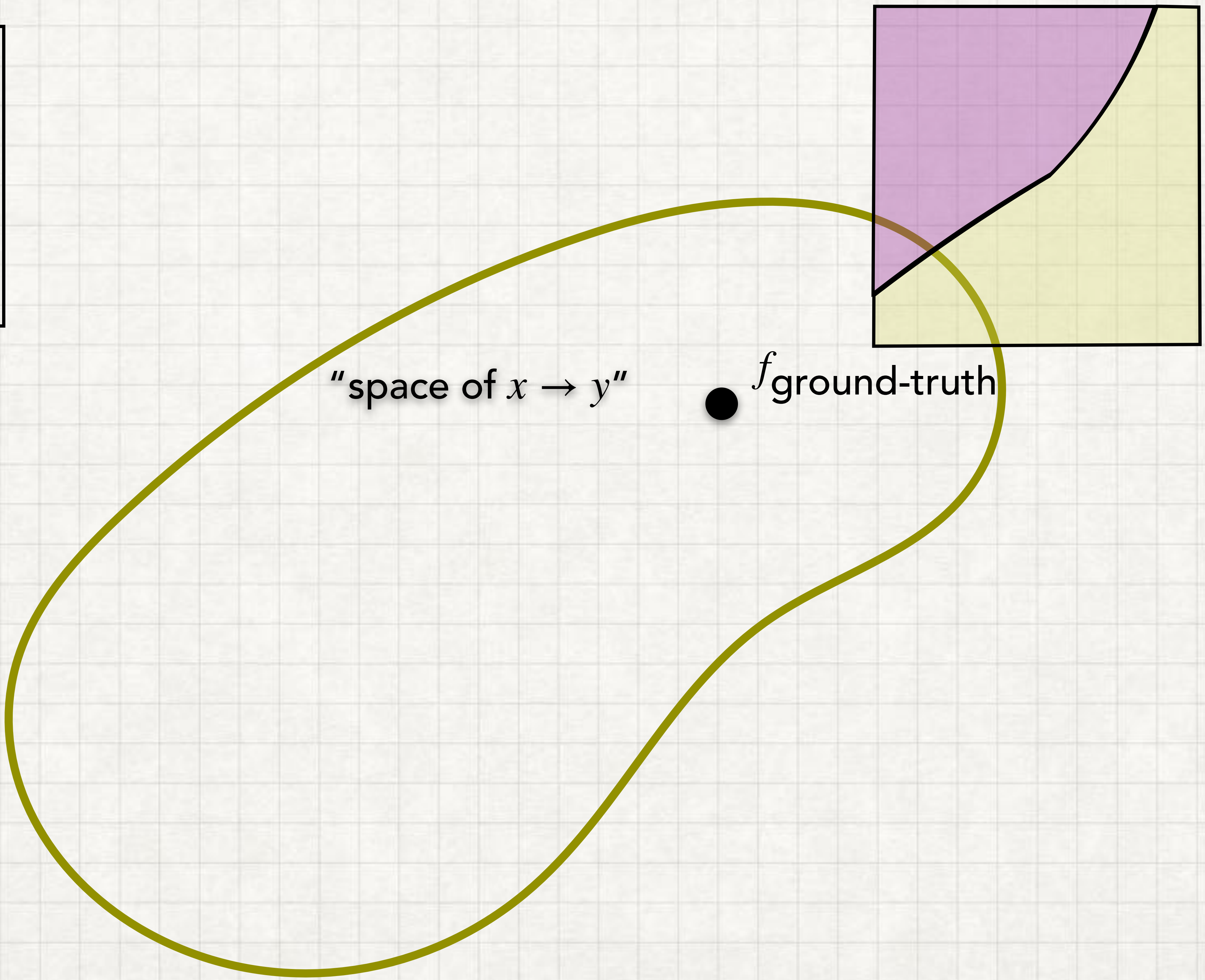
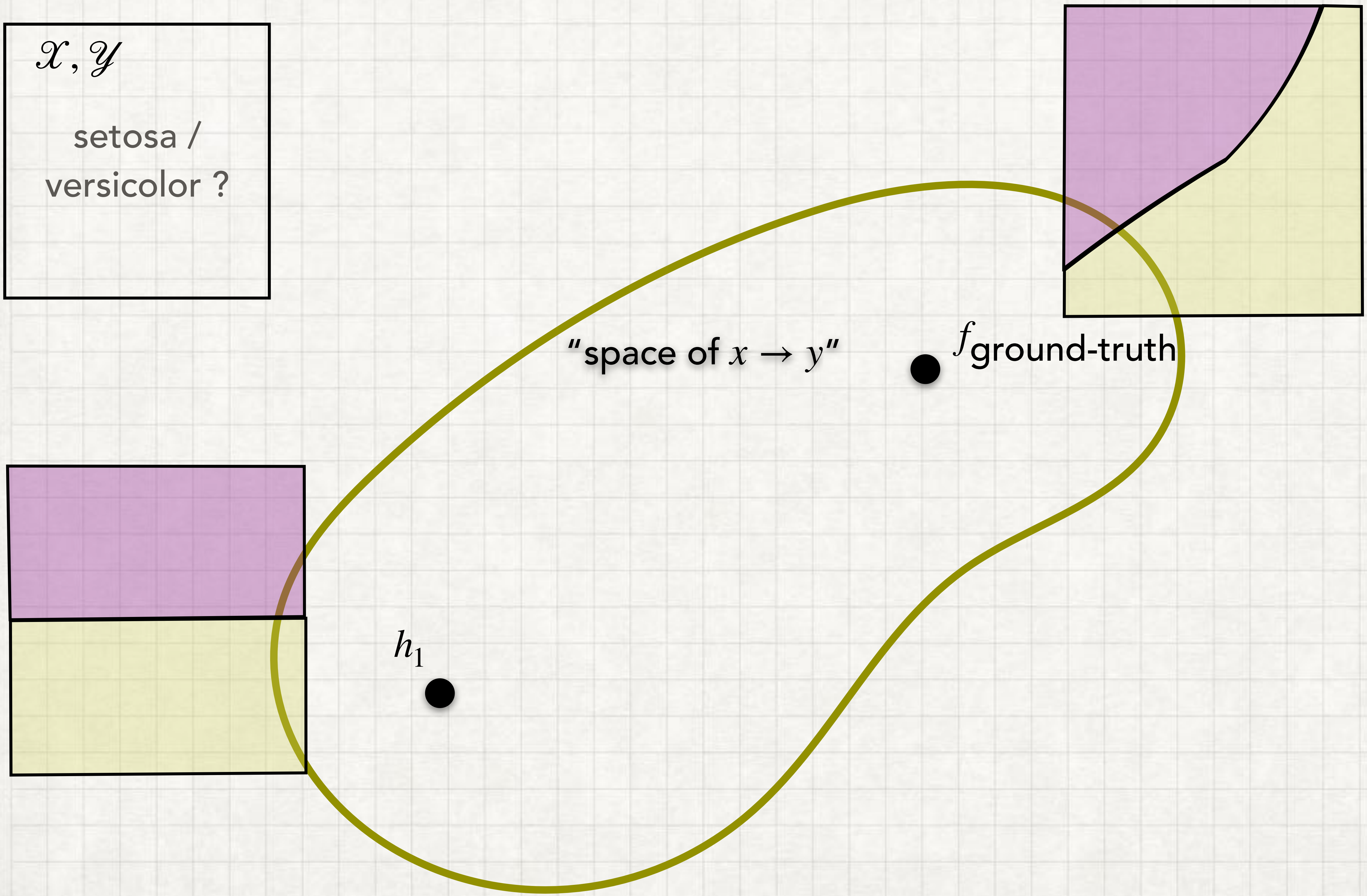
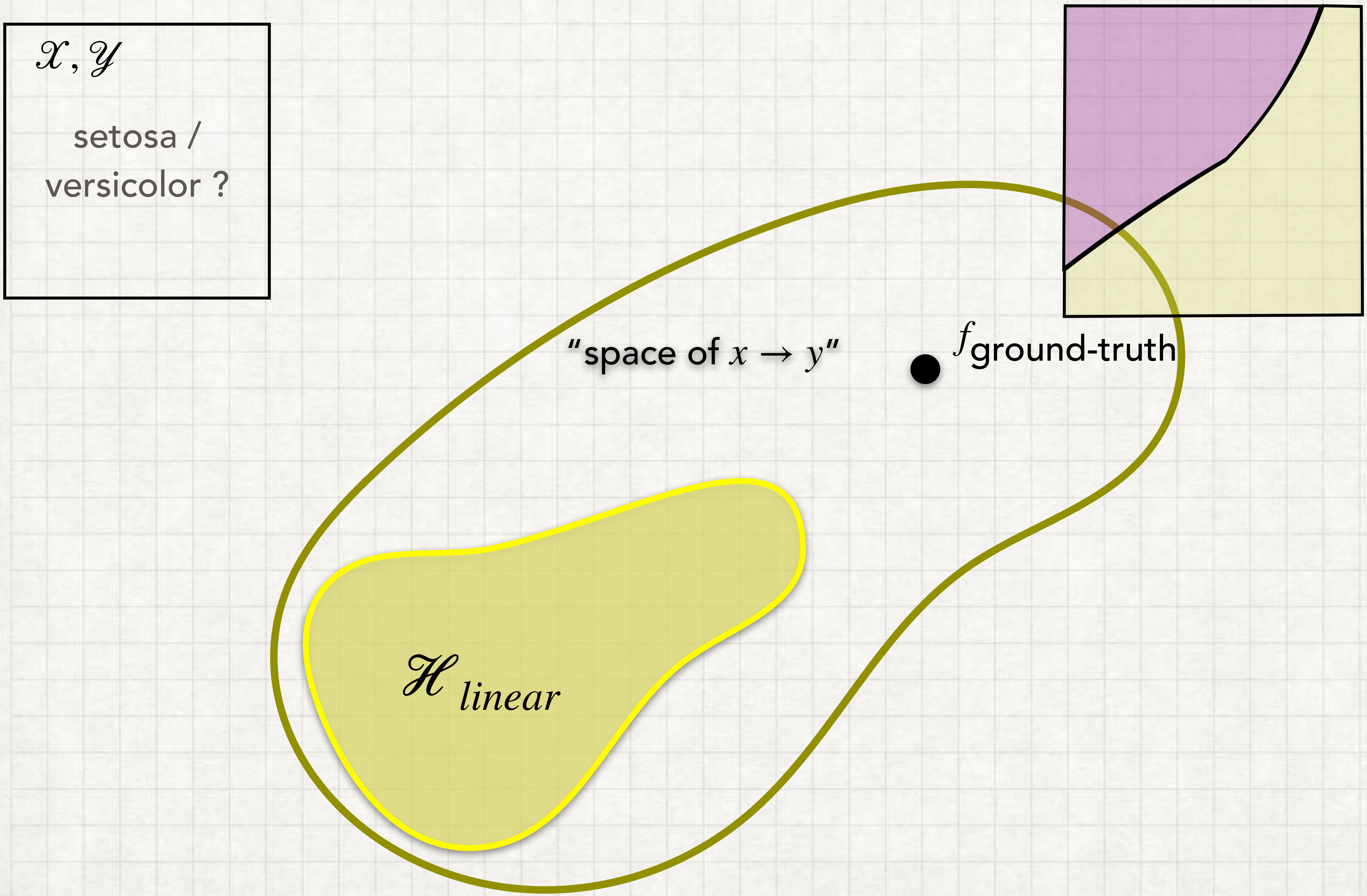


FUNCTIONAL SPACE …



FUNCTIONAL SPACE.



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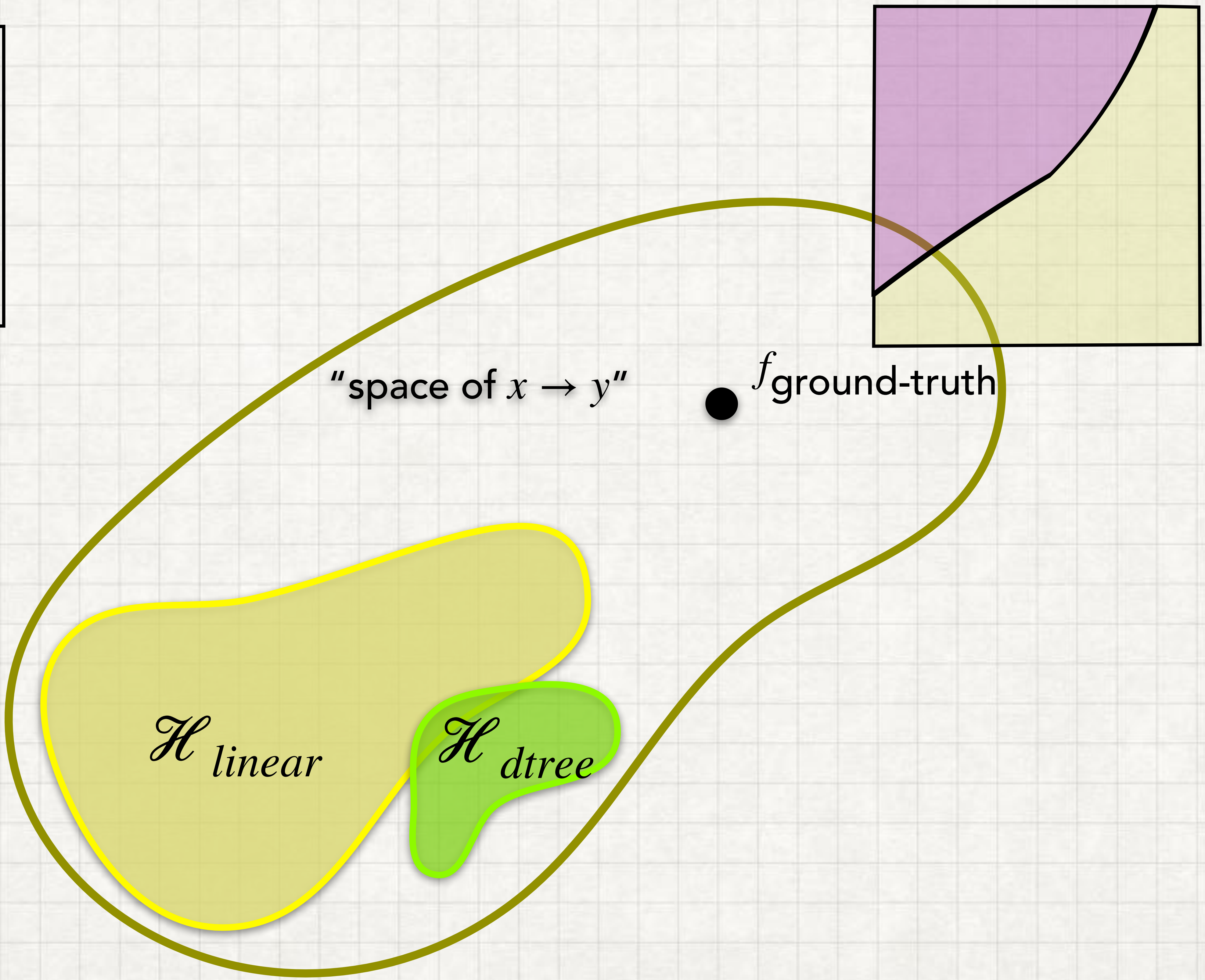
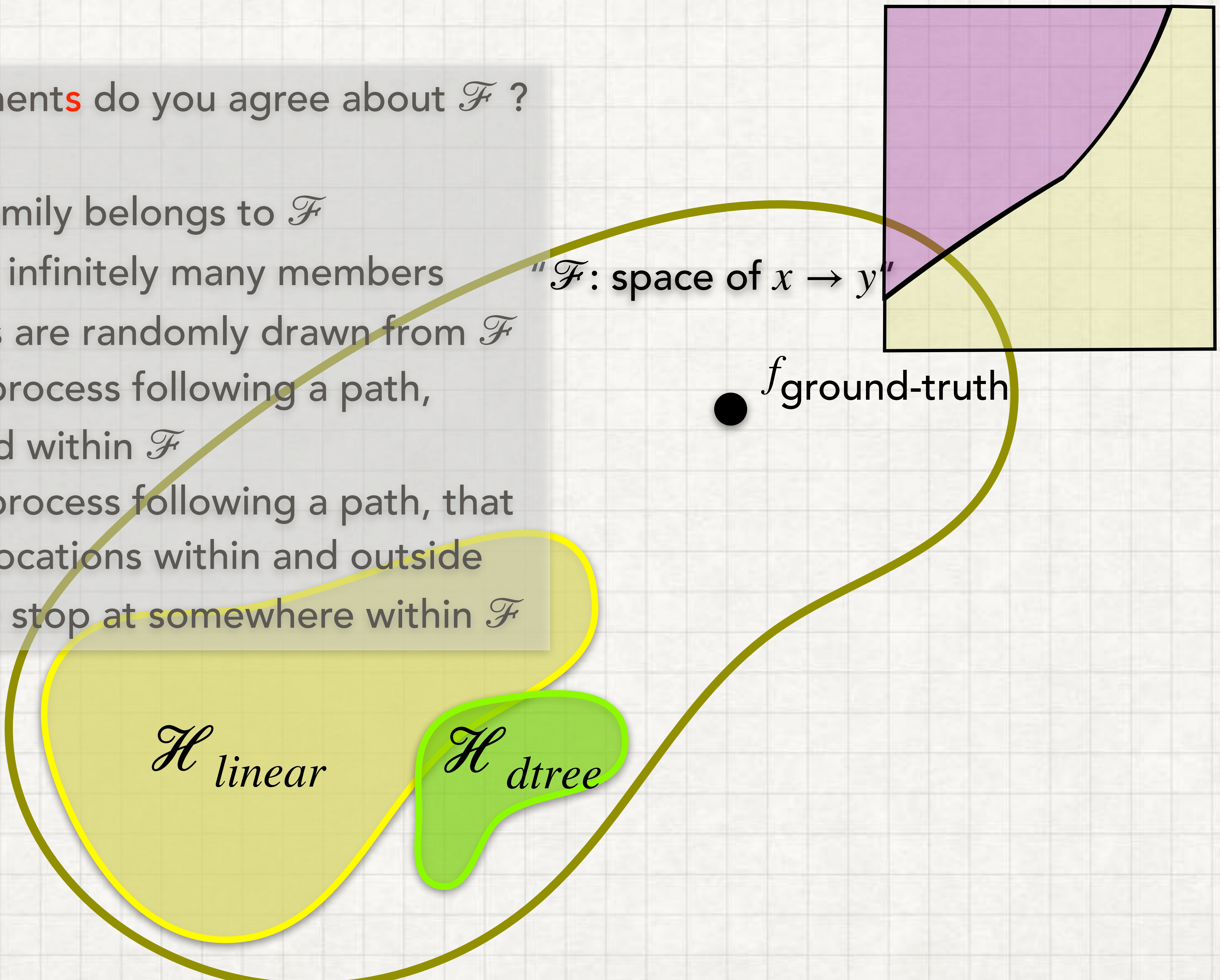


FUNCTIONAL SPACE.

Q: Which statements do you agree about $\mathscr{F}$?

A. Any model family belongs to $\mathscr{F}$
B. $\mathscr{F}$ must have infinitely many members
C. Data samples are randomly drawn from $\mathscr{F}$
D. Training is a process following a path, strictly limited within $\mathscr{F}$
E. Training is a process following a path, that may lead to locations within and outside $\mathscr{F}$, but finally stop at somewhere within $\mathscr{F}$



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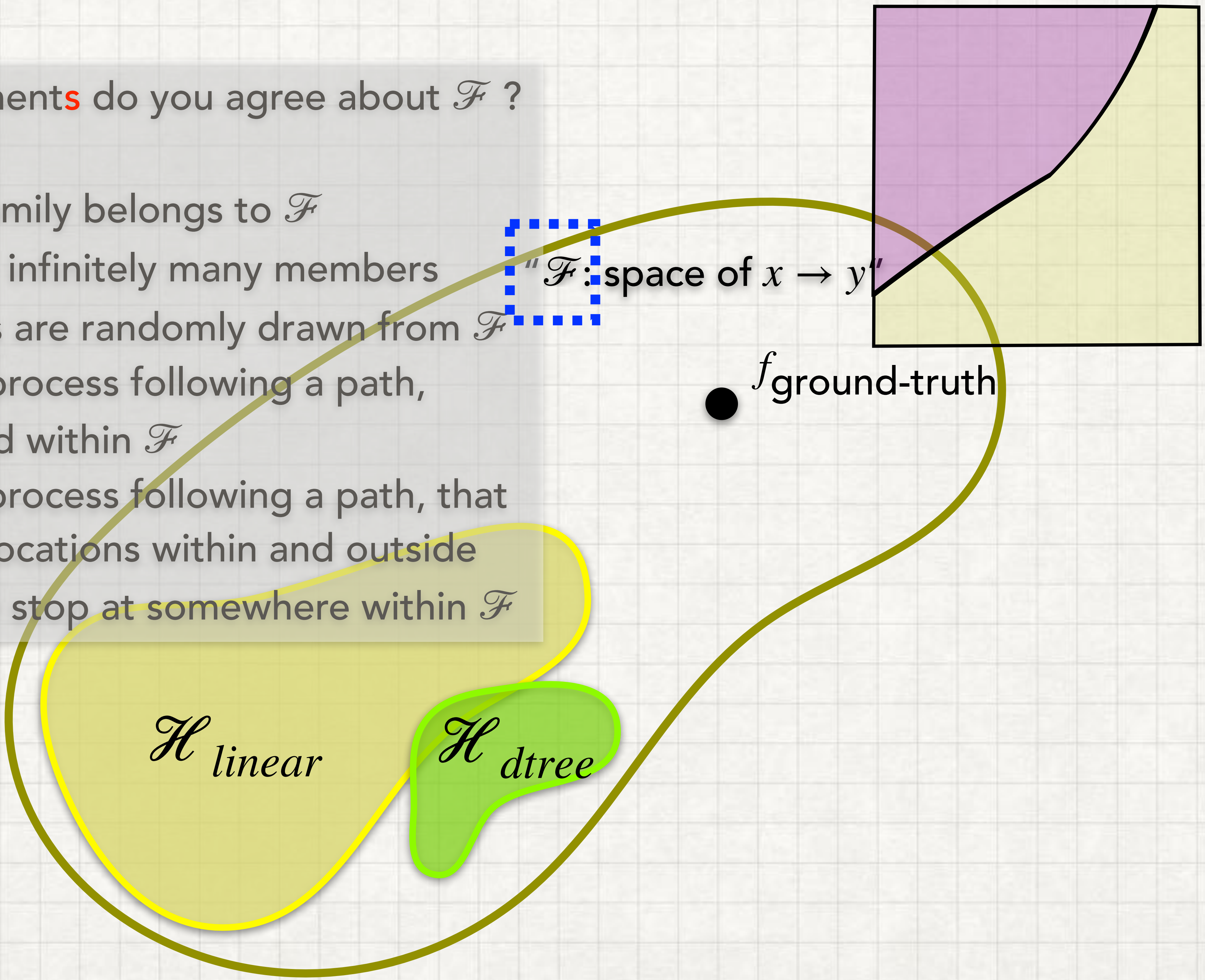

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"$\mathscr{F}$: space of $x \rightarrow y$"

