Exercise Sheet 8

1 One-Class SVM

a) $-\xi_i \geq 0$ if we set $\xi_i \geq 0$

$- \langle \phi(x_i), w \rangle \geq \rho - \xi_i$ can be achieved by setting ξ_i very large

b) $\mathcal{L}(w, \rho, \xi, \lambda_1, \lambda_2) = \frac{1}{2}\|w\|^2 - \rho + \frac{1}{N\nu}\sum_{i=1}^{N} \xi_i$

$+ \sum_i \lambda_{1_i} \underbrace{(\rho - \xi_i - \langle \phi(x_i), w \rangle)}_{\leq 0} + \sum_i \lambda_{2_i} \underbrace{(-\xi_i)}_{\leq 0}$

c) $\max_{\alpha \geq 0, \beta \geq 0} \min_{w, \rho, \xi} \mathcal{L}(\cdot)$ $\quad \lambda_1 \to \alpha, \lambda_2 \to \beta$

$\frac{\partial \mathcal{L}(\cdot)}{\partial w} = w - \sum_i \alpha_i \phi(x_i) \stackrel{!}{=} 0 \iff w = \sum_i \alpha_i \phi(x_i)$

$\frac{\partial \mathcal{L}(\cdot)}{\partial \rho} = -1 + \sum_i \alpha_i \phi(x_i) \stackrel{!}{=} 0 \iff \sum_i \alpha_i \stackrel{!}{=} 1$

$\frac{\partial \mathcal{L}(\cdot)}{\partial \xi_i} = \frac{1}{N\nu} - \alpha_i - \beta_i = 0$

$\max_{\alpha \geq 0, \cancel{\beta \geq 0}} \frac{1}{2}\|\sum_i \alpha_i \phi(x_i)\|^2 - \sum_i \alpha_i \langle \phi(x_i), \sum_j \alpha_j \phi(x_j) \rangle$

$= \max_{\alpha \geq 0} -\frac{1}{2}\sum_{ij} \alpha_i \alpha_j \underbrace{\phi(x_i)^\top \phi(x_j)}_{k(x_i, x_j)}$

$\sum_i \alpha_i = 1$, $\alpha_i \geq 0$, $\frac{1}{N\nu} - \alpha_i - \beta_i = 0 \iff \frac{1}{N\nu} \geq \alpha_i$

$\to 0 \leq \alpha_i \leq \frac{1}{N\nu}$

d) $\max_{\alpha} -\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j k(x_i, x_j) \rightarrow \min_{\alpha} \underbrace{\frac{1}{2} \sum_{ij} \alpha_i \alpha_j \underbrace{k(x_i, x_j)}_{K_{ij}}}_{\alpha^T K \alpha}$

$\sum_{i=1}^{N} \alpha_n = 1 \rightarrow 1^T \alpha = 1$

$0 \leq \alpha_i \leq \frac{1}{N\nu} \rightarrow \begin{pmatrix} -I \\ I \end{pmatrix} \alpha \leq \begin{pmatrix} 0 \\ \frac{1}{N\nu} \end{pmatrix}$

e) $\langle \phi(x), w \rangle < \langle \phi(x_{SV}), w \rangle$

$w = \sum_i \alpha_i \phi(x_i)$

$\langle \phi(x), \sum_i \alpha_i \phi(x_i) \rangle < \langle \phi(x_{SV}), \sum_i \alpha_i \phi(x_i) \rangle$

$= \sum_i \alpha \underbrace{\langle \phi(x), \phi(x_i) \rangle}_{k(x, x_i)} < \sum_i \alpha_i \underbrace{\langle \phi(x_{SV}), \phi(x_i) \rangle}_{k(x_{SV}, x_i)}$

$= \sum_i \alpha_i k(x, x_i) < \sum_i \alpha_i k(x_{SV}, x_i)$