Deep Reinforcement Learning: A brief introduction

A/Prof Richard Yi Da Xu
Yida.Xu@uts.edu.au
Wechat: aubedata
https://github.com/roboticcam/machine-learning-notes

University of Technology Sydney (UTS)

March 22, 2018

Deep Reinforcement Learning

- A video from Google DeepMind's Deep Q-learning playing Atari Breakout: `https://www.youtube.com/watch?v=TmPfTpjtdgg`
- *Mnih, Volodymyr, et al. "Playing atari with deep reinforcement learning." arXiv preprint arXiv:1312.5602 (2013).*
- code is also available `https://github.com/kuz/DeepMind-Atari-Deep-Q-Learner`

N.B.

- Apologies for those have seen it before

significance of this demo shows it's possible to use Neural Network to learn how to play a game, based on:

- sequences of screen images
- scores the game receives
- goal is to learn the best policy for **actions** to take

Surely you **don't need a menu** to learn how to play Atari. i.e., it's **model-free**!

Reinforcement Learning (RL)

Forget about the Neural network for a second, how is Reinforcement Learning (RL) different to conventional supervised learning?

- No data label like supervised learning, i.e., no *"best-action-for-that-screen"* label
- only **reward** signal
- feedback in **delayed**, not instantaneous
- data are not i.i.d., (consecutive frames are similar)
- agent's actions affects the subsequent data it receives.

Let's get started with some RL background.

Reinforcement Learning (RL)

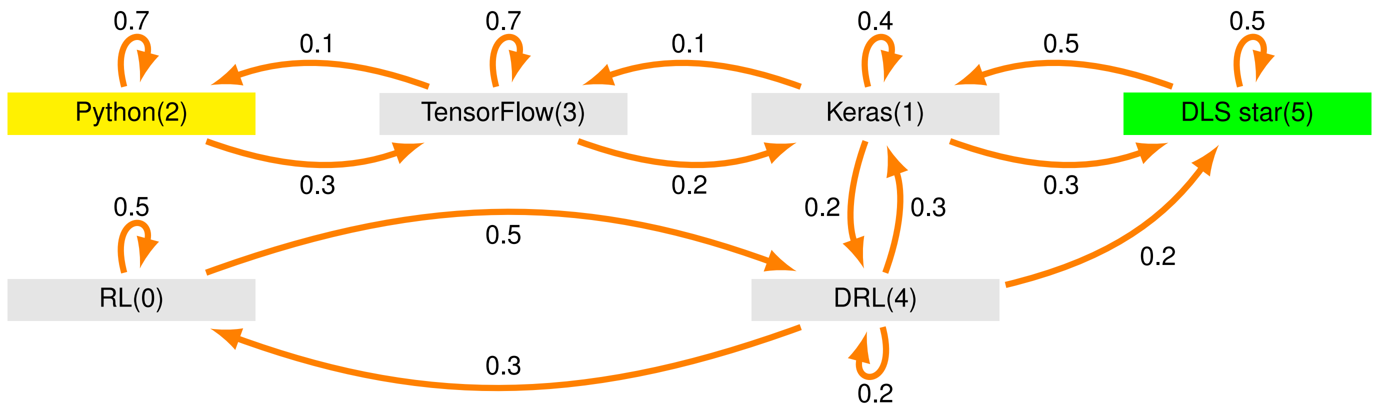
another way to look at it:

- RL uses training information that **evaluates the actions** taken rather than **instructs by giving correct actions**.
- a need for active exploration: explicit trial-and-error search for good behavior.
- **purely evaluative feedback** indicates how good the action taken is, but not whether it is the best or the worst action possible.
- **purely instructive feedback** indicates correct action to take, independently of the action actually taken. supervised learning

Application of RLs

- **marketing**
  customer's attributes s, marketing actions a, customer signs up r
- **drone control**
  all avaiable sensor data a, controls s, not crashing r
- **chatbot**
  conversations to-date s, things that a robot will say a, customer satisfaction r