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WRONG

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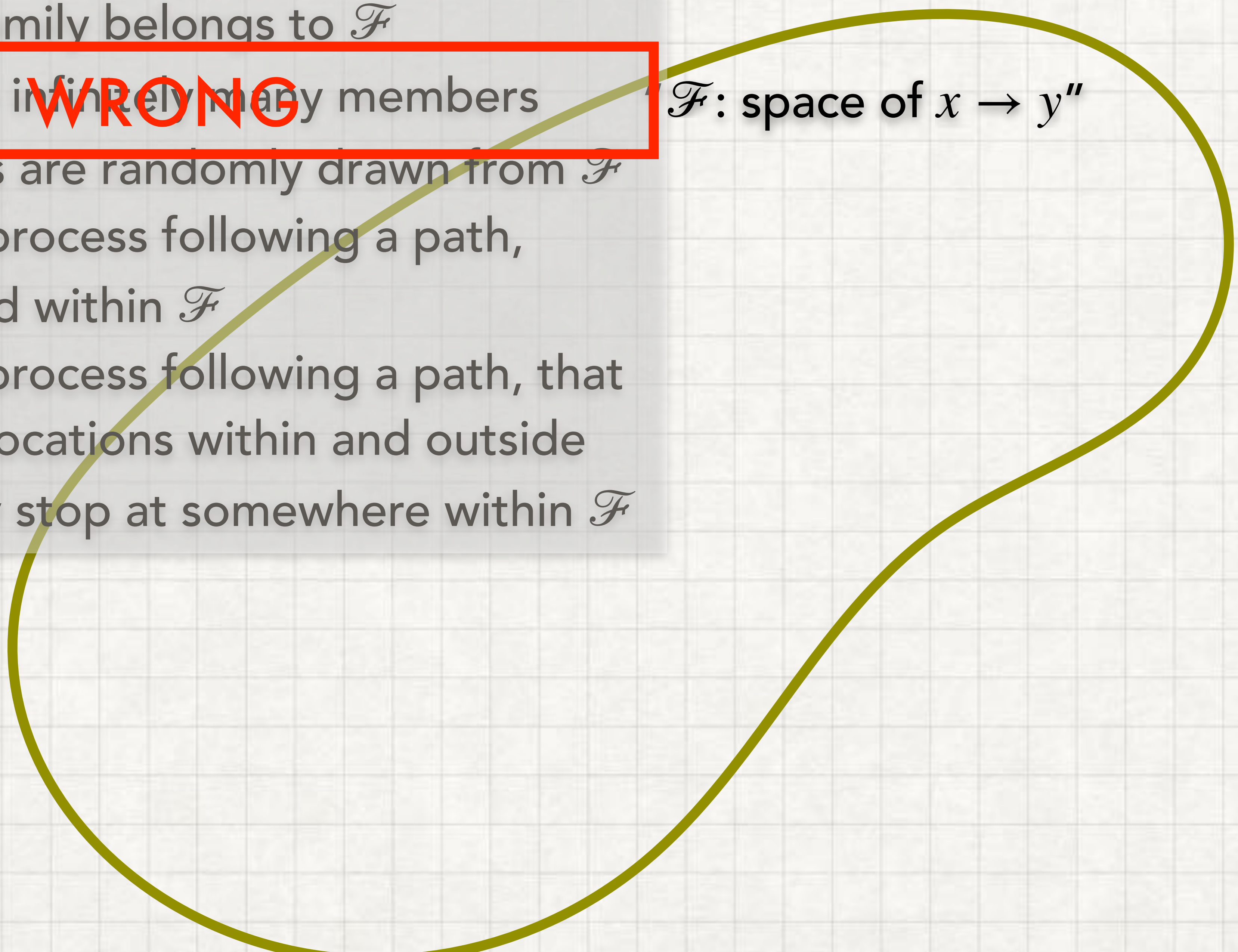

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3758280234548012036833624189723865048677365517592586770565238397822316814983377085357327257526588443337024577495260577603092278913516177656519073109687802364646940433162365621467244164785911318325937291112215801805317492327775155799698990751422139691179948773438020494216249544022145293907816475633395350247725849016076668629825679186228496361602088773658349501637901885230262474405073903820321888923861099058697067531432439211984822120754440224333665547868565593896895856381265823772240377217022399914414660261857526515029364722809110185003203754963367499515695215418504417479258440662952796718726052857925526601307020479982183347493563216774695296825517658582675027158940078877272500707803502629523772140288422974862635978797921763382209326194895093 76

	X1	L	L	L	M	M	M	H	H	H
Y|X	X2	L	M	H	L	M	H	L	M	H
g0		0	0	0	0	0	0	0	0	0
g1		0	0	0	0	0	0	0	0	1
g2		0	0	0	0	0	0	0	1	0
g3		0	0	0	0	0	0	0	1	1
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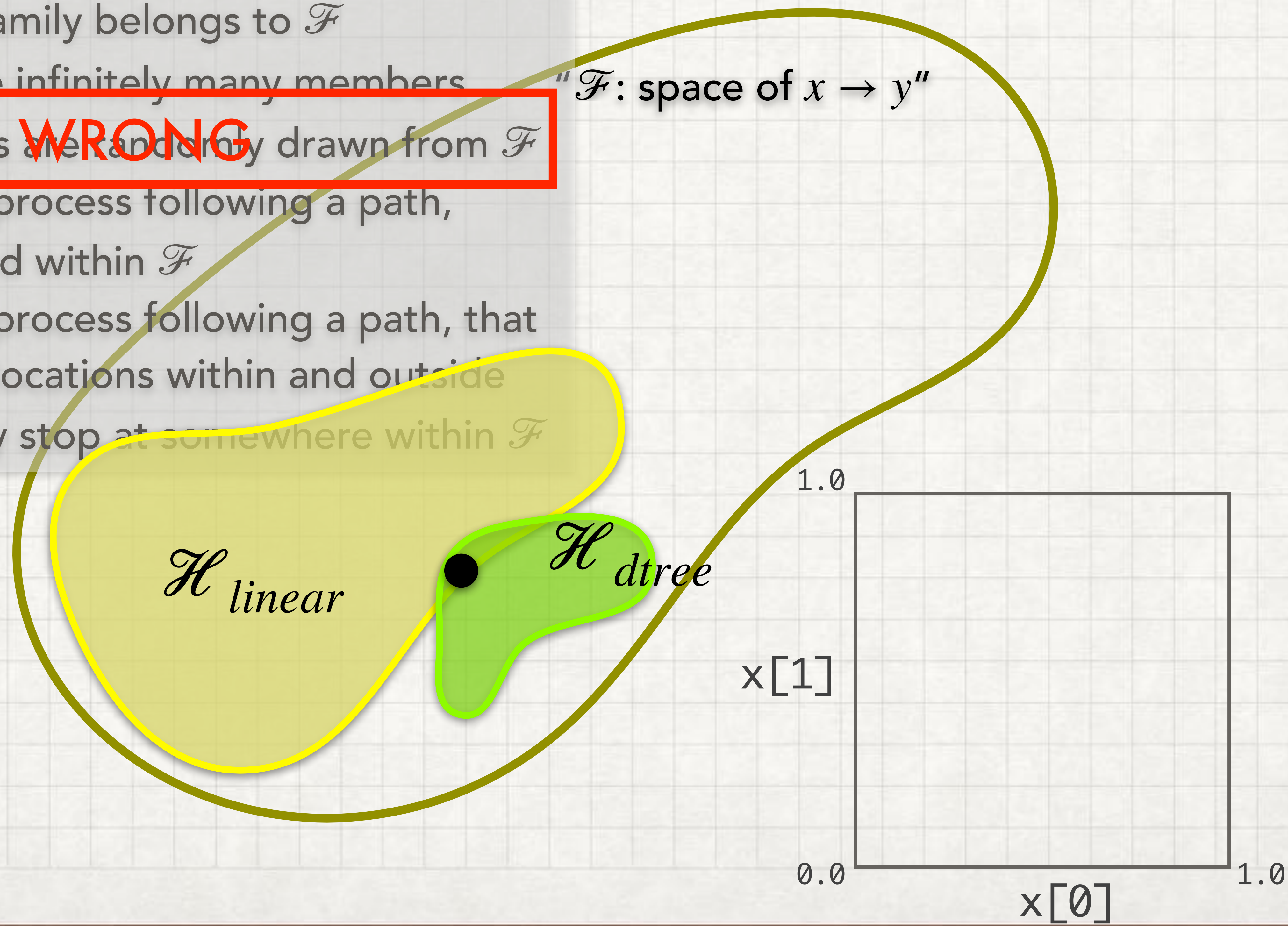

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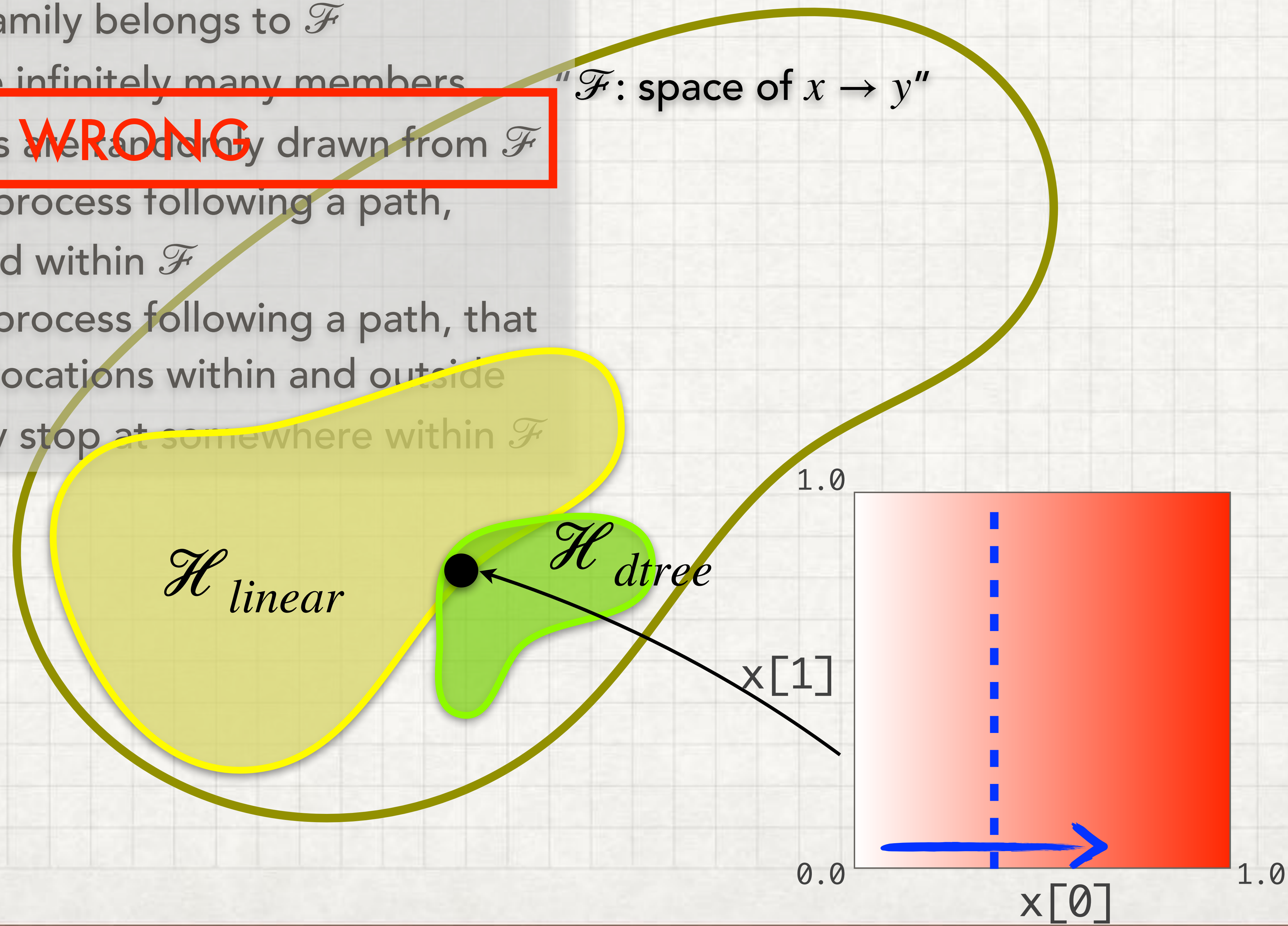

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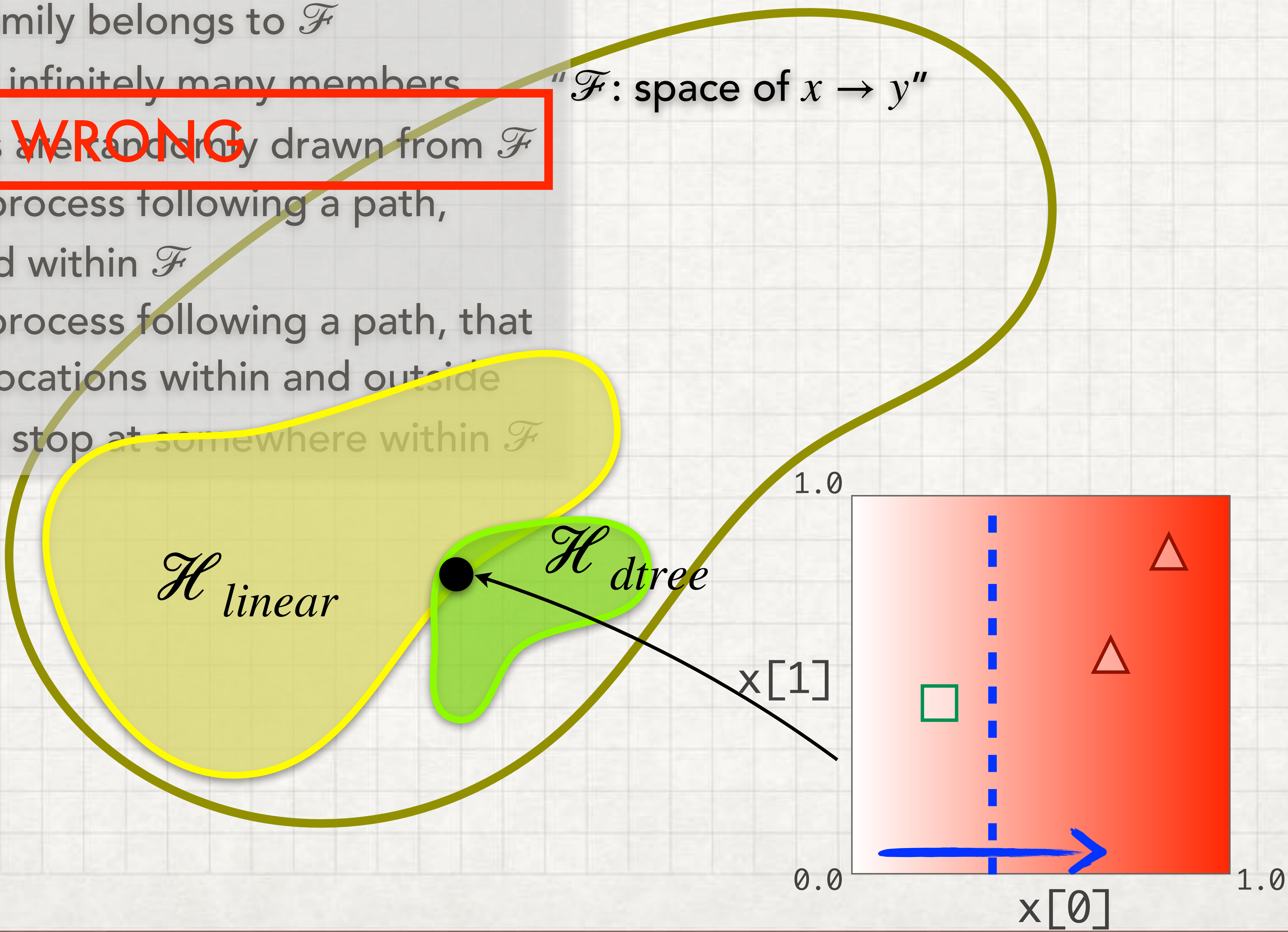

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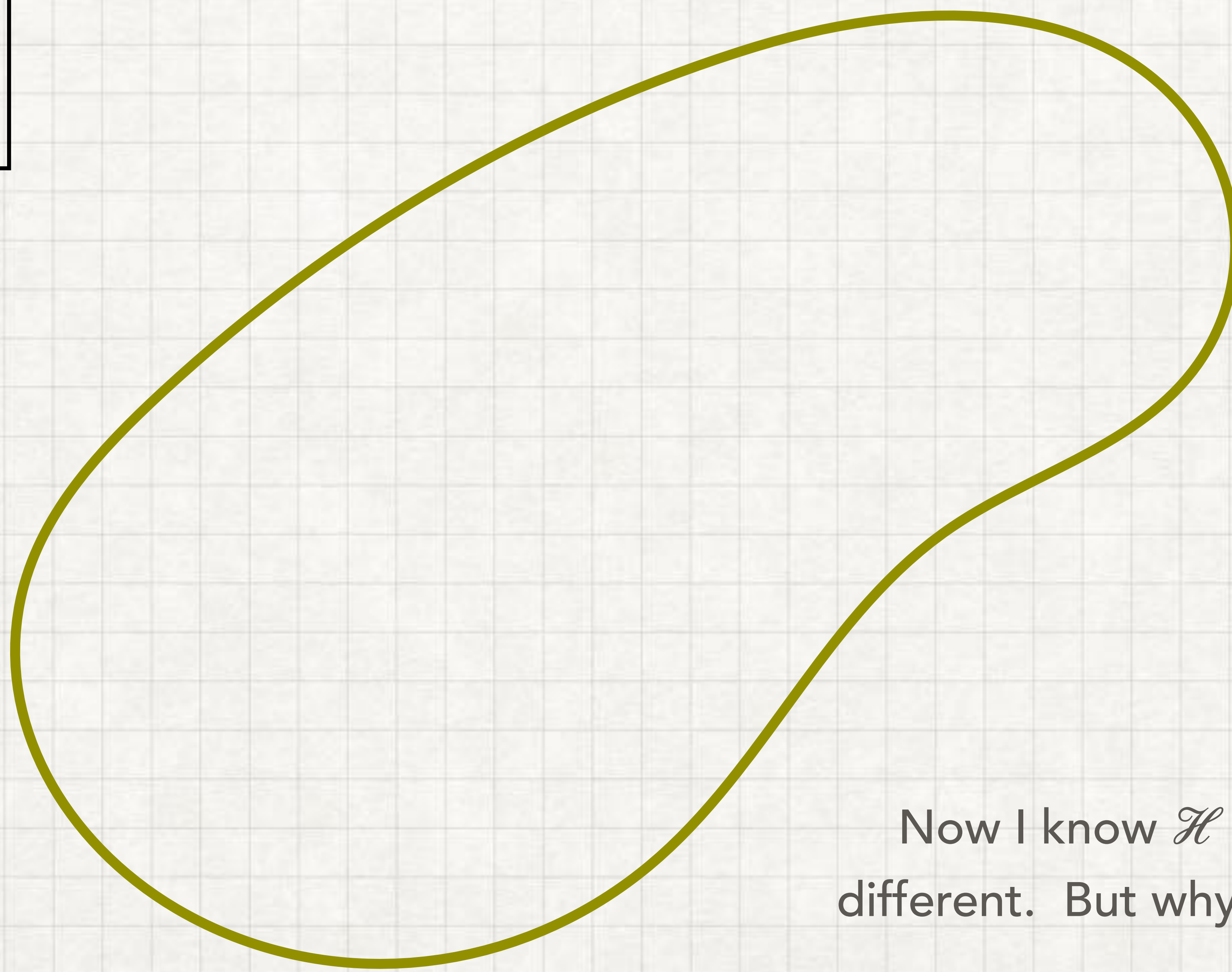
WRONG

"$\mathscr{F}$: space of $x \to y$"

WHY FUNCTIONAL SPACE?

$\mathscr{X}, \mathscr{Y}$

setosa /
versicolor ?



Now I know $\mathscr{H}$ and $\mathscr{F}$ are different. But why we need $\mathscr{F}$?

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$\mathscr{X}, \mathscr{Y}$

setosa /
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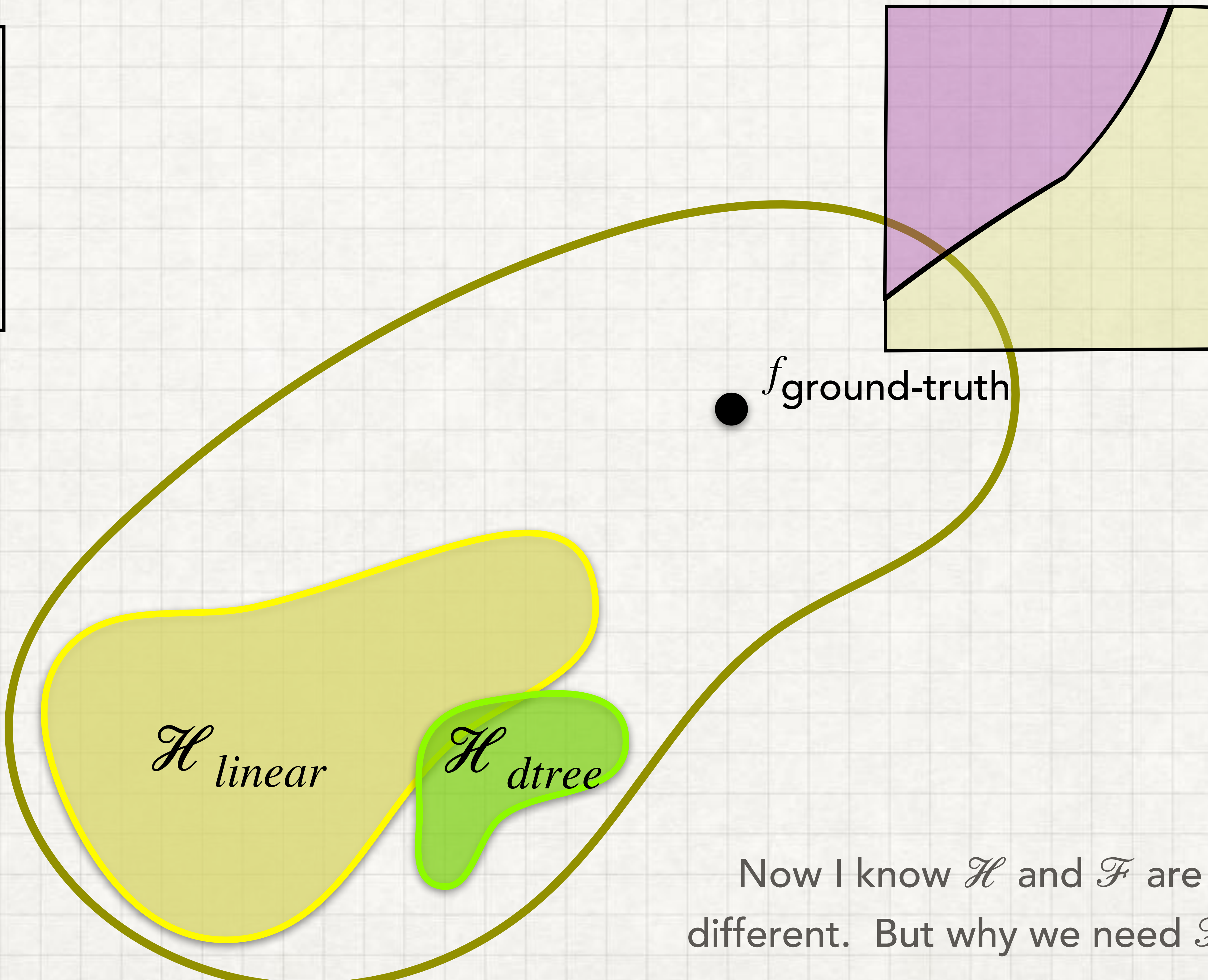

