# Deep Reinforcement Learning: A brief introduction

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## Deep Reinforcement Learning

- A video from Google DeepMind's Deep Q-learning playing Atari Breakout: `https://www.youtube.com/watch?v=TmPfTpjtdgg`
- *Mnih, Volodymyr, et al. "Playing atari with deep reinforcement learning." arXiv preprint arXiv:1312.5602 (2013).*
- code is also available `https://github.com/kuz/DeepMind-Atari-Deep-Q-Learner`

**N.B.**

- Apologies for those have seen it before

**significance** of this demo shows it's possible to use Neural Network to learn how to play a game, based on:

- sequences of screen images
- scores the game receives
- goal is to learn the best policy for **actions** to take

Surely you **don't need a menu** to learn how to play Atari. i.e., it's **model-free**!

## Reinforcement Learning (RL)

Forget about the Neural network for a second, how is Reinforcement Learning (RL) different to conventional supervised learning?

- No data label like supervised learning, i.e., no *“best-action-for-that-screen”* label
- only **reward** signal
- feedback in **delayed**, not instantaneous
- data are not i.i.d., (consecutive frames are similar)
- agent’s actions affects the subsequent data it receives.

Let’s get started with some RL background.

## Reinforcement Learning (RL)

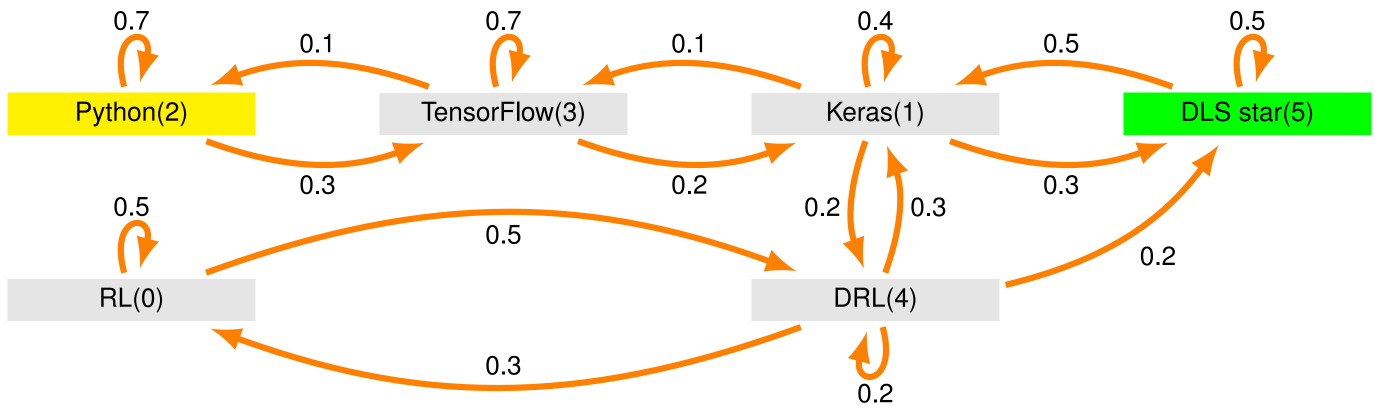
another way to look at it:

- RL uses training information that **evaluates the actions** taken rather than **instructs by giving correct actions**.
- a need for active exploration: explicit trial-and-error search for good behavior.
- **purely evaluative feedback** indicates how good the action taken is, but not whether it is the best or the worst action possible.
- **purely instructive feedback** indicates correct action to take, independently of the action actually taken. supervised learning

# Application of RLs

- **marketing**
  customer's attributes $s$, marketing actions $a$, customer signs up $r$
- **drone control**
  all avaiable sensor data $a$, controls $s$, not crashing $r$
- **chatbot**
  conversations to-date $s$, things that a robot will say $a$, customer satisfaction $r$

