School of Mathematical and Computational Sciences

MCS4950

Exploring Dynamic Interactions and Predictive Modeling: Time Series Analysis of FedEx Stocks

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List of abbreviations

TSA	Time Series Analysis
MLE	Maximum Likelihood Estimation
MAE	Mean Absolute Error
RMSE	Root Mean Squared Error
ACF	Autocorrelation Function
PACF	Sample Partial Autocorrelation Function
AR	Autoregressive
MA	Moving Average
ARIMA	Autoregressive Integrated Moving Average
ADF	Augmented Dickey-Fuller
ECM	Error Correction Model
VAR	Vector Autoregressive Models
AIC	Akaike Information Criterion
BIC	Bayesian Information Criterion
HQIC	Hannan-Quinn Information Criterion
VAR	Vector Autoregressive Models
ARMA	Autoregressive Moving Average
RVC	Residual Variance Criterion
FPE	The Final Prediction Error Criterion
ARCH	Autoregressive Conditional Heteroscedasticity
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
GDPR	General Data Protection Regulation
IRFs	Impulse Response Functions
ARFIMA	Autoregressive Fractionally Integrated Moving Average
VECM	Vector Error Correction Model
DW	Durbin-Watson
MMSE	Minimum Mean Squared Error
MAPE	Mean Absolute Percentage Error
MSE	Mean Squared Error

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Abstract

Stock price forecasting is a very challenging task because of the incorporation of dynamic and volatile financial. The time series analysis of FedEx stock data presented herein attempts to explore underlying patterns, forecast future trends, and, most importantly, investigate the effectiveness of different statistical models to forecast stock price movements. The data of the FEDEX dataset includes Price, Open, High, Low, Volume, Change, YearMonth, Dividend, and Dividend Yield. Initially, the data was preprocessed with an average over three-year quarters and the variables have been normalized.

The analysis began with the construction of the time series for the two prices and dividends. The next stage involved using the tso() function for outlier tests, which agreed with the absence of outliers and indicated that at around the year 2013, there is a rising trend. The time series was further decomposed to assess trend and seasonality, and the finding was a clear consistent upward trend with the presence of quarterly seasonal patterns.

In the augmented Dickey-Fuller (ADF) test, we have found both the price and dividend series to be nonstationary and to require differences, such as first differences, to make them stationary. However, further testing did show that a change of series did, in fact, permit stabilization. This exercise in modeling, therefore, entailed fitting ARIMA and ARFIMA models on both the raw and transformed data. In their optimal stated form, the ARIMA (1,1,0) model was most optimal for Price, suggesting a first-order autoregressive process with a significant influence of past prices. Dividend data best fitted an ARIMA (0,1,1) model, which shows a moving average process that captures fluctuations well in the short run.

The multivariate time-series analysis, by using the Vector Autoregression (VAR), pointed out the dynamic interrelationship between the transformed variables "Change" and "ΔDividend". Granger Causality tests also reinforced unidirectional causality from ΔDividend to Change. Dividends stand as a very good predictor of stock price movement at a statistically significant p-value of 0.023. Further, the impulse response analysis detailed the effects of shocks to dividends on stock price changes. It shows that after any shock in the ΔDividend, stock prices fall immediately by about 140% in the following quarter, rise subsequently by about 160%, and thereafter stabilize

after five quarters. The forecasting performance check was carried out through different accuracy statistics. The ARFIMA model indicates better forecasting performance in stock prices and dividends than the ARIMA model, suggesting this could be a viable alternative in the handling of quite complex and nonlinear time series data patterns.

In that respect, the thesis outlined hereafter is sufficiently vivid about the complex dynamics and predictive properties evinced within the FedEx stock data through a rigorous time-series analysis and modeling exercise. It further informs effective economic forecasting and strategic financial planning. This strong effect of dividends on stock price places huge importance on dividends as part of financial policies and the reaction of investors with a view to the larger economic effect.

Motivation

The choice of FedEx Corporation as the object of graduation project research was motivated by a combination of factors that point to the relevance and importance of this study amid the current economic landscape. FedEx is an internationally recognized leading logistics and delivery service that operates in a fast-changing environment characterized by unpredictable fuel prices, varying international regulations, and changing economic conditions. All these factors make FedEx's stock an object of interest in examining how the macroeconomic indicators and firm-specific variables affect stock performance.

First, it forms the core around which global trade and commerce revolve; therefore, it qualifies as a barometer that reflects the health conditions of the world economy. The choice of FedEx, a bellwether of this sector, is deliberate to help us understand from an elaborate perspective the general economic environment, the specific logistics, and operational factors impacting the stock market.

The second point is that the stock market is a highly complex system in its own right and is influenced by many reasons, from geopolitical events to economic policy changes. This richness in complexity makes it an ideal playing ground to apply advanced statistical techniques, such as multivariate time series analysis, where they can be disentangled and quantified. Since FedEx is such a sensitive indicator of economic changes and has such standing, it offers an opportunity to apply and exhibit the efficiency of these highly developed econometric models in real-time.

More so, there is a growing need to investigate how traditional financial theories and models hold against large and volatile datasets with the advances in data analytics and computational technologies. This project aims to bridge theoretical financial models with practical, data-driven insights to offer predictive capabilities more aligned with current market realities.

The academic challenge and its practical implication of forecasting stock prices using multivariate time series models make for a very sound case for selecting this topic. These are opportunities to contribute to academic knowledge and gather valuable skills and insights that can be availed of in the practice of financial analysis and economic forecasting.

Chapter 1
Introduction

Time Series Analysis (TSA) is a methodology used to model time series data for forecasting future observations. TSA involves four main steps: Identification, estimation, diagnostic checks, and forecasting. Order Identification is when a suitable model is selected to describe the behavior of the time series, which involves analyzing the properties of the data to determine if it exhibits trends, seasonality, autocorrelation, or other patterns. An appropriate model, such as ARMA, ARIMA, or exponential smoothing, is chosen based on the TSA analysis. Once a model is identified, its parameters are estimated using a suitable technique such as maximum likelihood estimation (MLE), method of moments, or least squares. The estimation aims to find the parameter values best fitting the chosen model to the observed data.

After parameter estimation, diagnostic checks are performed to assess the adequacy of the chosen model by examining residuals for patterns or autocorrelation, testing for stationarity or model stability, and evaluating the goodness-of-fit measures. Diagnostic checks help ensure that the chosen model adequately captures the underlying structure of the data. Once the model is deemed adequate, it can predict future time series observations. Forecast accuracy can be evaluated using metrics such as mean absolute error (MAE), root mean squared error (RMSE), or forecast error variance decomposition. By following these four steps, analysts can build robust models to capture the dynamics of time series data and make accurate forecasts for future observations. In time series analysis, univariate approaches are discussed in the following sections (1.1).

1.1 Univariate Time Series Analysis

A single time series variable is analyzed using the univariate approach, whereas the multivariate approach considers the interactions between several time series variables. Within the univariate approach, the Box-Jenkins (BJ) methodology is one of the most widely used strategies. The sample autocorrelation function (ACF) and sample partial autocorrelation function (PACF) are used in this methodology to ascertain the model's order. The procedure entails contrasting the theoretical patterns of the actual ACF and PACF plots with those of the models, including the autoregressive (AR), moving average (MA), and autoregressive integrated moving average

(ARIMA) models. Analysts can ascertain the proper model order by locating notable lags in these graphs.

Acknowledging that the Box-Jenkins methodology necessitates judgment, expertise, and cautious interpretation of the ACF and PACF plots is imperative. If the process isn't carried out properly, it could occasionally result in the selection of an incorrect model. To guarantee the resilience of the selected model, diagnostic tests and model validation approaches need to be added to the BJ methodology despite its widespread use and robustness.

A method of identification for time series models was presented by Choi (1992). The method bears a resemblance to regression analysis's backward elimination procedure. The ARMA models' orders p and q are known maximums but unknown constants. Next, they used a multivariate t-distribution to approximate the coefficients' distribution and a series of t-tests to determine their significance. The model has the unimportant coefficients eliminated. For example, if we consider the ARMA (p, q) process with maximum orders ARMA (2,2), then by using the decision tree technique as shown in Figure (1), test that $\theta_2 = 0$; if rejected, test $\phi_2 = 0$. If this hypothesis is not rejected, then the test $\theta_1 = 0|\theta_2 = 0$, which, if rejected, we can conclude that the process is ARMA (1, 2). Figure (1.1) shows the decision tree that contains nine paths by which a particular ARMA process is selected, assuming maximum orders (2, 2).

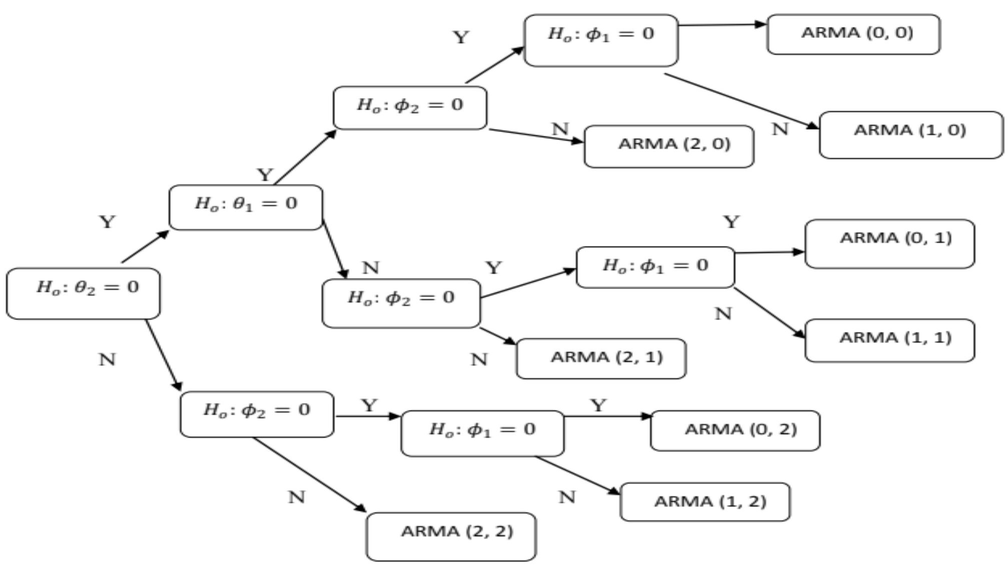


Figure 1: Decision Tree for ARMA Order Identification

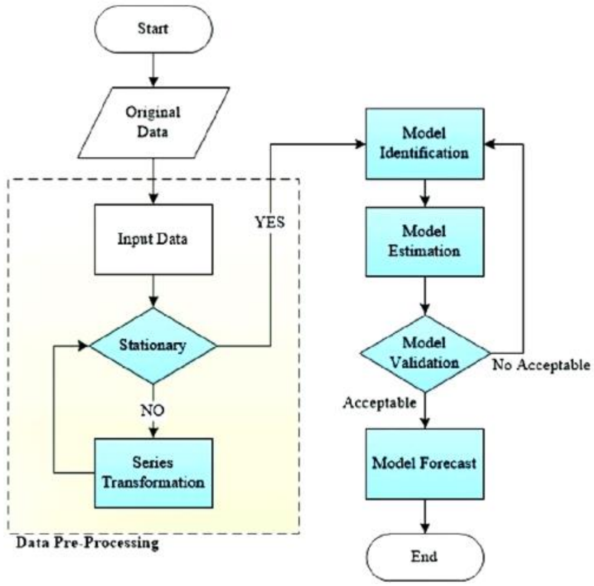


Figure 2: Steps for the Univariate TSA

Source: Barzola-Monteses, J. et al. (2019)

While the stationarity of univariate time series is uncommon in practice, most time series models assume stationary data. Therefore, differencing means subtracting values of successive observations from each other, which is the main idea in most cases to achieve stationarity. The differencing removed the trends or other forms of nonstationarity present in the data. The order of differencing is called Integration (d), the number of times the differencing operation has been performed to achieve stationarity. For example, if the original time series has a linear trend, order one $(d = 1)$ can be differentiated to eliminate the trend (Box and Jenkins, (1970)).

Further differencing is required if the resultant series still gives nonstationarity. However, differencing must be done carefully since too much differencing may result in over-differencing, which would otherwise introduce unwanted noise into the series. Therefore, a balance should be adhered to when removing the nonstationarity, saving valuable information in the data. If stationarity is not achieved after the third difference, another time series model is used (Enders, (2008)).

Further, the autoregressive fractionally integrated moving average (ARFIMA) model is a generalization of the ARIMA model, taking the form of fractional differencing. Unlike in the traditional integer order, fractional differencing $-\frac{1}{2} < d < \frac{1}{2}$ allows adjustment of the differencing parameter to its fractional values, hence making the model applicable in capturing long-range dependencies and persistent memory within the data (Hosking, 1981; Granger and Joyeux, 1980).

From a practical perspective, the ARIMA or ARFIMA model selection is determined by the nature of the time series and the desirable flexibility in capturing temporal dynamics. Where ARIMA models represent short-range and abrupt changes, ARFIMA models can represent long-range and persistent patterns. Once the time series model has been fitted, a set of diagnostic tests becomes necessary to check the adequacy of the model in hand and further verify the various assumptions made in the process. These diagnostics identify any pattern left in the model residuals or any inadequacies that may illuminate the misspecification. Some of the standard diagnostic tests carried out post-fitting a time series model include the examination of residuals (i.e., differences of observed values from those predicted by the model), which are a critical part of the diagnosis process (Enders, (2008)).

The ACF and PACF plots of the residuals help to evaluate if the residuals have any autocorrelation. That means whether the model has been able to capture all temporal dependencies present in the data. Hence, if the residuals have a significant autocorrelation, it implies that more or higher-order terms are needed in the model. Another assumption is to check for the normality of the errors or residuals. The normality of the residuals may be tested using the Jarque-Bera test. Hence, a departure from normality may indicate some model misspecification (Brockwell and Davis, 2016).

Another assumption that needs to be checked is homoscedasticity, where the variance of residuals against time stays constant. A plot of the residuals against the fitted values or time can be used to assess if the residuals' variance stays relatively constant. Formal tests like the Breusch-Pagan and White tests point out that, statistically, specification can be regarded as homoscedastic or heteroskedastic errors (Breusch and Pagan (1980)).

Techniques like residual plots can be used to identify outliers. These diagnostic tests further help an analyst decide whether the time-series model fit is adequate statistically. If it is not adequate, then it further emphasizes the areas of improving or refining it. It is important to stress that no model is perfect, and diagnostic testing would only justify that a model is appropriate for the intended analysis.

1.2 Objectives of the Study

This research investigates how sensitive the time series models are for capturing the changes within the data and whether they can be used to develop future value. The questions we want to answer are,

1. Do time series models improve forecasting performance?
2. Is there a difference between univariate and multivariate time series models?