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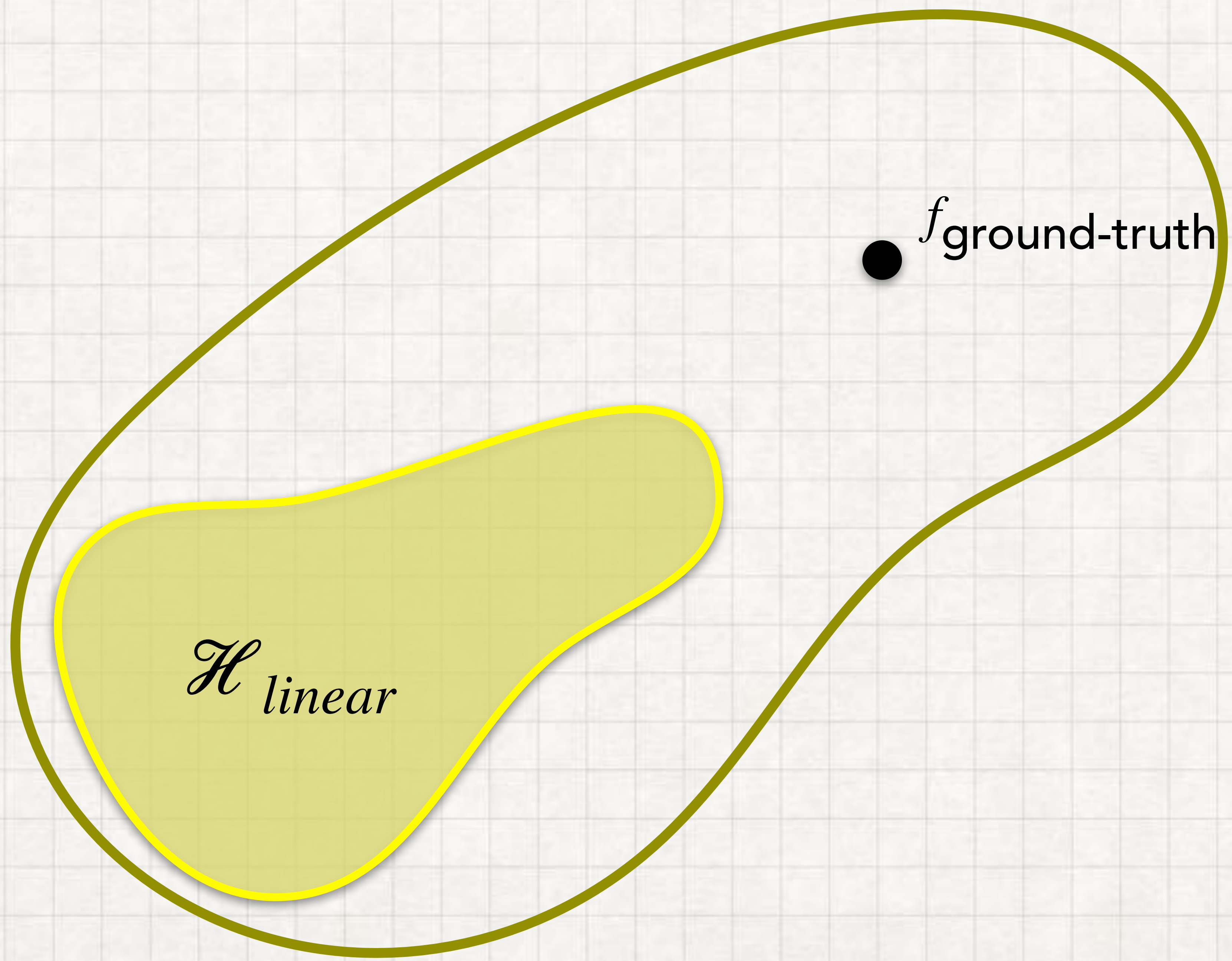
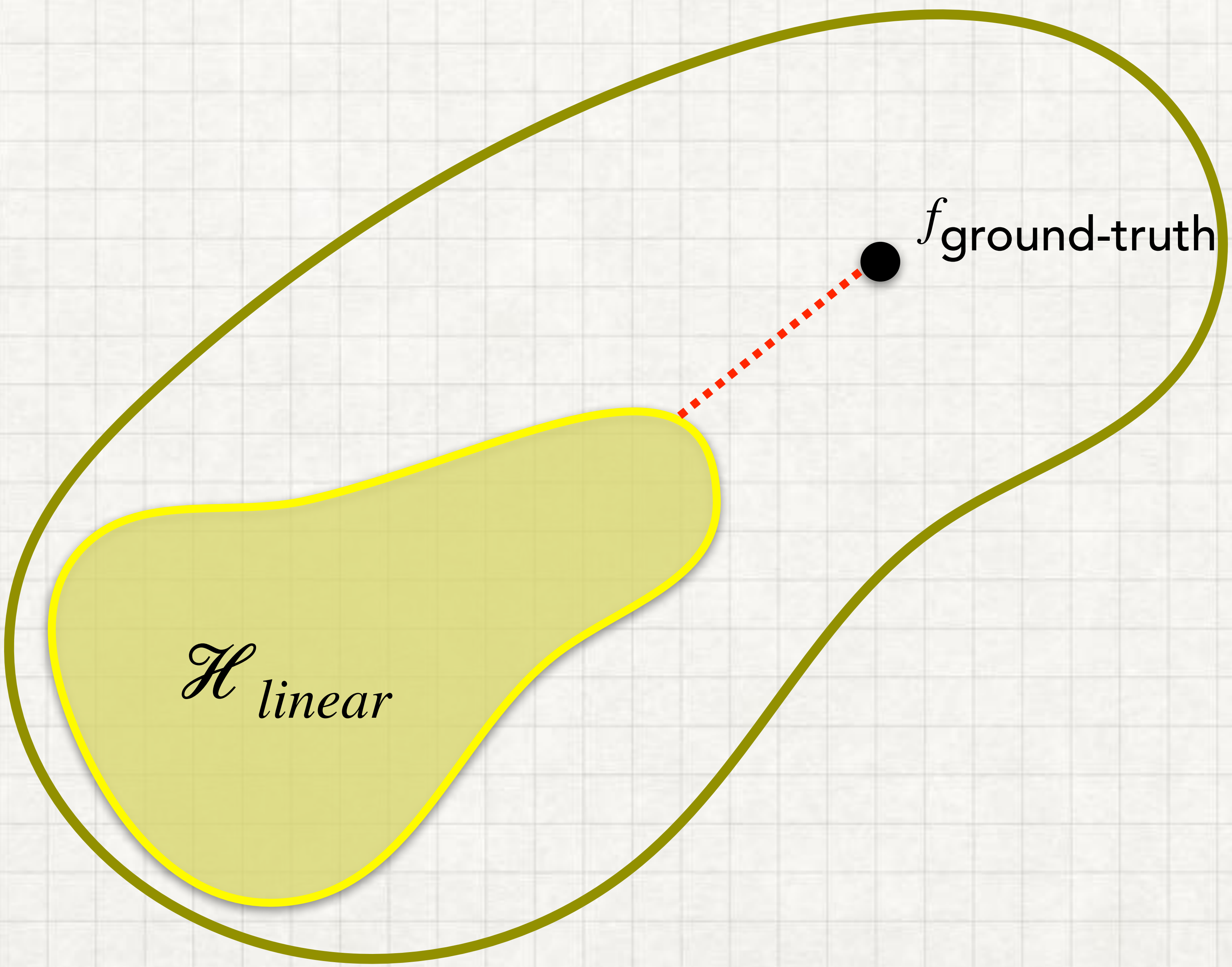
WHY FUNCTIONAL SPACE – GIVEN $\mathscr{H}$?



WHY FUNCTIONAL SPACE – GIVEN $\mathscr{H}$?



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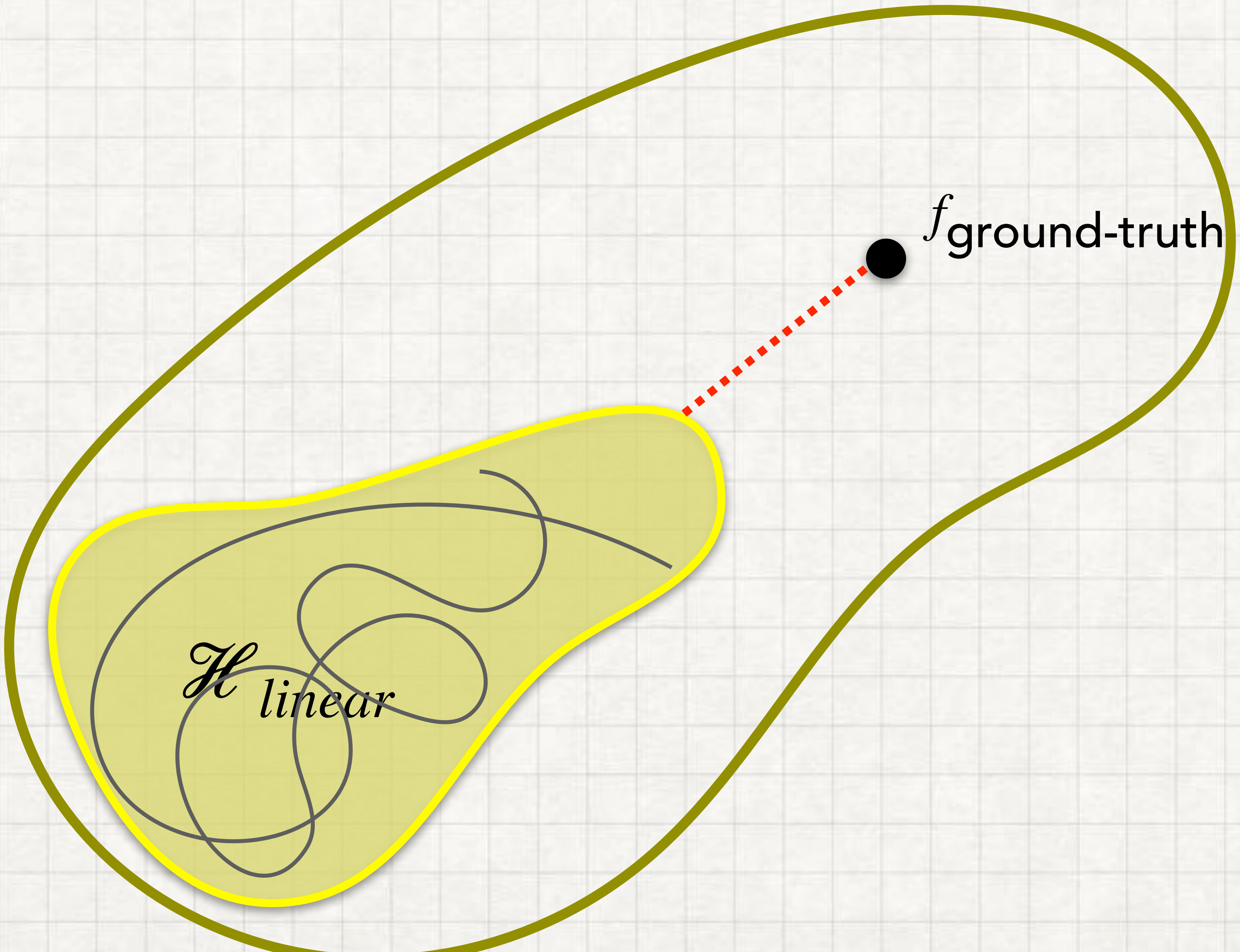
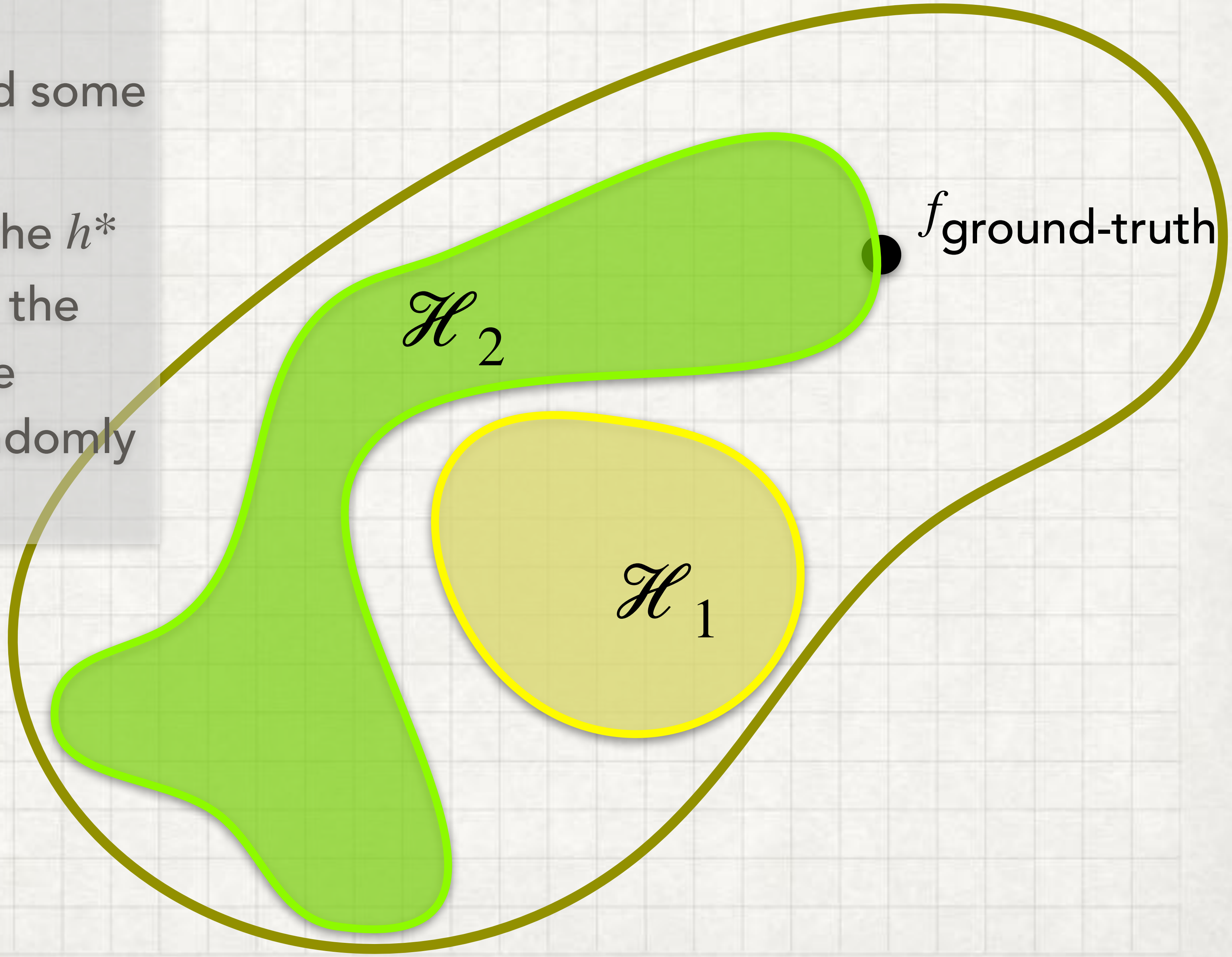


WHICH ONE IS BETTER?

Q: Which hypothesis family will result in less error?

A. $\mathscr{H}_2$, it (almost) includes the ground-truth, so one can find some good predictor h^* from $\mathscr{H}_2$
B. Not sure, because whether the h^* selected from $\mathscr{H}_2$ is close to the ground-truth depends on the training dataset, which is randomly generated.



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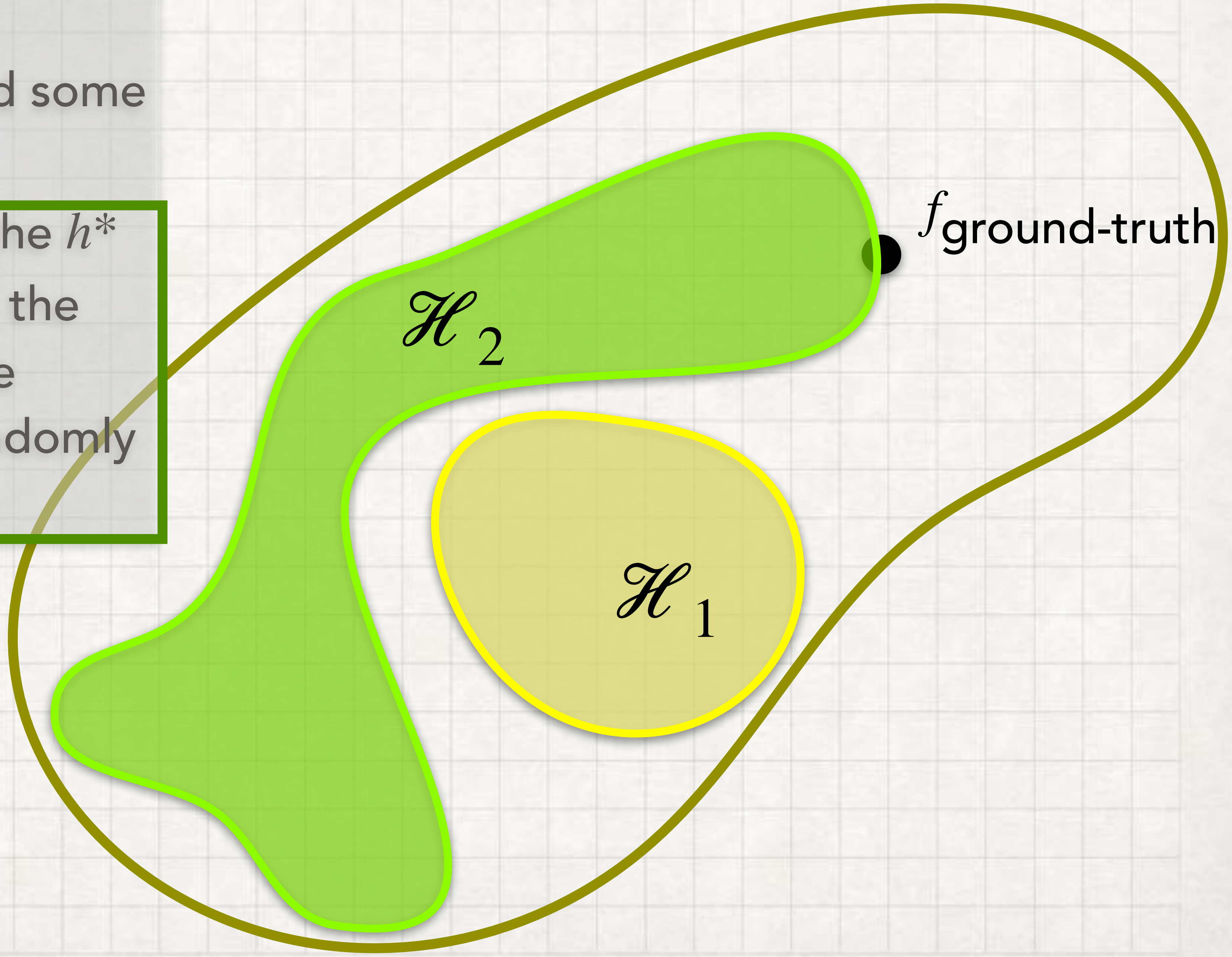


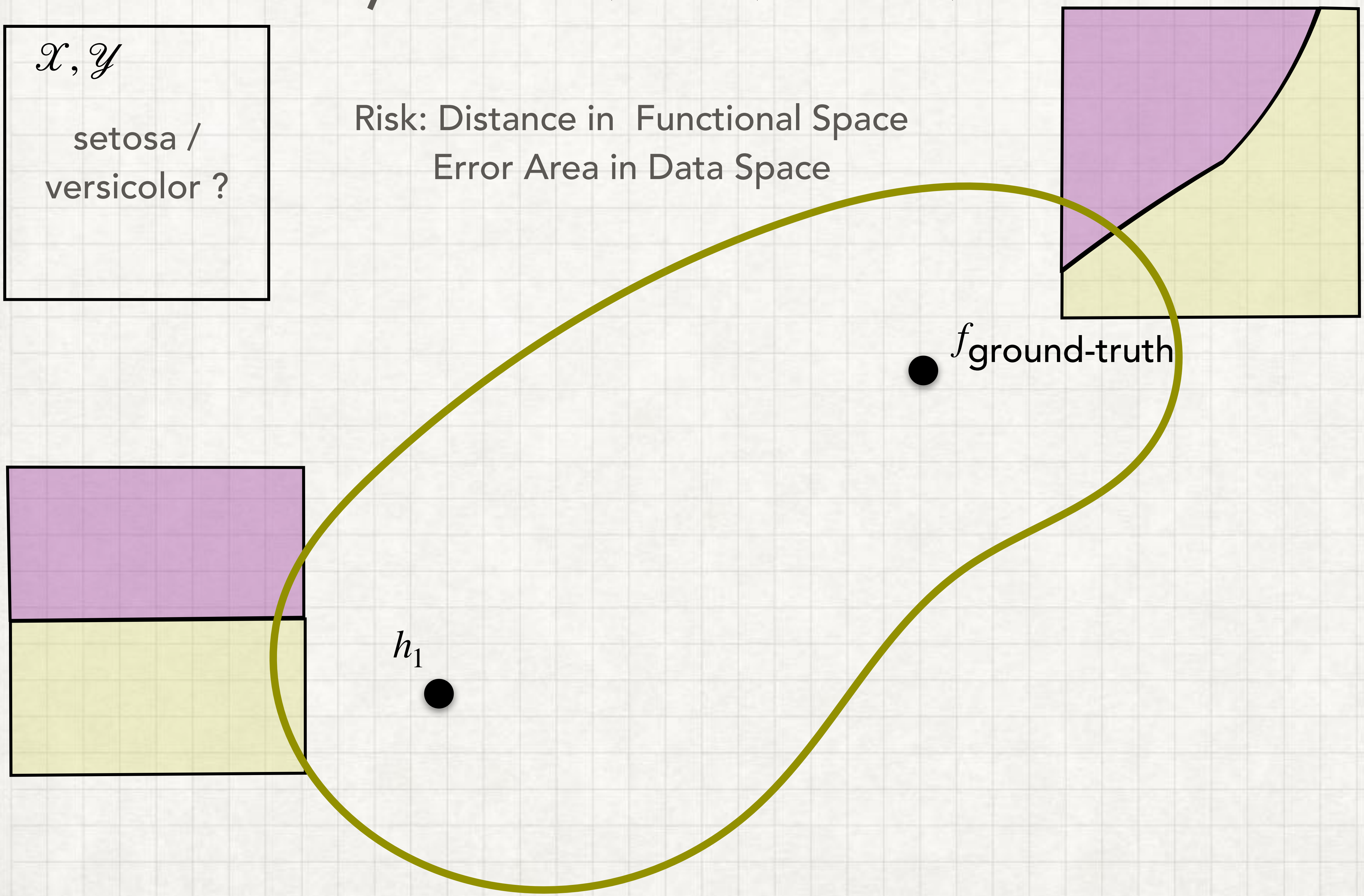
ERROR/RISK IN FUNCTIONAL SPACE

$\mathcal{X}, \mathcal{Y}$

setosa / versicolor ?