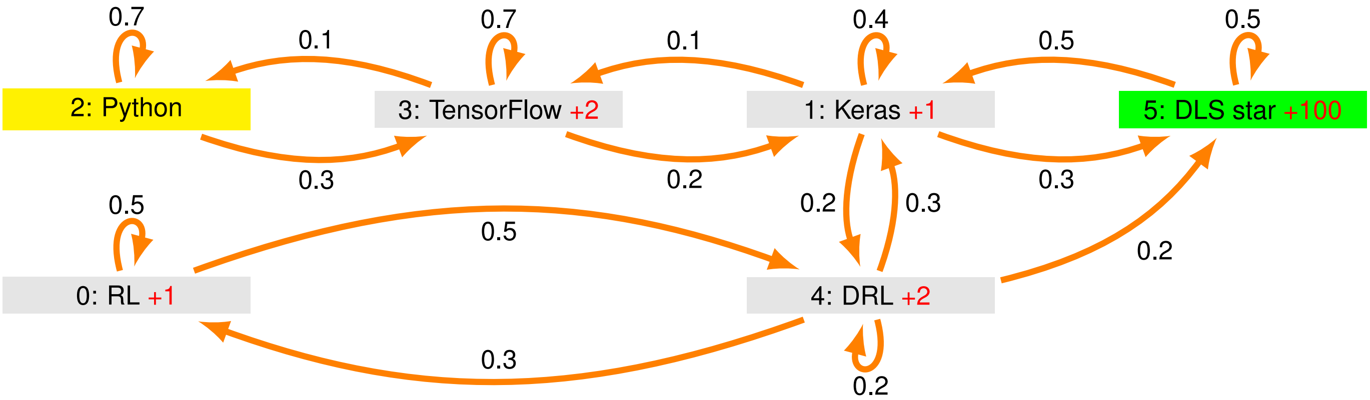
# Markov Process




- one may start from **python** and generate sequences with transition probabilities to end up in **DLS star**. examples:
  - Python, Python, Python, TensorFlow, Keras, DLS star
  - Python, Python, Python, TensorFlow, TensorFlow, Keras, DRL, DRL RL, DLS star
  - Python, Python, TensorFlow, TensorFlow, Keras, DRL, DLS star
  - The question is: how we may able to measure "how good" each path? . . .

# Markov Reward Process

Let's add some rewards to being at each of the state:




What we care is the **total return** $G_t$: sum of **discounted** reward from time-step $t$

$$G_t = R_{t+1} + \gamma R_{t+2} + \cdots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \qquad \text{where } \gamma \in [0, 1]$$

note that $G_t$ is a random variable
**exercise** what happens when $\gamma = 0$ and $\gamma = 1$

# Markov Random Process: Bellman Equation (new)

- **state value function** $V(s)$ of MRP is expected total return starting from state $s$

$$\begin{aligned} V(s) &= \mathbb{E}_{s_{t+1}, s_{t+2}, \dots, r_{t+1}, r_{t+2}, \dots}[G_t | s_t = s] \\ &= \mathbb{E}_{s_{t+1}, s_{t+2}, \dots, r_{t+1}, r_{t+2}, \dots}\big[R_{t+1} + \gamma \underbrace{(R_{t+2} + \gamma R_{t+3} + \dots)}_{G_{t+1}}\big] \end{aligned}$$

- $\mathbb{E}[.]$ needs the integrate over $(s_1, s_2, \dots \in \mathcal{S}, r_1, r_2, \dots \in \mathcal{R})$:
- $s_1, s_2, \dots$ and $r_1, r_2, \dots$ are generated in the following fashion:

$$s_0 \rightarrow (s_1, r_1) \qquad s_1 \rightarrow (s_2, r_2) \dots$$

- for clarity, we let $s_t \rightarrow s_0$ and $s_{t+k} \rightarrow s_k$:

# Markov Random Process: Bellman Equation (new)

- suppose we have a **universal state value function** $V(.)$:

$$V(.) = \sum_{s_0} \Pr(s_0) \sum_{s_1, r_1} \Pr(s_1, r_1 | s_0) \sum_{s_2, r_2} \Pr(s_2, r_2 | s_1) \sum_{s_3, r_3} \dots [r_1 + \gamma(r_2 + \gamma r_3 + \dots)]$$

- however, we usually specify value of $v_\pi(s_0)$ to evaluate:

$$\begin{aligned} V(s_0) &= \underbrace{\sum_{s_1, r_1} \Pr(s_1, r_1 | s_0) \left( r_1 + \gamma \underbrace{\sum_{s_2, r_2} \Pr(s_2, r_2 | s_1) \sum_{s_3, r_3} \dots [r_2 + \gamma(r_3 + \gamma r_4 + \dots]}_{V(s_1) \triangleq \mathbb{E}[G_{t+1} | s_1]} \right)}_{V(s_0) \triangleq \mathbb{E}[G_t | s_0]} \\ &= \mathbb{E}_{s_1, r_1} [r_1 + \gamma V(s_1) | s_0] \\ &= \mathbb{E}_{s_1} [R_1 + \gamma V(s_1) | s_0] \text{ if } R_1 \text{ is deterministic} \end{aligned}$$

# Markov Random Process: Bellman Equation (new)

$$V(s_0) = \mathbb{E}_{s_1}\left[R_1 + \gamma V(s_1)|s_0\right]$$

- **Bellman equations**: value of the current state, $v(s)$ breaks up into (1) **immediate** and (2) **future** rewards.
- state value function $V(s)$ is written in a consecutive time steps
- difficult to estimate: because $V(s)$ also depends on various other $V(s')$ which occur at different times

# Bellman Equation in matrix form

- to simplify, making $R_t$ deterministic

$$V(s_0) = \mathbb{E}_{s_1}\left[R_1 + \gamma V(s_1)|s_0\right]$$

- say $s \in \{1, \dots, n\}$:

$$\underbrace{V(s_0 = 1)}_{v(1)} = \mathbb{E}_{s_1}\left[\underbrace{R_1(s_0 = 1)}_{R_1} + \gamma V(s_1)|s_0 = 1\right]$$

$$V(s_0 = 2) = \mathbb{E}_{s_1}\left[R_1(s_0 = 2) + \gamma v(s_1)|s_0 = 2\right]$$

$$\dots$$

take the first line,

$$\begin{aligned} v(1) &= \mathbb{E}_{s_1}\left[R_1 + \gamma V(s_1)|s_0 = 1\right] \\ &= R_1 + \gamma \mathbb{E}\left[V(s_1)|s_0 = 1\right] \\ &= R_1 + \gamma\left(\sum_{s_1=1}^{n} v(s_1)\Pr(1 \to s_1)\right) \\ &= R_1 + \gamma\left(\sum_{j=1}^{n} v(j)\Pr(1 \to j)\right) \\ &\dots \\ \implies v(n) &= R_n + \gamma\left(\sum_{j=1}^{n} v(j)\Pr(k \to j)\right) \end{aligned}$$

