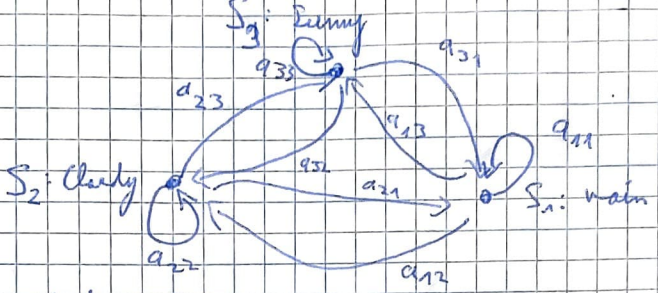
Lecture 6

Hidden Markov Models



States S_i → weather

transition probabilities: a_{xy} ⇒ $S_x \to S_y$

$$A = \{a_{ij}\} = \begin{pmatrix} 0,4 & 0,3 & 0,3 \\ 0,2 & 0,6 & 0,2 \\ 0,1 & 0,1 & 0,8 \end{pmatrix} \qquad a_{ij} \geq 0 \qquad \sum_{j=1}^{N} a_{ij} = 1$$

No memory: only the previous state counts!

$P[q_t = S_j \mid q_{t-1} = S_i]$ → probability that current state is S_j if previous state was S_i

q_t = current state at time t

$P[\text{sun}, \text{sun}, \text{sun}, \text{rain}, \text{rain}, \text{sun}, \text{cloudy}, \text{sun}]$

$= P[\text{observation} \mid \text{Model}] = P[S_3, S_3, S_3, S_1, S_1, S_3, S_2, S_3 \mid \text{Model}]$

$= P[S_3] \cdot P[S_3 \mid S_3] \cdot P[S_3 \mid S_3] \cdot P[S_1 \mid S_3] \cdot P[S_1 \mid S_1]$

$\cdot P[S_3 \mid S_1] \cdot P[S_2 \mid S_3] \cdot P[S_3 \mid S_2]$

$= \pi_3 \cdot a_{33} \cdot a_{33} \cdot a_{31} \cdot a_{11} \cdot a_{13} \cdot a_{32} \cdot a_{23} = 1,5 \cdot 10^{-4}$

Computation for $N = 3$ states and $T = 100$ observations

computations $= 2T \cdot N^T = 2 \cdot 100 \cdot 3^{100} \approx 10^{50}$

Click 4 Colors

Viterbi Algorithm:

only consider largest probabilities and discard the rest!