Risk: Distance in Functional Space

Error Area in Data Space



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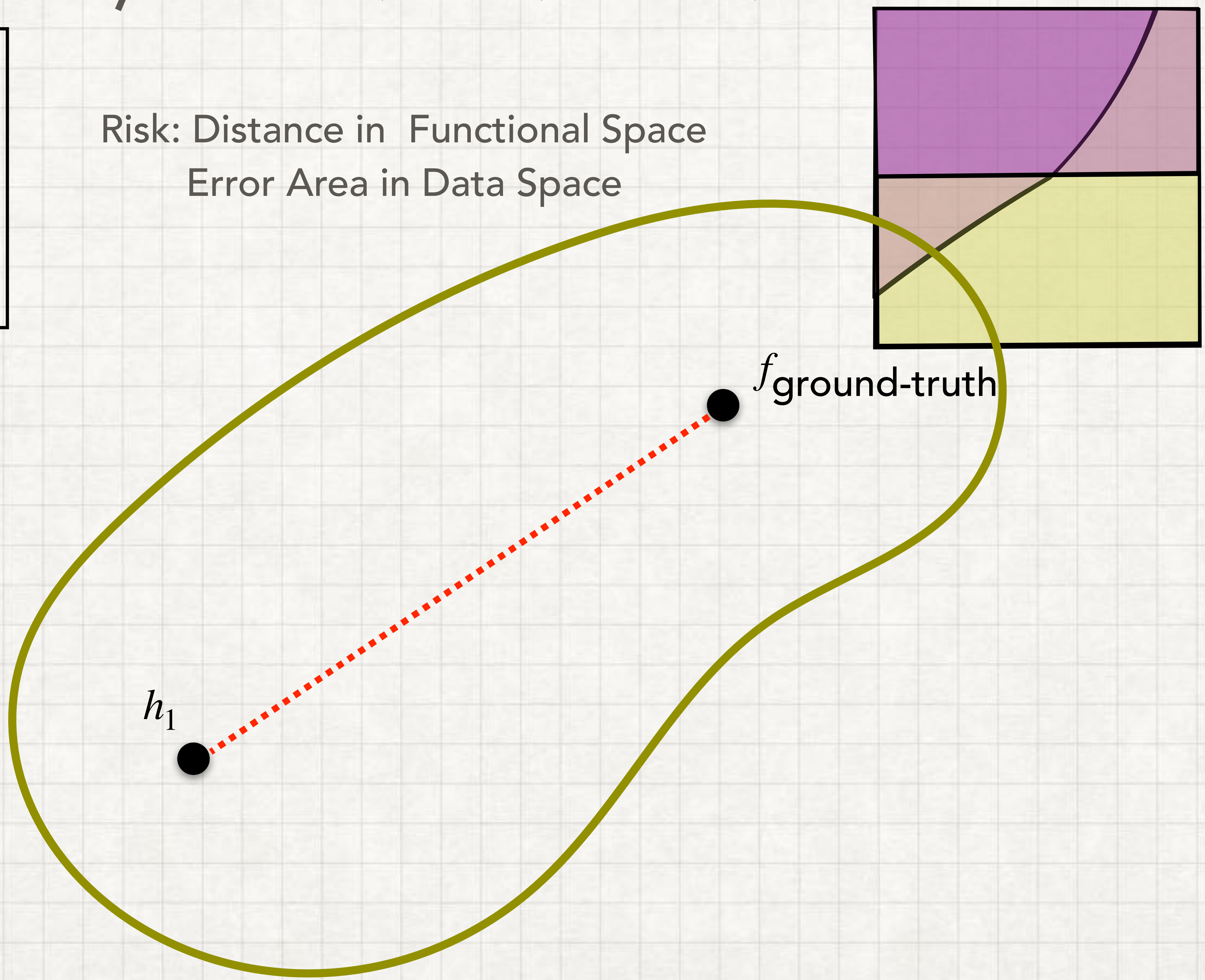

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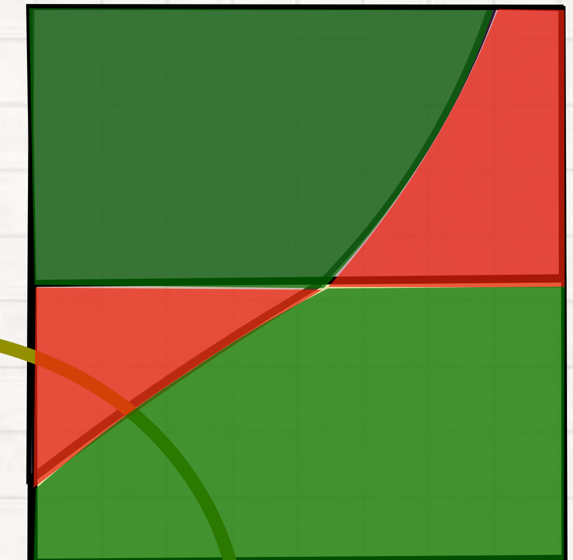

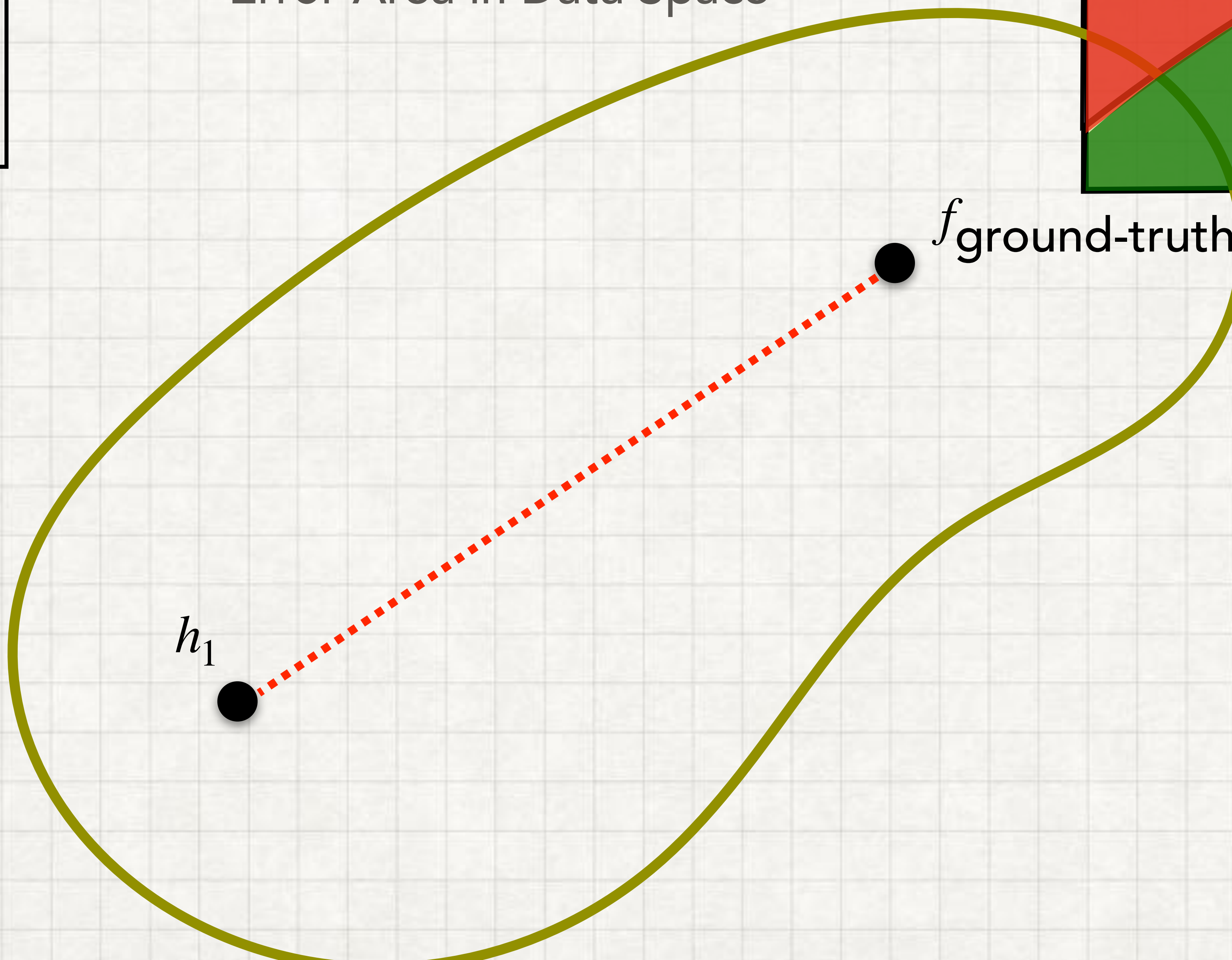

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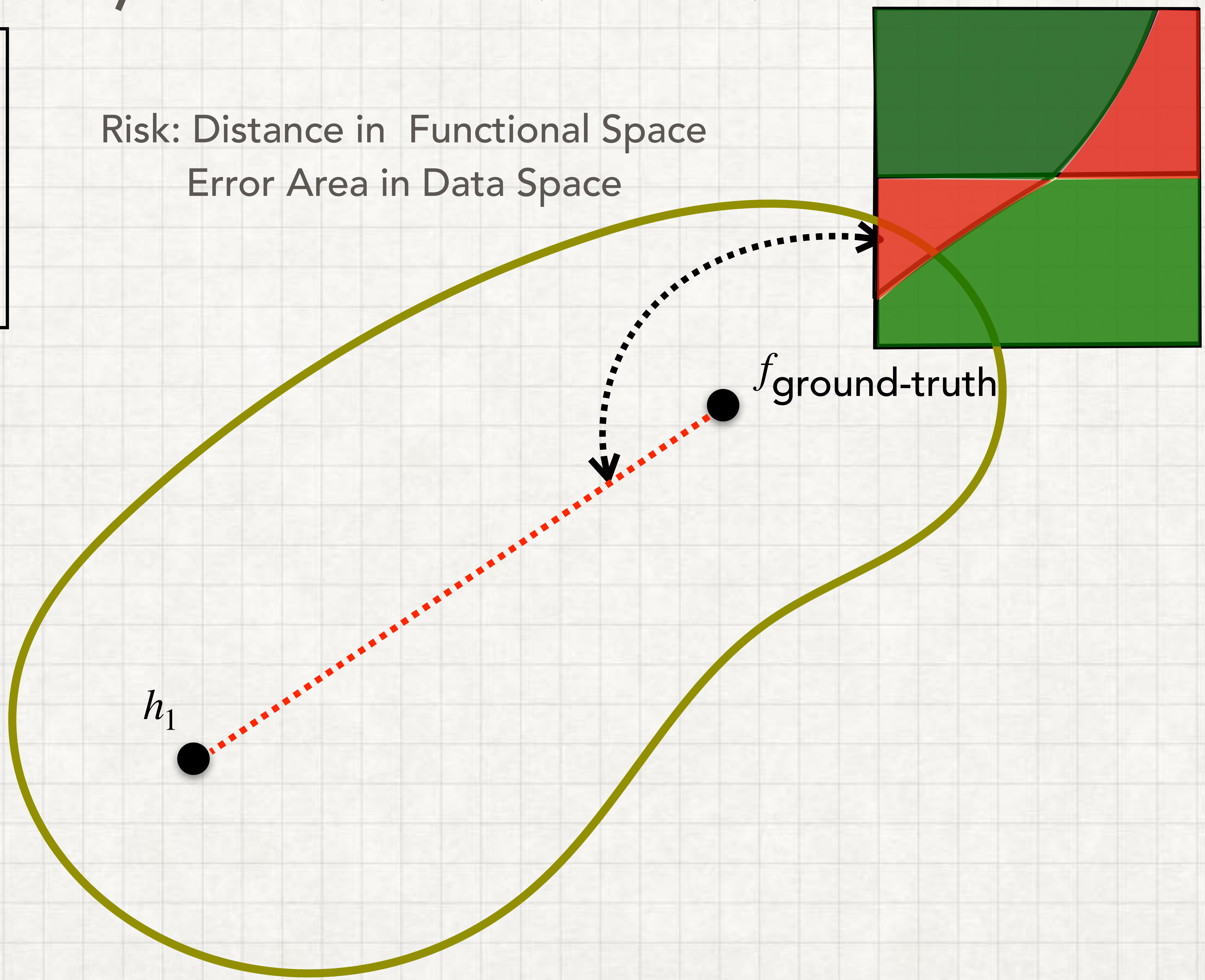
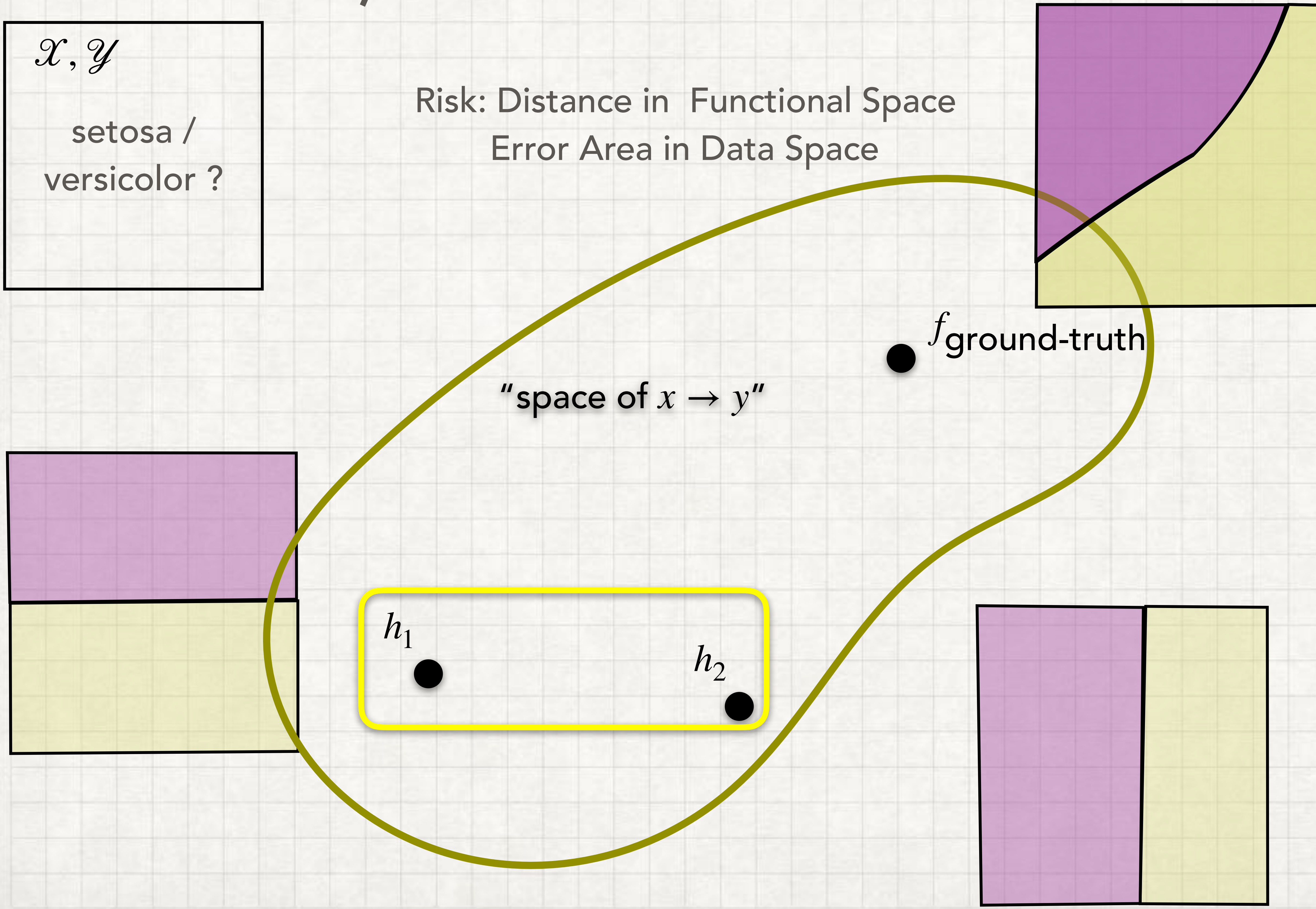


ERROR/RISKS OF TWO HYPOTHESES

Risk: Distance in Functional Space

Error Area in Data Space



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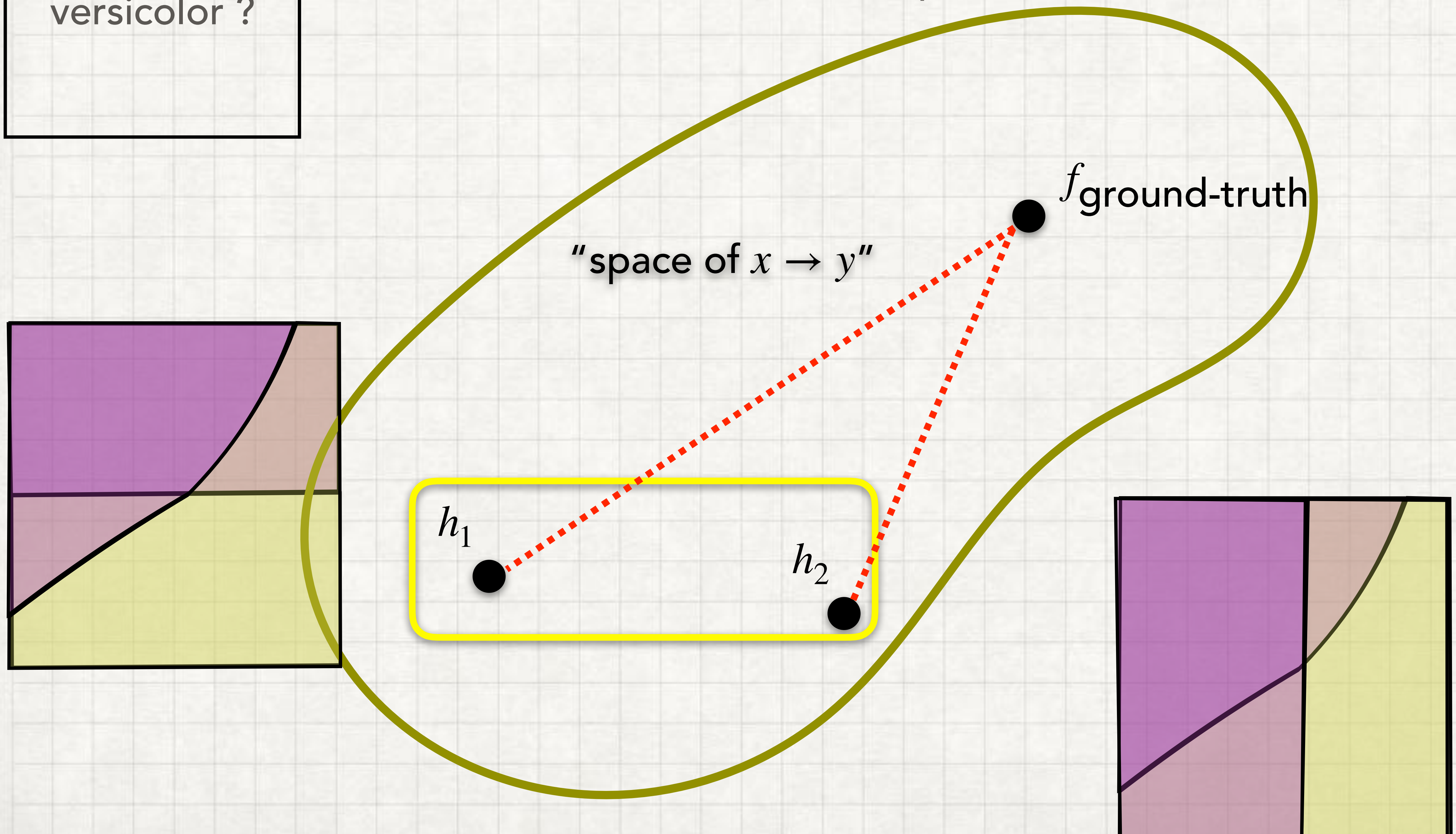

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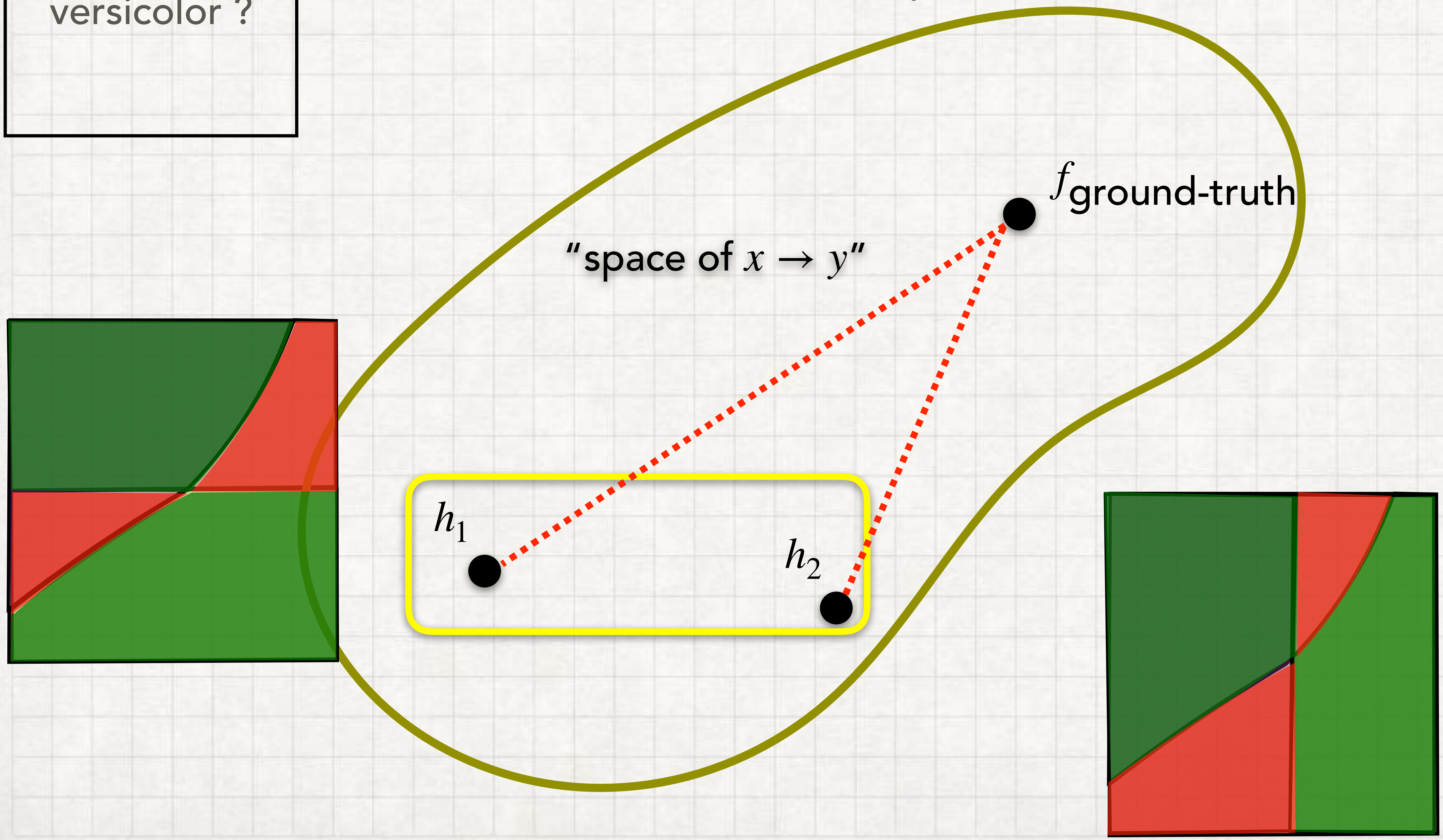
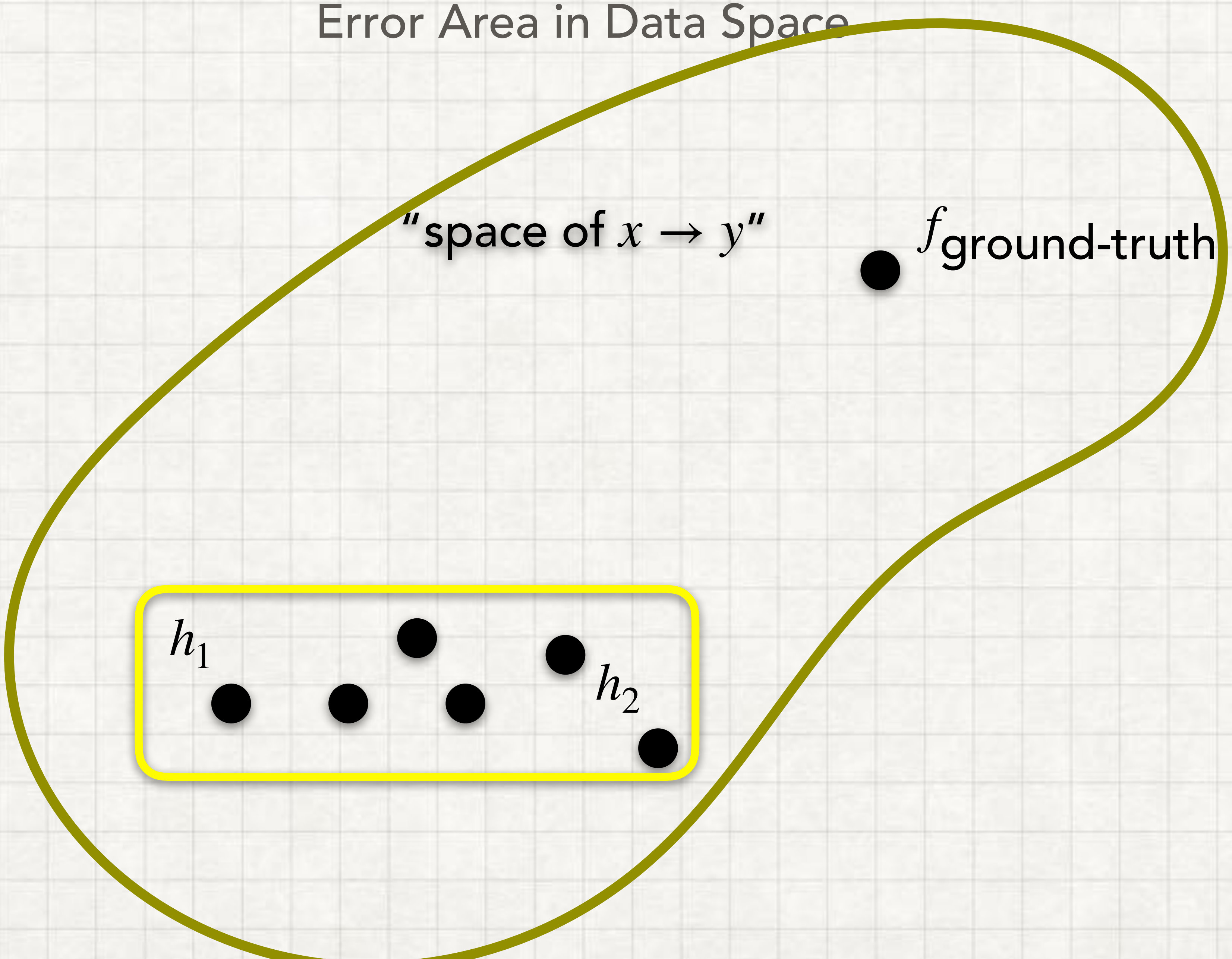