# Bellman Equation in matrix form (2)

$$
\begin{aligned}
v(k) &= R_k + \gamma \Big( \sum_{j=1}^{n} v(j) \Pr(k \to j) \Big) \\
&= R_k + \gamma \mathcal{P}_{k,:}^{\top} \mathbf{v} \\
\implies \mathbf{v} &= \mathbf{R} + \gamma \mathcal{P} \mathbf{v}
\end{aligned}
$$

$$
\implies \begin{bmatrix} v(1) \\ \vdots \\ v(n) \end{bmatrix} = \begin{bmatrix} R_1 \\ \vdots \\ R_n \end{bmatrix} + \gamma \begin{bmatrix} \mathcal{P}_{1,1} & \dots & \mathcal{P}_{1,n} \\ \vdots & & \vdots \\ \mathcal{P}_{n,1} & \dots & \mathcal{P}_{n,n} \end{bmatrix} \begin{bmatrix} v(1) \\ \vdots \\ v(n) \end{bmatrix}
$$

the solution to **MRP** is straight forward:

$$
\begin{aligned}
\mathbf{v} &= \mathbf{R} + \gamma \mathcal{P} \mathbf{v} \\
(I - \gamma \mathcal{P}) v &= R \\
\mathbf{v} &= (I - \gamma \mathcal{P})^{-1} R
\end{aligned}
$$

# Markov Decision Process (MDP)

- now agent has **actions**
- concept of **policy** $\pi$: take a state $s_t$ as input and decides and action $a_t$

$$\pi(a|s) = \Pr(A_t = a | S_t = s)$$

- a policy is time-invariant (or stationary) and stochastic
- next state for an agent, now also depends on its action taken:

$$\mathcal{P}^{a_0}_{s_0 \to s_1} = \Pr(S_1 = s_1 | S_0 = s_0, A_0 = a)$$

- multiple transition matrix $\mathcal{P}$ each depends on the $a$ taken
- once fixed $\pi$, MDP becomes MRP with transition probability $\mathcal{P}^{\pi}_{s \to s'}$:

$$\mathcal{P}^{\pi}_{s_0 \to s_1} = \sum_{a_0 \in \mathcal{A}} \pi(a_0 | s_0) \mathcal{P}^{a_0}_{s_0 \to s_1}$$

# Markov Decision Process: Bellman Equation (new)

- given a policy $\pi$, **state value function** $v(s)$ is expected total return starting from state $s$

$$
\begin{aligned}
v_\pi(s) &= \mathbb{E}_\pi[G_t | s_t = s] \\
&= \mathbb{E}_\pi\Big[R_{t+1} + \gamma \underbrace{\big(R_{t+2} + \gamma R_{t+3} + \dots\big)}_{G_{t+1}}\Big]
\end{aligned}
$$

- $\mathbb{E}_\pi[.]$ needs the integrate over $(a_0, a_1, \dots \in \mathcal{A}, s_0, s_1, \dots \in \mathcal{S}, r_1, r_2, \dots \in \mathcal{R})$:
- for clarity, we let $s_t \to s_0$ and $s_{t+k} \to s_k$:
- $s_0 \to a_0, \qquad (s_0, a_0) \to (s_1, r_1), \qquad s_1 \to a_1, \qquad (s_1, a_1) \to (s_2, r_2), \dots$

# Markov Decision Process: Bellman Equation (new)

- suppose we have a **universal state value function** $V_\pi(.)$, i.e., no matter what the current state and action is:

$$
\begin{aligned}
&v_\pi(.)\\
&= \sum_{s_0} \Pr(s_0) \sum_{a_0} \pi(a_0|s) \sum_{s_1,r_1} \Pr(s_1, r_1|s_0, a_0) \sum_{a_1} \pi(a_1|s_1) \sum_{s_2,r_2} \Pr(s_2, r_2|s_1, a_1) \sum_{a_2} \cdots \sum_{s_3,r_3} \dots\\
&\qquad \big[r_1 + \gamma(r_2 + \gamma r_3 + \dots)\big]
\end{aligned}
$$

- however, we do know the value $v_\pi(s_0)$:

$$
\begin{aligned}
&v_\pi(s_0)\\
&= \underbrace{\sum_{a_0} \pi(a_0|s) \sum_{s_1,r_1} \Pr(s_1, r_1|s_0, a_0) \Big(r_1 + \gamma \underbrace{\sum_{a_1} \pi(a_1|s_1) \sum_{s_2,r_2} \Pr(s_2, r_2|s_1, a_1) \sum_{a_2} \cdots \sum_{s_3,r_3} \dots [r_2 + \gamma(r_3 + \gamma r_4 + \dots]}_{v_\pi(s_1) \triangleq \mathbb{E}_\pi[G_{t+1}|s_1]} \Big)}_{v_\pi(s_0) \triangleq \mathbb{E}_\pi[G_t|s_0]}\\
&= \sum_{a_0} \pi(a_0|s) \sum_{s_1,r_1} \Pr(s_1, r_1|s_0, a_0) (r_1 + \gamma \mathbb{E}_\pi[G_{t+1}|s_1])
\end{aligned}
$$