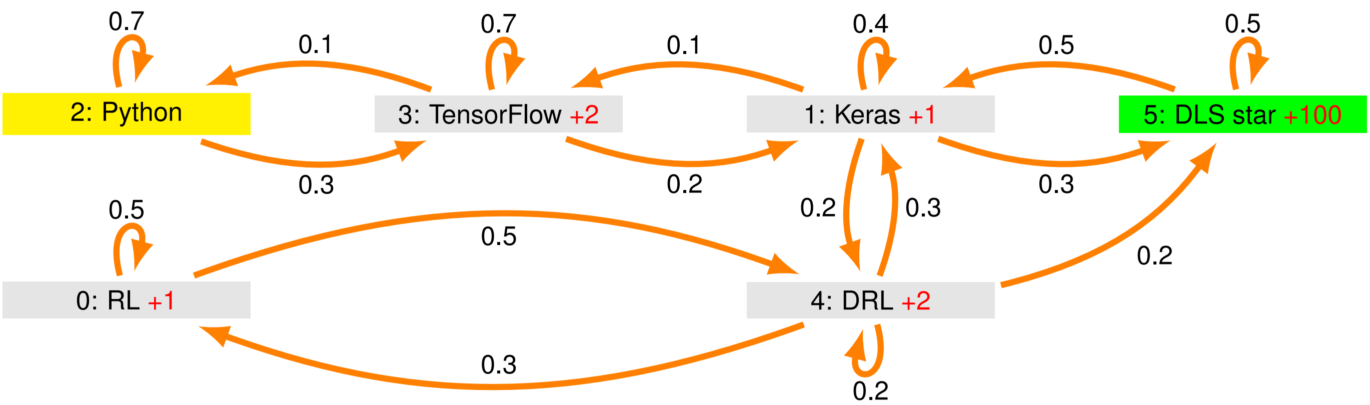
Markov Process



- one may start from **python** and generate sequences with transition probabilities to end up in **DLS star**. examples:
  - Python, Python, Python, TensorFlow, Keras, DLS star
  - Python, Python, Python, TensorFlow, TensorFlow, Keras, DRL, DRL RL, DLS star
  - Python, Python, TensorFlow, TensorFlow, Keras, DRL, DLS star
  - The question is: how we may able to measure “how good” each path? . . .

Markov Reward Process

Let's add some rewards to being at each of the state:



What we care is the **total return** G_t: sum of **discounted** reward from time-step t

$$G_t = R_{t+1} + \gamma R_{t+2} + \cdots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \qquad \text{where } \gamma \in [0, 1]$$

note that G_t is a random variable

exercise what happens when $\gamma = 0$ and $\gamma = 1$

Markov Random Process: Bellman Equation (new)

- **state value function** $V(s)$ of MRP is expected total return starting from state s

$$
\begin{aligned}
V(s) &= \mathbb{E}_{s_{t+1}, s_{t+2}, \dots, r_{t+1}, r_{t+2}, \dots}[G_t | s_t = s] \\
&= \mathbb{E}_{s_{t+1}, s_{t+2}, \dots, r_{t+1}, r_{t+2}, \dots}\Big[R_{t+1} + \gamma \underbrace{\left(R_{t+2} + \gamma R_{t+3} + \dots\right)}_{G_{t+1}}\Big]
\end{aligned}
$$

- $\mathbb{E}[.]$ needs the integrate over $(s_1, s_2, \dots \in \mathcal{S}, r_1, r_2, \dots \in \mathcal{R})$:
- $s_1, s_2, \dots$ and $r_1, r_2, \dots$ are generated in the following fashion:

$$
s_0 \rightarrow (s_1, r_1) \qquad s_1 \rightarrow (s_2, r_2) \dots
$$

- for clarity, we let $s_t \rightarrow s_0$ and $s_{t+k} \rightarrow s_k$:

Markov Random Process: Bellman Equation (new)

- suppose we have a **universal state value function** $V(.)$:

$$V(.) = \sum_{s_0} \Pr(s_0) \sum_{s_1, r_1} \Pr(s_1, r_1 | s_0) \sum_{s_2, r_2} \Pr(s_2, r_2 | s_1) \sum_{s_3, r_3} \dots [r_1 + \gamma(r_2 + \gamma r_3 + \dots)]$$

- however, we usually specify value of $v_\pi(s_0)$ to evaluate:

$$\begin{aligned} V(s_0) &= \underbrace{\sum_{s_1, r_1} \Pr(s_1, r_1 | s_0) \left(r_1 + \gamma \underbrace{\sum_{s_2, r_2} \Pr(s_2, r_2 | s_1) \sum_{s_3, r_3} \dots [r_2 + \gamma(r_3 + \gamma r_4 + \dots]}_{V(s_1) \triangleq \mathbb{E}[G_{t+1} | s_1]} \right)}_{V(s_0) \triangleq \mathbb{E}[G_t | s_0]} \\ &= \mathbb{E}_{s_1, r_1} \left[r_1 + \gamma V(s_1) | s_0 \right] \\ &= \mathbb{E}_{s_1} \left[R_1 + \gamma V(s_1) | s_0 \right] \text{ if } R_1 \text{ is deterministic} \end{aligned}$$

Markov Random Process: Bellman Equation (new)

$$V(s_0) = \mathbb{E}_{s_1}\left[R_1 + \gamma V(s_1) | s_0\right]$$

- **Bellman equations**: value of the current state, $v(s)$ breaks up into (1) **immediate** and (2) **future** rewards.
- state value function $V(s)$ is written in a consecutive time steps
- difficult to estimate: because $V(s)$ also depends on various other $V(s')$ which occur at different times