ERROR/RISKS OF MANY HYPOTHESES

Risk: Distance in Functional Space
Error Area in Data Space



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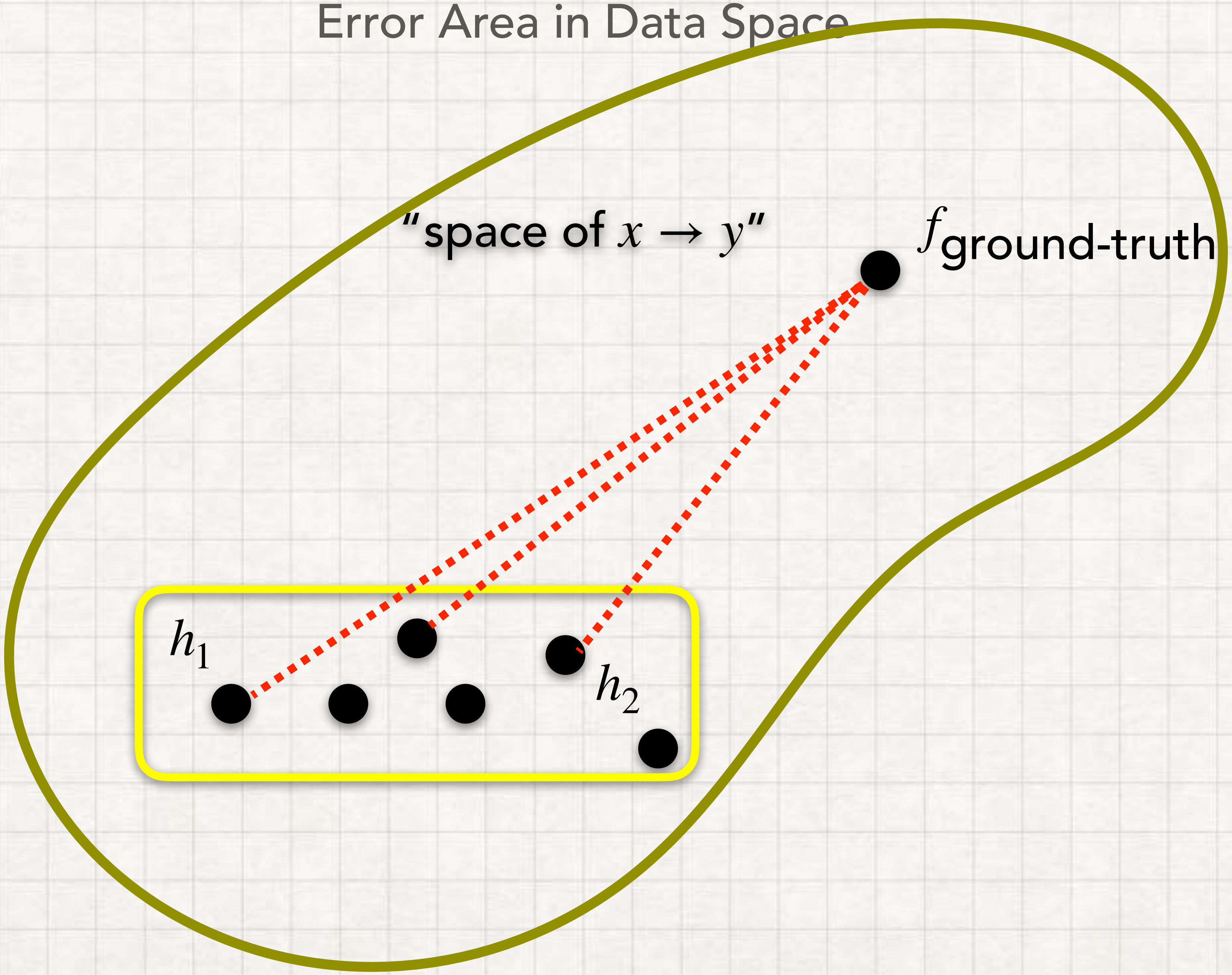
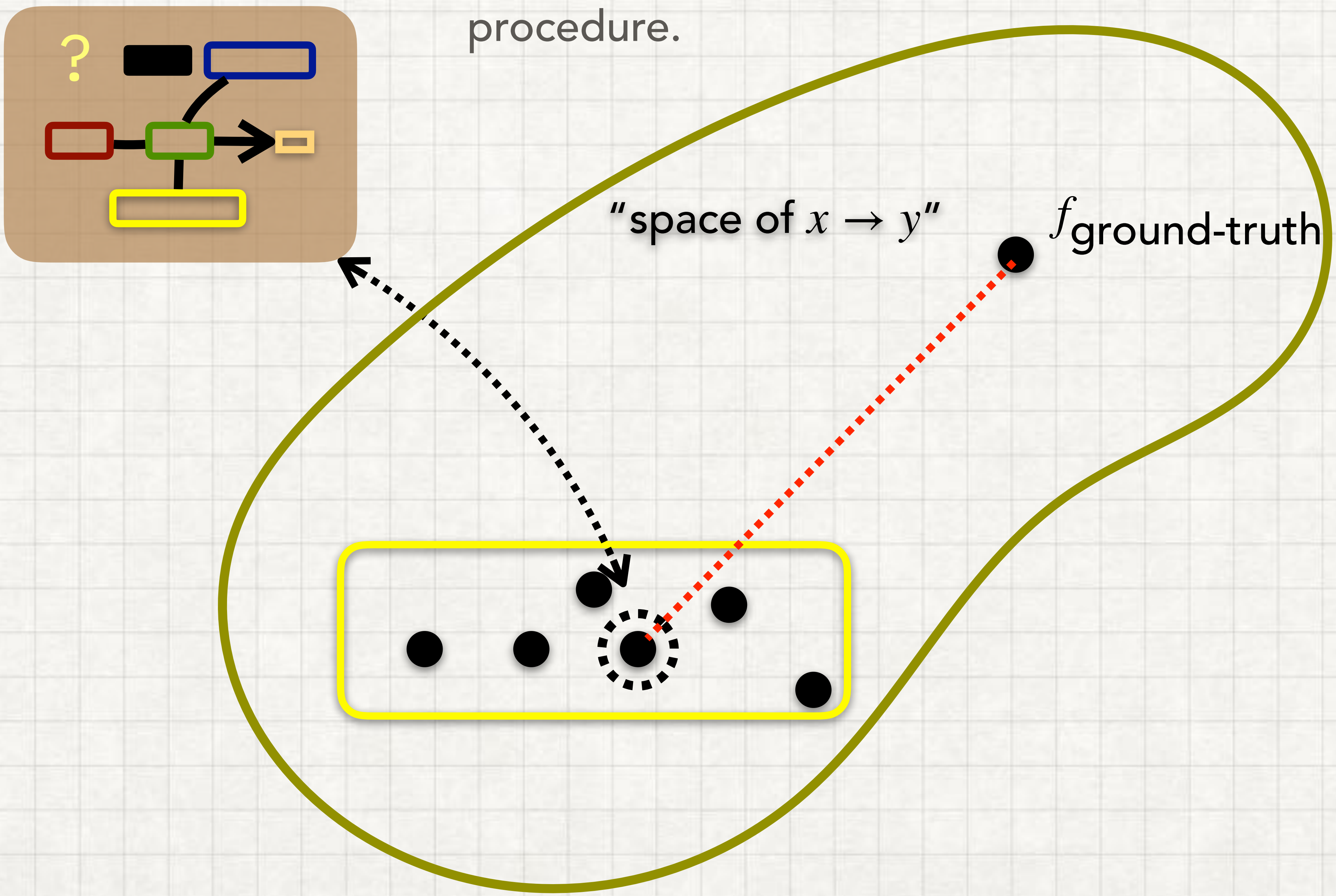


ERROR/RISKS OF SELECTED HYPOTHESES FROM MANY

The learning problem is concerned with the hypothesis selected by the learning procedure.



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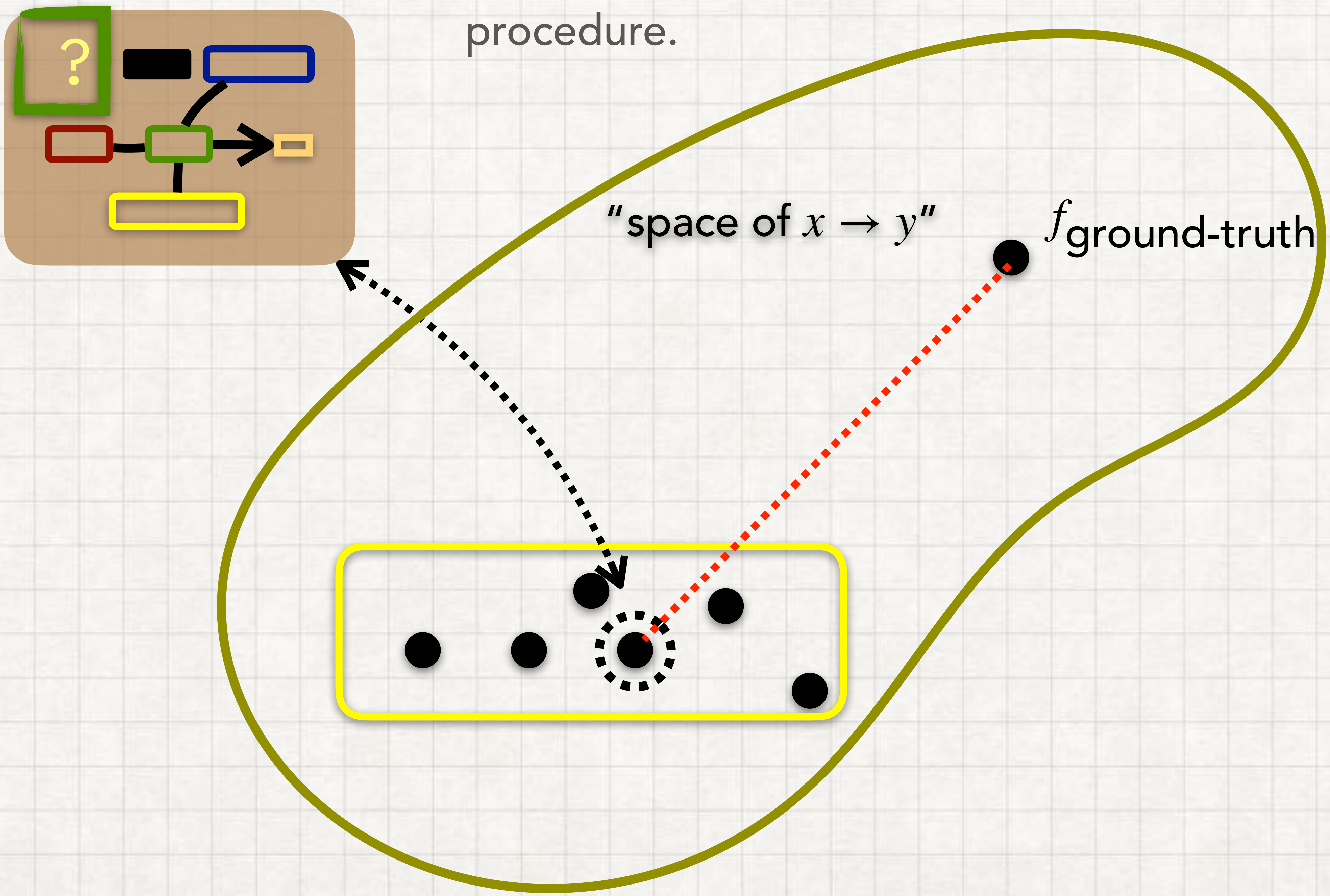
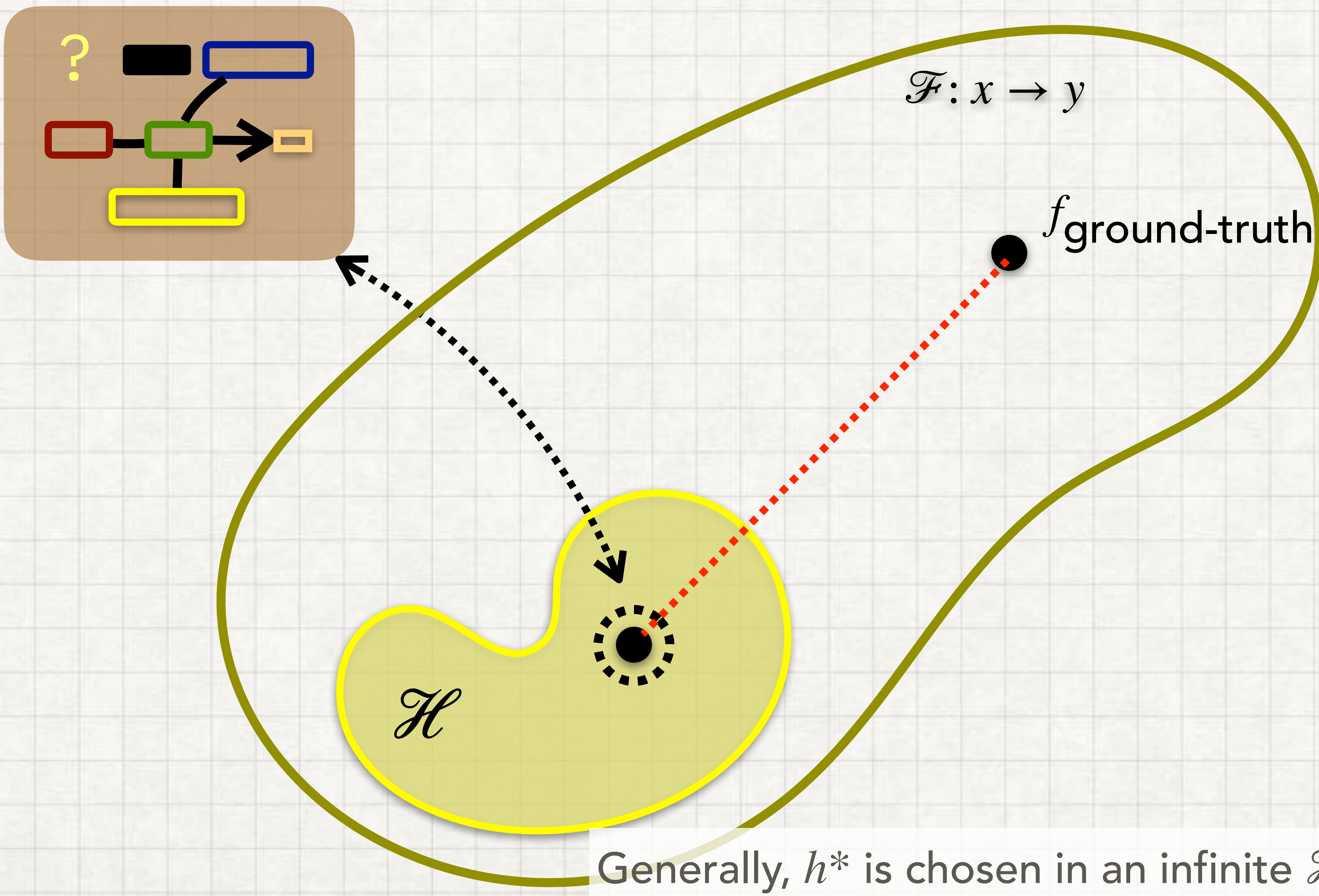


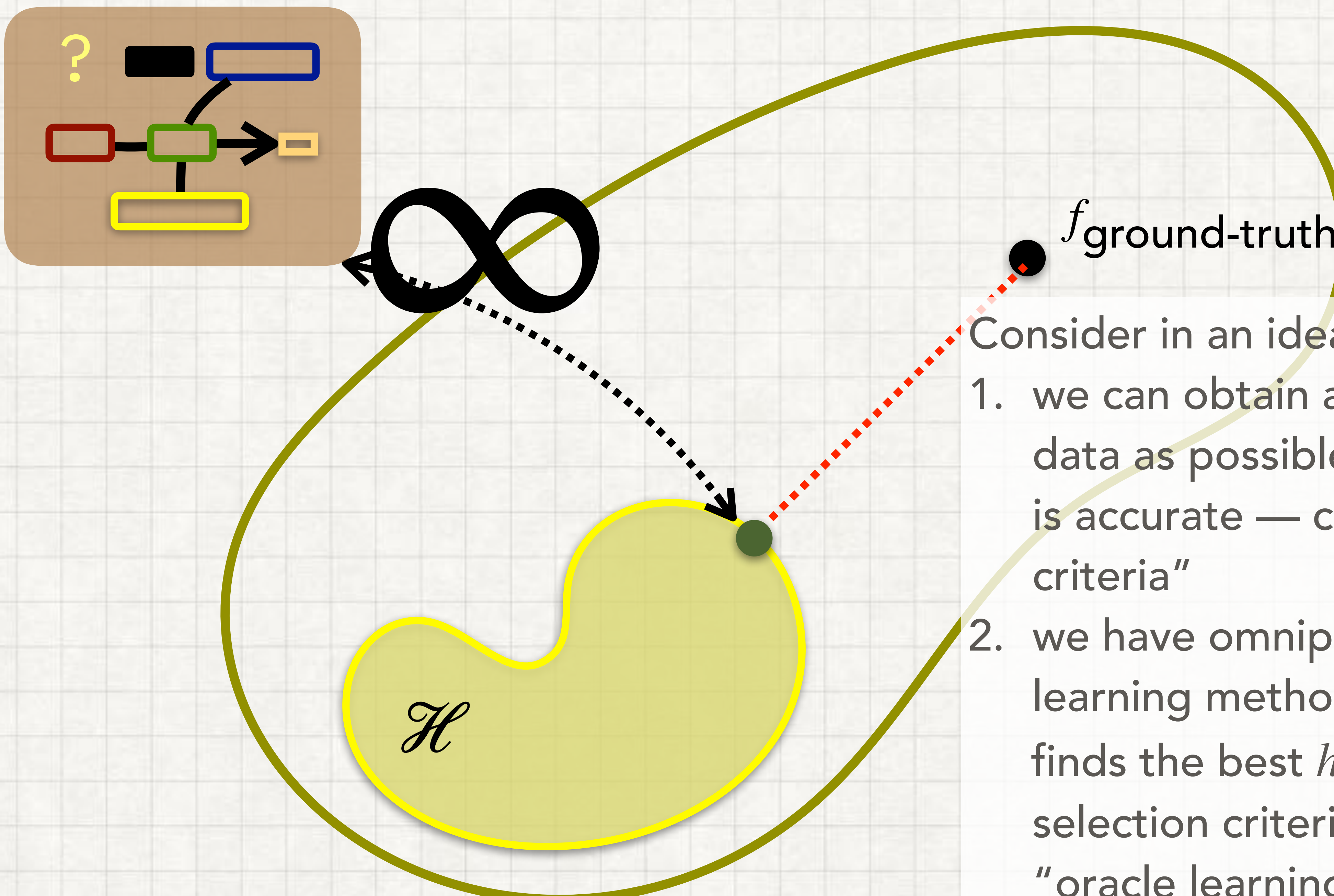
ERROR/RISKS OF SELECTED HYPOTHESIS FROM $\mathscr{H}$



Generally, h^* is chosen in an infinite $\mathscr{H}$.

MOD2: DISCRIMINATE TWO TYPES OF RISKS WHEN FACING RANDOM DATA BIAS AND VARIANCE

OPTIMUM WITHIN $\mathscr{H}$



Consider in an ideal world,

1. we can obtain as many training data as possible — the evaluation is accurate — call this "oracle criteria"
2. we have omnipotently powerful learning method, which always finds the best $h^* \in \mathscr{H}$ for any selection criteria — call this "oracle learning algorithm"

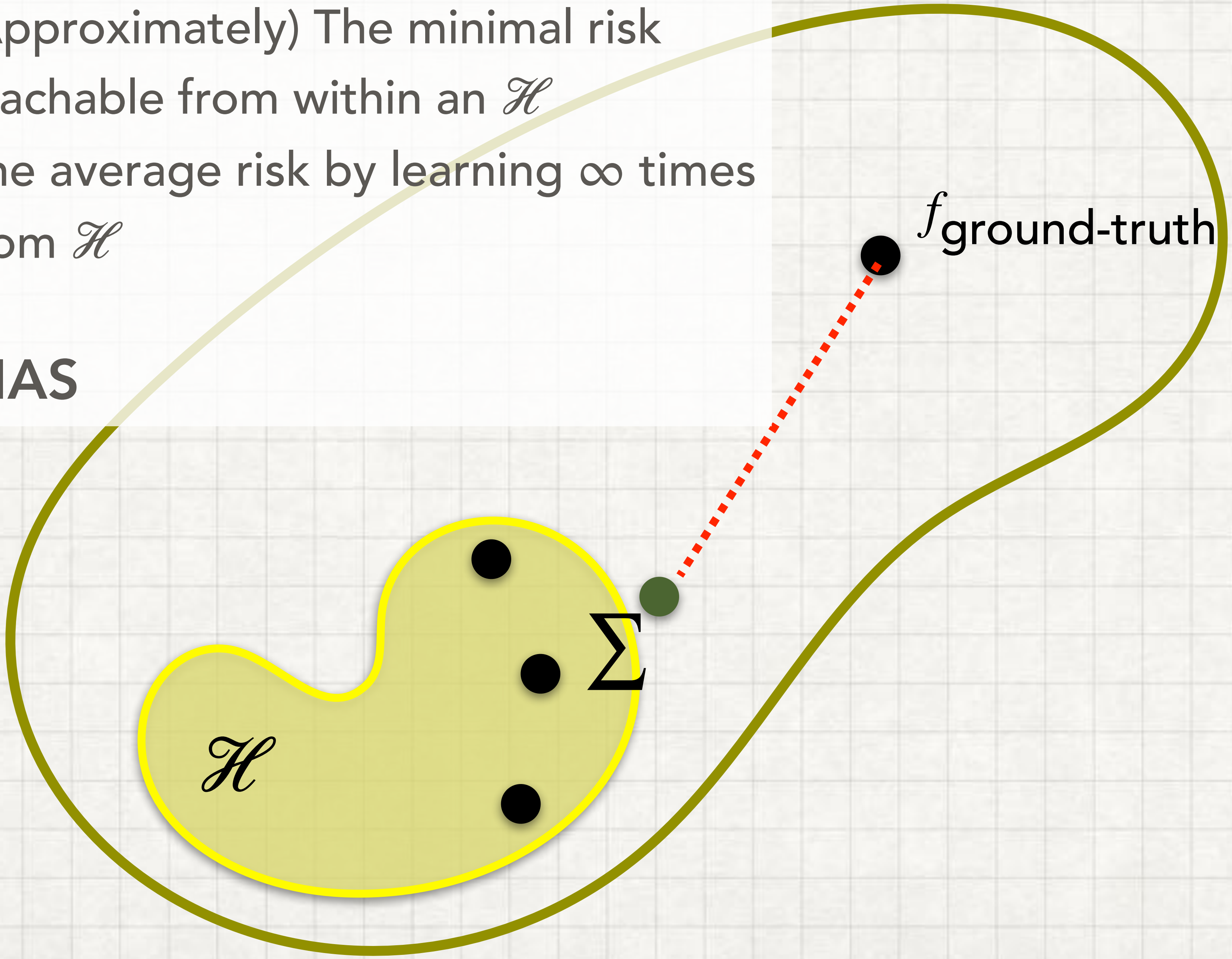
Putting together, we then consider the minimal risk of an $\mathscr{H}$.