# Bellman equation extends to $Q(s, a)$

summarise slides from before:

$$\begin{aligned}
V_\pi(s_0) &= \sum_{a_0} \pi(a_0|s_0) \sum_{s_1, r_1} \Pr(s_1, r_1|s_0, a_0)(r_1 + \gamma v_\pi(s_1)) \\
&= \sum_{a_0} \pi(a_0|s_0) \sum_{s_1, r_1} \Pr(s_1, r_1|s_0, a_0)(r_1 + \gamma \mathbb{E}_\pi[G_{t+1}|s_1]) \\
&= \sum_{a_0} \pi(a_0|s_0) \mathbb{E}_{(s_1, r_1)\sim}[r_1 + \gamma v_\pi(s_1)]
\end{aligned}$$

insert $a_0$ to obtain $Q$ function:

$$\begin{aligned}
Q_\pi(s_0, a_0) &= \sum_{s_1, r_1} \Pr(s_1, r_1|s_0, a_0)(r_1 + \gamma v_\pi(s_1)) \\
&= \sum_{s_1, r_1} \Pr(s_1, r_1|s_0, a_0)(r_1 + \gamma \mathbb{E}_\pi[G_{t+1}|s_1]) \\
&= \mathbb{E}_{(s_1, r_1)\sim}[r_1 + \gamma v_\pi(s_1)]
\end{aligned}$$

since any policy $\pi$ works, then:

$$Q_{\pi*}(s_0, a_0) = \mathbb{E}_{(s_1, r_1)\sim}[r_1 + \gamma v_{\pi*}(s_1)]$$

# Bellman optimality

- we know best $V_*(s)$ must be the best action from an optimal (state, action) pair: $Q_{\pi*}(s_0, a_0)$:

$$V_*(s_0) = \max_{a_0} Q_{\pi*}(s_0, a_0)$$

- and from before:

$$\begin{aligned}
V_*(s_0) &= \max_{a_0} Q_{\pi_*}(s_0, a_0) \\
&= \max_{a_0} \mathbb{E}_{(s_1, r_1)\sim}[r_1 + \gamma v_{\pi_*}(s_1)] \qquad \text{from previous page} \\
&= \sum_{s_1, r_1} \Pr(s_1, r_1 | s_0, a_0)(r_1 + \gamma v_{\pi_*}(s_1)) \\
&= \sum_{s_1, r_1} \Pr(s_1, r_1 | s_0, a_0)(r_1 + \gamma \max_{a_1} Q_{\pi*}(s_1, a_1)) \\
&= \max_{a_0} \mathbb{E}_{(s_1, r_1)\sim}\left[r_1 + \gamma \max_{a_1} Q_{\pi*}(s_1, a_1)\right] \\
&= \max_{a_0} \mathbb{E}_{(s_1, r_1)\sim}\left[r_1 + \gamma \max_{a_1} Q_{\pi*}(s_1, a_1) \middle| s_0\right] \text{ removed } |s_0 \text{ for clarity previously} \\
\implies Q_*(s_0) &= \mathbb{E}_{(s_1, r_1)\sim}\left[r_1 + \gamma \max_{a_1} Q_{\pi*}(s_1, a_1) \middle| s_0\right]
\end{aligned}$$

- also $r_1 \triangleq r_1(s_0, \pi(s_0), s_1)$

# Solve Bellman's equation using Temporal Difference

$$V_\pi(s_0) = \sum_{a_0} \pi(a_0|s_0) \sum_{s_1,r_1} \Pr(s_1, r_1|s_0, a_0)(r_1 + \gamma v_\pi(s_1))$$

- drop $|s$ again for clarity:

$$
\begin{aligned}
V^\pi(s) &= \mathbb{E}_{s'}\left[r(s,\pi(s),s') + \gamma V^\pi(s')\right] \\
\implies V^\pi(s) + \eta V^\pi(s) &= V^\pi(s) + \eta\left(\mathbb{E}_{s'}\left[r(s,\pi(s),s') + \gamma V^\pi(s')\right]\right) \\
\implies V^\pi(s) &= V^\pi(s) + \eta\left(\mathbb{E}_{s'}\left[r(s,\pi(s),s') + \gamma V^\pi(s')\right] - V^\pi(s)\right)
\end{aligned}
$$

- instead of compute this expectation, in **each iteration** $t$, we sample a new state $\tilde{s'} \sim \Pr(s'|\dots)$

$$V^\pi_{t+1}(s) = V^\pi_t(s) + \eta\left(r(s,\pi(s),\tilde{s'}) + \gamma V^\pi_t(\tilde{s'}) - V^\pi_t(s)\right)$$