Bellman Equation in matrix form

- to simplify, making R_t deterministic

$$V(s_0) = \mathbb{E}_{s_1}\left[R_1 + \gamma V(s_1)|s_0\right]$$

- say $s \in \{1, \dots, n\}$:

$$\underbrace{V(s_0 = 1)}_{v(1)} = \mathbb{E}_{s_1}\left[\underbrace{R_1(s_0 = 1)}_{R_1} + \gamma V(s_1)|s_0 = 1\right]$$
$$V(s_0 = 2) = \mathbb{E}_{s_1}\left[R_1(s_0 = 2) + \gamma v(s_1)|s_0 = 2\right]$$
$$\dots$$

take the first line,

$$\begin{aligned}
v(1) &= \mathbb{E}_{s_1}\left[R_1 + \gamma V(s_1)|s_0 = 1\right] \\
&= R_1 + \gamma \mathbb{E}\left[V(s_1)|s_0 = 1\right] \\
&= R_1 + \gamma \left(\sum_{s_1=1}^{n} v(s_1) \Pr(1 \to s_1) \right) \\
&= R_1 + \gamma \left(\sum_{j=1}^{n} v(j) \Pr(1 \to j) \right) \\
&\dots \\
\implies v(n) &= R_n + \gamma \left(\sum_{j=1}^{n} v(j) \Pr(k \to j) \right)
\end{aligned}$$

Bellman Equation in matrix form (2)

$$
\begin{aligned}
v(k) &= R_k + \gamma \left(\sum_{j=1}^{n} v(j) \Pr(k \rightarrow j) \right) \\
&= R_k + \gamma \mathcal{P}_{k,:}^{\top} \mathbf{v} \\
\implies \mathbf{v} &= \mathbf{R} + \gamma \mathcal{P} \mathbf{v}
\end{aligned}
$$

$$
\implies \begin{bmatrix} v(1) \\ \vdots \\ v(n) \end{bmatrix} = \begin{bmatrix} R_1 \\ \vdots \\ R_n \end{bmatrix} + \gamma \begin{bmatrix} \mathcal{P}_{1,1} & \dots & \mathcal{P}_{1,n} \\ \vdots & & \vdots \\ \mathcal{P}_{n,1} & \dots & \mathcal{P}_{n,n} \end{bmatrix} \begin{bmatrix} v(1) \\ \vdots \\ v(n) \end{bmatrix}
$$

the solution to **MRP** is straight forward:

$$
\begin{aligned}
\mathbf{v} &= \mathbf{R} + \gamma \mathcal{P} \mathbf{v} \\
(I - \gamma \mathcal{P}) v &= R \\
\mathbf{v} &= (I - \gamma \mathcal{P})^{-1} R
\end{aligned}
$$

Markov Decision Process (MDP)

- now agent has **actions**
- concept of **policy** π: take a state s_t as input and decides and action a_t

$$\pi(a|s) = \Pr(A_t = a|S_t = s)$$

- a policy is time-invariant (or stationary) and stochastic
- next state for an agent, now also depends on its action taken:

$$\mathcal{P}^a_{s\to s'} = \Pr(S_{t+1} = s'|S_t = s, A_t = a)$$

- multiple transition matrix $\mathcal{P}$ each depends on the a taken
- once fixed π, MDP becomes MRP with transition probability $\mathcal{P}^\pi_{s\to s'}$:

$$\mathcal{P}^\pi_{s\to s'} = \sum_{a\in\mathcal{A}} \pi(a|s)\mathcal{P}^a_{s\to s'}$$

Markov Decision Process: Bellman Equation (new)

- given a policy π, **state value function** $v(s)$ is expected total return starting from state s

$$
\begin{aligned}
v_\pi(s) &= \mathbb{E}_\pi[G_t | s_t = s] \\
&= \mathbb{E}_\pi\Big[R_{t+1} + \gamma \underbrace{\big(R_{t+2} + \gamma R_{t+3} + \dots\big)}_{G_{t+1}}\Big]
\end{aligned}
$$

- $\mathbb{E}_\pi[.]$ needs the integrate over $(a_0, a_1, \dots \in \mathcal{A}, s_0, s_1, \dots \in \mathcal{S}, r_1, r_2, \dots \in \mathcal{R})$:

- for clarity, we let $s_t \to s_0$ and $s_{t+k} \to s_k$:

- $s_0 \to a_0, \qquad (s_0, a_0) \to (s_1, r_1), \qquad s_1 \to a_1, \qquad (s_1, a_1) \to (s_2, r_2), \dots$

Markov Decision Process: Bellman Equation (new)

- suppose we have a **universal state value function** $V_\pi(.)$, i.e., no matter what the current state and action is:

$$
\begin{aligned}
&v_\pi(.)\\
&= \sum_{s_0} \Pr(s_0) \sum_{a_0} \pi(a_0|s) \sum_{s_1, r_1} \Pr(s_1, r_1|s_0, a_0) \sum_{a_1} \pi(a_1|s_1) \sum_{s_2, r_2} \Pr(s_2, r_2|s_1, a_1) \sum_{a_2} \cdots \sum_{s_3, r_3} \dots\\
&\qquad [r_1 + \gamma(r_2 + \gamma r_3 + \dots)]
\end{aligned}
$$