DEFINITION OF BIAS

(Approximately) The minimal risk reachable from within an $\mathscr{H}$

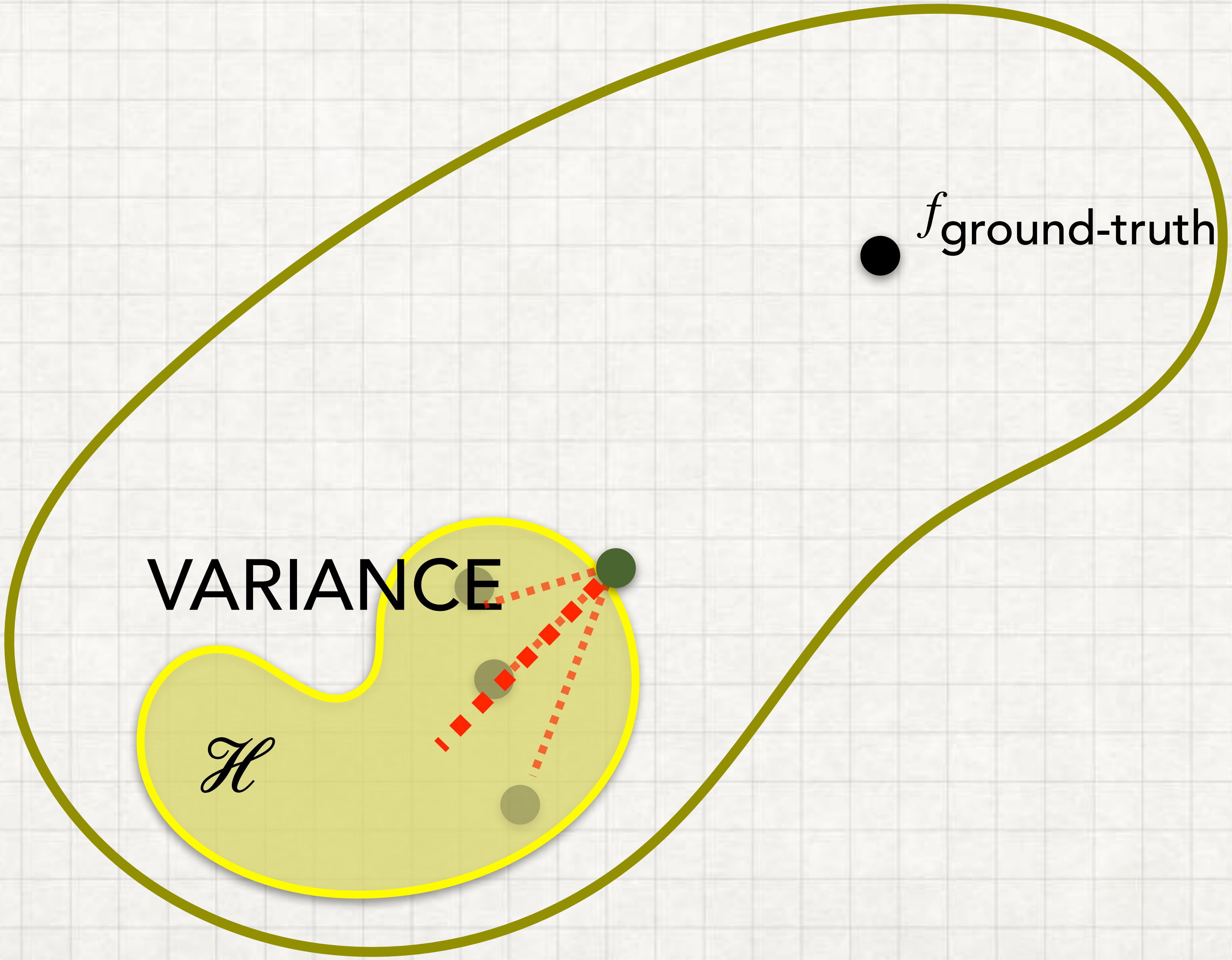
The average risk by learning ∞ times from $\mathscr{H}$

∞ **BIAS**



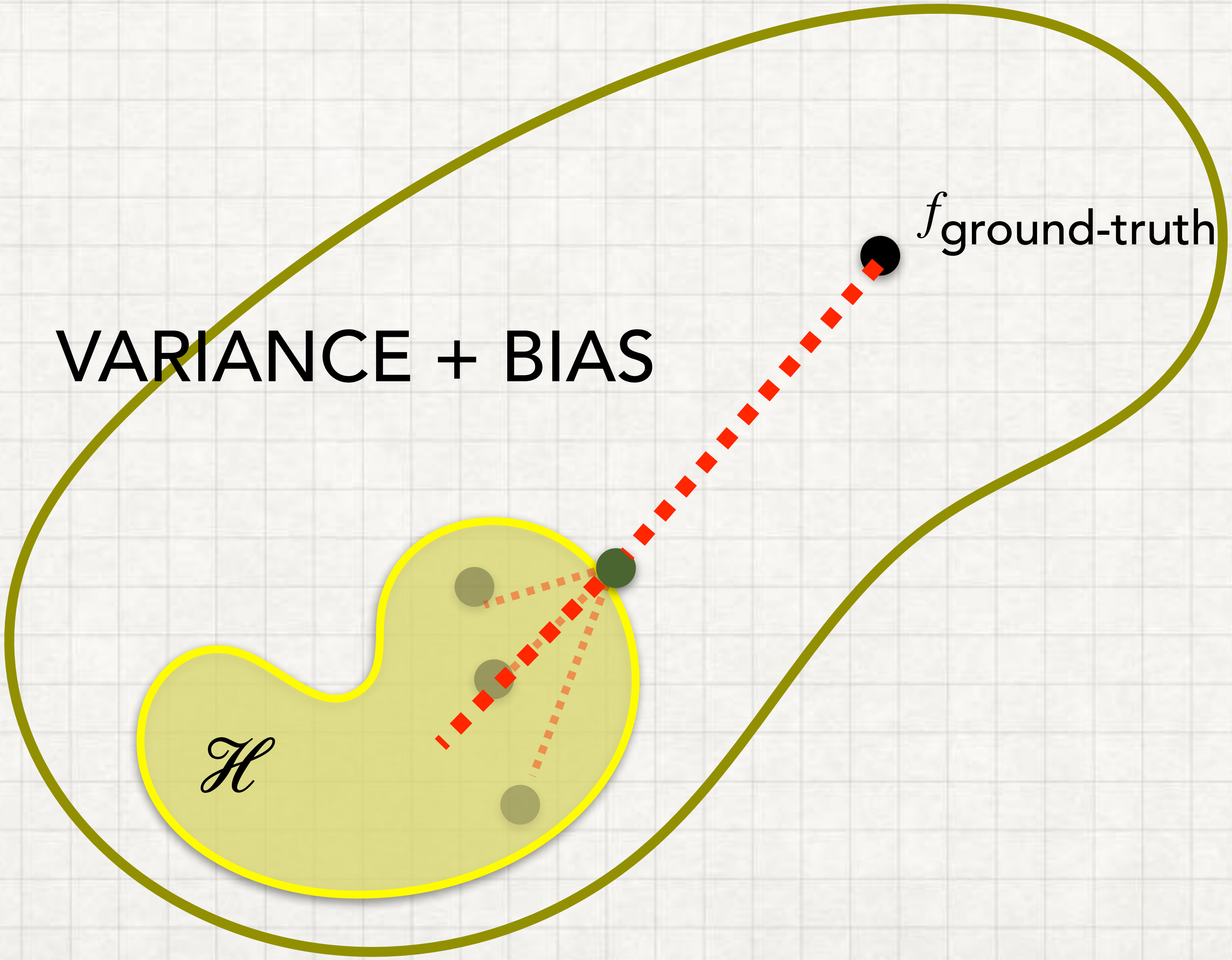
RISK OF TRAINING WITH ONE, RANDOM

D_{train}



TOTAL RISK OF A TRAINING

- The expected risk of the hypothesis selected by the learning framework consists of the following two parts.

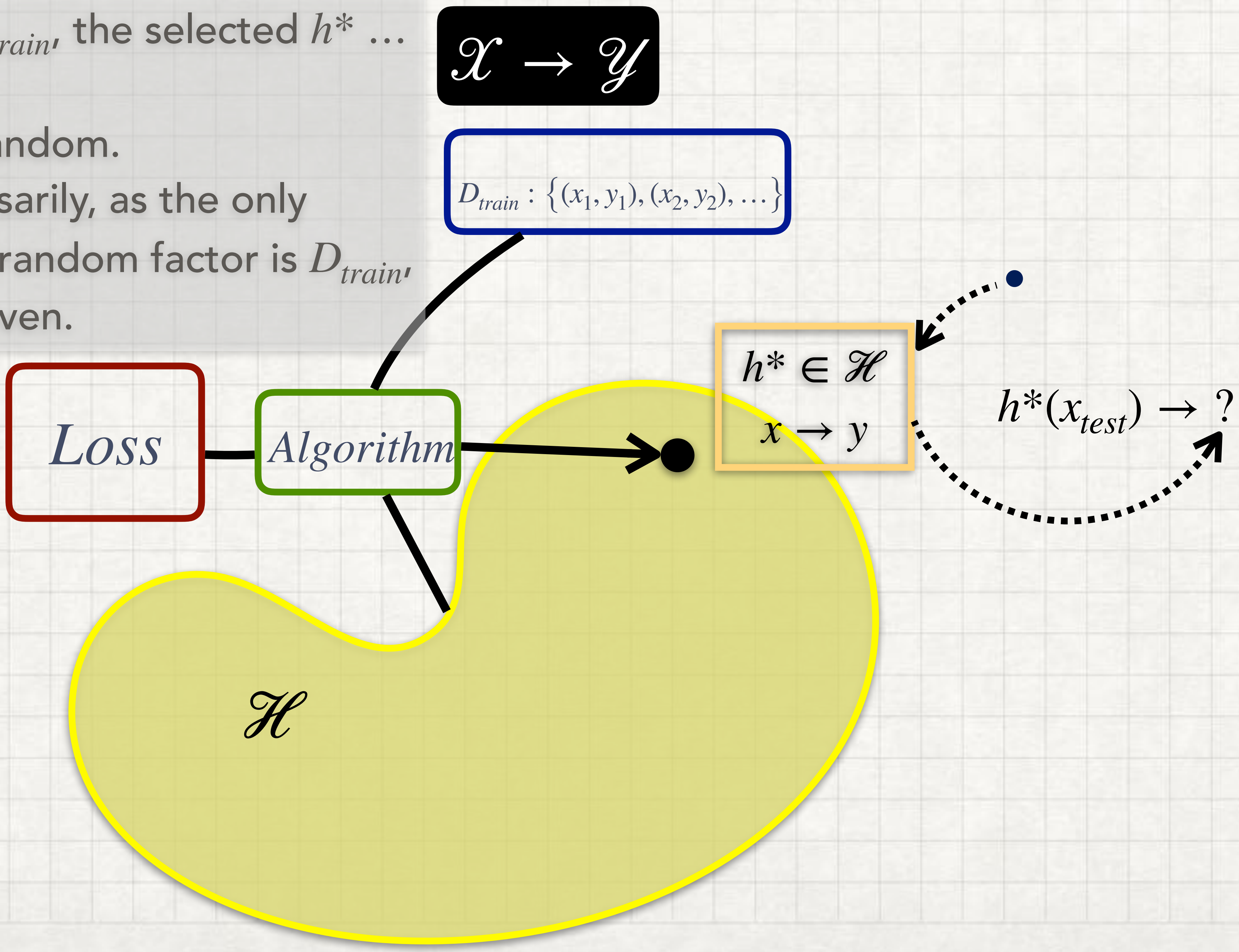


SUMMARISE: RANDOMISATION OF THE PREDICTION

- "Microscope of risk", consider the generalisation error at one x_{test}.

Q: Given a D_{train}, the selected h^* …

A. must be random.
B. Not necessarily, as the only inevitable random factor is D_{train}, which is given.



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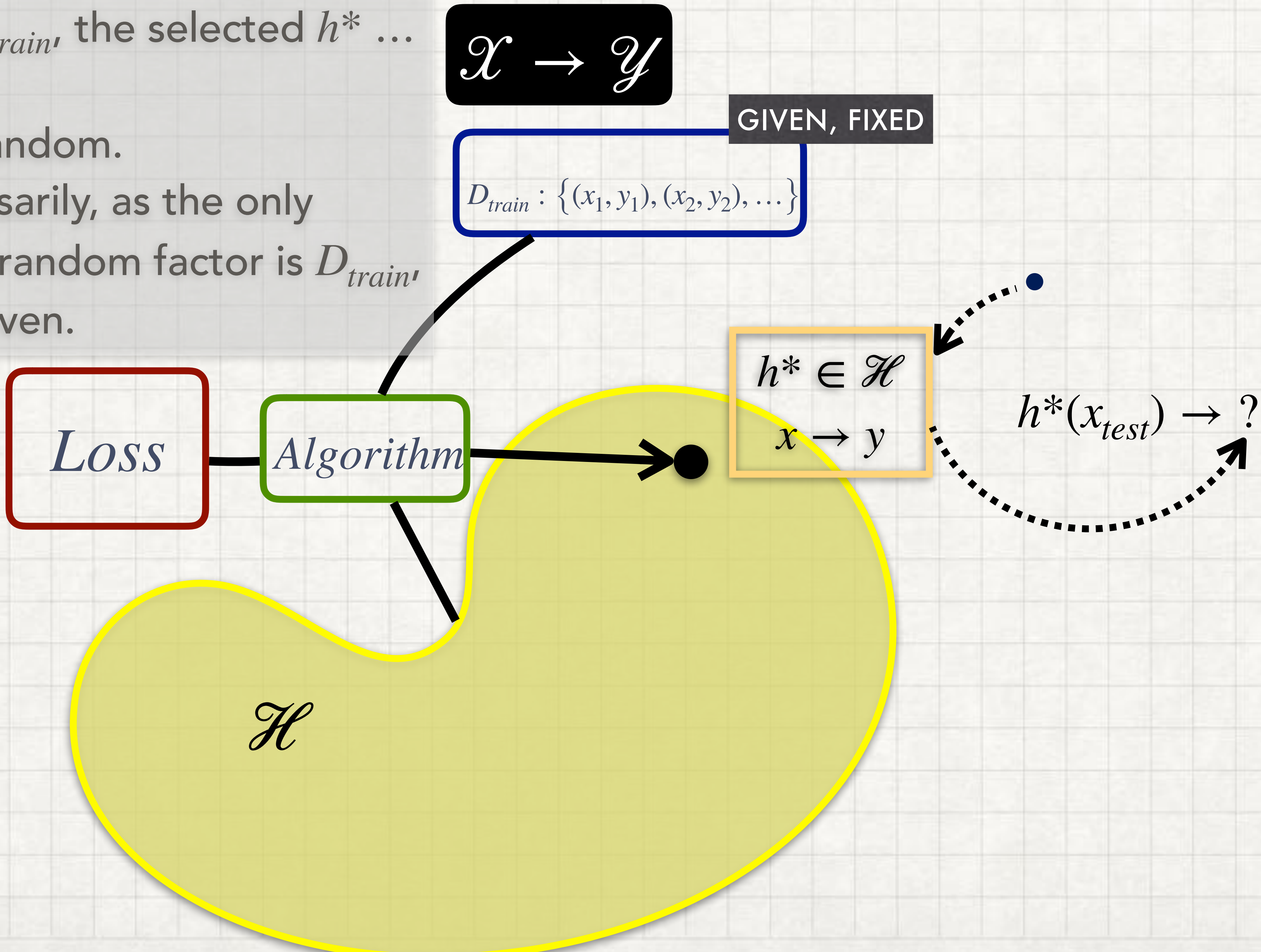

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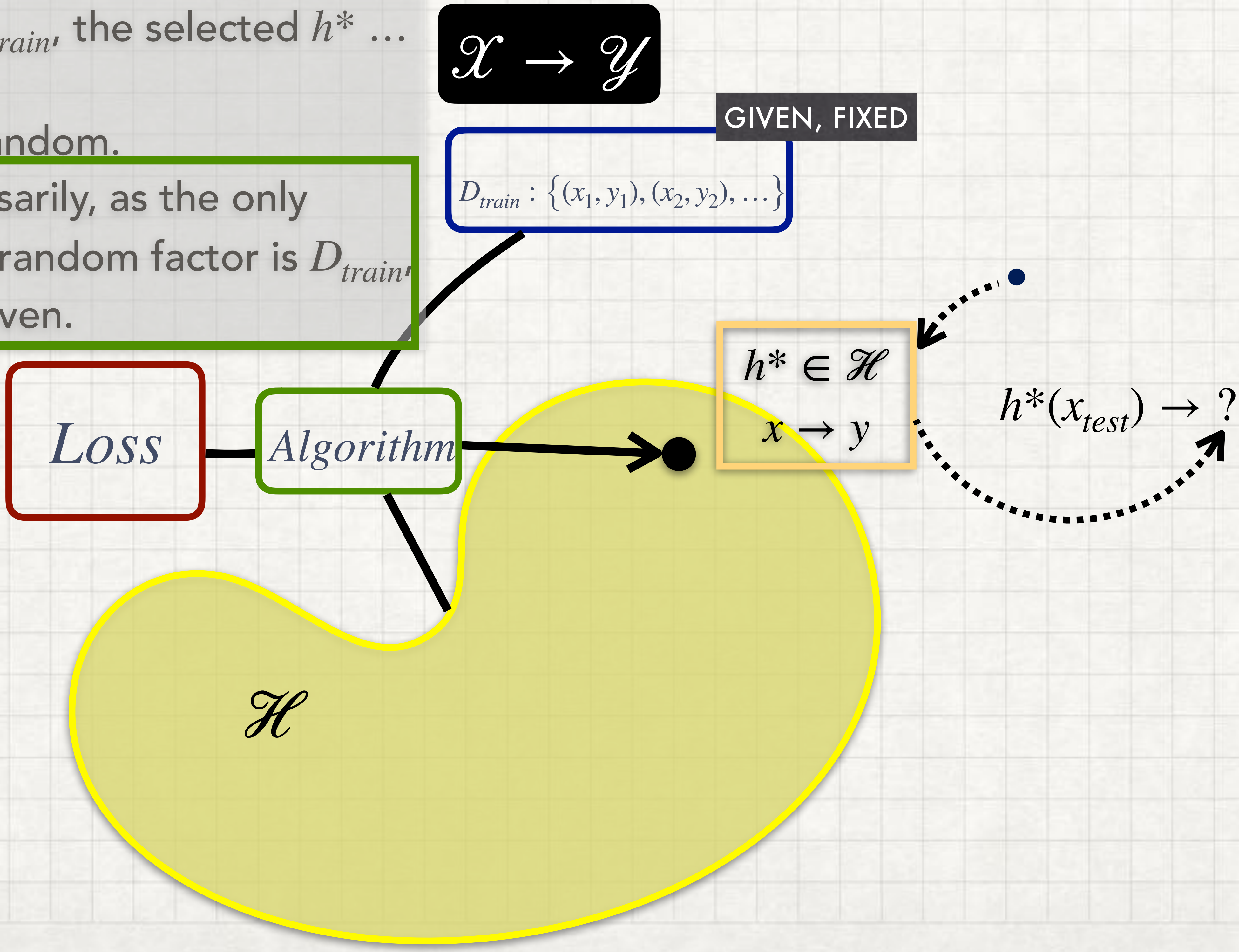

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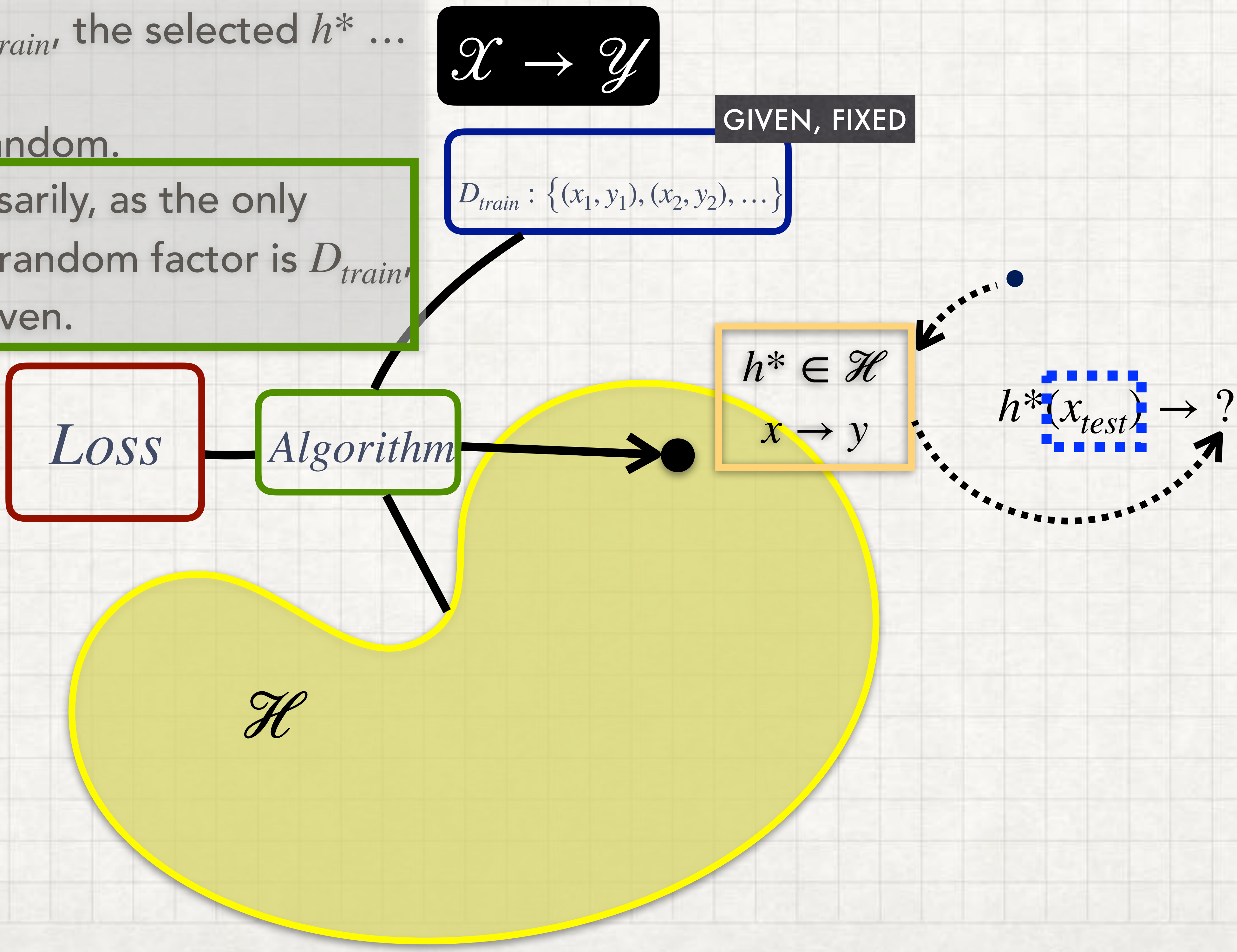
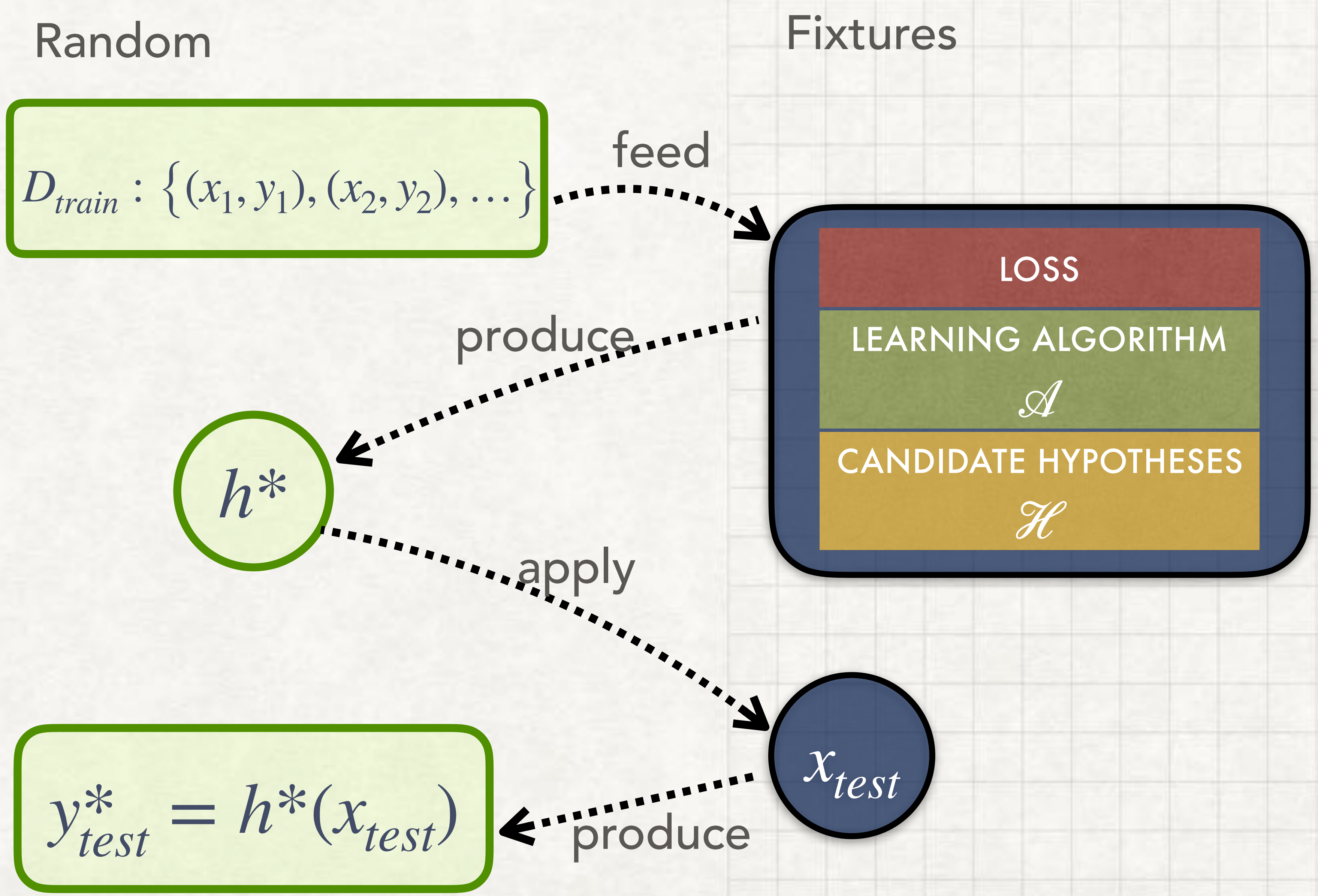