- note that the last equation is called **temporal difference**

# Bellman's equation: Three ways of solving it

$$V_\pi(s_0) = \mathbb{E}_\pi\left[G_t|s_0\right]$$

— could be approximated by Monte-carlo, i.e., sample $s_1, s_2, \ldots$ and compute $G_t$

$$= \mathbb{E}_\pi\left[r(s_0, \pi(s_0), s_1) + \gamma V_\pi(s_1)\right]$$

— could be approximated by Temporal Difference

$$= \sum_{a_0} \pi(a_0|s_0) \sum_{s_1} \mathcal{P}^{a_0}_{s_0 \to s_1}\left[r(s_0, \pi(s_0), s_1) + \gamma V_\pi(s_1)\right]$$

— could be solved exactly by Dynamic programming

# Policy Iteration

- choose an arbitrary policy $\pi'$
- **while** before some stopping criteria:

  $\pi = \pi'$

  compute the value function $V_\pi(1), \dots V_\pi(n)$ using policy $\pi$:

$$V_\pi(s_0) = R(s, \pi(s_0)) + \gamma \sum_{s_1 \in \mathbb{S}} \mathcal{P}^{a_0}_{s_0 \to s_1} V_\pi(s_1)$$

  improve the policy at each state:

$$\pi'(s_0) = \arg\max_{a_0} \left[ R(s, a_0) + \gamma \sum_{s_1 \in \mathbb{S}} \mathcal{P}^{a_0}_{s_0 \to s_1} V_\pi(s_1) \right]$$

# Value Iteration

**loop**$\forall s \in \mathbb{S}$

**loop**$\forall a \in \mathcal{A}$

$$Q(s_0, a_0) = R(s, a_0) + \gamma \sum_{s_1 \in \mathbb{S}} \mathcal{P}_{s_0 \to s_1}^{a_0} V_\pi(s_1)$$

$$V(s_0) = \max_{a_0} Q(s_0, a_0)$$

# Action-value (Q) function

- action-valued function $Q^{\pi}(s, a) = \mathbb{E}\big[G_t | S_t = s, A_t = a, \pi\big]$:
- expected total return starting from state $s$, taking action $a$, and then follow policy $\pi$
- Stochastic policy $\pi$:

$$V^{\pi}(s) = \mathbb{E}_{a \sim \pi(s)}[Q^{\pi}(s, a)]$$

- deterministic policy:

$$v^*(s) = \max_{a'} Q^*(s, a')$$

- from before;

$$\begin{aligned} V^*(s) &= \max_a \left( \mathbb{E}_{s'} \left[ r(s, a, s') + \gamma \underbrace{V^*(s')} \Big| s \right] \right) \\ &= \max_a \underbrace{\left( \mathbb{E}_{s'} \left[ r(s, a, s') + \gamma \big( \max_{a'} Q^*(s', a') \big) \Big| s \right] \right)}_{Q^*(s', a') \text{ by definition}} \end{aligned}$$

- therefore:

$$Q^*(s, a) = \mathbb{E}_{s'} \left[ r(s, a, s') + \gamma \big( \max_{a'} Q^*(s', a') \big) \big| s, a \right] \Big)$$

# Action-value (Q) function

$$Q^*(s,a) = \mathbb{E}_{s'}\left[ r(s,a,s') + \gamma\big(\max_{a'} Q^*(s', a')\big) | s, a \right] \Big)$$

- drop $|s, a$, let's solve this by **temporal difference**:

$$Q^\pi(s,a) = \mathbb{E}_{s'}\left[ r(s,\pi(s),s') + \gamma\big(\max_{a'} Q^\pi(s', a')\big) \right]$$

$$\implies Q^\pi(s,a) + \eta Q^\pi(s,a) = Q^\pi(s,a) + \eta\left( \mathbb{E}_{s'}\left[ r(s,a,s') + \gamma\big(\max_{a'} Q^\pi(s', a')\big) \right] \right)$$

$$\implies Q^\pi(s,a) = Q^\pi(s,a) + \eta\left( \mathbb{E}_{s'}\left[ r(s,a,s') + \gamma\big(\max_{a'} Q^\pi(s', a')\big) \right] - Q^\pi(s,a) \right)$$

- instead of compute this expectation, in **each iteration** $t$, we sample a new state $(\tilde{s'}, \tilde{a}) \sim \Pr(s', a| \dots)$.

  Q-Learning: recursively:

$$Q(s,\tilde{a}) = Q(s,\tilde{a}) + \eta\Big( \underbrace{r(s,\tilde{a},\tilde{s'}) + \gamma\big(\max_{a'} Q(\tilde{s'}, a')\big)}_{y} - Q(s,\tilde{a}) \Big)$$

- let $\eta = 1$:

$$Q(s,\tilde{a}) = r(s,\tilde{a},\tilde{s'}) + \gamma\big(\max_{a'} Q(\tilde{s'}, a')\big)$$