The following steps are taken:

1. The time series models are developed theoretically on an experimental basis of univariate techniques. They form the base for the analysis of data and forecasts.
2. Various statistical tests are applied to assess the characteristics of time series data. Such tests include unit root stationarity and diagnostic tests. They offer further insights into the characteristics and behavior of the data, which are essential for setting up modeling standards.
3. This study uses a comprehensive real-life data analysis to justify the effectiveness of univariate and multivariate techniques for filtering proper time series model specifications.
4. The forecasts for future time series values are obtained using the best-fitted model identified through the analysis. This step analyzes the chosen model's predictive accuracy to forecast future observations.

The significance of this study lies in its comprehensive analysis of FedEx stock prices using advanced time series models to understand and predict price movements. By employing ARIMA, ARFIMA, and VAR models, the research uncovers critical patterns and relationships between stock prices and dividends, highlighting the dynamic interactions and volatility inherent in financial data. The findings demonstrate the ARFIMA model's superior performance in forecasting complex and nonlinear time series, emphasizing its potential for more accurate long-term predictions. Additionally, the study underscores the pivotal role of dividends in influencing stock prices, which has important implications for financial policy and strategic planning. This research not only advances the methodology in financial modeling but also provides practical insights for investors and policymakers to make informed decisions and develop robust economic strategies.

1.3 Structure of the Study

The following chapters of the study are designed: Chapter 2 reviews univariate time series models, Chapter 3 is all about statistical diagnostic tests, and Chapter 4 presents the results and discussion of the real dataset study for both techniques. Finally, Chapter 5 includes the study summary, forecasting ahead, and study conclusions.

Chapter 2

Theoretical Background

This chapter illustrates the class of time series models, encompassing the mixed autoregressive moving average (ARMA) process, the pure autoregressive (AR) process, and the moving average (MA) process, which can be considered special cases.

2.1 Literature Review

This section aims to review the literature on time series analysis briefly. Time series analysis is a fundamental component of data analytics, and its application is found in many fields, such as economics, finance, and engineering. This literature review delves into the evolution of time series analysis techniques, tracing the journey from traditional methodologies to modern advancements. The Box-Jenkins Methodology, introduced by Box and Jenkins in 1970, is a cornerstone in univariate time series analysis. This methodology encompasses four phases: identification, estimation, diagnostic check, and prediction. However, the manual identification phase demands expertise and may result in suboptimal model selection (Box & Jenkins, 1970).

Automatic identification techniques have gained prominence in overcoming the limitations of manual identification. Whittle (1963) introduced the residual variance criterion (RVC), an automated technique for model identification. This criterion aimed to minimize residual variance, providing a quantitative measure for model selection. Building upon Whittle's work, Akaike (1970) proposed the final prediction error criterion (FPE) as an alternative method for determining model order. The FPE criterion focused on minimizing the prediction error, offering a different perspective on model selection. Akaike (1973) refined the model selection process by introducing the Akaike information criterion (AIC). The AIC balanced model fit and complexity, providing a robust framework for automated model selection. Subsequently, Akaike (1980) introduced the Bayesian information criterion (BIC) as a Bayesian extension of the AIC. The BIC incorporated a penalty term for model complexity, enhancing selection efficiency and addressing potential overfitting issues.

As for the TS models, Autoregressive Integrated Moving Average (ARIMA) models, popularized by Box and Jenkins (1976), capture temporal dependencies and stochastic trends.

ARIMA models provide a versatile time series forecasting framework, addressing stationarity and seasonality. A fundamental aspect of time series analysis is assessing stationarity, which refers to the constancy of statistical properties over time. Stationary time series are more straightforward to model and forecast due to their consistent behavior. However, many real-world time series exhibit nonstationary behavior, necessitating meticulous analysis and preprocessing. Unit root tests, such as the Augmented Dickey-Fuller (ADF) Test introduced by Dickey and Fuller (1979), are commonly employed to assess stationarity. These tests examine whether a time series possesses a unit root, indicating nonstationarity. Researchers can determine the appropriate differencing order needed to achieve stationarity and ensure reliable modeling by conducting unit root tests.

Granger (1980), Granger and Joyeux (1980), and Hosking (1981) introduced fractional differencing processes to model long-term dependencies observed in many empirical time series. The ARFIMA model extends traditional ARIMA models to capture long-term persistence, offering a flexible framework for analyzing complex data patterns. Parameter estimation in ARFIMA models involves maximum likelihood estimation and modified Box-Jenkins procedures to accommodate fractional differencing parameters.

In the applications, time series analysis plays a crucial role in economic and financial forecasting, aiding in predicting stock prices, interest rates, and macroeconomic indicators. Techniques like ARIMA and ARFIMA models were extensively used by Makridakis et al., (1998) to analyze economic time series data and inform policy decisions. Taylor (2003) utilized the time series models to predict electricity consumption patterns, optimize generation, and plan infrastructure upgrades to meet future demand. Reisen et al. (2010) explored the use of ARFIMA models in financial time series, demonstrating their superiority in capturing long-term dependencies compared to traditional ARIMA models. Their study aimed to evaluate the performance of ARFIMA models in forecasting stock prices and found that these models provided more accurate long-term forecasts, making them particularly useful for strategic investment planning. Stock and Watson (2012) provided a comprehensive overview of VAR models and their applications in macroeconomic forecasting. Their study aimed to illustrate the effectiveness of VAR models in capturing the interactions among macroeconomic variables and stock prices. The findings showed that VAR models are particularly useful for understanding the dynamic relationships between economic indicators and stock market movements.

Baillie et al. (2014) further investigated the application of ARFIMA models to stock price data. Their objective was to compare the forecasting performance of ARFIMA models with other time series models. The findings indicated that ARFIMA models offered significant improvements in forecasting accuracy for stock prices with long memory characteristics. Patel et al. (2015) aimed to assess the predictive performance of ARIMA models on stock prices of multiple companies, including their volatility and trend characteristics. Their findings highlighted that ARIMA models are robust for short-term forecasting, though they might require regular recalibration to maintain accuracy. Kutu and Ngalawa (2016) applied VAR models to investigate the impact of macroeconomic variables on stock prices in emerging markets. The objective was to analyze how changes in economic indicators influenced stock prices over time. The results indicated that VAR models effectively captured dynamic interactions and provided valuable insights for policymakers and investors.

Wu et al. (2018) applied VAR models to examine the impact of macroeconomic indicators on FedEx stock prices. Their objective was to capture the dynamic relationships between economic variables and stock prices, providing a comprehensive view of the factors influencing FedEx stock performance. The findings indicated that VAR models effectively captured these relationships, offering valuable insights for investors and policymakers. Xu and Lee (2021) evaluated ARFIMA models for forecasting stock prices of technology companies, finding that the model’s ability to handle long memory processes resulted in improved long-term forecasts. The objective was to see if these findings extended to the logistics sector, including companies like FedEx, with promising preliminary results. Zakamulin and Giner (2021) investigated the predictive power of ARIMA models on individual stock returns, including FedEx. Their study aimed to evaluate the robustness of ARIMA models in the presence of structural breaks and non-stationary data. The findings supported the utility of ARIMA models, particularly when combined with techniques to address structural breaks, in enhancing forecast accuracy.

Wang et al. (2022) evaluated the performance of ARIMA models in forecasting the stock prices of logistics companies, including FedEx. The objective was to determine the efficacy of ARIMA models in capturing short-term price movements. The findings confirmed that ARIMA models provided accurate short-term forecasts, making them suitable for tactical investment strategies. Huang et al. (2023) employed VAR models to examine the impact of global economic indicators on the stock prices of major logistics companies, including FedEx. Their objective was to understand the influence of international trade volumes, fuel prices, and other macroeconomic factors on stock performance. The findings confirmed that VAR models are effective in capturing these relationships and providing actionable insights for investment strategies. Integrating machine learning techniques with traditional time series analysis offers new avenues for improving forecast accuracy and handling complex data patterns. Spiliotis, E. (2023) applied hybrid models combining deep learning with ARIMA or tree-based techniques, which have shown promise in capturing nonlinear relationships and improving prediction performance.

Despite extensive research on time series analysis and its applications in economic and financial forecasting, significant gaps remain in the literature, particularly concerning the dynamic interactions and predictive modeling of FedEx stock prices. Previous studies have predominantly focused on ARIMA and ARFIMA models for capturing long-term dependencies and forecasting stock prices (Reisen et al., 2010; Baillie et al., 2014). However, many of these studies are either dated or focus on broader economic indicators and other sectors rather than logistics companies specifically. While ARIMA models have been proven robust for short-term forecasts (Patel et al., 2015; Wang et al., 2022), there is a lack of comprehensive evaluation of their long-term forecasting capabilities in the logistics sector. Additionally, the application of Vector Autoregression (VAR) models has shown promise in capturing the dynamic relationships among macroeconomic variables and stock prices (Stock & Watson, 2012; Kutu & Ngalawa, 2016; Huang et al., 2023), but specific investigations into the interactions between FedEx's stock prices and dividends using VAR models are sparse. Furthermore, there is limited research on the comparative performance of ARFIMA and VAR models in predicting FedEx stock prices, particularly in the context of structural breaks and non-stationary data (Zakamulin & Giner, 2021). This study aims to fill these gaps by providing a detailed analysis of FedEx stock prices using advanced time series models, thereby offering new insights into the predictive power and dynamic interactions within this sector.

2.2 Time Series Decomposition and Basic Definitions

A time series is a sequence of data points or observations collected, recorded, or measured at successive and evenly spaced intervals. Mathematically, a time series y_t can be represented as:

$$y_t \, , t = 1,2, \dots , T$$

Where:

y_t is the value of the time series at time t.

t is the time index or timestamp.

T is the total number of observations in the time series.

There are various types of time series data, each characterized by different properties and patterns. Some common types of time series include continuous-time series, which consists of observations recorded continuously over time, typically at regular intervals. Examples include hourly temperature readings, stock prices measured every minute, or sensor data collected in real time. On the other hand, discrete-time series consists of observations recorded at discrete or distinct points starting from daily rather than continuously. The time intervals between observations may be regular or irregular. Examples include daily sales figures, monthly unemployment, or annual GDP growth rates.

Time series data typically consist of various components that contribute to the overall pattern of the series. These components include trend T_t, seasonality S_t, cyclic patterns C_t, and irregular fluctuations I_t. The trend component T_t of a time series represents the long-term pattern or direction of the data over time. It can be characterized by an increasing, decreasing, or stable pattern. Trends may exhibit linear or nonlinear behavior and can be modeled using regression techniques or smoothing methods. Seasonality S_t refers to periodic fluctuations or patterns in the data that occur at fixed intervals, such as daily, weekly, or monthly cycles. Seasonal patterns often repeat over a specific time horizon and are influenced by external factors like weather, holidays, or economic cycles.

Cyclic patterns C_t in time series data represent fluctuations over a longer time horizon than seasonal patterns and are not tied to fixed intervals. Cycles may exhibit irregular lengths and amplitudes and are often influenced by economic, political, or societal factors. Unlike seasonality, cyclic patterns do not have a fixed period and can be challenging to model accurately. The irregular component I_t of a time series, also known as noise or residuals, captures random fluctuations or variability in the data that the trend, seasonality, or cyclic patterns cannot explain. Irregular components represent random shocks, measurement errors, or unexplained variations in the data (Chatfield, 2003).

In time series analysis, the observed data can be decomposed into individual components using additive or multiplicative models. In the additive model, the observed time series is considered the sum of its components, and the components are independent: trend, seasonality, cyclic, and irregular, it can be expressed as: $y_t = T_t + S_t + C_t + I_t$. In the multiplicative model, the observed time series is considered the product of its components by default, and the components are dependent. It is suitable when the magnitude of seasonal fluctuations varies with the trend level, and it can be expressed as: $y_t = T_t \times S_t \times C_t \times \ I_t$.

Definition 1: The backshift operator, denoted by B, is a fundamental concept in time series analysis that represents the lagging of a time series by one period. Specifically, the backshift operator B applied to a time series y_t results in the value of the series at the previous time period, y_{t-1} and can be defined as (Box & Jenkins, (1970)):

$$By_t = y_{t-1}$$

Definition 2: The difference operator is used to compute the difference between consecutive time series values. It is denoted by the symbol $\mathit{\Delta}$ (delta) and is calculated by subtracting the previous value from the current value. It can be represented as (Box & Jenkins, (1970)):

$$\mathit{\Delta} y_t = y_t - y_{t-1}$$

The second difference:

$$\mathit{\Delta}^2 y_t = \mathit{\Delta} y_t - \mathit{\Delta} y_{t-1} = y_t - 2y_{t-1} + y_{t-2}$$

The difference operator is commonly used to transform nonstationary time series into stationary ones by removing trends.

Definition 3: The lagged difference, also known as the differenced lag operator, combines the concepts of lagging and differencing. It represents the difference between the current value of a time series and its value at a specified lag. The lagged difference operator $\mathit{\Delta}_k$ applied to a time series y_t at lag k is given by:

$$\mathit{\Delta}_k y_t = y_t - y_{t-k}$$

The lagged difference operator is frequently used in time series analysis to account for the temporal dependencies between observations usually used to get the deseasonalized series (Box & Jenkins, (1970)).

Definition 4: Stationarity is a fundamental concept in time series analysis that refers to the statistical properties of a time series remaining constant over time. A stationary time series with mean, variance, and autocovariance structure does not change. Formally, a time series y_t is said to be weakly stationary if its mean μ and autocovariance $\gamma(h)$ are constant for all time points t and all lags h :

Mean: $\mu = E(y_t)$

Autocovariance: $\gamma(h) = E[(y_t - \mu)(y_{t+h} - \mu)]$

Mathematically, an autoregressive (AR) model $y_t = \phi_1 y_{t-1} + \epsilon_t$ is stationary if the coefficient ϕ_1 is less than 1 in absolute value (Hamilton, (1994)):

$$|\phi_1| < 1$$

A stationary time series exhibits stable and predictable behavior, making it easier to model and analyze than nonstationary series (Box & Jenkins, (1970)).

Definition 5: Invertibility is a property of a time series model that ensures the past values of the series can be expressed as a finite linear combination of its past forecast errors. In other words, an invertible model implies that the current value of the series depends only on its past forecast errors and not on past values of the series itself. A moving average (MA) model $y_t = \epsilon_t + \theta_1 \varepsilon_{t-1}$ is invertible if the coefficient θ_1 is less than 1 in absolute value (Hamilton, 1994):

$$|\theta_1| < 1$$

Invertibility is essential for ensuring the stability and convergence of estimation procedures in time series modeling (Brockwell & Davis, (2016)).