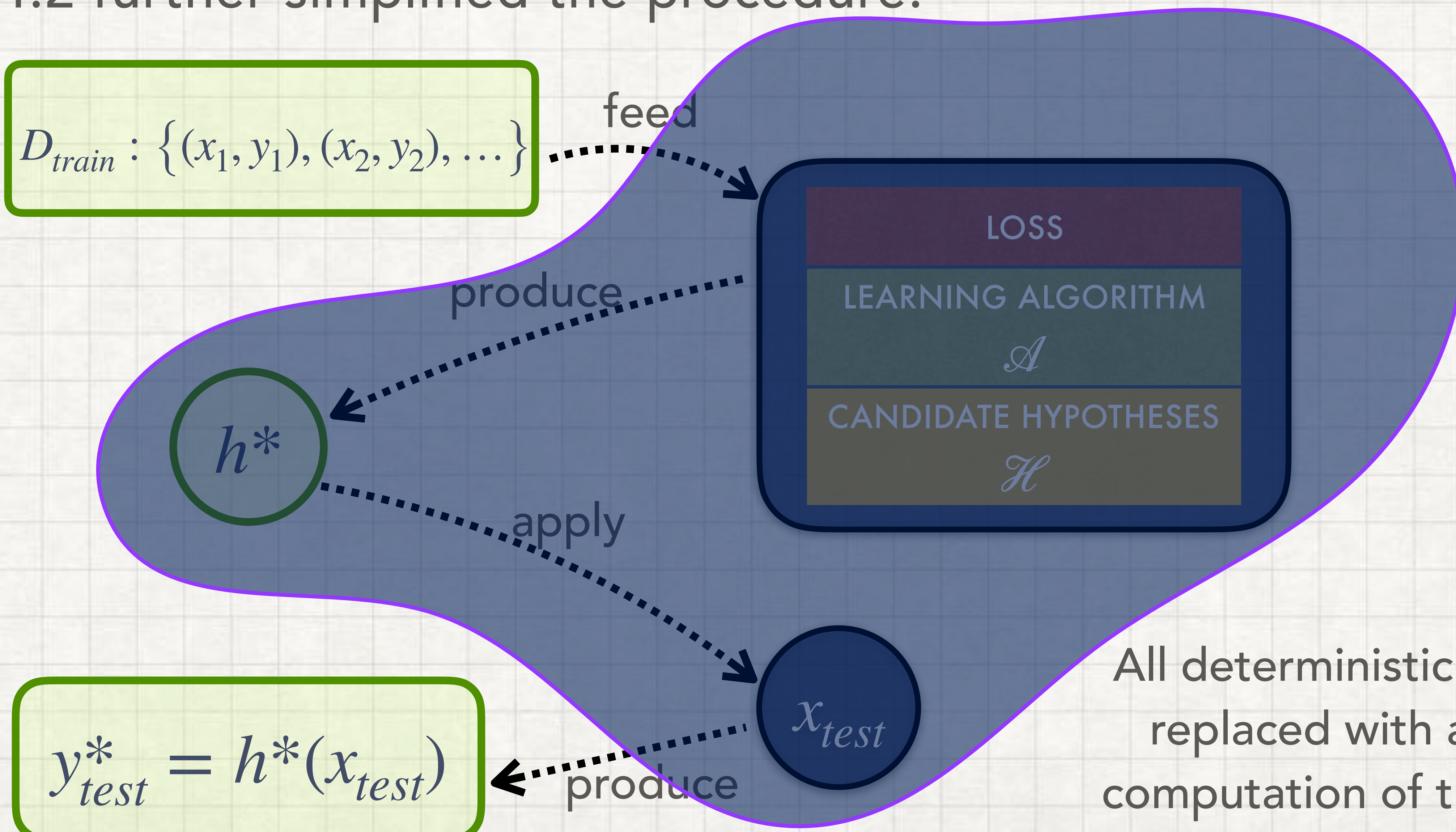
RANDOMISATION ANALYSIS

- The only factor that is inevitably random is the data sample D_{train}, so the final result of the following flow is random.



NOTEBOOK STUDY

- 4.2 and 4.3, Skip the "bootstrapping" parts (the notebook contains Python implementation, which must respect the structure of the contents, rather than the progress of ideas.)
    - 4.2 further simplified the procedure:



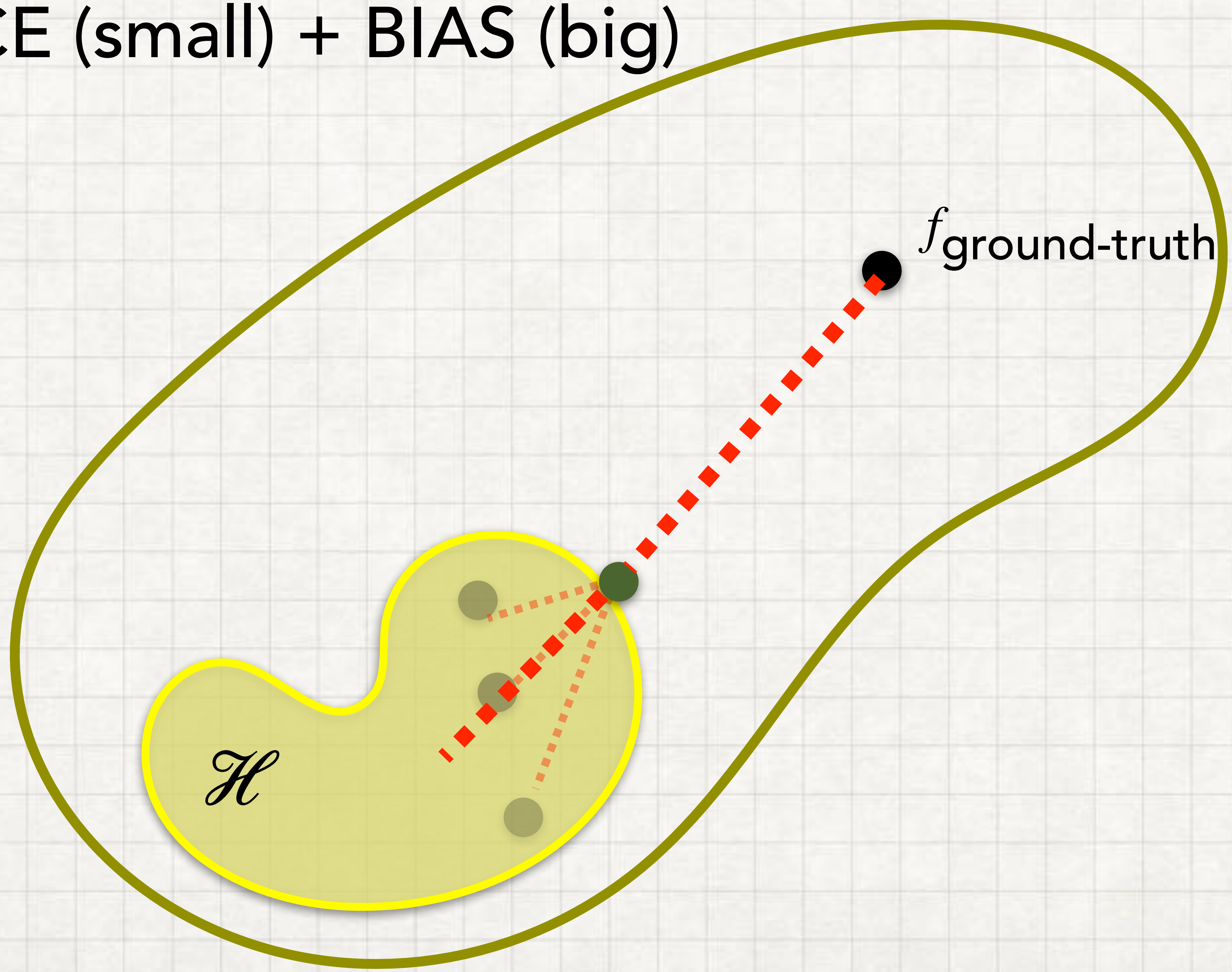
All deterministic steps are replaced with a simple computation of the sample median.

NOTEBOOK STUDY

- 4.2 and 4.3, Skip the “bootstrapping” parts (the notebook contains Python implementation, which must respect the structure of the contents, rather than the progress of ideas.)
  - 4.2 simplify -> median computation.
  - 4.3 Experiment with the first Exercise in “Bias and Variance Explained”.

TOTAL RISK OF A TRAINING
SMALL $\mathscr{H}$

VARIANCE (small) + BIAS (big)

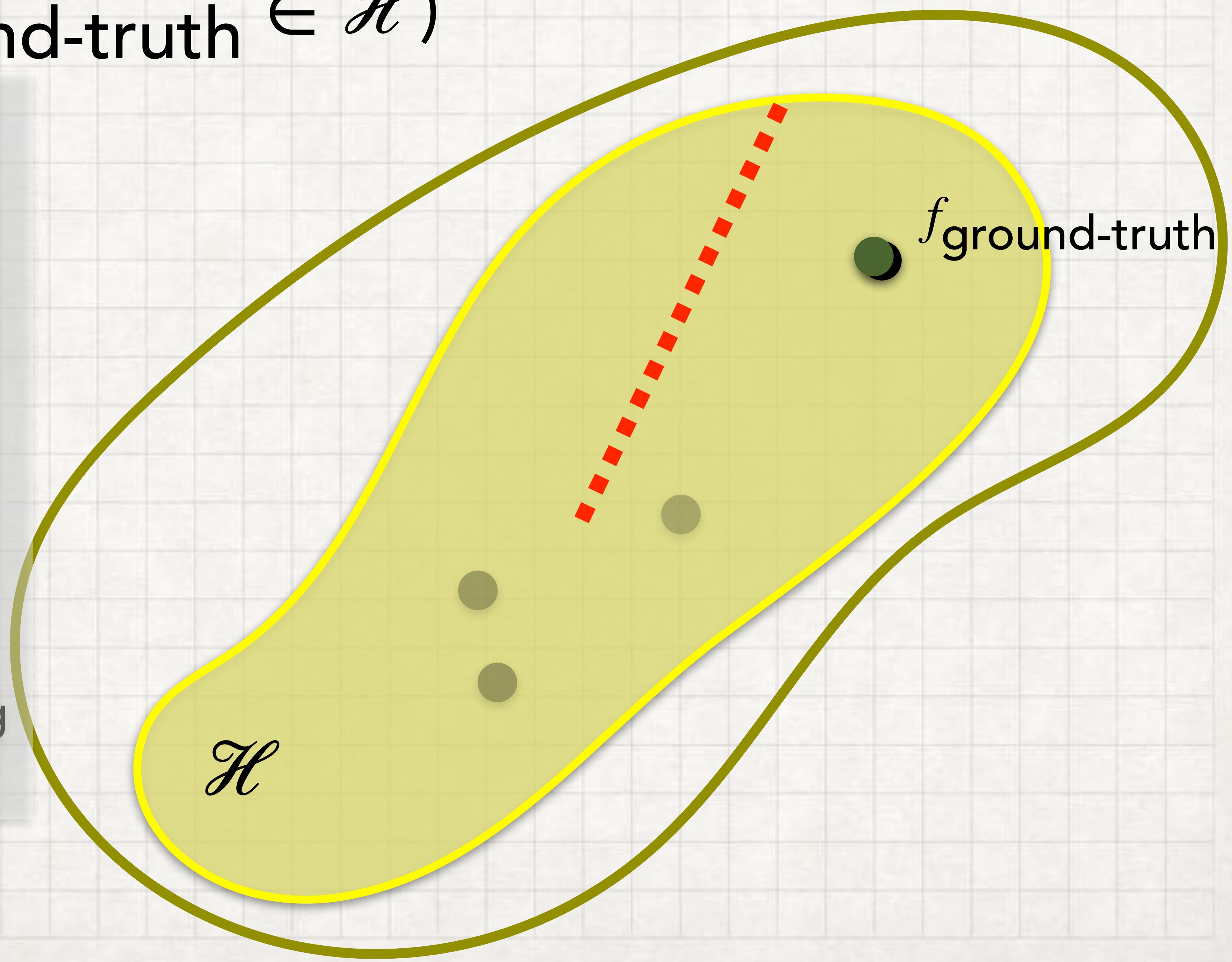


TOTAL RISK OF A TRAINING
BIG $\mathscr{H}$

VARIANCE (big) + BIAS (small/zero — when $f_{\text{ground-truth}} \in \mathscr{H}$)

Q: Given $\mathscr{H}$, can we reduce the variance?

A. No, variance is an intrinsic property of $\mathscr{H}$.
B. Yes, by using less training samples, so the training result will not change as much.
C. Yes, by using more training samples.



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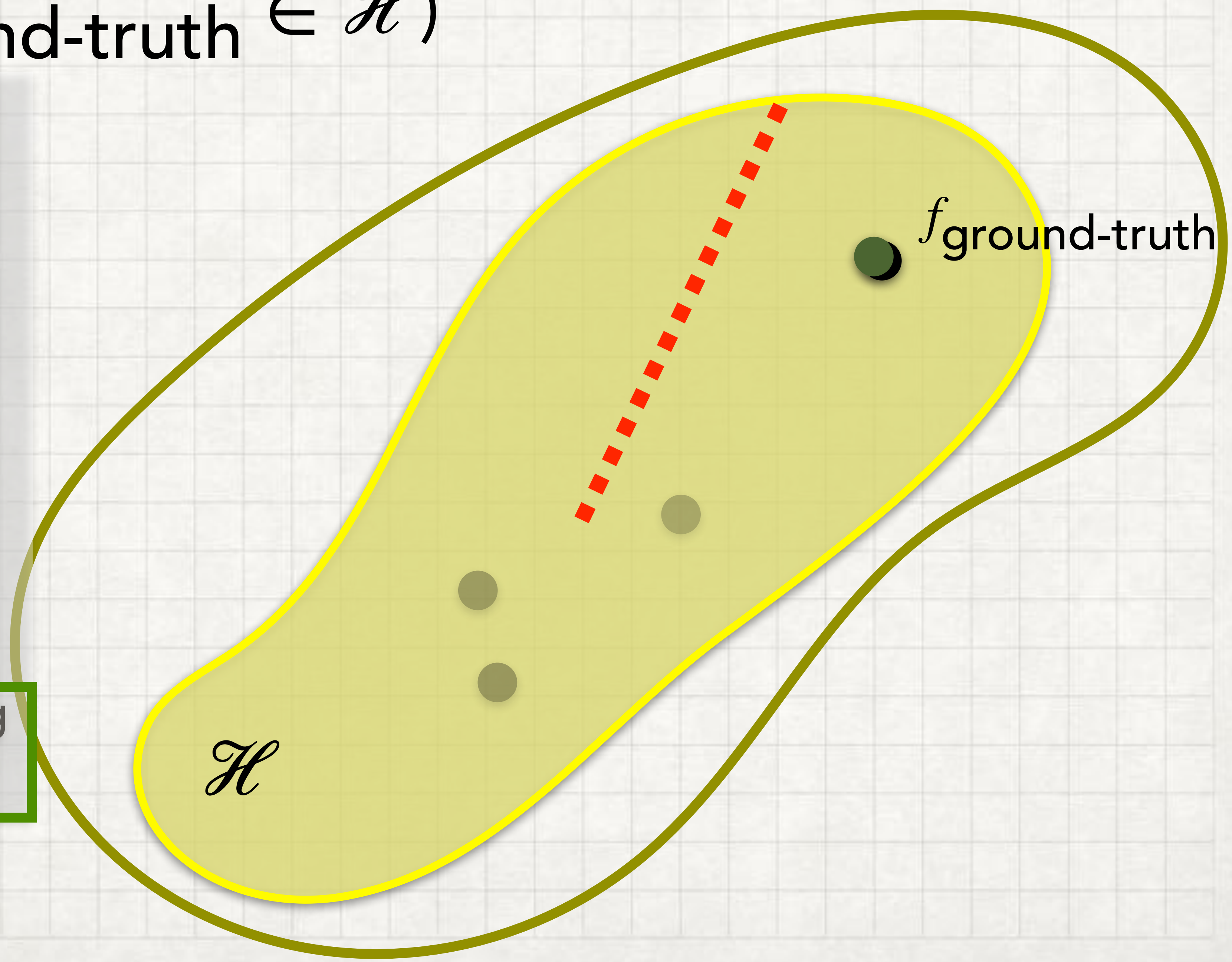

TOTAL RISK OF A TRAINING
BIG $\mathscr{H}$

VARIANCE (big) + BIAS (small/zero — when $f_{\text{ground-truth}} \in \mathscr{H}$)

Q: Given $\mathscr{H}$, can we reduce the bias?

A. Very little, bias is an asymptotical property of $\mathscr{H}$, can not be easily changed by providing more training samples.
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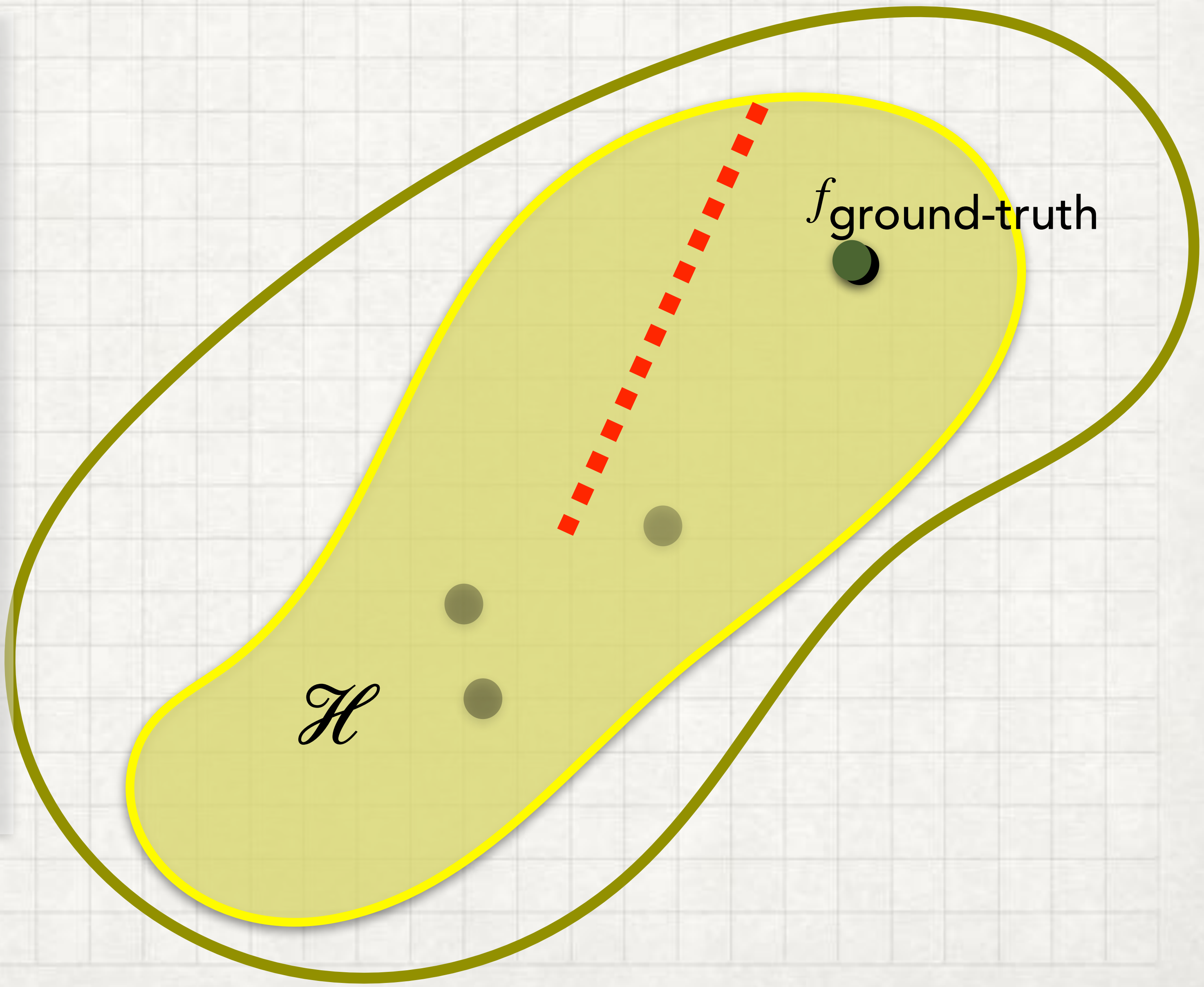