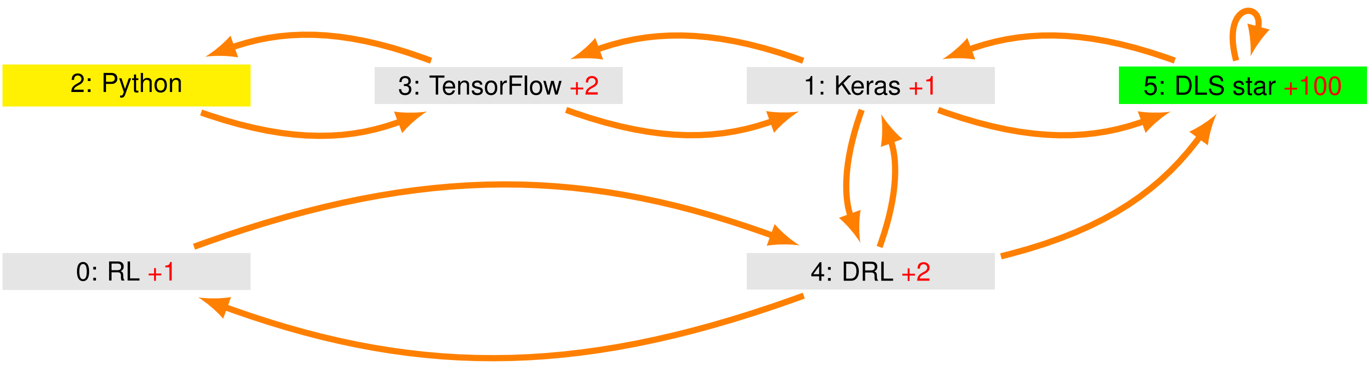
# Q-Learning algorithm

**Require:** choice of $\gamma$ Rewards matrix $R$

1: $Q \leftarrow \mathbf{0}$
2: **for** each episode **do**
3: randomise initiate state $s_0$
4: **while** goal state not reached **do**
5: select $(a, s') \sim \Pr(a, s'|.)$
6: compute $\max_{a'} Q(s', a')$
7: $Q(s, a) \leftarrow r(s, a, s') + \gamma\big(\max_{a'} Q(s', a')\big)$
8: $s_t \leftarrow s_{t+1}$
9: **end while**
10: **end for**

# Q-Learning example

We took the example from the Markov Reward Process example earlier:




- there is small immediate rewards by going from one module to another
- you get a final large reward by becoming DLS star
- let $\gamma = 0.5$
- in this special example, $a = s'$, i.e., the action is to turn into the next state (module of studies).
- assume equal probabilities for all edges.

# Q-Learning example: episode 1

$$R = \begin{array}{c|cccccc} S\downarrow, A\rightarrow & RL(0) & Ke(1) & Py(2) & TF(3) & DRL(4) & DLS^*(5) \\ \hline RL(0) & - & - & - & - & 2 & - \\ Ke(1) & - & - & - & 2 & 2 & 100 \\ Py(2) & - & - & - & 2 & - & - \\ TF(3) & - & 1 & 0 & - & - & - \\ DRL(4) & 1 & 1 & - & - & - & 100 \\ DLS^*(5) & - & 1 & - & - & - & 100 \end{array}$$

- $s \sim \Pr(s|.) = 1$, i.e, Keras
- at $s = 1$, it has **allowable actions**: go to state $\{3, 4, 5\}$, i.e., $a \in \{3, 4, 5\}$
- $(a, s') \sim \Pr(a, s'|.) = (5, 5)$
- at $s' = 5$, it has **allowable actions**: $a' \in \{1, 5\}$:

$$Q(s, a) = r(s, a, s') + \gamma(\max_{a'} Q(s', a'))$$
$$= R(1, s' = 5) + 0.5 \max[Q(s' = 5, 1), Q(s' = 5, 5)]$$
$$= 100 + 0.5 \times 0 = 100$$

- set $s \leftarrow s' \implies s = 5$, i.e., goal state, end

**before**

$$Q = \begin{array}{c|cccccc} S\downarrow, A\rightarrow & RL(0) & Ke(1) & Py(2) & TF(3) & DRL(4) & DLS^*(5) \\ \hline RL(0) & 0 & 0 & 0 & 0 & 0 & 0 \\ Ke(1) & 0 & 0 & 0 & 0 & 0 & 0 \\ Py(2) & 0 & 0 & 0 & 0 & 0 & 0 \\ TF(3) & 0 & 0 & 0 & 0 & 0 & 0 \\ DRL(4) & 0 & 0 & 0 & 0 & 0 & 0 \\ DLS^*(5) & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

**after**

$$Q = \begin{array}{c|cccccc} S\downarrow, A\rightarrow & RL(0) & Ke(1) & Py(2) & TF(3) & DRL(4) & DLS^*(5) \\ \hline RL(0) & 0 & 0 & 0 & 0 & 0 & 0 \\ Ke(1) & 0 & 0 & 0 & 0 & 0 & 100 \\ Py(2) & 0 & 0 & 0 & 0 & 0 & 0 \\ TF(3) & 0 & 0 & 0 & 0 & 0 & 0 \\ DRL(4) & 0 & 0 & 0 & 0 & 0 & 0 \\ DLS^*(5) & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