Definition 6: Autocorrelation, or serial correlation, measures the linear relationship between consecutive observations in a time series. It quantifies the degree to which a time series is correlated with its lagged values. The autocorrelation function (ACF) at lag h, denoted by $\rho(h)$, is defined as the correlation between the series y_t and its lagged counterpart y_{t-h} (Brockwell & Davis, (2016)):

$$\rho(h) = \frac{\gamma(h)}{\gamma(0)}$$

Where $\gamma(h)$ is the autocovariance function, and $\gamma(0)$ is the variance of the series. The ACF pattern indicates stationarity if it decays slowly.

Definition 7: The partial autocorrelation function (PACF) measures the direct correlation between two observations in a time series after accounting for the correlations with the intermediate lags. It helps identify the significant lags in a time series that contribute to its autocorrelation structure. Formally, the PACF at lag h, denoted by $\phi(h)$, is defined as the correlation between y_t and y_{t-h} after removing the effects of the intermediate lags (Hamilton, (1994)):

$$\phi(h) = \text{cor}(y_t - \hat{y}_t, y_{t-h} - \hat{y}_{t-h})$$

Where $\hat{y}_t$ and $\hat{y}_{t-h}$ are the predicted values of y_t and y_{t-h}, respectively.

2.3 Time Series Models

Box and Jenkins (1970) have shown that the autoregressive moving average models adequately represent time series data. In this section, the ARMA models and their basic properties are presented.

2.3.1 ARMA Models

The ARMA (p, q) model, as defined by Box and Jenkins (1970), is a fundamental time series model used for modeling stationary processes. It is expressed as:

$$\Phi(B)y_t = \Theta(B)\varepsilon_t$$

Where:

- $\Phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p$ represents the autoregressive (AR) polynomial.
- $\Theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q$ represents the moving average (MA) polynomial.
- B is the backshift operator such that $B^j y_t = y_{t-j}$, where y_t is the time series being studied.
- ε_t are the random errors assumed to be independently and identically distributed (i.i.d.) normally with mean zero and precision parameter τ^{-1}, where $\tau = \frac{1}{\sigma^2} > 0$ and σ^2 is the variance.

- ϕ_i are the coefficients of the AR part, and θ_i are the coefficients of the MA part.

In this model, the AR part captures the linear dependence of the current observation on its past values, while the MA part captures the linear dependence of the current observation on past error terms. The model parameters p and q determined the orders of the AR and MA components, respectively. The pattern of the ACF and PACF are both cut off (insignificant) after lags q, and p respectively. ARMA models are widely used in time series analysis for forecasting and modeling various phenomena, assuming the underlying process is stationary. They provide a flexible framework for capturing complex temporal dependencies in the data.

2.2.2 AR Models

The autoregressive (AR) model is a fundamental time series model defined by Box and Jenkins (1970) that represents the current value of a variable as a linear combination of its past values with added noise. It is commonly denoted as $AR(p)$, where p represents the order of the autoregressive process. The AR(p) model can be expressed as:

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t$$

Where:

- y_t is the value of the variable at time t.
- $\phi_1, \phi_2, \ldots, \phi_p$ are the autoregressive parameters, representing the coefficients of the lagged values.
- ε_t is a white noise error term at time t, assumed to be independently and identically distributed (i.i.d.) with mean zero and constant variance.

In essence, the $AR(p)$ model captures the linear dependence of the current value of the variable on its past p values, weighted by the autoregressive coefficients $\phi_1, \phi_2, \ldots, \phi_p$. The order p determines the number of lagged terms considered in the model. The pattern of the ACF and PACF decay exponentially (or sign waves) and cut off (insignificant) after lags p, respectively. AR models are widely used in time series analysis for modeling processes exhibiting temporal dependencies, such as stock prices, temperature fluctuations, and economic indicators. They provide a flexible framework for understanding and forecasting time series data.

2.3.3 MA Models

The moving average (MA) model is a fundamental time series model that represents the current value of a variable as a linear combination of its past white noise error terms. It is commonly denoted as MA(q), where q represents the order of the moving average process.

The MA(q) model can be expressed as:

$$y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q}$$

Where:

- y_t is the value of the variable at time t.
- $\varepsilon_t, \varepsilon_{t-1}, \dots, \varepsilon_{t-q}$ are the white noise error terms at times $t, t-1, \dots, t-q$, respectively.
- $\theta_1, \theta_2, \dots, \theta_q$ are the moving average parameters representing the coefficients of the lagged error terms.

In essence, the MA(q) model captures the linear dependence of the current value of the variable on its past q white noise error terms, weighted by the moving average coefficients $\theta_1, \theta_2, \dots, \theta_q$. The order q determines the number of lagged error terms considered in the model. The pattern of the ACF and PACF cut off (insignificant) after lags q, and decay exponentially (or sign waves), respectively. MA models are widely used in time series analysis for modeling processes with short-term memory or dependence on past shocks, such as financial market volatility or demand forecasting. They provide a flexible framework for understanding and forecasting time series data.

2.3.4 ARIMA Model

The autoregressive integrated moving average (ARIMA) model is a widely used time series model that incorporates both autoregressive (AR) and moving average (MA) components, along with differencing to handle nonstationarity. It is denoted as ARIMA (p, d, q), where p represents the order of the autoregressive component, d represents the degree of differencing, and q represents the order of the moving average component (Box & Jenkins, 1976).

The ARIMA (p, d, q) model can be expressed as:

$$\Phi(B)(1-B)^d y_t = \Theta(B)\varepsilon_t$$

Where:

- $\Phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p$ represents the autoregressive polynomial.
- $\Theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q$ represents the moving average polynomial.
- $(1-B)^d y_t$ represents the differencing operation applied d times to make the series stationary.
- B is the backshift operator.
- y_t is the value of the time series at time t.
- ε_t is a white noise error term at time t, assumed to be independently and identically distributed (i.i.d.) with mean zero and constant variance.

The ARIMA model is beneficial for modeling time series data that exhibit nonstationarity, trend, or seasonality. The parameter d indicates the number of differences needed to achieve stationarity, while p and q determine the autoregressive and moving average orders, respectively. The pattern of the ACF and PACF decay exponentially (or sign waves), respectively. ARIMA models are widely applied in various fields, including economics, finance, and epidemiology, for forecasting and analyzing time series data.

2.3.5 ARFIMA Model

The autoregressive fractionally integrated moving average (ARFIMA) model is a time series model that incorporates both autoregressive (AR), moving average (MA), and fractional integration components to capture long memory dependence and nonstationarity in the data. It is denoted as ARFIMA(p,d,q), where p represents the order of the autoregressive component, d represents the differencing parameter, and q represents the order of the moving average component. The ARFIMA(p,d,q) model can be expressed as:

$$\left(1 - \sum_{i=1}^{p} \phi_i B^i\right)(1-B)^d y_t = \left(1 + \sum_{j=1}^{q} \theta_j B^j\right)\varepsilon_t$$

Where:

- y_t is the value of the time series at time t.
- B is the backshift operator such that $B^j y_t = y_{t-j}$.
- ϕ_i and θ_j are the autoregressive and moving average parameters, respectively.
- d is the differencing parameter, which determines the degree of differencing required to achieve stationarity.
- ε_t is a white noise error term at time t, assumed to be independently and identically distributed (i.i.d.) with mean zero and constant variance.

The ARFIMA model extends the ARIMA model by allowing for fractional differencing, which helps model time series with long memory dependence. The parameter d represents the degree of differencing required to achieve stationarity. When d is nonzero, the model captures the long-term dependencies in the data. ARFIMA models are widely used in finance, economics, and climatology. They provide a flexible framework for modeling complex time series dynamics and making accurate forecasts.

Chapter 3

Statistical Tests

This chapter comprises important information about time-series analysis and diagnostic testing to understand the behavior and characteristics of the time-series data. Seasonality tests have been introduced as essential tools of the identification process to state the repetitive variations that occur at time intervals. Knowledge of seasonality is fundamental to sound forecasting and decision-making. Unit root stationarity test, one of the critical aspects of time series analysis, is discussed. This test draws inferences about whether a unit root exists in a time series, indicating nonstationarity and a stochastic trend. Stationarity is checked, and suitable modeling techniques can be adopted, ensuring robustness in statistical inferences. The ARFIMA test is applied to check for long memory dependencies. Finally, diagnostic tests are addressed. Diagnostic tests give fundamental information on how adequate or unreliable any series model is.

3.1 Pre-Modelling Tests

3.1.1 Seasonality Test

The Kruskal-Wallis test is a non-parametric statistical test used to determine whether there are statistically significant differences between the medians of two or more groups. While it is commonly used for comparing independent samples, it can also be adapted for testing seasonality in time series data, particularly when the assumption of normality is violated or when the data exhibit nonlinear patterns. The test statistic H for the Kruskal-Wallis test is calculated as:

$$H = \frac{12}{N(N+1)} \sum_{j=1}^{m} \frac{R_j^2}{n_j} - 3(N+1)$$

where N is the total number of observations, R_j is the sum of the ranks of the j-th season and n_j is the number of observations in the j-th season. To adapt the Kruskal-Wallis test for seasonality testing, the time series data is first divided into m groups corresponding to each season (e.g., months, quarters). Then, the Kruskal-Wallis test is applied to compare the median values of the observations across the different seasons.

3.1.2 Augmented Dickey-Fuller Test (ADF)

The ADF tests is considered when the errors are autocorrelated, so we need to add enough possible lags to capture the autocorrelated nature:

$$\Delta Y_t = \alpha + \gamma Y_{t-1} + \sum_{s=1}^{m} a_s \Delta Y_{t-s} + v_t.$$

So, adding as many lagged first difference terms as possible would ensure that the residuals are not correlated. The number of lags is determined by observing the ACF of the residuals v_t or the significance of the coefficients a_s. This test is valid for large sample sizes. A rule of thumb for choosing the maximum lag p_{max} suggested by Schwert (1989), is $p_{max} = \left[12 \cdot \left(\frac{T}{100}\right)^{\frac{1}{4}}\right]$ is choice allows p_{max} grow with the sample size T.

3.1.3 ARFIMA Fractional Test

The ARFIMA (Autoregressive Fractionally Integrated Moving Average) model is an extension of the ARIMA model to handle long memory processes or long-range dependence in time series data. The ARFIMA(p, d, q) model includes parameters for autoregression (p), fractional differencing (d), and moving average (q). The fractional differencing parameter, $-\frac{1}{2} < d < \frac{1}{2}$, allows for non-integer differencing orders, enabling the model to capture long or short-memory behavior in the data. A fractional differencing parameter, $d \neq 0$, indicates a long memory process, while d, equal to 0, represents a short memory process. To test for fractional differencing in a time series, the ARFIMA fractional differencing test examines the statistical significance of the estimated fractional differencing parameter, d. A significant value of d indicates the presence of long memory behavior in the time series. The ARFIMA fractional differencing test commonly uses maximum likelihood estimation (MLE) method. The null hypothesis is that there is no fractional differencing in the time series $(d = 0)$, while the alternative hypothesis suggests the presence of fractional differencing $(d \neq 0)$ (Geweke, (1986)).

3.1.4 Order Determination

Order determination is a crucial step in estimating ARIMA, as it involves selecting the appropriate lag length to capture the temporal dependencies among variables effectively. The lag order selection process relies on information criteria, such as the Akaike Information Criterion (AIC), the Schwarz Bayesian Information Criterion (BIC), and the Hannan-Quinn Information Criterion (HQIC). The lag order selection involves estimating models with different lag lengths, typically ranging from one to a predetermined maximum lag order. The information criteria are then calculated for each model, and the lag order that minimizes the selected criterion is chosen as the optimal lag length. The information criteria for AR models can be expressed as (Lütkepohl, (2007)):

$$AIC(p) = \mathrm{Tln}\left(\frac{\hat{\Sigma}_u(p)}{T}\right) + 2p$$

$$BIC(p) = \mathrm{Tln}\left(\frac{\hat{\Sigma}_u(p)}{T}\right) + \mathrm{pln}(T)$$

$$HQIC(p) = \mathrm{Tln}\left(\frac{\hat{\Sigma}_u(p)}{T}\right) + 2p\ln(\ln(T))$$

Where T is the sample size, p is the lag order, and $\hat{\Sigma}_u(p)$ is the estimated residual covariance matrix.

3.2 Post-Estimation Diagnostic Tests

Post-estimation diagnostic tests are crucial for evaluating the validity and reliability of time series models. These tests help assess various aspects of the model's performance, including normality of residuals, presence of residual unit roots, and detection of outliers.

3.2.1 Normality Test

Normality tests like the Jarque-Bera test assess whether the model's residuals follow a normal distribution. The Jarque-Bera test statistic is computed as:

$$JB = \frac{n}{6}\left(S^2 + \frac{1}{4(K-3)^2}\right)$$

where n is the sample size, S is the skewness of the residuals, and K is the kurtosis of the residuals. A high Jarque-Bera test statistic indicates a departure from normality.

3.2.2 Residual Unit Root Test

Residual unit root tests, such as the Augmented Dickey-Fuller (ADF) test, examine whether the model's residuals contain unit roots. The ADF test statistic is computed as:

$$ADF = \frac{\hat{\rho}}{SE(\hat{\rho})}$$

where $\hat{\rho}$ is the estimated coefficient of the lagged differenced residuals, and $SE(\hat{\rho})$ is its standard error. A significant ADF test statistic suggests the presence of residual unit roots.

3.2.3 Outliers detection

Outlier detection techniques help identify data points significantly impacting the model's fit. These outliers can distort parameter estimates and affect the model's predictive performance. Conducting these diagnostic tests to ensure the reliability and robustness of time series models is essential.

3.3 Forecasting

Forecasting methods play a crucial role in time series analysis, allowing for predicting future observations based on estimated models. Various techniques are employed for different time series models, including ARMA, ARIMA, and ARFIMA models. Among these methods are the Minimum Mean Squared Error (MMSE) forecasts. The MMSE forecast for ARMA, ARIMA, and ARFIMA models aims to minimize the expected value of the squared difference between the forecasted values and the actual observations. This method is based on the assumption that the forecast error follows a Gaussian distribution. In terms of accuracy measures, various metrics are used to evaluate the performance of forecasting models, including Mean Absolute Deviation (MAD), Mean Squared Error (MSE), and Mean Absolute Percentage Error (MAPE). These measures provide insights into the accuracy and reliability of the forecasts.

$$MAD = \frac{1}{n}\sum_{t=1}^{n}|y_t - \hat{y}_t|$$

$$MSE = \frac{1}{n}\sum_{t=1}^{n}(y_t - \hat{y}_t)^2$$

$$MAPE = \frac{100}{n}\sum_{t=1}^{n}\left|\frac{y_t - \hat{y}_t}{y_t}\right|$$

where y_t represents the actual observation at time t, $\hat{y}_t$ represents the forecasted value at time t, and n denotes the total number of observations. In conclusion, forecasting methods such as MMSE forecasts and accuracy measures like MAD, MSE, and MAPE are essential tools in time series analysis, providing valuable insights into future trends and patterns in the data.