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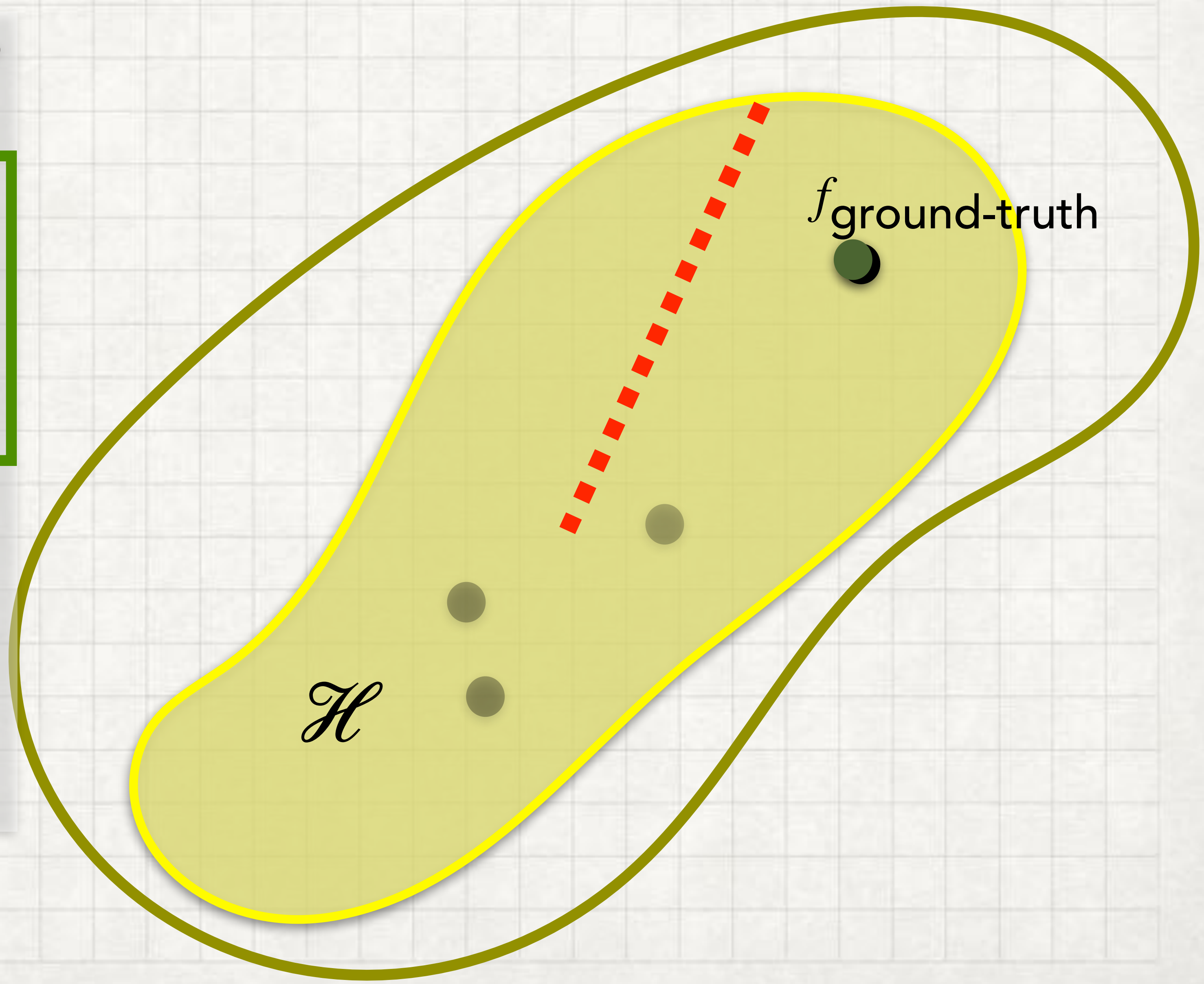
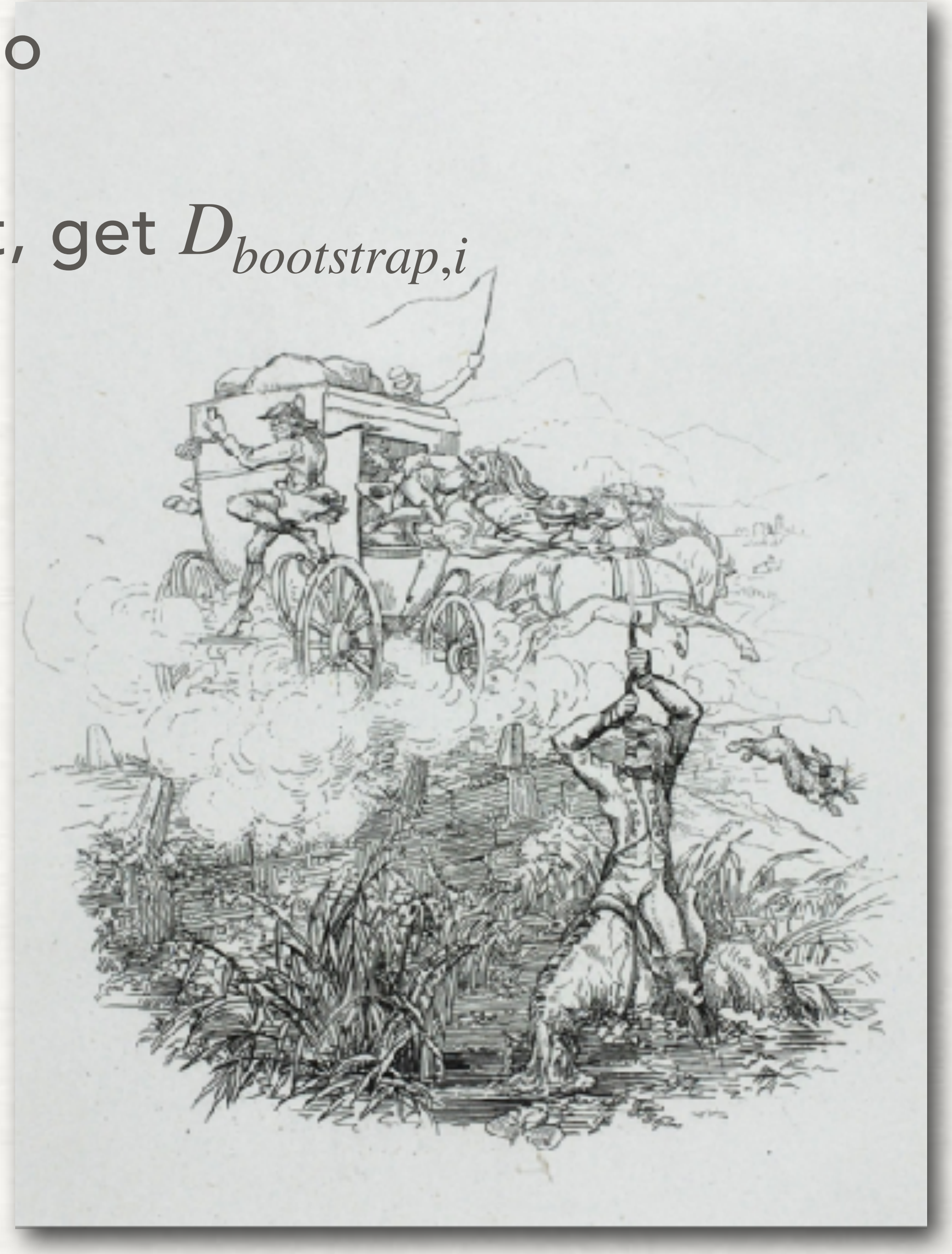


MOD3: ASSESSING AND ADDRESSING VARIANCE USING A SINGLE D_{train}

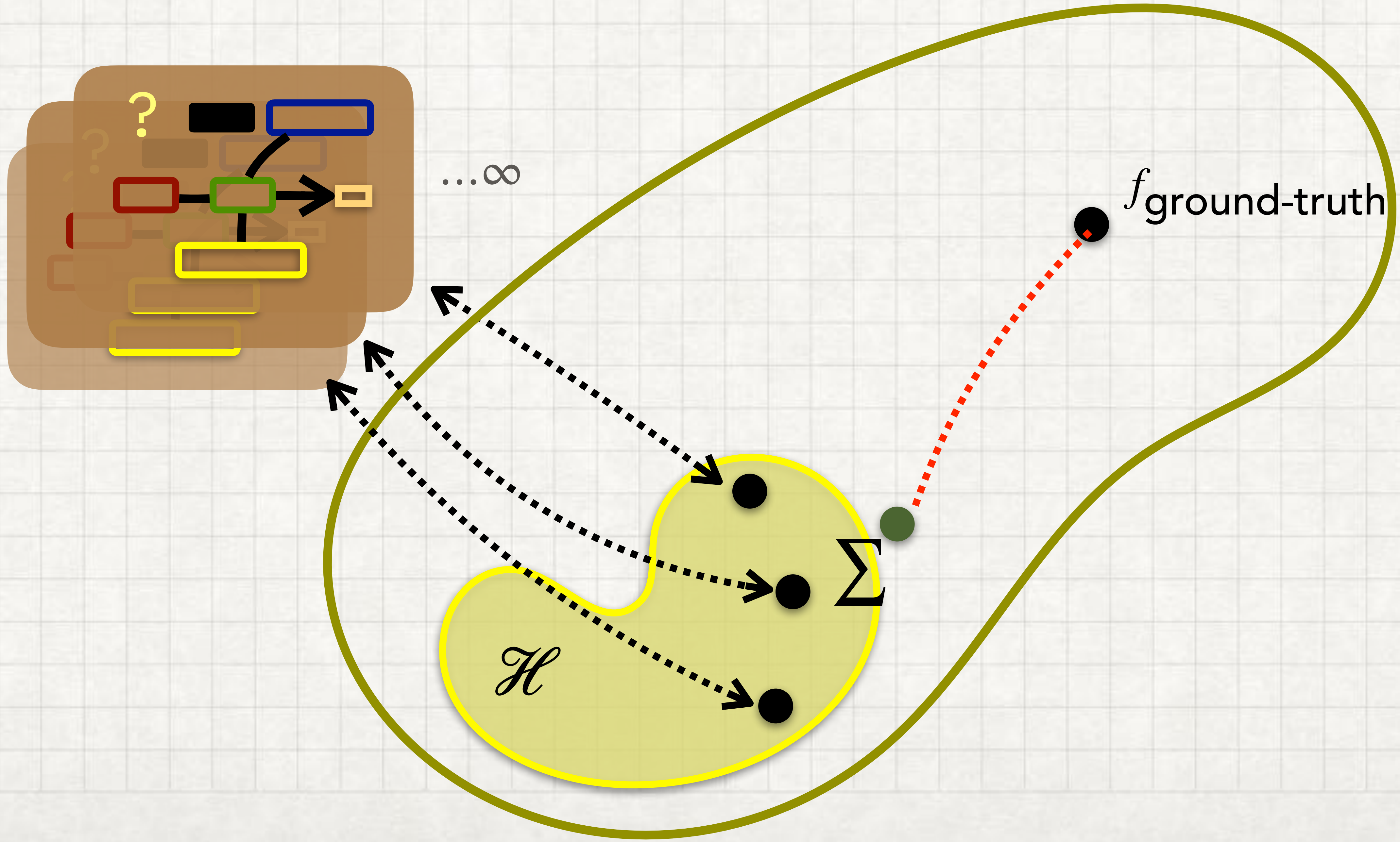
REVIEW: BOOTSTRAP / BAGGING

- IN: Data D_{train}, Model Family and Learning Method
- Repeat $bootstrap-number times: do
  - Re-sample D_{train} with replacement, get $D_{bootstrap,i}$
  - Train model, get h_i^B
- Prediction: Aggregate $h_i^B(x_{test})$'s



BAGGING IS AN ATTEMPT TO ACHIEVE "INFINITE" MODEL AVERAGING

- The bootstrap samples are used to assess statistical information and reduce the variance due to the randomisation of D_{train}



BOOTSTRAP SAMPLES

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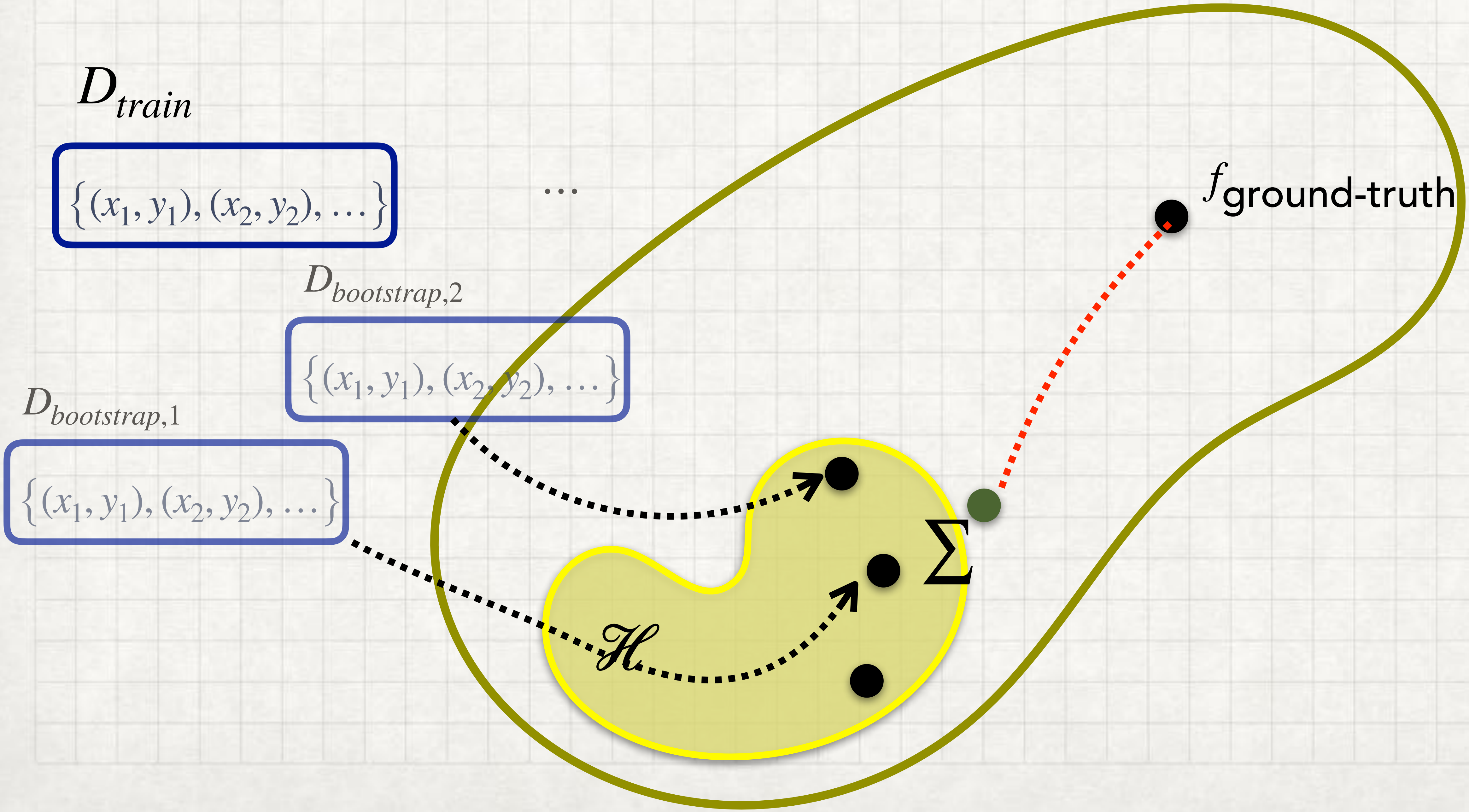
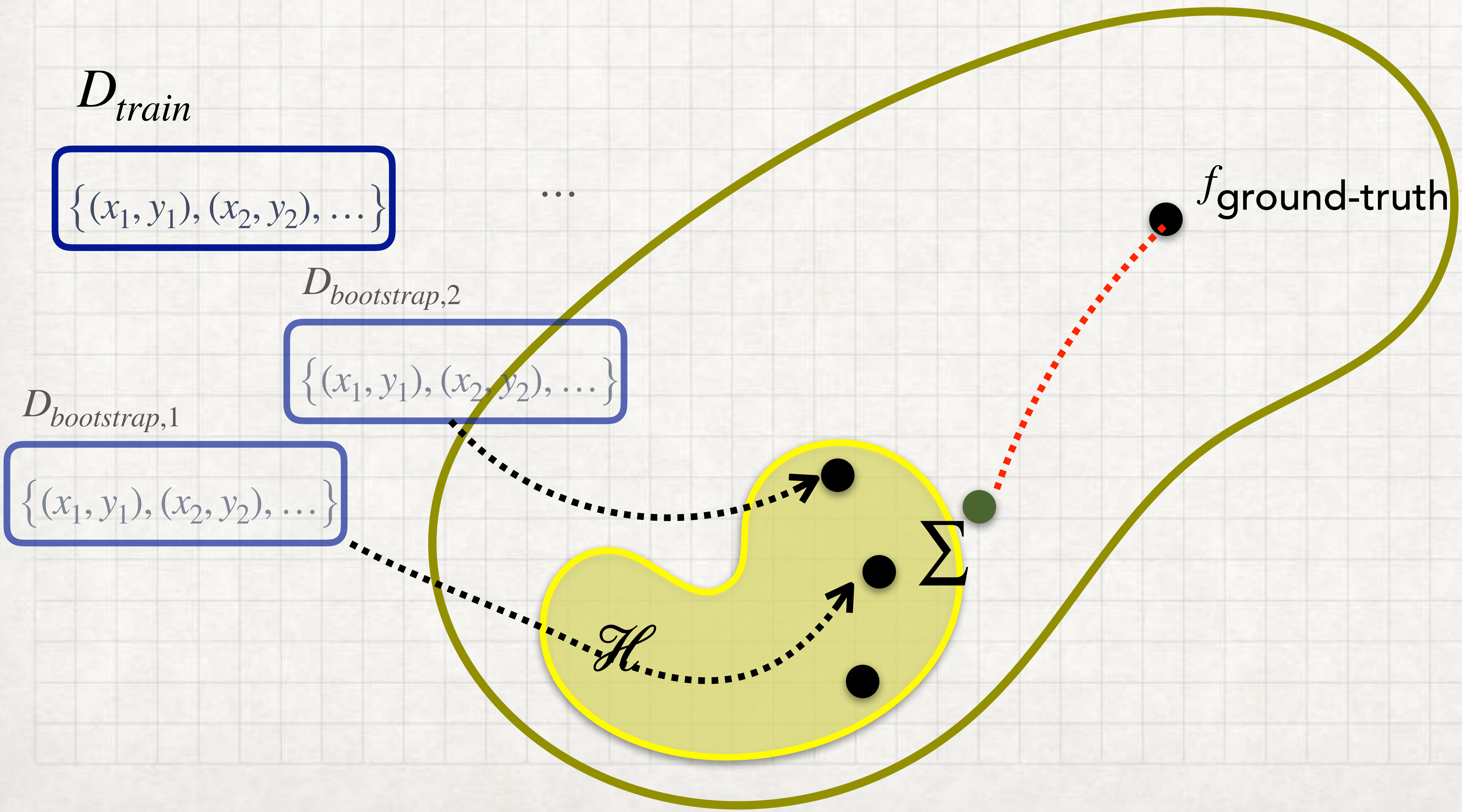


BOOTSTRAP: FREE LUNCH?

- Bootstrap cannot change the bias. But it can reduce variance given right settings at the cost of computation.



NOTEBOOK STUDY

- 4.2 “bootstrap” parts
  - What is bootstrap samples.
  - Experiment and contemplate: For the estimation of median from samples, how bootstrap (resampling) helps reduce variance.

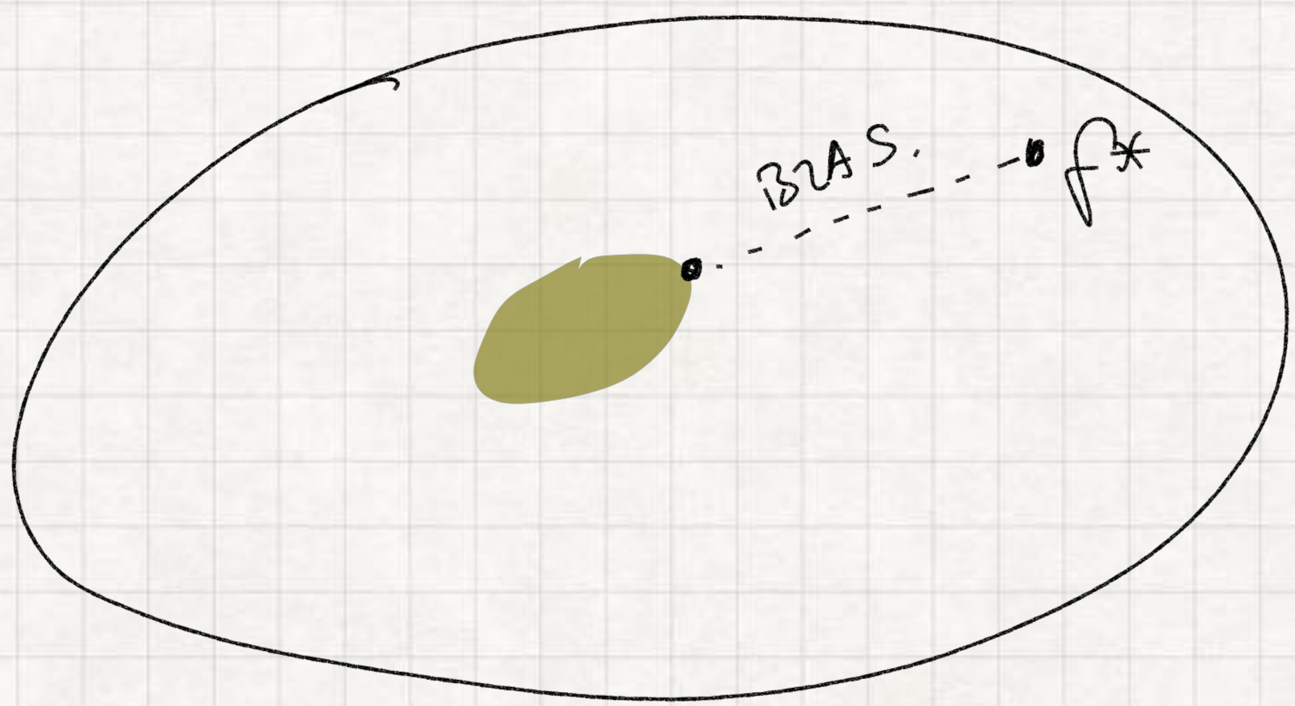
- 4.3 Observe Bias and Variance in sample model (if not done in the previous notebook study).

- 4.4 Bootstrap helps reduce variance in some settings.

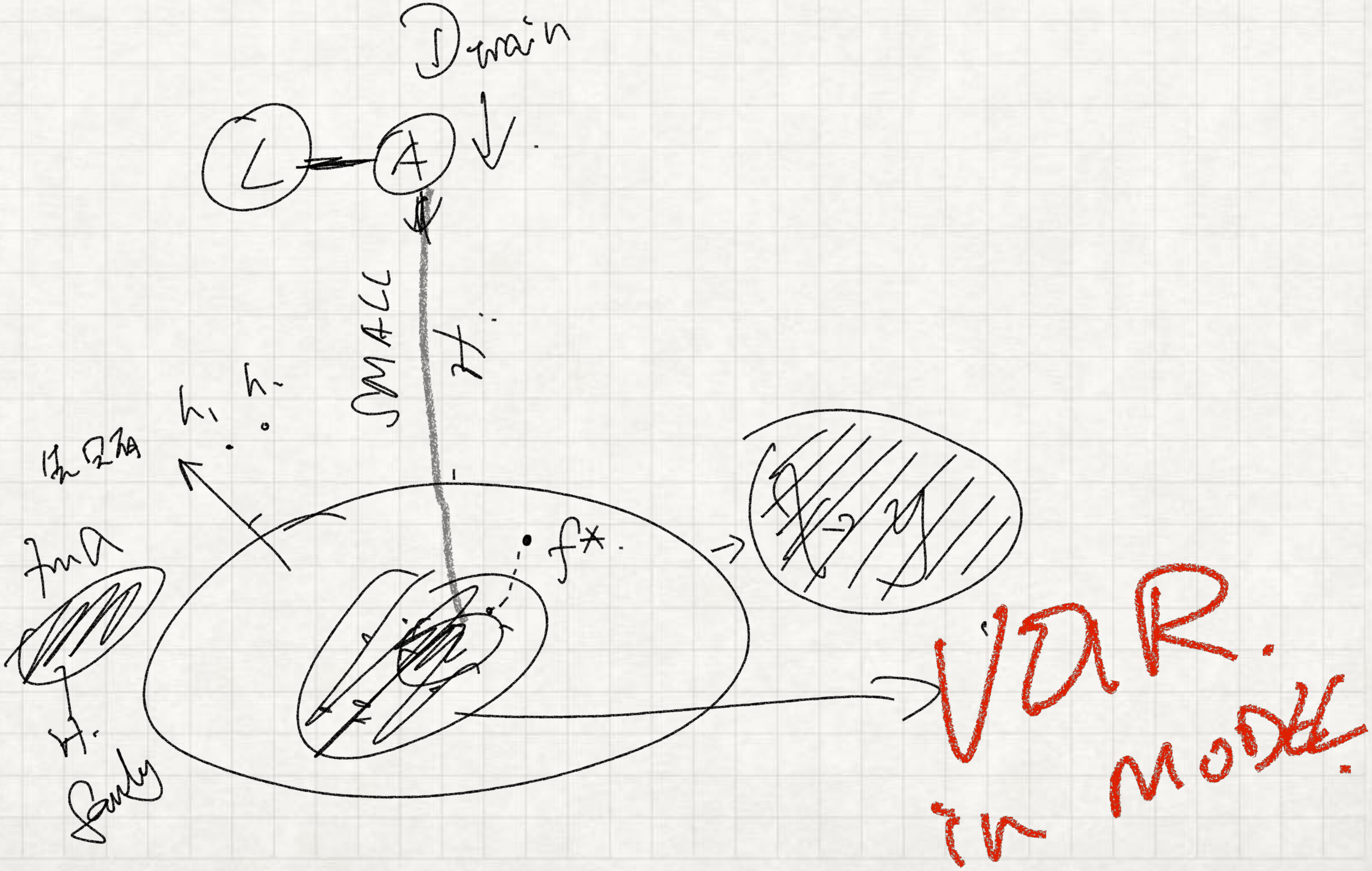
THANKS

WHAT WE HAVE LEFT?

- observation noise
- Training capability — partially why neural network works.



VARIANCE: BIG $\mathscr{H}$, BIG VARIANCE



NOTEBOOK TIME

- Compare bias/variance of two models