- however, we do know the value $v_\pi(s_0)$:

$$
\begin{aligned}
&v_\pi(s_0)\\
&= \underbrace{\sum_{a_0} \pi(a_0|s) \sum_{s_1, r_1} \Pr(s_1, r_1|s_0, a_0) \Big(r_1 + \gamma \underbrace{\sum_{a_1} \pi(a_1|s_1) \sum_{s_2, r_2} \Pr(s_2, r_2|s_1, a_1) \sum_{a_2} \cdots \sum_{s_3, r_3} \dots [r_2 + \gamma(r_3 + \gamma r_4 + \dots]}_{v_\pi(s_1) \triangleq \mathbb{E}_\pi[G_{t+1}|s_1]} \Big)}_{v_\pi(s_0) \triangleq \mathbb{E}_\pi[G_t|s_0]}\\
&= \sum_{a_0} \pi(a_0|s) \sum_{s_1, r_1} \Pr(s_1, r_1|s_0, a_0)(r_1 + \gamma \mathbb{E}_\pi[G_{t+1}|s_1])
\end{aligned}
$$

Bellman equation extends to $Q(s, a)$

summarise slides from before:

$$\begin{aligned} V_\pi(s_0) &= \sum_{a_0} \pi(a_0|s_0) \sum_{s_1, r_1} \Pr(s_1, r_1|s_0, a_0)(r_1 + \gamma v_\pi(s_1)) \\ &= \sum_{a_0} \pi(a_0|s_0) \sum_{s_1, r_1} \Pr(s_1, r_1|s_0, a_0)(r_1 + \gamma \mathbb{E}_\pi[G_{t+1}|s_1]) \\ &= \sum_{a_0} \pi(a_0|s_0) \mathbb{E}_{(s_1, r_1)\sim}[r_1 + \gamma v_\pi(s_1)] \end{aligned}$$

insert a_0 to obtain Q function:

$$\begin{aligned} Q_\pi(s_0, a_0) &= \sum_{s_1, r_1} \Pr(s_1, r_1|s_0, a_0)(r_1 + \gamma v_\pi(s_1)) \\ &= \sum_{s_1, r_1} \Pr(s_1, r_1|s_0, a_0)(r_1 + \gamma \mathbb{E}_\pi[G_{t+1}|s_1]) \\ &= \mathbb{E}_{(s_1, r_1)\sim}[r_1 + \gamma v_\pi(s_1)] \end{aligned}$$

since any policy π works, then:

$$Q_{\pi*}(s_0, a_0) = \mathbb{E}_{(s_1, r_1)\sim}[r_1 + \gamma v_{\pi*}(s_1)]$$

Bellman optimality

- we know best $V_*(s)$ must be the best action from an optimal (state, action) pair: $Q_{\pi*}(s_0, a_0)$:

$$V_*(s_0) = \max_{a_0} Q_{\pi*}(s_0, a_0)$$

- and from before:

$$\begin{aligned}
V_*(s_0) &= \max_{a_0} Q_{\pi_*}(s_0, a_0) \\
&= \max_{a_0} \mathbb{E}_{(s_1, r_1)\sim} [r_1 + \gamma v_{\pi_*}(s_1)] \qquad \text{from previous page} \\
&= \sum_{s_1, r_1} \Pr(s_1, r_1 | s_0, a_0)(r_1 + \gamma v_{\pi_*}(s_1)) \\
&= \sum_{s_1, r_1} \Pr(s_1, r_1 | s_0, a_0)(r_1 + \gamma \max_{a_1} Q_{\pi*}(s_1, a_1)) \\
&= \max_{a_0} \mathbb{E}_{(s_1, r_1)\sim} \left[r_1 + \gamma \max_{a_1} Q_{\pi*}(s_1, a_1) \right] \\
&= \max_{a_0} \mathbb{E}_{(s_1, r_1)\sim} \left[r_1 + \gamma \max_{a_1} Q_{\pi*}(s_1, a_1) \middle| s_0 \right] \text{ removed } |s_0 \text{ for clarity previously} \\
\implies Q_*(s_0) &= \mathbb{E}_{(s_1, r_1)\sim} \left[r_1 + \gamma \max_{a_1} Q_{\pi*}(s_1, a_1) \middle| s_0 \right]
\end{aligned}$$

- also $r_1 \triangleq r_1(s_0, \pi(s_0), s_1)$

Solve Bellman's equation using Temporal Difference

$$V_{\pi}(s_0) = \sum_{a_0} \pi(a_0|s_0) \sum_{s_1, r_1} \Pr(s_1, r_1|s_0, a_0)(r_1 + \gamma v_{\pi}(s_1))$$

- drop $|s$ again for clarity:

$$\begin{aligned} V^{\pi}(s) &= \mathbb{E}_{s'}\Big[r(s, \pi(s), s') + \gamma V^{\pi}(s')\Big] \\ \implies V^{\pi}(s) + \eta V^{\pi}(s) &= V^{\pi}(s) + \eta\bigg(\mathbb{E}_{s'}\Big[r(s, \pi(s), s') + \gamma V^{\pi}(s')\Big]\bigg) \\ \implies V^{\pi}(s) &= V^{\pi}(s) + \eta\bigg(\mathbb{E}_{s'}\Big[r(s, \pi(s), s') + \gamma V^{\pi}(s')\Big] - V^{\pi}(s)\bigg) \end{aligned}$$