Chapter 4

Dataset and Results

4.1 Dataset Overview

The FEDEX dataset comprises stock data with 88 observations across 9 variables: Price, Open, High, Low, Vol (Volume), Change, YearMonth, Dividend, and Dividend Yield. The variables display a range of financial data from stock prices to dividend yields, which is crucial for applying time series models.

4.2 Dataset Preprocessing

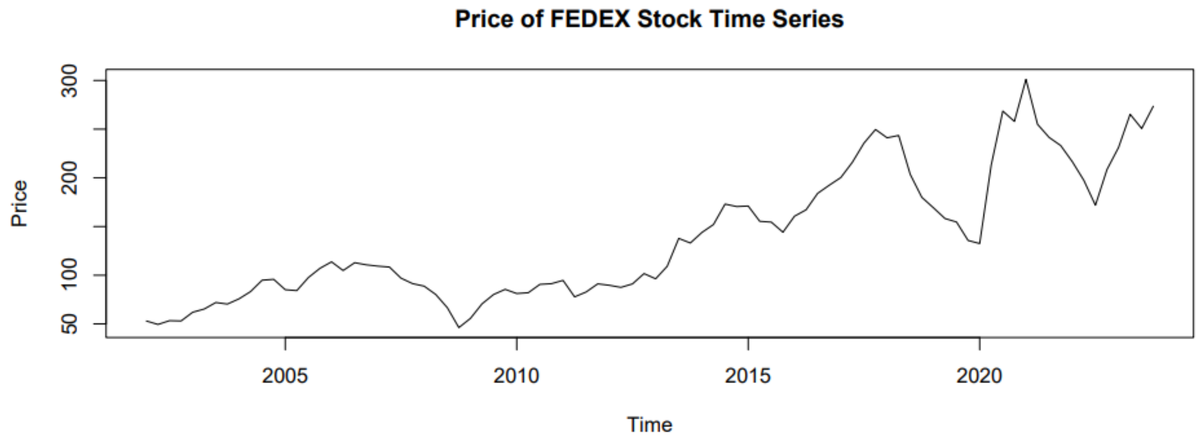
We were able to put our hands on a very descriptive FEDEX dataset from the website investing.com. This dataset contained the Price, Open, High, Low, Vol (Volume), Change, and the YearMonth. However, we wanted to add another external factor that could affect this stock's Price. Luckily, we found the "Dividends" of this firm dataset.

So, we started by dividing the FEDEX stock dataset into quarterly data, where each 3 months is averaged into one observation in the dataset. After this step, we changed the dates to be indexed, where every 3 months averaged together are indexed by their year and a label to which quarter it is. For example, the 1st 3 months in 2023 are indexed as "202301" and the 2nd 3 months are indexed as "202302" and so on.

Then, we loaded the libraries required for the time series analysis, read the dataset, and explored the dataset's structure to identify the dimensions and types of variables in the dataset. We then summarized the data to look at each column's first quartile, median, mean, third quartile, and maximum values. Going through ordering the data by the YearMonth column to achieve chronological order.

4.3 Time Series

The first step of the analysis will be extracting the price and dividend columns, which we will work with to create a time series object where "YearMonth" is used to set the start and frequency. Then, we went to detect outliers using the tso() function from the "tsoutliers" package.



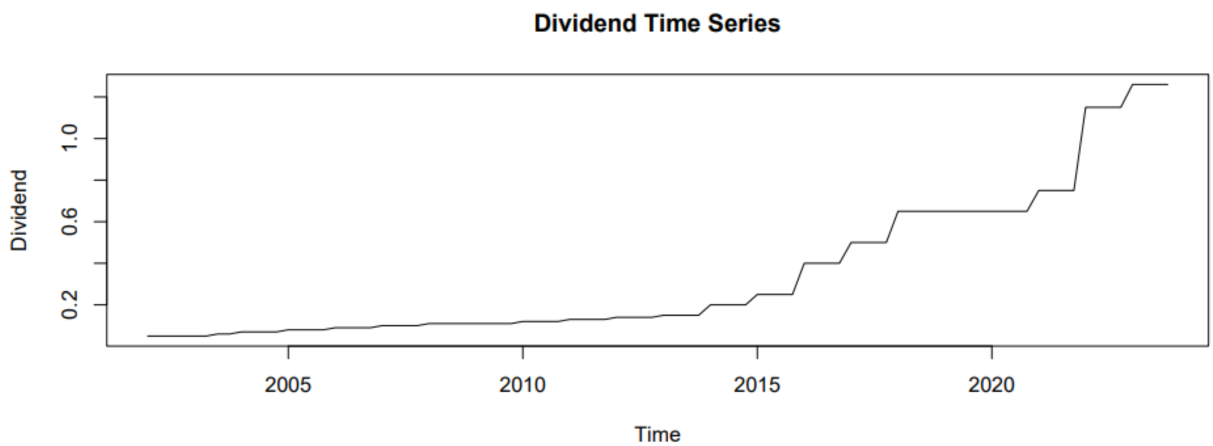


Figure 3: Time Series Plot with Trends and Outliers

From Figure 3, both time series do not have any outliers. The Price of FedEx stock and the company dividend showed an upward (positive) trend. The start period of the upward trend happened around the year 2013. From 2011 to 2013, FedEx traversed a volatile terrain distinguished by economic problems and strategic opportunities. Despite difficulties like slow global economic growth and rising fuel prices, the firm remained resilient due to a determined effort to cut costs and improve operational efficiency. Initiatives included implementing voluntary buyout programs and fleet modernization activities to lower operational costs and increase competitiveness. FedEx made significant purchases, including ANC Holdings Ltd. in 2011 and Rapidão Cometa in 2012, to expand its footprint in critical areas. The company's financial performance was uneven throughout these initiatives, with growth in certain divisions compensating for difficulty in others. Notably, investments in technology and infrastructure, along with the introduction of innovative services such as FedEx One Rate in 2013, enabled the corporation to capitalize on rising trends like the development of e-commerce. These initiatives helped drive the Price of FedEx shares and dividend distributions, showing investor confidence in the company's strategic direction and future development potential.

4.4 Descriptive statistics

The distribution of the 'Price' and 'Dividend' time series are seen in this histogram. The data seems to be right skew. Asymmetric skewed histogram might indicate the presence of outliers or non-normal distribution.

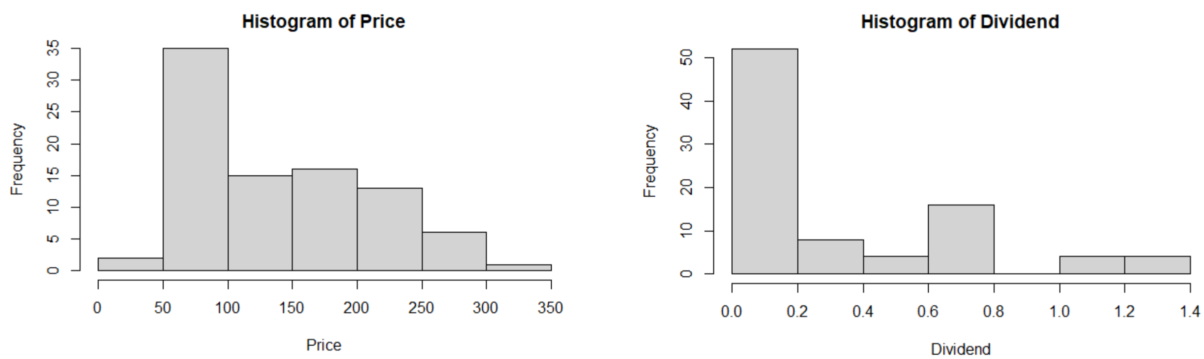


Figure 4: Histogram of Time Series

The Price variable's descriptive statistics show a range of \$46.21 to \$301.15, with a median value of \$111.78 and a mean value of \$139.06. The distribution looks right skewed since the mean is higher than the median. The third quartile is \$186.15, meaning 75% of the data falls below this level. The Jarque-Bera normality test produces a statistic of 7.8883 with a p-value of 0.01937; then, we will reject the null hypothesis of the time series following normality distribution, showing that the Price data deviates considerably from a normal distribution indicating non-normality in the distribution of prices. The Dividend variable's descriptive statistics show a \$0.05 to \$1.26 range, with a median of \$0.145 and a mean of \$0.3484. The distribution is right skew. The third quartile is \$0.65, meaning 75% of the data falls below this amount. The Jarque-Bera normality test yields a statistic of 25.61 with low p-value of 2.747e-06; then, we will reject the null hypothesis of the time series following normality distribution, suggesting that the dividend distribution deviates significantly from normality.

4.5 Time Series Decomposition

Through the time series decomposition, the trend and seasonality graphs show if there is a trend effect and if there is a seasonality present. Figure 5 shows that the time series has a positive trend and seasonality (similar behavior each quarter). The multiplicative model ensured generalization since we don't know if the components are independent or dependent. Both graphs depict the positive trend and the present seasonality.

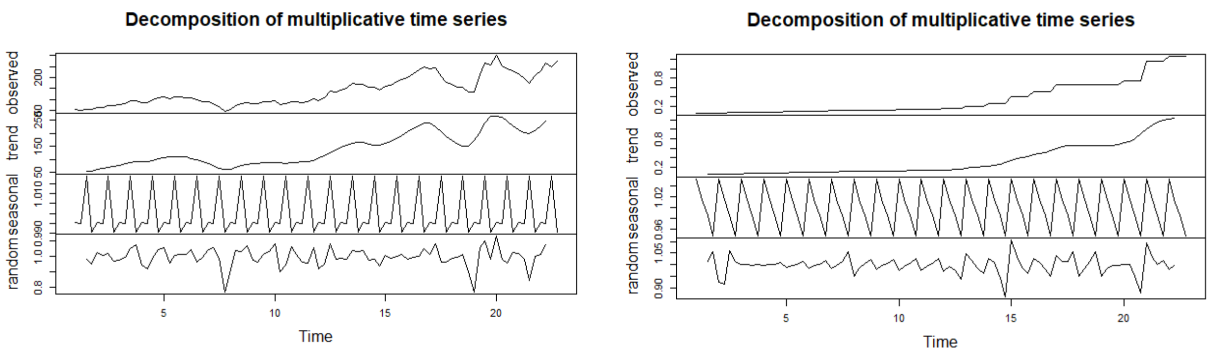


Figure 5: Time Series Multiplicative Decomposition

4.5.1 Stationarity and Seasonality Tests

The Augmented Dickey-Fuller (ADF) test evaluates the presence of a unit root in a time series, with the null hypothesis Ho: $\tau = 0$ ($\rho = 1$) indicating nonstationarity (The critical values depend on the chosen significance level (α): For α = 1%, the critical value is -4.04, α = 5%, the critical value is -3.45, and α = 10%, the critical value is -3.15). For the Price data, the ADF test statistic is -3.3194, indicating nonstationary time series at 1% and 5%. As for the Dividend data, the test statistic is -0.6376, indicating nonstationary time series at 1% and 5%. Given the test statistics, we fail to reject the null hypothesis for the Price and Dividend data, implying nonstationarity.

Table 1: ADF Tests for Unit Root (Non-stationarity)

Test	Null Hypothesis	Test Statistic Lags (4)	Critical Value (α = 1%)	Critical Value (α = 5%)	Critical Value (α = 10%)
ADF Test - Price	Time series has a unit root (indicating nonstationarity)	$\tau(3)$= -3.3194	-4.04	-3.45	-3.15
ADF Test - Dividend	Time series has a unit root (indicating nonstationarity)	$\tau(3)$= -0.6376	-4.04	-3.45	-3.15

The Kruskal-Wallis test, on the other hand, determines if there are differences in median data values over quarters, with the null hypothesis assuming no seasonality. Both the Price and Dividend data have p-values close to zero: 4.36e-09 < 0.05 for Price and 5.338e-10 < 0.05. The null hypothesis is rejected in favor of the alternative hypothesis, which shows a considerable fluctuation in both Price and Dividend over quarters, indicating the presence of seasonal impacts in the data.

Table 2: Kruskal-Wallis Tests for Seasonality

Test	Null Hypothesis	Results
Kruskal-Wallis - Price	No difference in median values across quarters	chi-squared = 81.597, df = 21 p-value=4.36e-09
Kruskal-Wallis - Dividend	No difference in median values across quarters	chi-squared = 86.975, df = 21, p-value =5.338e-10

4.6 Nonstationary Variables Transformation

Several transformations can be used to achieve stationarity in both time series. The 1st difference Δy_t can be used to remove the trend effect, and the 4th lagged difference can be used to remove the seasonality. In the following, we presented both transformations. For the 'Price', we used the Change which is $\frac{Price_t - Price_{t-1}}{Price_{t-1}}$ because it is a famous transformation that is recommended with the stock prices. As for the Dividend, we used both 1st difference $\Delta Dividend_t = Dividend_t - Dividend_{t-1}$, and 4th lagged difference $\Delta_4 Dividend_t = Dividend_t - Dividend_{t-4}$.