# Q-Learning example: episode 2, Iteration 1

$$R = \begin{array}{l} \\ \text{RL(0)} \\ \text{Ke(1)} \\ \text{Py(2)} \\ \text{TF(3)} \\ \text{DRL(4)} \\ \text{DLS*(5)} \end{array} \begin{array}{c} \text{S} \downarrow, \text{A} \rightarrow \quad \text{RL(0)} \;\; \text{Ke(1)} \;\; \text{Py(2)} \;\; \text{TF(3)} \;\; \text{DRL(4)} \;\; \text{DLS*(5)} \\ \begin{pmatrix} - & - & - & - & 2 & - \\ - & - & - & 2 & 2 & 100 \\ - & - & - & 2 & - & - \\ - & 1 & 0 & - & - & - \\ 1 & 1 & - & - & - & 100 \\ - & 1 & - & - & - & 100 \end{pmatrix} \end{array}$$

- $s \sim \Pr(s|.) = 3$
- at $s = 3$, it has **allowable actions**: go to state $\{1, 2\}$, i.e., $a \in \{1, 2\}$
- $(a, s') \sim \Pr(a, s'|.) = (1, 1)$
- at $s' = 1$, it has **allowable actions**: $a' \in \{3, 4, 5\}$:

$$\begin{aligned} Q(s, a) &= r(s, a, s') + \gamma(\max_{a'} Q(s', a')) \\ &= R(3, 1) + 0.5 \max[Q(1, 3), Q(1, 4), Q(1, 5)] \\ &= 1 + 0.5 \times 100 = 51 \end{aligned}$$

- set $s \leftarrow s' \implies s = 1$, i.e., **not** a goal state, keep on going

**before**

$$Q = \begin{array}{l} \\ \text{RL(0)} \\ \text{Ke(1)} \\ \text{Py(2)} \\ \text{TF(3)} \\ \text{DRL(4)} \\ \text{DLS*(5)} \end{array} \begin{array}{c} \text{S} \downarrow, \text{A} \rightarrow \quad \text{RL(0)} \;\; \text{Ke(1)} \;\; \text{Py(2)} \;\; \text{TF(3)} \;\; \text{DRL(4)} \;\; \text{DLS*(5)} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

**after**

$$Q = \begin{array}{l} \\ \text{RL(0)} \\ \text{Ke(1)} \\ \text{Py(2)} \\ \text{TF(3)} \\ \text{DRL(4)} \\ \text{DLS*(5)} \end{array} \begin{array}{c} \text{S} \downarrow, \text{A} \rightarrow \quad \text{RL(0)} \;\; \text{Ke(1)} \;\; \text{Py(2)} \;\; \text{TF(3)} \;\; \text{DRL(4)} \;\; \text{DLS*(5)} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 51 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

# Q-Learning example: episode 2, Iteration 2

$$R = \begin{array}{c|cccccc} S\downarrow, A\rightarrow & RL(0) & Ke(1) & Py(2) & TF(3) & DRL(4) & DLS^*(5) \\ \hline RL(0) & - & - & - & - & 2 & - \\ Ke(1) & - & - & - & 2 & 2 & 100 \\ Py(2) & - & - & - & 2 & - & - \\ TF(3) & - & 1 & 0 & - & - & - \\ DRL(4) & 1 & 1 & - & - & - & 100 \\ DLS^*(5) & - & 1 & - & - & - & 100 \end{array}$$

**before**

$$Q = \begin{array}{c|cccccc} S\downarrow, A\rightarrow & RL(0) & Ke(1) & Py(2) & TF(3) & DRL(4) & DLS^*(5) \\ \hline RL(0) & 0 & 0 & 0 & 0 & 0 & 0 \\ Ke(1) & 0 & 0 & 0 & 0 & 0 & 100 \\ Py(2) & 0 & 0 & 0 & 0 & 0 & 0 \\ TF(3) & 0 & 51 & 0 & 0 & 0 & 0 \\ DRL(4) & 0 & 0 & 0 & 0 & 0 & 0 \\ DLS^*(5) & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

**after**

$$Q = \begin{array}{c|cccccc} S\downarrow, A\rightarrow & RL(0) & Ke(1) & Py(2) & TF(3) & DRL(4) & DLS^*(5) \\ \hline RL(0) & 0 & 0 & 0 & 0 & 0 & 0 \\ Ke(1) & 0 & 0 & 0 & 0 & 0 & 100 \\ Py(2) & 0 & 0 & 0 & 0 & 0 & 0 \\ TF(3) & 0 & 51 & 0 & 0 & 0 & 0 \\ DRL(4) & 0 & 0 & 0 & 0 & 0 & 0 \\ DLS^*(5) & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

- $s = 1$ from previous iteration
- at $s = 1$, it has **allowable actions**: go to state $\{3, 4, 5\}$, i.e., $a \in \{3, 4, 5\}$
- $(a, s') \sim \Pr(a, s' | .) = (5, 5)$
- at $s' = 5$, it has **allowable actions**: $a' \in \{1, 5\}$:

$$\begin{aligned} Q(s, a) &= r(s, a, s') + \gamma \left( \max_{a'} Q(s', a') \right) \\ Q(1, 5) &= R(1, 5) + 0.5 \max[Q(5, 1), Q(5, 5)] \\ &= 100 + 0.5 \times 0 = 100 \end{aligned}$$