- instead of compute this expectation, in **each iteration** t, we sample a new state $\tilde{s'} \sim \Pr(s'|\dots)$

$$V^{\pi}_{t+1}(s) = V^{\pi}_t(s) + \eta\Big(r(s, \pi(s), \tilde{s'}) + \gamma V^{\pi}_t(\tilde{s'}) - V^{\pi}_t(s)\Big)$$

- note that the last equation is called **temporal difference**

Bellman's equation: Three ways of solving it

$$V^{\pi}(s) = \mathbb{E}_{\pi}\left[G_t | s_t = s\right]$$

– could be approximated by Monte-carlo, i.e., sample $s_{t+1}, s_{t+2}, \dots$ and compute G_t

$$= \mathbb{E}_{\pi}\left[r(s, \pi(s), s') + \gamma V^{\pi}(s')\right]$$

– could be approximated by Temporal Difference

$$= \sum_{a} \pi(a|s) \sum_{s'} \mathcal{P}^{a}_{s \to s'}\left[r(s, \pi(s), s') + \gamma V^{\pi}(s')\right]$$

– could be solved exactly by Dynamic programming

Action-value (Q) function

- action-valued function $Q^\pi(s,a) = \mathbb{E}\big[G_t | S_t = s, A_t = a, \pi\big]$:
- expected total return starting from state s, taking action a, and then follow policy π
- Stochastic policy π:

$$V^\pi(s) = \mathbb{E}_{a\sim\pi(s)}[Q^\pi(s,a)]$$

- deterministic policy:

$$v^*(s) = \max_{a'} Q^*(s,a')$$

- from before;

$$\begin{aligned} V^*(s) &= \max_a \left(\mathbb{E}_{s'} \left[r(s,a,s') + \gamma \underbrace{V^*(s')}\Big| s \right] \right) \\ &= \max_a \underbrace{\left(\mathbb{E}_{s'} \left[r(s,a,s') + \gamma \big(\max_{a'} Q^*(s',a') \big) \Big| s \right] \right)}_{Q*(s',a') \text{ by definition}} \end{aligned}$$

- therefore:

$$Q^*(s,a) = \mathbb{E}_{s'} \left[r(s,a,s') + \gamma \big(\max_{a'} Q^*(s',a') \big) \big| s,a \right] \Big)$$

Value iteration vs Policy iteration

- policy iteration
  - choose an arbitrary policy π
  - **loop** compute the value function of policy π for all states
  - improve the policy at each state:
- value iteration
  - **loop** $\forall s \in \mathbb{S} \quad \forall a \in \mathcal{A}$
  - compute $Q(s, a)$ then $V(s) = \max_a Q(s, a)$

Action-value (Q) function

$$Q^*(s,a) = \mathbb{E}_{s'}\left[r(s,a,s') + \gamma\big(\max_{a'} Q^*(s',a')\big) \,|\, s,a \right]\Big)$$

- drop $|s,a$, let's solve this by **temporal difference**:

$$Q^\pi(s,a) = \mathbb{E}_{s'}\left[r(s,\pi(s),s') + \gamma\big(\max_{a'} Q^\pi(s',a')\big) \right]$$

$$\Longrightarrow \; Q^\pi(s,a) + \eta Q^\pi(s,a) = Q^\pi(s,a) + \eta\left(\mathbb{E}_{s'}\left[r(s,a,s') + \gamma\big(\max_{a'} Q^\pi(s',a')\big) \right] \right)$$

$$\Longrightarrow \; Q^\pi(s,a) = Q^\pi(s,a) + \eta\left(\mathbb{E}_{s'}\left[r(s,a,s') + \gamma\big(\max_{a'} Q^\pi(s',a')\big) \right] - Q^\pi(s,a) \right)$$

- instead of compute this expectation, in **each iteration** t, we sample a new state $(\tilde{s'}, \tilde{a}) \sim \Pr(s', a|\ldots)$.

  Q-Learning: recursively:

$$Q(s,\tilde{a}) = Q(s,\tilde{a}) + \eta\Big(\underbrace{r(s,\tilde{a},\tilde{s'}) + \gamma\big(\max_{a'} Q(\tilde{s'},a')\big)}_{y} - Q(s,\tilde{a}) \Big)$$

- let $\eta = 1$:

$$Q(s,\tilde{a}) = r(s,\tilde{a},\tilde{s'}) + \gamma\big(\max_{a'} Q(\tilde{s'},a')\big)$$