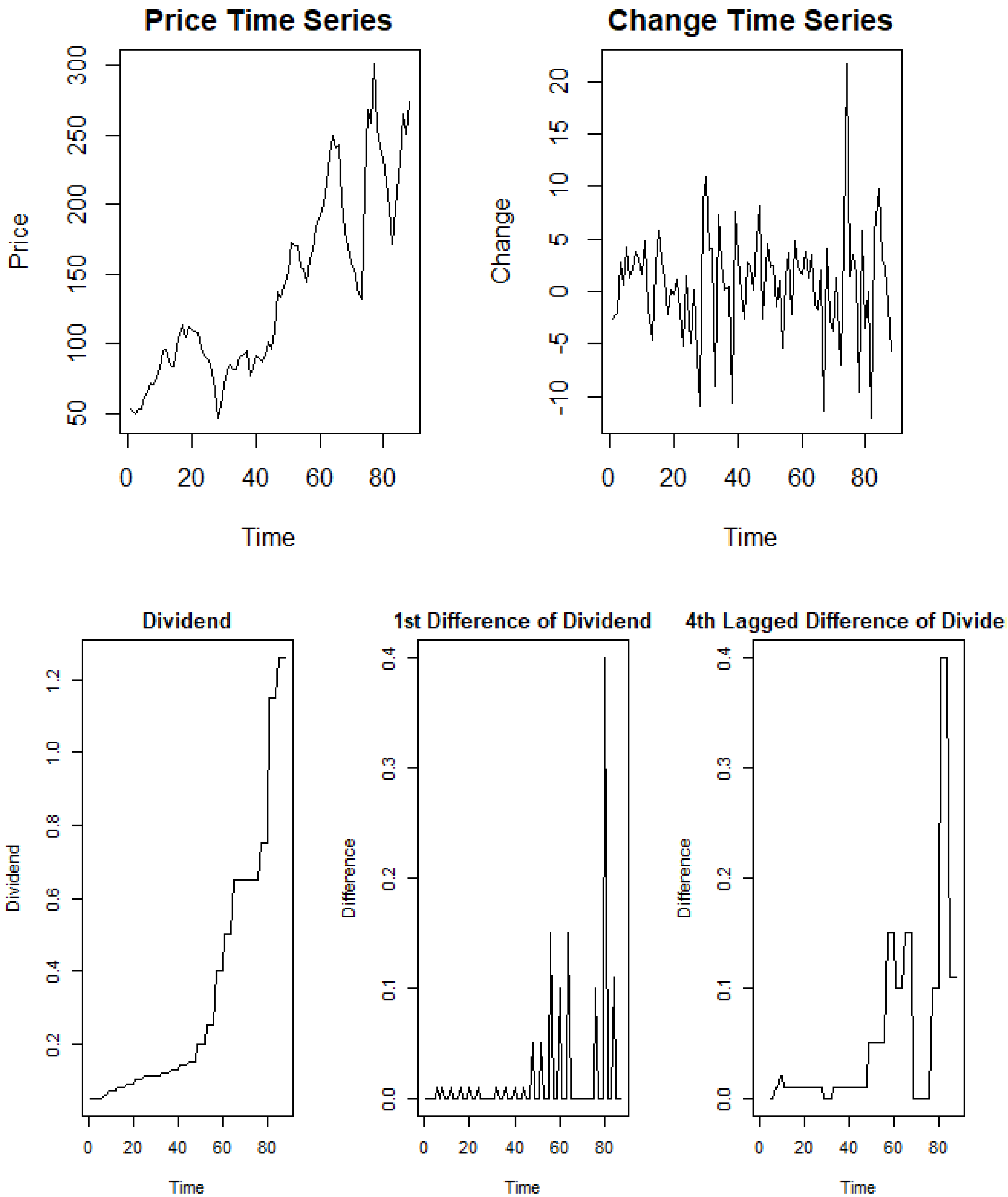


Figure 6: Time Series Transformation Plots

After applying the transformations, the change in Price showed stationarity in the center line (mean) but with one or two peaks. After performing the ADF and Kruskal Wallis tests, The Augmented Dickey-Fuller (ADF) test was conducted to assess the stationarity of the 'Change' variable. The test statistic obtained was -4.0929. Comparing this statistic to critical values at various significance levels (1% critical value: -3.51, 5% critical value: -2.89, 10% critical value: -2.58), the test statistic absolute value was larger than all critical values, which leads us to reject the null hypothesis of a unit root, indicating that the 'Change' variable is stationary. Therefore, the 'Change' variable does not exhibit a trend, suggesting that it is not driven solely by past values and instead displays a stable pattern over time. As for Kruskal-Wallis's test statistic, chi-squared = 27.593, df = 21, p-value = 0.1521 > 0.05, which indicates no seasonal effect.

The transformations of the Dividend both showed stationarity; Table 3 presents the results of various statistical tests conducted on the dataset. The test statistics of the ADF test are reported. Additionally, Kruskal-Wallis tests were conducted to examine the presence of seasonality in the dataset. The p-values from these tests indicate whether there is significant evidence to reject the null hypothesis of no seasonality. Notably, the Kruskal-Wallis test yielded statistically insignificant results for the 4th lagged differences, suggesting the no seasonality. Overall, these tests provide valuable insights into the temporal dynamics and potential trends present in the dataset. So, both variables need the first differences to achieve stationarity and remove seasonality. We will use 'Change' instead of 'price, and the Δ Dividend instead of Dividend.

Table 3: ADF and Kruskal-Wallis for the Transformed Dividend

Test	Test Statistic	P value
Δ Dividend ADF	-5.045375	
Δ_4 Dividend ADF	-4.63755	
Kruskal-Wallis (Δ Dividend)		0.997 > 0.05
Kruskal-Wallis (Δ_4 Dividend)		3.92E-10 < 0.05

4.7 ARIMA for Price and Dividend

This section discussed the univariate modeling of both time series Price and Dividend before and after the transformation. Subsection 4.7.1 shows the univariate time series analysis for the Price, and 4.7.2 discusses the same analysis for the Dividend.

4.7.1 ARIMA for 'Price'

This section discusses the model identification, estimation, and diagnostic checking for the time series 'Price'. We started with model identification, fitting ARIMA with different orders for p, d, q, calculating AIC, BIC, and HQ, and then choosing the order with the lowest criteria (Measure of Error). Also, Figures 7 show the ACF and PACF of Price and Change.

Table 4:ADF and Kruskal-Wallis for the Transformed Price

	Order	AIC	BIC	HQ
$Price_t$	ARIMA 1,0,0	767.2804	772.2351	766.2785
	ARIMA 0,1,1	750.9693	755.9011	749.9674
	ARIMA 1,1,0	750.2403*	755.1722*	749.2384*
	ARIMA 1,1,1	751.9078	759.3055	751.9039
$Change_t = \frac{Price_t - Price_{t-1}}{Price_{t-1}}$	ARIMA 1,0,0	541.2519	546.2066	540.2499
	ARIMA 0,0,1	544.2511	549.2057	543.2491
	ARIMA 1,1,0	576.0305	580.9624	575.0286
	ARIMA 0,1,1	543.4228	548.3547	544.4209
	ARIMA 1,1,1	543.4189	550.8166	543.4150

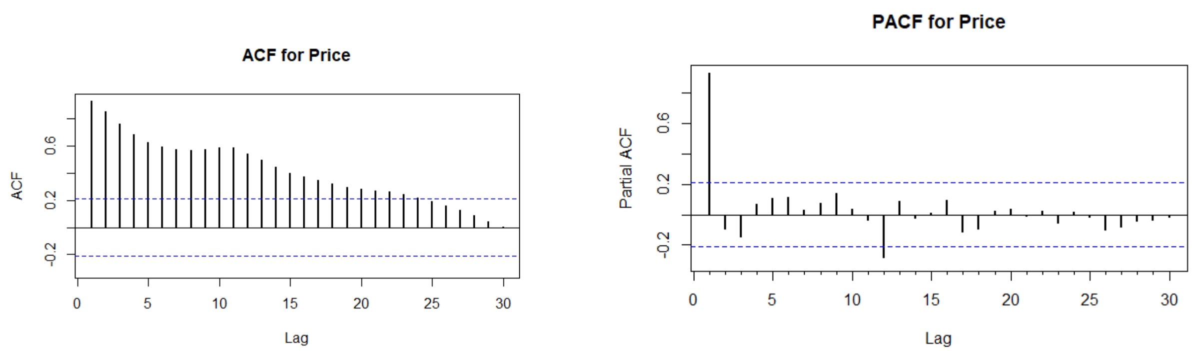


Figure 7:ACF and PACF for Price

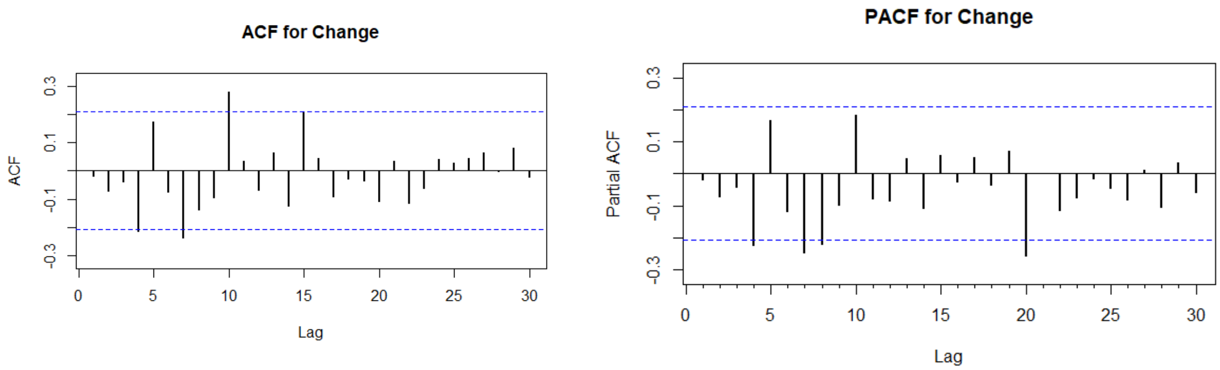


Figure 8: ACF and PACF for Change

Based on the lowest values of AIC, BIC, and HQ, the best ARIMA order for modeling the price series appears to be ARIMA(1,1,0), as it has the lowest values across all three criteria compared to the other orders. This order suggests a first-order differencing to achieve stationarity, indicating that the series exhibits a first-order autoregressive pattern with no moving average component, which implies that the previous period's Price influences the current Price with a time lag of one unit, and the series requires differencing to stabilize its mean.

Similarly, for the change in price series, calculated as the percentage change from the previous period, the ARIMA(1,0,0) order emerges as the most suitable based on the lowest AIC, BIC, and HQ values. This order indicates a first-order autoregressive model without differencing, suggesting that the current change in Price is directly related to the previous period's change. It suggests that the change in Price exhibits a persistent autoregressive pattern without needing differencing to achieve stationarity.

Table 5: ARIMA Model for Price and Change

	ARIMA(1,1,0)	ARIMA(1,0,0)
	D.Price	Change
_cons	2.574	0.82
p-value	-0.323	-0.129
ARMA		
Lagged AR	0.195***	0.183***
p-value	0.009	0.008
N	87	88
AIC	750	541
BIC	758.4	546.1

p-values are in parentheses. *p<0.05, **p<0.01, ***p<0.001

Table 5 presents the ARIMA model results for both Price and Change. Two different ARIMA configurations, ARIMA (1,1,0) and ARIMA(1,0,0) were applied to the datasets. For the 'Price' variable, the constant term (_cons) was estimated at 2.574 for ARIMA (1,1,0) and 0.82 for ARIMA(1,0,0). The p-values associated with these estimates were -0.323 and -0.129, respectively, suggesting statistical insignificance for both models. Moving to the ARMA component, the lagged autoregressive term (Lagged AR) was found to be 0.195 and 0.183 for ARIMA(1,1,0) and ARIMA(1,0,0) models, respectively, with p-values of 0.009 and 0.008, indicating significant autocorrelation effects in the time series. The Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) were utilized for model comparison, with lower values indicating better model fit. The AIC was 750 for ARIMA(1,1,0) and 541 for ARIMA(1,0,0), while the BIC was 758.4 and 546.1, respectively. Overall, these results suggest that both ARIMA configurations provide valuable insights into the dynamics of the Price and Change variables, with the ARIMA(1,0,0) model demonstrating a superior fit based on the AIC and BIC metrics. The diagnostics analysis for the ARIMA(1,0,0) = AR(1) for the variable Change is shown in the combined figure (4).

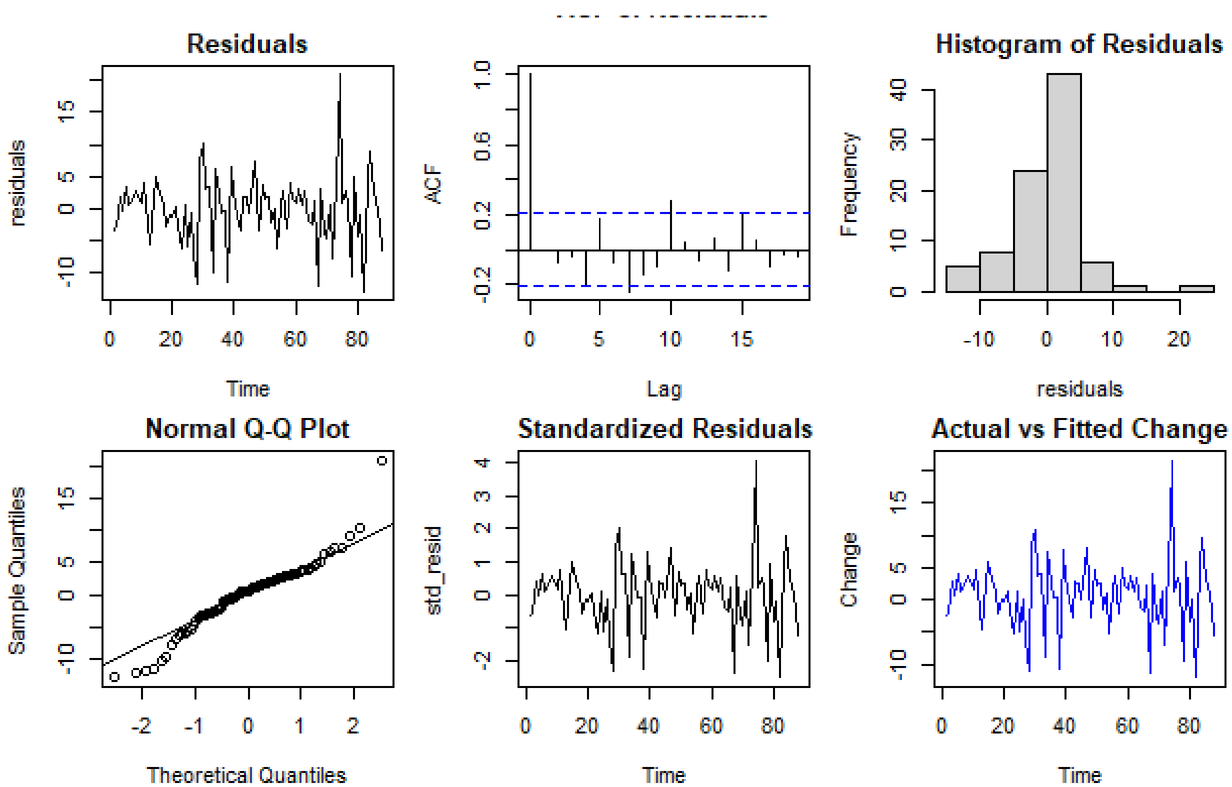


Figure 9: Residual Analysis for Change

Analyzing the residuals from the ARIMA(1,0,0) model for the change variable reveals several noteworthy patterns. Firstly, the time series plot of the residuals indicates that they are predominantly centered around zero, suggesting that the model captures much of the variability in the data. However, there appears to be one outlier that deviates substantially from the rest of the observations, warranting further investigation into its potential impact on the model's performance.

Secondly, the standardized residuals exhibit a seemingly random pattern, with most values clustering near zero. However, one outlier exceeds the typical threshold of three standard deviations, indicating a potential anomaly in the data that may require attention. This outlier suggests the presence of an unusual observation that could influence the model's predictive accuracy. Additionally, the normal probability plot of the residuals reveals deviations from the expected diagonal line, particularly at the beginning and end of the plot. These deviations indicate non-normal behavior in the residuals, suggesting that the model may not fully capture all underlying patterns in the data.