- set $s \leftarrow s' \implies s = 5$, i.e., goal state, end

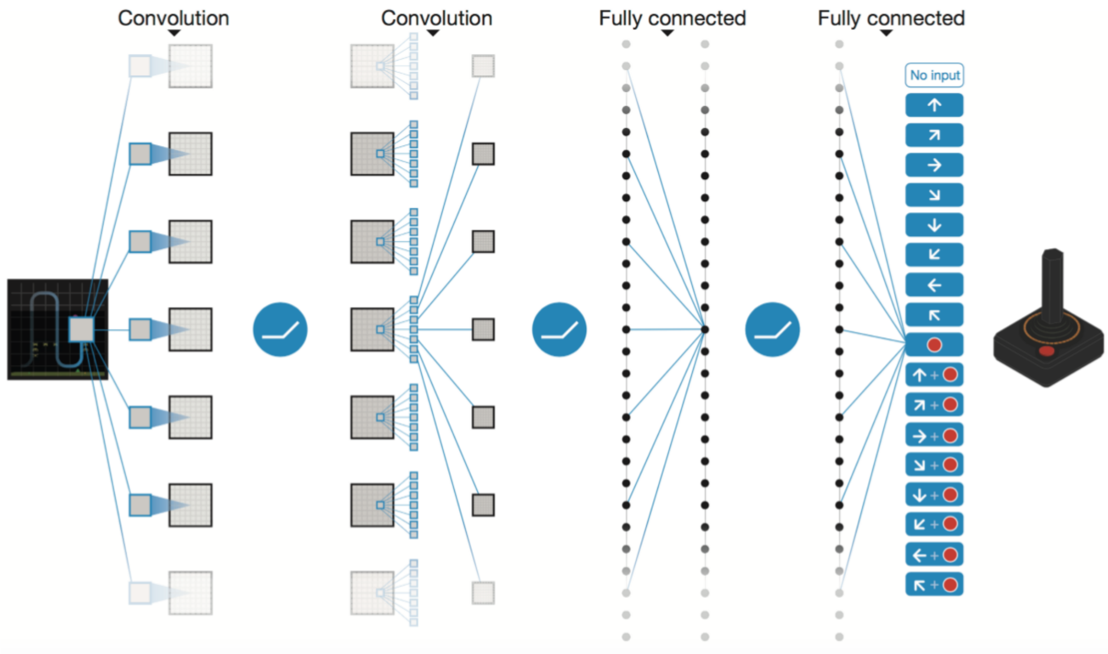
the state-action table gets updated until convergence.

## On the setting of Atari

- the states are far too many!
- need a **function approximator** to estimate the action-value function, $Q(s, a|\theta) \approx Q^*(s, a)$
- guess what? Deep Neural Network helps!

## Represent $Q(s, a)$ using neural networks

- The figure below represents a row of the Q function table earlier:




Conv [16] $\rightarrow$ ReLU $\rightarrow$ Conv [32] $\rightarrow$ ReLU $\rightarrow$ FC [256] $\rightarrow$ ReLU $\rightarrow$ FC [|A|]

- these are **not** softmax functions.

## abstracted algorithm for Deep Q-Learning

**Require:** Initialize an empty replay memory
**Require:** Initialize the DQN weights $\theta$

**for** each episode **do**
  **for** $t = 1, \dots T$ **do**
    with probability $\epsilon$ select $\tilde{a}$ random action
    otherwise, select:

$$\tilde{a} = \max_{a} \left( Q^{*}(s, a|\theta) \right)$$

    perform $\tilde{a}$ and receive rewards $r_t$ and state $s'$.
    add tuple $(s, \tilde{a}, r_t, s')$ into replay memory
    Sample a mini-batch of tuples $(s_j, a_j, r_j, s'_j)$ from the replay memory
    and perform stochastic gradient descent on the DQN, based on the loss function:

$$\Bigg( \underbrace{r_j + \gamma \big( \max_{a'} Q(s'_j, a'|\theta^{-}) \big)}_{y_j} - Q(s_j, a_j|\theta) \Bigg)^2$$

  **end for**
**end for**

innovation

- freeze parameters of target network $Q(s'_j, a'|\theta^{-})$ for fixed number of iterations
- while updating the online network $Q(s; a; \theta_i)$ by gradient descent

# double Deep Q-Leanring

- same values $\theta$ both to select and to evaluate an action:

$$\begin{aligned} y_j &= r_j + \gamma\big(\max_{a'} Q(s'_j, a'|\theta)\big) \\ &= r_j + \gamma\big(Q(s'_j, \arg\max_a Q(s'_j, a, \theta)|\theta)\big) \end{aligned}$$

- more likely to select overestimated values
- resulting in overoptimistic value estimates
- the solution is:

$$y_j = r_j + \gamma\big(\max_{a'} Q(s'_j, \arg\max_a Q(s'_j, a, \theta)|\theta')\big)$$

- still estimating value of policy according to current values defined by $\theta$
- use second set of weights $\theta'$ to **fairly** evaluate value of this policy

## In summary

- CNN and RNN are two of the building blocks in Deep Learning
- People have been putting them into many existing machine learning frameworks, and have generated many interesting stuff
- but there is plenty still needs to be explored!