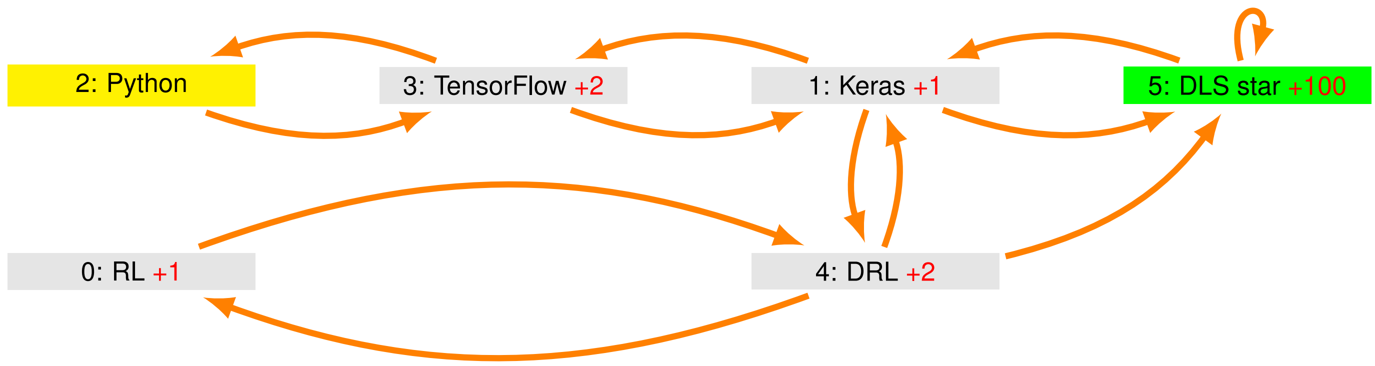
Q-Learning algorithm

Require: choice of γ Rewards matrix R

```
Q ← 0
for each episode do
    randomise initiate state s_0
    while goal state not reached do
        select (a, s') ~ Pr(a, s'|.)
        compute max_{a'} Q(s', a')
        Q(s, a) ← r(s, a, s') + γ( max_{a'} Q(s', a'))
        s_t ← s_{t+1}
    end while
end for
```

Q-Learning example

We took the example from the Markov Reward Process example earlier:



- there is small immediate rewards by going from one module to another
- you get a final large reward by becoming DLS star
- let $\gamma = 0.5$
- in this special example, $a = s'$, i.e., the action is to turn into the next state (module of studies).
- assume equal probabilities for all edges.

Q-Learning example: episode 1

$$R = \begin{array}{c} \text{S} \downarrow, \text{A} \rightarrow \\ \text{RL(0)} \\ \text{Ke(1)} \\ \text{Py(2)} \\ \text{TF(3)} \\ \text{DRL(4)} \\ \text{DLS*(5)} \end{array} \begin{array}{c} \begin{array}{cccccc} \text{RL(0)} & \text{Ke(1)} & \text{Py(2)} & \text{TF(3)} & \text{DRL(4)} & \text{DLS*(5)} \end{array} \\ \begin{pmatrix} - & - & - & - & 2 & - \\ - & - & - & 2 & 2 & 100 \\ - & - & - & 2 & - & - \\ - & 1 & 0 & - & - & - \\ 1 & 1 & - & - & - & 100 \\ - & 1 & - & - & - & 100 \end{pmatrix} \end{array}$$

- $s \sim \Pr(s|.) = 1$, i.e, Keras
- at $s = 1$, it has **allowable actions**: go to state $\{3, 4, 5\}$, i.e., $a \in \{3, 4, 5\}$
- $(a, s') \sim \Pr(a, s'|.) = (5, 5)$
- at $s' = 5$, it has **allowable actions**: $a' \in \{1, 5\}$:

$$\begin{aligned} Q(s, a) &= r(s, a, s') + \gamma(\max_{a'} Q(s', a')) \\ &= R(1, s' = 5) + 0.5 \max[Q(s' = 5, 1), Q(s' = 5, 5)] \\ &= 100 + 0.5 \times 0 = 100 \end{aligned}$$

- set $s \leftarrow s' \implies s = 5$, i.e., goal state, end

before

$$Q = \begin{array}{c} \text{S} \downarrow, \text{A} \rightarrow \\ \text{RL(0)} \\ \text{Ke(1)} \\ \text{Py(2)} \\ \text{TF(3)} \\ \text{DRL(4)} \\ \text{DLS*(5)} \end{array} \begin{array}{c} \begin{array}{cccccc} \text{RL(0)} & \text{Ke(1)} & \text{Py(2)} & \text{TF(3)} & \text{DRL(4)} & \text{DLS*(5)} \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

after

$$Q = \begin{array}{c} \text{S} \downarrow, \text{A} \rightarrow \\ \text{RL(0)} \\ \text{Ke(1)} \\ \text{Py(2)} \\ \text{TF(3)} \\ \text{DRL(4)} \\ \text{DLS*(5)} \end{array} \begin{array}{c} \begin{array}{cccccc} \text{RL(0)} & \text{Ke(1)} & \text{Py(2)} & \text{TF(3)} & \text{DRL(4)} & \text{DLS*(5)} \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Q-Learning example: episode 2, Iteration 1

$$
R = \begin{array}{c} \text{S} \downarrow, \text{A} \rightarrow \\ \text{RL(0)} \\ \text{Ke(1)} \\ \text{Py(2)} \\ \text{TF(3)} \\ \text{DRL(4)} \\ \text{DLS*(5)} \end{array}
\begin{array}{c} \begin{array}{cccccc} \text{RL(0)} & \text{Ke(1)} & \text{Py(2)} & \text{TF(3)} & \text{DRL(4)} & \text{DLS*(5)} \end{array} \\
\begin{pmatrix} - & - & - & - & 2 & - \\ - & - & - & 2 & 2 & 100 \\ - & - & - & 2 & - & - \\ - & 1 & 0 & - & - & - \\ 1 & 1 & - & - & - & 100 \\ - & 1 & - & - & - & 100 \end{pmatrix} \end{array}
$$