Furthermore, the autocorrelation function (ACF) of the residuals displays a cutoff pattern, with only the first lag showing statistical significance, which suggests that there may still be some temporal dependence in the residuals that the model has not adequately accounted for. Lastly, the histogram of the residuals displays a non-normal distribution, with a noticeable gap and outlier. This departure from normality could indicate that the model may benefit from further refinement to capture the underlying distribution of the data better and improve its predictive performance.

The Augmented Dickey-Fuller (ADF) test for unit root indicates that the test regression drift model fits reasonably well. The coefficient estimates show statistical significance for the lagged first difference term with a t-value of 8.2833. The test statistic of 8.2833 falls well above the critical values for significance, providing strong evidence against the presence of a unit root in the residuals. Furthermore, the Breusch-Pagan test for heteroscedasticity yields a insignificant result with a p-value of 0.3427, indicating the presence of homoscedasticity in the residuals.

4.7.2 ARIMA for Dividend

This section discusses the model identification, estimation, and diagnostic checking for the time series 'Dividend'. We started with model identification, fitting ARIMA with different orders for p, d, q, calculating AIC, BIC, and HQ, and then choosing the order with the lowest criteria (Measure of Error). Also, Figures (10) and (11) show the ACF and PACF of Dividend and Δ Dividend.

Table 6: ADF and Kruskal-Wallis for the Transformed Dividend

	Order	AIC	BIC	HQ
$Dividend_t$	ARIMA 1,0,0	-258.2086	-253.2540	-259.2106
	ARIMA 0,1,1	-260.6945*	-255.7495*	-261.6832*
	ARIMA 1,1,0	-260.6813	-255.7495*	-261.6832*
	ARIMA 1,1,1	-258.6813	-251.2835	-258.6852
$\Delta Dividend_t = Dividend_t - Dividend_{t-1}$	ARIMA 1,0,0	-260.6813	-254.2134	-261.6838*
	ARIMA 0,0,1	-260.6945*	-255.7495*	-261.6838*
	ARIMA 1,1,0	-221.5142	-216.6055	-222.5212
	ARIMA 0,1,1	-260.1280	-255.2193	-261.1350
	ARIMA 1,1,1	-259.3536	-251.9905	-259.3677

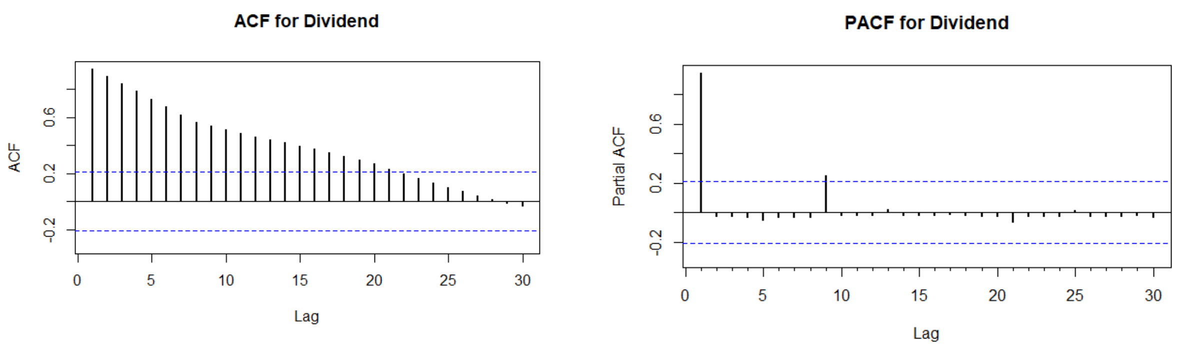


Figure 10: ACF and PACF for 'Dividend'

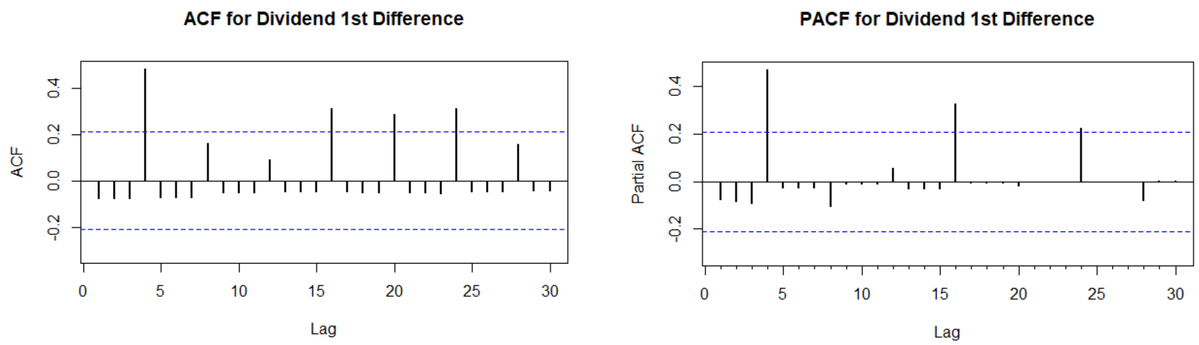


Figure 11: ACF and PACF for Δ Dividend

The determination of the appropriate ARIMA order for modeling the Dividend and its first difference ΔDividend involves evaluating several information criteria, including the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Hannan-Quinn (HQ) criterion. For the Dividend variable, considering AIC, BIC, and HQ collectively suggests that the ARIMA(0,1,1) model provides the best fit among the tested orders. This order indicates a non-seasonal differencing of 1 with a moving average component of order 1, reflecting a model that captures the trend in the data while accounting for short-term dependencies. In contrast, for the first difference ΔDividend, the AIC, BIC, and HQ criteria collectively favor the ARIMA(0,0,1) model. This order signifies a non-seasonal moving component of order 1. This model is indicative of a process where the changes in the Dividend variable exhibit a consistent relationship with its immediate shock in the system, highlighting a potentially stronger memory effect.

Table 7: ARIMA Model for ΔDividend

	ARIMA (0,1,1) for Dividend = MA(1) for **ΔDividend**
_cons	0.0139
p-value	0.124
ARMA	
L.ma	0.0508***
p-value	0.000
N	87
aic	-260.7
bic	-255.8

p-values are in parentheses. *p<0.05, **p<0.01, ***p<0.001

Table 7 presents the ARIMA model specifications for ΔDividend, denoting the first differences of the Dividend variable. The constant term ('_cons') is estimated to be 0.0139, indicating the expected change in ΔDividend when no other variables are considered. Although the associated p-value of 0.124 suggests marginal statistical significance, further exploration is warranted to assess its robust. In terms of the autoregressive moving average (ARMA) component, the model incorporates a moving average (MA) term ('L.ma') with a coefficient estimate of 0.0508, which is statistically significant with a p-value of 0.000. This underscores the influence of past forecast errors on the current value of ΔDividend, highlighting the importance of short-term fluctuations in its dynamic. Overall, the ARIMA (0,0,1) model for ΔDividend exhibits a strong fit, as evidenced by the AIC and BIC values of -260.7 and -255.8, respectively. These metrics reflect the model's ability to balance goodness of fit with parsimony, indicating its suitability for capturing the underlying patterns in the data while avoiding overfitting. The diagnostics analysis for the ARIMA (0,0,1) = MA(1) for the variable ΔDividend is shown in the combined figure (12).

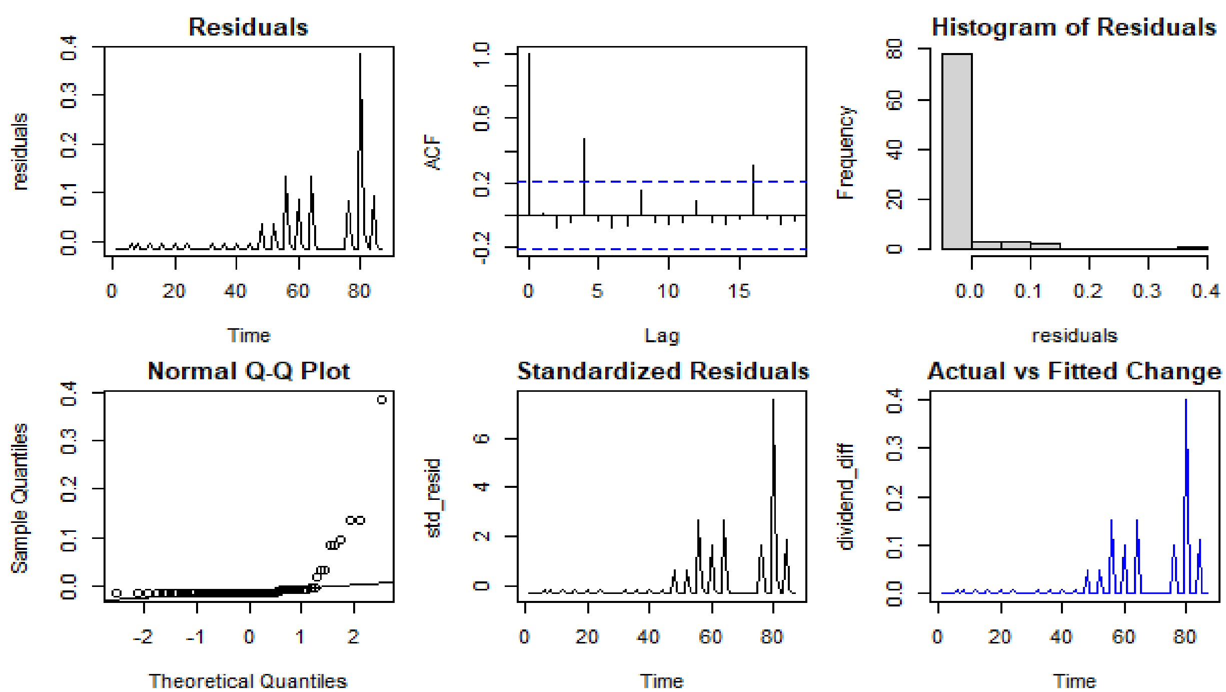


Figure 12: Residual Analysis for ΔDividend

Analyzing the residuals from the ARIMA (0,0,1) model for the ΔDividend variable reveals several noteworthy patterns. Firstly, the time series plot of the residuals indicates that they are predominantly centered around zero, suggesting that the model captures much of the variability in the data. However, there appears to be several outliers that deviate substantially from the rest of the observations, warranting further investigation into its potential impact on the model's performance like the model volatility analysis.

Secondly, the standardized residuals exhibit a seemingly random pattern, with most values clustering near zero. However, several outliers exceed the typical threshold of three standard deviations, indicating a potential anomaly in the data that may require attention. These outliers suggest the presence of unusual observations that could influence the model's predictive accuracy. Additionally, the normal probability plot of the residuals reveals deviations from the expected diagonal line, particularly at the end of the plot. These deviations indicate non-normal behavior in the residuals, suggesting that the model may not fully capture all underlying patterns in the data.

Furthermore, the autocorrelation function (ACF) of the residuals displays a cutoff pattern, with only the first lag showing statistical significance, which suggests that there may still be some temporal dependence in the residuals that the model has not adequately accounted for. Lastly, the histogram of the residuals displays a non-normal distribution, with a noticeable gap and outlier. This departure from normality could indicate that the model may benefit from further refinement to capture the underlying distribution of the data better and improve its predictive performance.

The Augmented Dickey-Fuller (ADF) test is conducted to examine whether the residuals exhibit a unit root, indicating nonstationarity. The results of the ADF test suggest that the residuals do not contain a unit root (test statistic: -4.8546), providing evidence against the presence of non-stationarity. Additionally, the Breusch-Pagan test is employed to assess the presence of heteroscedasticity in the residuals. The test yields a highly significant result (p-value $< 2.2e-16$), suggesting that heteroscedasticity exists, indicating that the variance of the residuals is not constant across observations. These results imply that the ARIMA model adequately captures the temporal dynamics in the data, as evidenced by the absence of a unit root in the residuals. However, the significant Breusch-Pagan test suggests that the variance of the residuals varies across observations, indicating potential model misspecification or the presence of additional explanatory variables not included in the model. Therefore, while the ARIMA model provides a reasonable fit to the data in terms of temporal dynamics, further investigation into the sources of heteroscedasticity may be warranted to improve model performance.

4.8 ARFIMA for Price and Dividend

This section discussed the fractional ARFIMA modeling of both time series Price and Dividend before the transformation. The models of ARFIMA are fitted to the nonstationary time series so we will fit them to the Price with AR (1) part and Dividend with MA (1) part as supported by the ARIMA results in the previous section. Table (8) depicts the ARFIMA results and their significance.

Table 8: ARFIMA Model for Price and Dividend

	Model 1 Price	Model 2 Dividend
Main		
_cons	147.4*	0.431
	-0.017	-0.578
ARFIMA		
$Price_{t-1}$	0.833***	
	0.000	
d	0.349**	0.496***
	0.006	0.000
ε_{t-1}		0.553***
		0.000
N	88	88
AIC	765.1	-184.1
BIC	775	-174.1

p-values in parentheses * p<0.05, ** p<0.01, *** p<0.001

The ARFIMA model was applied to both the Price and Dividend datasets, yielding insightful results. For the Price series, the estimated long memory parameter (d) is 0.349, indicating a persistent dependence between observations, slightly less pronounced compared to the Dividend series ($d = 0.496$). This suggests that both series exhibit long-range dependence, which is a common characteristic in financial time series data. Additionally, the estimated autoregressive parameter ($Price_{t-1}$) is highly significant (p<0.001) for both Price, highlighting the presence of strong autocorrelation in the data. In the Dividend model, the estimated coefficient for ε_{t-1} is 0.553, and it is highly significant with p<0.001. This indicates that the moving average component significantly influences the short-term dynamics of the Dividend series. The positive coefficient suggests that past deviations from the mean in the Dividend series have a persistent effect on the current value, indicating some level of autocorrelation in the series. This finding emphasizes the importance of considering the moving average component in modeling the Dividend series accurately. The estimated mean for the Price series (147.4) is statistically significant (*p*<0.05), indicating a non-zero mean in the series. Conversely, the mean for the Dividend series is not statistically significant, suggesting that the series may be centered around zero. These results provide valuable insights into the underlying dynamics of the Price and Dividend series, highlighting their long memory properties and autocorrelation structure, which are essential considerations for forecasting and risk management purposes in financial analysis.

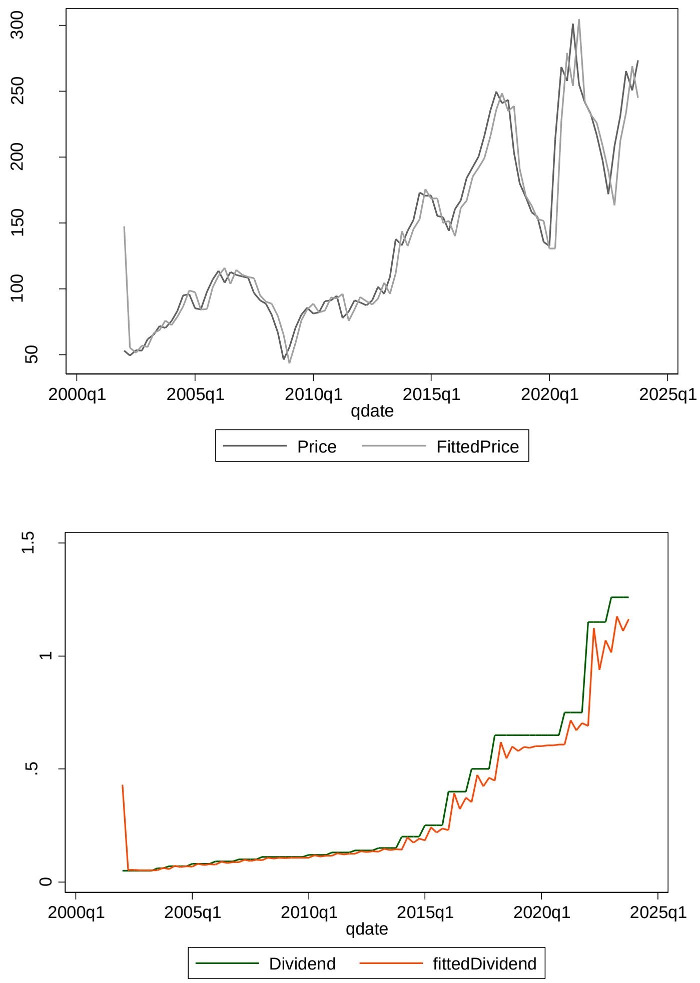


Figure 13: Actual VS Fitted for ARFIMA Models

For the Price, the Jarque-Bera test indicates that the residuals are not normally distributed, as evidenced by a JB statistic of 112.35 with a p-value less than 2.2e-16, suggesting significant departure from normality. The Augmented Dickey-Fuller (ADF) provided evidence of stationarity in the residuals. The ADF test yields a test statistic of -4.5278 with a p-value of 0.01, rejecting the null hypothesis of a unit root, indicating stationarity. As for the dividend, Jarque-Bera test for normality of the residuals indicates a significant departure from normality, with a JB statistic of 5178.2 and a p-value less than 2.2e-16. The Augmented Dickey-Fuller test suggests stationarity of the residuals, with a test statistic of -3.7319 and a p-value of 0.02643.

4.9 Multivariate Time Series Analysis

The following section is dedicated to analyzing the relationship between both time series and classifying the time effect duration. This section is divided into 3 subsections.

4.9.1 Short-Run Causality Granger Test

The Granger causality test is a valuable tool used to assess the causal relationship between two variables. Specifically, the Short-Run Causality Granger Test, often referred to simply as the Granger causality test, evaluates whether the past values of one variable help predict the current values of another variable. It's an essential method for exploring the direction and strength of causal relationships within a system of stationary variables. That's why we will use Change and ΔDividend instead of their nonstationary counterparts.

Table 9: Order Selection for VAR Models

lag	AIC	HQ	BIC
0	3.135	3.15856*	3.19343*
1	3.162	3.232	3.337
2	3.02638*	3.335	3.509
3	3.275	3.439	3.683
4	3.126	3.237	3.551

The table (9) presents various lag orders for VAR models along with corresponding values of three different criteria: Akaike Information Criterion (AIC), Hannan-Quinn Information Criterion (HQIC), and Schwarz Bayesian Information Criterion (SBIC). Each criterion provides a measure of model fit while penalizing the number of parameters, thereby aiding in lag order selection. Upon examination, it's evident that the optimal lag order differs depending on the criterion used. For instance, according to the AIC criterion, lag order 2 is favored as it exhibits the lowest AIC value. Conversely, lag order 0 stands out as the preferred choice when considering the HQIC and SBIC criteria, as it yields the lowest values for both. Ultimately, we will choose according to AIC which is lag 2. Table 10 shows Granger Causality tests for Change and ΔDividend using lag (2).

Table 10: Granger causality Wald tests

Equation Dependent	Independent	chi2	df	Prob>Chi2
change	ddividend	5.159	1	0.023**
ddividend	change	2.273	1	0.132

p-values in parentheses * p<0.05, ** p<0.01, *** p<0.001

The Granger causality Wald tests presented in the table assess the directional causal relationship between the variable's Change and ΔDividend, considering each variable as both the dependent and independent variable in separate tests. For the equation where Change is the dependent variable and ΔDividend is the independent variable, the chi-square statistic is 5.159 with 1 degree of freedom, resulting in a p-value of 0.023. This indicates a statistically significant result at the 0.05 significance level, suggesting that ΔDividend Granger causes Change. Conversely, when ΔDividend is the dependent variable and "change" is the independent variable, the chi-square statistic is 2.273 with 1 degree of freedom, yielding a p-value of 0.132. In this case, the result is not statistically significant at the 0.05 significance level, indicating that there is insufficient evidence to conclude that Change Granger causes ΔDividend. Overall, these findings provide insights into the causal dynamics between the variables, which indicate a unilateral causality from the direction of ΔDividend towards change highlighting the importance of considering the directionality of causal relationships in understanding the underlying mechanisms of the data.

4.9.2 VAR Model (2)

Fitting a Vector Autoregression (VAR) model is a common approach in time series analysis to explore the dynamic relationships between multiple variables over time. In this context, we aim to fit a VAR (2) model to the variables Change and ΔDividend. The VAR model is a multivariate extension of the autoregressive (AR) model, where each variable in the system is regressed on its own lagged values as well as the lagged values of all other variables in the system. By incorporating lagged values of multiple variables, VAR models capture the interdependencies and feedback mechanisms among them, allowing for the examination of both short-term and long-term dynamics. Table 11 examines the VAR model for both variables.

Table 11: VAR Model Results

VARIABLES	(1) $Change_t$	(2) $\Delta dividend_t$
$Change_{t-1}$	-0.0261	-0.000480
	(0.109)	(0.00110)
$Change_{t-2}$	-0.0787	0.00156
	(0.106)	(0.00106)
$\Delta dividend_{t-1}$	-24.71**	-0.0803
	(10.64)	(0.107)
$\Delta dividend_{t-2}$	1.800	-0.0891
	(10.99)	(0.110)
$Constant$	1.295**	0.0156**
	(0.620)	(0.00622)
Observations	85	85
chi2	16.0312	3.524
DF	4	4
P > chi2	0.003	0.0646
AIC	-0.156	3.172
BIC	-0.116	3.459
HQ	-0.398	3.287

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

The equations for both models can be represented as follows:

Model(1):

$$Change_t = 1.295 + (-0.0261 \times Change_{t-1}) + (-0.0787 \times Change_{t-2}$$
$$+ (-24.71 \times \Delta dividend_{t-1} + (1.800 \times \Delta dividend_{t-2}) + \epsilon_t$$

Model(2):

$$\Delta\ dividend_t = 0.0156 + (-0.000480 \times\ Change_{t-1}) + (0.00156 \times\ Change_{t-2})$$
$$+ (-0.0803 \times \Delta\ dividend_{t-1}) + (-0.0891 \times \Delta dividend_{t-2}) + \eta_t$$

The coefficients in both models play a crucial role in understanding the dynamics between the variables. In Model (1), the coefficient of $\Delta dividend_{t-1}$ indicates a significant negative impact on $Change_t$, suggesting that changes in dividends from the previous period strongly influence changes in the current period's variable. Similarly, Model (2) reveals the influence of lagged values of both variables on each other, providing insights into the short-term dynamics of the system. Significant testing of coefficients helps determine the reliability of the estimated relationships. A low p-value associated with a coefficient indicates a high level of confidence in its significance. In Model (1), the statistically significant coefficients highlight the importance of lagged dividend changes in predicting current changes in the variable $Change_t$, enhancing the interpretability and reliability of the model. Furthermore, information criteria such as AIC, BIC, and HQ aid in model selection by balancing goodness-of-fit and model complexity. Lower values of these criteria suggest a better trade-off between model fit and complexity. In this context, model (1) exhibits lower AIC, BIC, and HQ values compared to Model (2), indicating its superior fit to the data and thus its potential superiority in explaining the relationship between the variables.

Several diagnostics tests evaluating normality, stationarity, and root stability of the residuals was performed to evaluate the goodness of fitting of VAR (2) in predicting $Change_t$ using lagged $Change_t$ and lagged $\Delta Dividend_t$ in equation 1 and predicting $\Delta Dividend_t$ using lagged $Change_t$ and lagged $\Delta Dividend_t$ in equation 2. The ADF test evaluates autocorrelation at different lag orders to assess whether the residuals exhibit significant serial correlation. For the change equation, the test yields p-values of 0.73382 for lag 1 and 0.10902 for lag 2. These results suggest that the null hypothesis of no autocorrelation cannot be rejected, indicating that the model adequately captures the autocorrelation patterns in the data.

Moving to the Jarque-Bera test, significant chi-squared statistics indicate deviations from normality in the residuals. For the change equation, the test yields a chi-squared statistic of 17.978 with a p-value of 0.00012, rejecting the null hypothesis of normality. Similarly, for the ddividend equation, the chi-squared statistic is 4379.137 with a p-value of 0.00000.

4.9.3 IRF for VAR Model (2)

In section 4.9.3, the focus shifts to investigating Impulse Response Functions (IRFs) for the Vector Autoregressive (VAR) Model (2). IRFs offer valuable insights into the dynamic interactions between variables in a multivariate time series framework. By simulating the response of each variable to a shock in another variable, IRFs allow us to understand the short-term dynamics and transmission mechanisms within the system.

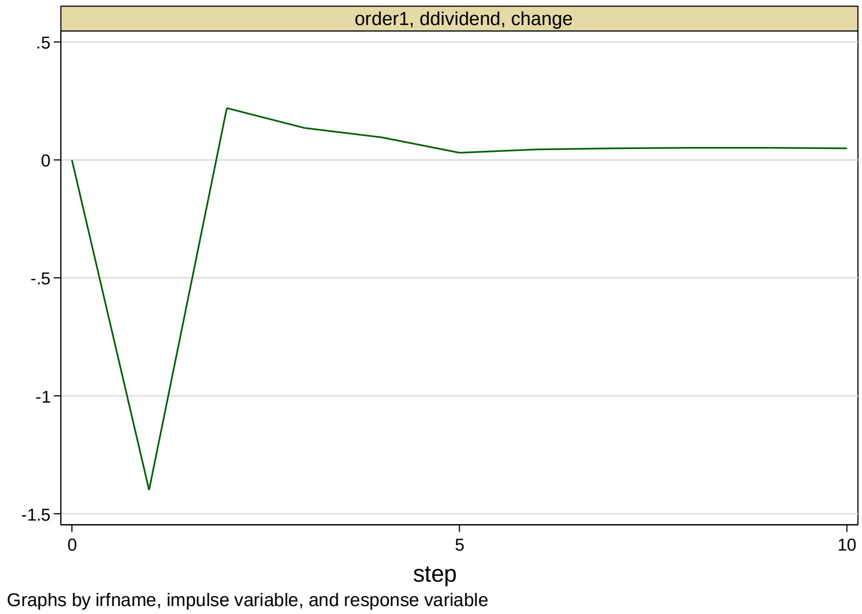


Figure 14: Effect of Shocks in $\Delta dividend_t$ on $Change_t$

Figure 14 shows the effect of any shock happening in dividend at time t, the stock price will decrease immediately 140% in time t+1, then increases by about 160% to compensate the decrease and raise the stock price above the starting point by 20%. The effect on the stock price decreases the price till the point t+5, everything converges to the starting stock price level or slightly above it.

4.10 Forecasting Change in FedEx Stock Price

4.10.1 Price Forecasting

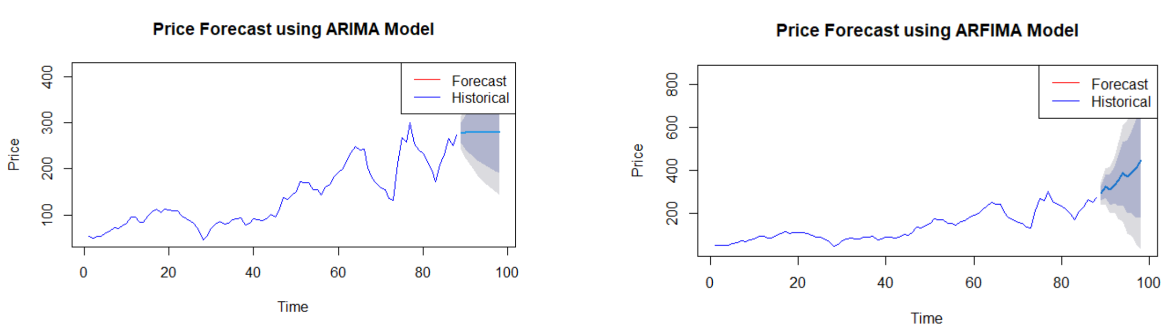


Figure 15: Forecasting Price using ARIMA (1,1,0), and ARFIMA (1,0.349,0)

Table 12: Accuracy Measures for forecasting Price

Accuracy Measure	ARIMA (1,1,0)	ARFIMA (1,0.349,0)
RMSE (Root MSE)	25.618	17.528
MAE (Mean Absolute Error)	15.812	11.841
MAPE (Mean Abs. % Error)	10.642%	8.418%

The ARFIMA (1,0.349,0) model outperforms the ARIMA (1,1,0) model in terms of most metrics, including RMSE, MAE, and MAPE. This indicates that the ARFIMA model provides more accurate forecasts compared to the ARIMA model for the given dataset. Specifically, the ARFIMA model has a lower RMSE, MAE, and MAPE, indicating smaller errors and better overall accuracy in forecasting the price. Overall, based on these metrics, we can conclude that the ARFIMA (1,0.349,0) model is more suitable for forecasting the price compared to the ARIMA (1,1,0) model.