before

$$
Q = \begin{array}{c} \text{S} \downarrow, \text{A} \rightarrow \\ \text{RL(0)} \\ \text{Ke(1)} \\ \text{Py(2)} \\ \text{TF(3)} \\ \text{DRL(4)} \\ \text{DLS*(5)} \end{array}
\begin{array}{c} \begin{array}{cccccc} \text{RL(0)} & \text{Ke(1)} & \text{Py(2)} & \text{TF(3)} & \text{DRL(4)} & \text{DLS*(5)} \end{array} \\
\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}
$$

after

$$
Q = \begin{array}{c} \text{S} \downarrow, \text{A} \rightarrow \\ \text{RL(0)} \\ \text{Ke(1)} \\ \text{Py(2)} \\ \text{TF(3)} \\ \text{DRL(4)} \\ \text{DLS*(5)} \end{array}
\begin{array}{c} \begin{array}{cccccc} \text{RL(0)} & \text{Ke(1)} & \text{Py(2)} & \text{TF(3)} & \text{DRL(4)} & \text{DLS*(5)} \end{array} \\
\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 51 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}
$$

- $s \sim \Pr(s|.) = 3$
- at $s = 3$, it has **allowable actions**: go to state $\{1, 2\}$, i.e., $a \in \{1, 2\}$
- $(a, s') \sim \Pr(a, s'|.) = (1, 1)$
- at $s' = 1$, it has **allowable actions**: $a' \in \{3, 4, 5\}$:

$$
\begin{aligned}
Q(s, a) &= r(s, a, s') + \gamma(\max_{a'} Q(s', a')) \\
&= R(3, 1) + 0.5 \max[Q(1, 3), Q(1, 4), Q(1, 5)] \\
&= 1 + 0.5 \times 100 = 51
\end{aligned}
$$

- set $s \leftarrow s' \implies s = 1$, i.e., **not** a goal state, keep on going

Q-Learning example: episode 2, Iteration 2

$$
R = \begin{array}{cc}
\text{S} \downarrow, \text{A} \rightarrow & \begin{array}{cccccc} \text{RL(0)} & \text{Ke(1)} & \text{Py(2)} & \text{TF(3)} & \text{DRL(4)} & \text{DLS*(5)} \end{array} \\
\begin{array}{c} \text{RL(0)} \\ \text{Ke(1)} \\ \text{Py(2)} \\ \text{TF(3)} \\ \text{DRL(4)} \\ \text{DLS*(5)} \end{array} &
\begin{pmatrix}
- & - & - & - & 2 & - \\
- & - & - & 2 & 2 & 100 \\
- & - & - & 2 & - & - \\
- & 1 & 0 & - & - & - \\
1 & 1 & - & - & - & 100 \\
- & 1 & - & - & - & 100
\end{pmatrix}
\end{array}
$$

before

$$
Q = \begin{array}{cc}
\text{S} \downarrow, \text{A} \rightarrow & \begin{array}{cccccc} \text{RL(0)} & \text{Ke(1)} & \text{Py(2)} & \text{TF(3)} & \text{DRL(4)} & \text{DLS*(5)} \end{array} \\
\begin{array}{c} \text{RL(0)} \\ \text{Ke(1)} \\ \text{Py(2)} \\ \text{TF(3)} \\ \text{DRL(4)} \\ \text{DLS*(5)} \end{array} &
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 100 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 51 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}
$$

after

$$
Q = \begin{array}{cc}
\text{S} \downarrow, \text{A} \rightarrow & \begin{array}{cccccc} \text{RL(0)} & \text{Ke(1)} & \text{Py(2)} & \text{TF(3)} & \text{DRL(4)} & \text{DLS*(5)} \end{array} \\
\begin{array}{c} \text{RL(0)} \\ \text{Ke(1)} \\ \text{Py(2)} \\ \text{TF(3)} \\ \text{DRL(4)} \\ \text{DLS*(5)} \end{array} &
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 100 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 51 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}
$$

- $s = 1$ from previous iteration
- at $s = 1$, it has **allowable actions**: go to state $\{3, 4, 5\}$, i.e., $a \in \{3, 4, 5\}$
- $(a, s') \sim \Pr(a, s' | .) = (5, 5)$
- at $s' = 5$, it has **allowable actions**: $a' \in \{1, 5\}$:

$$
\begin{aligned}
Q(s, a) &= r(s, a, s') + \gamma (\max_{a'} Q(s', a')) \\
Q(1, 5) &= R(1, 5) + 0.5 \max[Q(5, 1), Q(5, 5)] \\
&= 100 + 0.5 \times 0 = 100
\end{aligned}
$$

- set $s \leftarrow s' \implies s = 5$, i.e., goal state, end

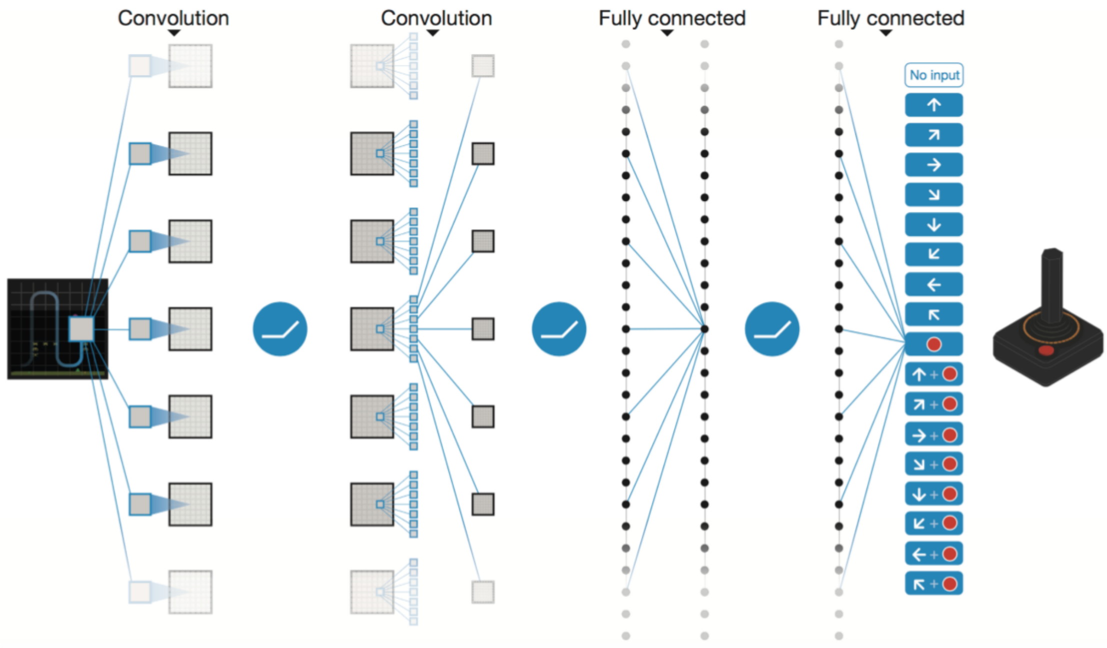
the state-action table gets updated until convergence.

On the setting of Atari

- the states are far too many!
- need a **function approximator** to estimate the action-value function, $Q(s, a|\theta) \approx Q^*(s, a)$
- guess what? Deep Neural Network helps!

Represent $Q(s, a)$ using neural networks

- The figure below represents a row of the Q function table earlier:



Conv [16] $\rightarrow$ ReLU $\rightarrow$ Conv [32] $\rightarrow$ ReLU $\rightarrow$ FC [256] $\rightarrow$ ReLU $\rightarrow$ FC [|A|]

- these are **not** softmax functions.

abstracted algorithm for Deep Q-Learning

Require: Initialize an empty replay memory
Require: Initialize the DQN weights θ

for each episode **do**
  for $t = 1, \dots T$ **do**
    with probability ϵ select $\tilde{a}$ random action
    otherwise, select:

$$\tilde{a} = \max_a \left(Q^*(s, a|\theta) \right)$$

    perform $\tilde{a}$ and receive rewards r_t and state s'.
    add tuple $(s, \tilde{a}, r_t, s')$ into replay memory
    Sample a mini-batch of tuples (s_j, a_j, r_j, s'_j) from the replay memory
    and perform stochastic gradient descent on the DQN, based on the loss function:

$$\left(\underbrace{r_j + \gamma \big(\max_{a'} Q(s'_j, a'|\theta^-) \big)}_{y_j} - Q(s_j, a_j|\theta) \right)^2$$

  end for
end for

innovation

- freeze parameters of target network $Q(s'_j, a'|\theta^-)$ for fixed number of iterations
- while updating the online network $Q(s; a; \theta_i)$ by gradient descent

double Deep Q-Leanring

- same values θ both to select and to evaluate an action:

$$
\begin{aligned}
y_j &= r_j + \gamma\big(\max_{a'} Q(s_j', a' | \theta)\big) \\
&= r_j + \gamma\big(Q(s_j', \arg\max_a Q(s_j', a, \theta) | \theta)\big)
\end{aligned}
$$

- more likely to select overestimated values
- resulting in overoptimistic value estimates
- the solution is:

$$
y_j = r_j + \gamma\big(\max_{a'} Q(s_j', \arg\max_a Q(s_j', a, \theta) | \theta')\big)
$$

- still estimating value of policy according to current values defined by θ
- use second set of weights θ' to **fairly** evaluate value of this policy

In summary

- CNN and RNN are two of the building blocks in Deep Learning
- People have been putting them into many existing machine learning frameworks, and have generated many interesting stuff
- but there is plenty still needs to be explored!