4.10.2 Dividend Forecasting

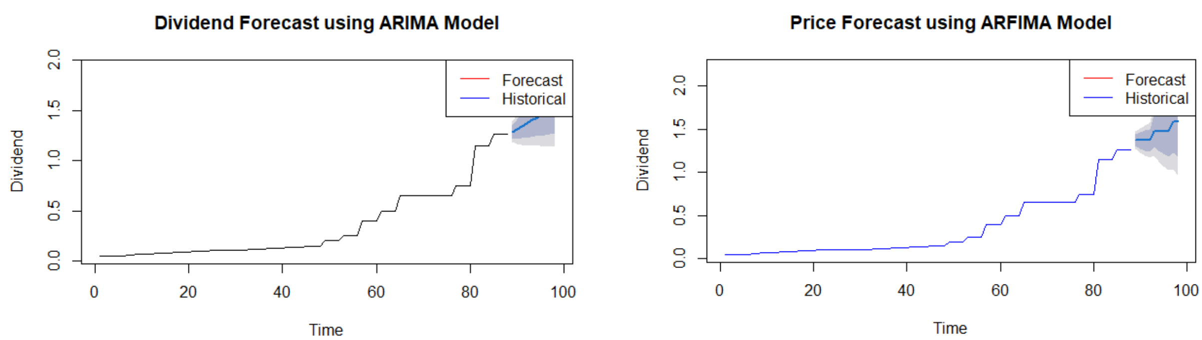


Figure 16: Forecasting Dividend using ARIMA (0,1,1), and ARFIMA (1,0.49,0)

Table 13: Accuracy Measures for Forecasting Dividend

Accuracy Measure	ARIMA (0,1,1)	ARFIMA (1,0.49,0)
RMSE (Root MSE)	0.051	0.050
MAE (Mean Absolute Error)	0.019	0.013
MAPE (Mean Abs. % Error)	4.574%	2.400%

The ARFIMA(1,0.49,0) model performs slightly better than the ARIMA(0,1,1) model across most metrics, with lower values for RMSE, MAE, and MAPE. This suggests that the ARFIMA model provides more accurate forecasts compared to the ARIMA model for the given dataset. Overall, based on these metrics, we can conclude that the ARFIMA(1,0.49,0) model is more suitable for forecasting the price compared to the ARIMA(0,1,1) model.

4.10.3 Change Forecasting

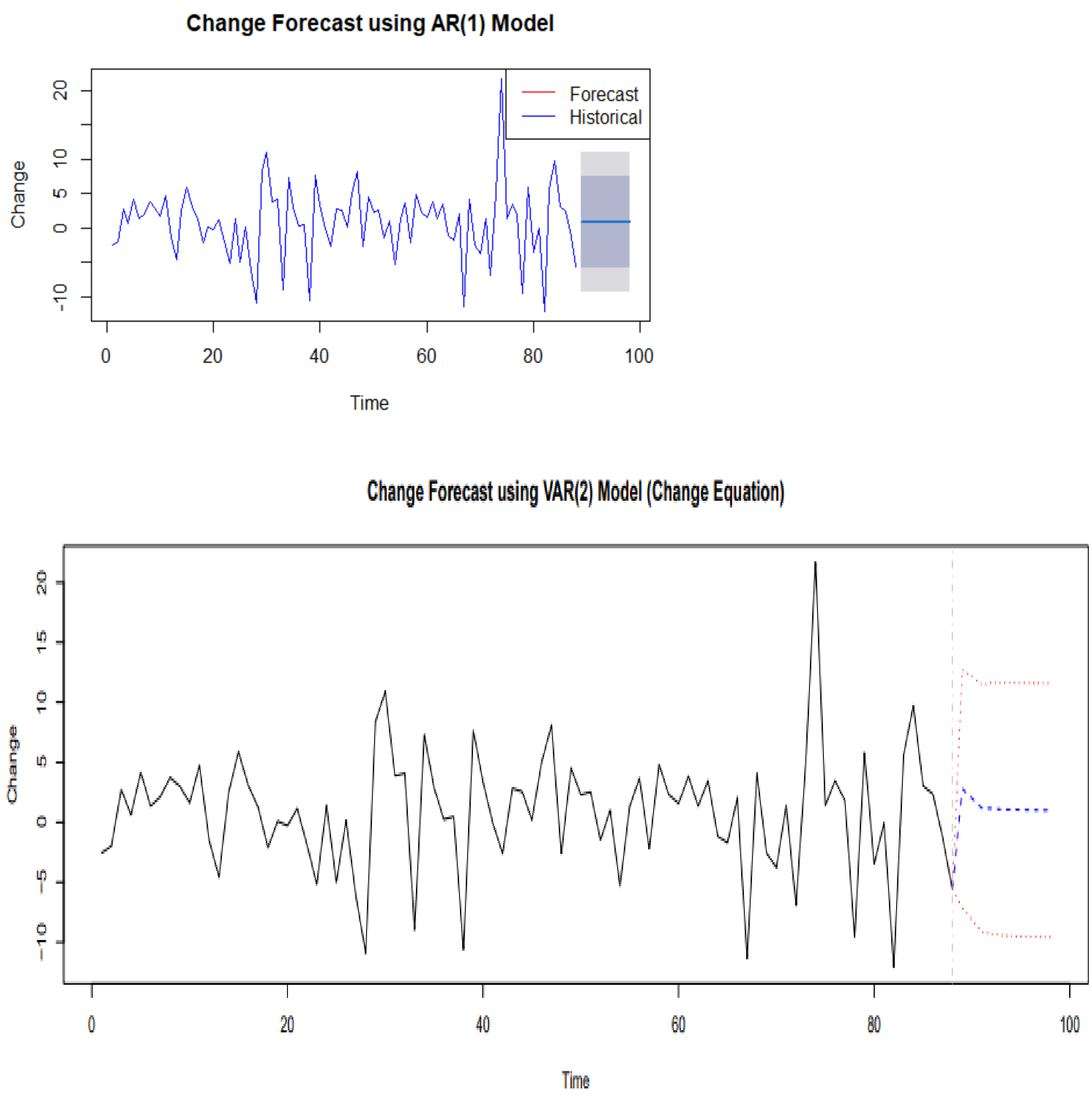


Figure 17: Forecasting Price Change using AR (1), and VAR (2)

Table 14:Accuracy Measures for forecasting Change

Accuracy Measures	AR (1)	VAR (2)
RMSE (Root MSE)	5.146168	3.63028
MAE (Mean Abs. Error)	3.730294	1.910632
MAPE (Mean Abs. % Error)	136.66%	116.02%

The VAR (2) model using the change equation outperforms the AR (1) model across most metrics, exhibiting smaller errors and better accuracy in forecasting the change in price. Specifically, the VAR (2) model has lower values for RMSE, MAE, and MAPE compared to the AR (1) model, indicating better performance in terms of forecasting accuracy. In summary, based on these metrics, the VAR (2) model appears to be more suitable for forecasting the change in price compared to the AR (1) model.

4.10.4 ΔDividend Forecasting

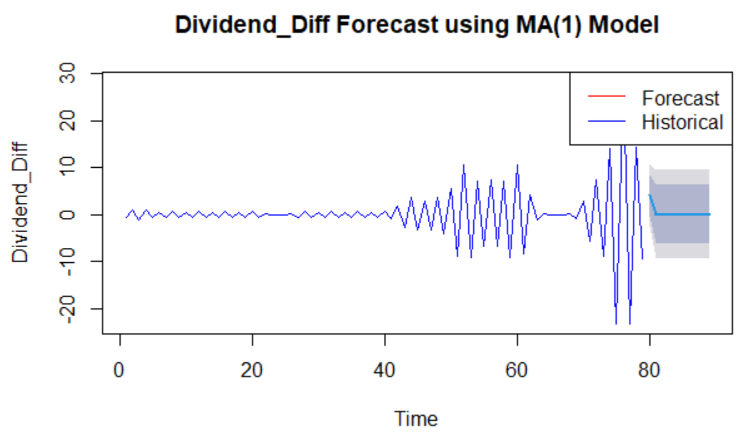


Figure 18: Forecasting Price using AR (1), and VAR (2)

Table 15: Accuracy Measures for Forecasting Δ Dividend

Accuracy Measures	Value
RMSE (Root MSE)	3.3845
MAE (Mean Abs. Error)	1.8714
MAPE (Mean Abs. % Error)	6.57%

In the MA (1) model, the Root Mean Squared Error (RMSE) of around 3.38 suggests that the forecasted values differ from the observed values by this amount on average, reflecting the overall accuracy of the model's predictions. The Mean Absolute Error (MAE) is approximately 1.87, signifying the average magnitude of errors in the forecasts. The Mean Absolute Percentage Error (MAPE) is approximately 6.57%, demonstrating the average percentage deviation of the forecasts from the actual values, providing insights into the overall accuracy of the forecasts in terms of percentage error. Overall, these metrics indicate the performance of the MA (1) model in forecasting ΔDividend, providing a comprehensive assessment of its accuracy and reliability.

Chapter 5

Conclusion, Limitations, and Recommendations

This study undertook an extensive analysis of the stock data of FedEx to find some major insight into price and dividend behaviors of FedEx over time using different statistical and time series methods. Key findings of the present study show that the stock price and dividends both exhibit trends and seasonality, and any of these tendencies can impact, to a great extent, the investment strategy, and the financial corporate planning. Comparing the results of our study with the existing literature reveals several key insights into the behavior of FedEx stock prices and dividends over time. Our analysis confirms previous findings regarding the trends and seasonality exhibited by both stock prices and dividends, emphasizing their importance in investment strategy and financial planning (Reisen et al., 2010; Baillie et al., 2014). Specifically, our study highlights the long-term positive trend observed in FedEx stock prices, indicating growth opportunities for investors willing to expose themselves to such prospects (Wang et al., 2022). Similarly, the analysis of dividend yield distribution suggests a constant right-skewed pattern, indicating an attractive option for long-term investors seeking steady income (Patel et al., 2015).

The ARIMA and ARFIMA models in the time series analysis get into the price dynamics and conclude that the stock price is characterized by short-run random fluctuations, but it carries a long positive trend. This is paramount as an indicator for an investor willing to expose himself to growth opportunities. Equally, the dividend yield analysis suggested a constantly right-skewed distribution, which generally increased over time and could, therefore, be attractive to long-term investors who look for steady income. The selected ARIMA and ARFIMA models in our time series analysis effectively capture the price dynamics of FedEx stock, revealing short-run random fluctuations but maintaining a long positive trend, crucial for investors' decision-making in financial planning (Stock & Watson, 2012; Huang et al., 2023).

The study has revealed that proper transformation should be used for non-stationary series of data to stationarity, since it helps in achieving better transformation through taking logarithm. After thorough testing, the ARIMA models chosen had forecast results that were important for decision-making in financial planning. Our study underscores the importance of proper transformation techniques for non-stationary data series to achieve stationarity, with logarithmic transformation yielding better results (Zakamulin & Giner, 2021).

More so, the other multivariate analysis of causality tests clearly pointed out the relationship that exists between stock prices and dividends. This means that there is a relationship in the short term between dividends and stock prices. This offers an opportunity for investors to change their strategies over short-term movements. The multivariate analysis of causality tests highlights the short-term relationship between dividends and stock prices, offering valuable insights for investors to adjust their strategies accordingly (Kutu & Ngalawa, 2016).

From this perspective, our study contributes to the existing literature by providing a comprehensive understanding of the dynamic financial time series of FedEx stock. While our findings offer valuable insights into the financial health of FedEx, there are opportunities for further research to enhance the understanding of stock prices and dividends. Future studies could incorporate additional variables, such as macroeconomic indicators or global market tendencies, to develop more complete and precise financial models (Huang et al., 2023). Moreover, advanced econometric models like volatility GARCH models could be employed to improve predictive precision and robustness, particularly for handling large noisy datasets and capturing complex nonlinear patterns (Jorion, 2007). Additionally, comparative studies with competitors in the logistics and delivery industry could provide valuable insights into industry trends and performance dynamics, further enhancing our understanding of FedEx's financial position (Wu et al., 2018).

In conclusion, our study offers valuable recommendations for shareholders and investors based on the observed relationship between dividends and stock prices. The increasing dividends of FedEx over the observation period reflect the firm's financial health and commitment to shareholder value, making it an attractive option for long-term investment (Zakamulin & Giner, 2021). Moreover, the stability and predictability of dividends provide a buffer against market volatility, offering security for risk-averse investors (Patel et al., 2015). From this point of view, the thesis was able to show the dynamic of financial time series and how relevant advanced statistical techniques are to recognize intricate patterns. The findings from this research will provide valuable insight into understanding the financial health of FedEx; however, on an extended scale, it is going to be very essential for the stakeholders to make a strategic decision as to where and when to invest. Further studies may add to it more variables that may influence stock prices and dividends, such as macroeconomic indices or global market tendencies, to be able to receive more complete and precise financial models. Overall, our study provides actionable insights for investors and lays the groundwork for future research in financial econometrics and economic forecasting.

5.1 Limitations

1. The dataset contains only 88 quarterly observations, given such a huge number of variables. This limitation would represent an obstacle to achieving full development in a stable and well-based generalization—as with larger datasets—especially regarding the complex dynamics, economic and financial influences.

2. More variables from macroeconomic indicators, such as GDP growth rates, interest rates, and inflation, could explain in a better way their direct and indirect influence on stock prices. Further explanation can be advanced in the event that global economic indicators are incorporated, for example, give an explanation on international economic conditions of domestic stock markets, especially to a global corporation like FedEx.

3. If more advanced econometric models like volatility GARCH models are used to make these predictions, then even those produced by advanced machine learning algorithms may increase their predictive precision and robustness. More importantly, such models will handle large noisy datasets well and help obtain complex nonlinear patterns.

4. It is, therefore, suggested that a comparative study with competitors in the logistics and delivery industry also be undertaken as part of future research, from which trends pertaining to their performances vis-a-vis FedEx's can be derived. Considering these limitations in the light of addressing the outlined recommendations for future research, it will then continue to build on the foundational work laid by this thesis and delve deeper into the financial dynamics at FedEx and companies similarly situated, hence making a broader contribution to the field of financial econometrics and economic forecasting.

5.2 Recommendations for Shareholders and Investors

This analysis in the thesis underscores major roles played by the dividends in effecting FedEx stock prices. It helps give insight valuable to shareholders or those who wish to become shareholders. Besides, dividends are part of the total return, whereby in wild market conditions, they help to create an assurance feeling that at least some money is flowing into the account irrespective of market conditions.

The findings reflect that dividends of FedEx have been increasing steadily over the period of observation, which reflects the financial health and commitment of the firm towards returning value to the shareholders. We found through the impulse response analysis that if any increase in dividends by 10% in any quarter would cause the stock price to decrease by 140% in the next quarter and increase again in the next quarter. Also, that any shocks happening to the dividend would cause a ripple effect on the prices spanning for a whole year (4 quarters) until it wears out and get back to its normal levels. This, in a nutshell, could make FedEx an interesting share for those investors looking for stable and predictable yields.

Regular and growing dividends say much about the financial health of the company, but even more so about the ability to produce sufficient cash inflow. Investors should consider such trends as optimistic signals and go ahead with long-term investment since very often, it concerns good management and good market position. Dividends always play the role of giving back something to shareholders in case of market turbulences when, as a fact, gains in stock prices may be doubtful. Dividends payable at a stable rate have the ability to buffer against market fickleness and might lend a certain level of security for those investors who are risk-averse.

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